SENSITIVITY ANALYSIS ON SMITH'S AMRV MODEL

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INTRODUCTION

The final report for the research in the area of "Sensitivity Analysis on Smith's AMRV Model" includes the following contributions:

A. Articles

(1) Ho, C.-H., 1995. Sensitivity in Volcanic Hazard Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data, Mathematical Geology: 27: 239-258.

(2) Ho, C.-H., Smith, E., and Yogodzinski, G. Volcanic Hazard Assessment Incorporating Expert Knowledge: Application to the Yucca Mountain Region, Nevada, U.S.A. (submitted for publication).

(3) Ho, C.-H. Volcanic Time Trend Analysis (submitted for publication).

B. Paper Presented

"Volcanic Hazard Analysis at the Yucca Mountain Nuclear Waste Repository Site," presented at the DOE/Geomatrix workshop on Alternative Hazard Models for the Probabilistic Volcanic Hazard Analysis (PVHA) project held on March 30-31, 1995 at Las Vegas, NV.

C. Abstracts submitted for presentation at the 30th Geological Congress to be held in Beijing, China, August 4-14, 1996 (currently undergoing review process).

(1) Ho, C.-H., Smith, E., and Yogodzinski, G. Volcanic Hazard Assessment Incorporating Multiple-Expert Knowledge.

(2) Ho, C.-H., and Smith, E. A 3-D Volcanic Hazard/Risk Assessment Model: Application to the Yucca Mountain Region, Nevada, U.S.A.

FUTURE WORK:

3-d Poisson Process for Volcanic Hazard Assessment: Application to the Yucca Mountain Region, Nevada, U.S.A.

Future work will concentrate on the following:

(1) To develop a 3-D Poisson model that permits identification and quantification of volcanic phenomena distributed through space and evolving in time (i.e. spatiotemporal data).

(2) Also, new model development for the volcanism at NTS will continue.

APPENDIX 1

Article (1): Sensitivity in Volcanic Hazard Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data

Mathematical Geology, Vol. 27, No. 2, 1995

Sensitivity in Volcanic Hazard Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data[1]

Chih-Hsiang Ho[2]

Both advocates and critics disagree on the significance and interpretation of critical geological features which relate to the safety and suitability of Yucca Mountain as a site for the construction of a high-level radioactive waste repository. Recent volcanism in the vicinity of Yucca Mountain is recognized readily by geologists and others with a knowledge of nuclear regulatory requirements as an important factor in determining future public and environmental safety. We regard basaltic volcanism as direct and unequivocal evidence of deep-seated geologic instability. Direct disruption of a repository site by basaltic volcanism therefore is a possibility. In this paper, sensitivity analysis of volcanic hazard assessment for the Yucca Mountain site is performed, taking into account some significant geological factors raised by experts. Three types of models are considered in the sensitivity data analysis. The first model assumes that both past and future volcanic activities follow a Homogeneous Poisson Process (HPP). The second model uses a Weibull Process (WP) to estimate the instantaneous recurrence rate based on the historical data at NTS (the Nevada Test Site). The model then switches from a WP of past events to a predictive HPP. The third model assumes that the prior historical trend based on a WP would continue for future activities. Hazards (at least one disruptive event during the next 10,000 years) using both classical and Bayesian approaches are evaluated based on the data for the following two observation periods: Pliocene and younger, and Quaternary. Combinations of various counts of events at volcanic centers of controversy and inclusion (or exclusion) of the youngest date at Lathrop Wells Center (= 0.01 Ma) generate 90 different data sets. Sensitivity analysis is performed for each data set and the minimum and the maximum hazards for each model are summarized. We conclude that the estimated overall probability of at least one disruption of a repository at the Yucca Mountain site by basaltic volcanism during the next 10,000 years is bounded between 2.02×10^{-5} and 6.57×10^{-3}.

KEY WORDS: nonhomogeneous Poisson process, volcanic hazard, Weibull distribution.

INTRODUCTION

In the ongoing national debate on nuclear power as a source of electricity, a key issue is the disposition of the high-level radioactive wastes produced in the

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process. At an earlier stage in this debate, Congress, aware of the importance of the waste issue, passed the Nuclear Waste Policy Act of 1982. This legislation required the federal government to develop a geologic repository for the permanent disposal of high-level radioactive waste from civilian nuclear power plants. This waste consists primarily of spent nuclear fuel. Congress designated the Department of Energy (DOE) to implement the provisions of the act.

The Department of Energy established the Office of Civilian Radioactive Waste Management (OCRWM) in 1983 in response to the legislation and set about to identify potential sites. After OCRWM had selected three potential sites to study, Congress enacted the Nuclear Waste Policy Amendments Act of 1987, which directed the DOE to characterize only one of those sites, Yucca Mountain, in southern Nevada.

To characterize the site, the DOE must study in detail the natural environment and the various natural processes to which a proposed deep geologic repository might be subject. For a site to be acceptable, these studies must demonstrate that the site could comply with regulations and guidelines established by the federal agencies that will be responsible for licensing, regulating, and managing the waste facility. The regulations, which were promulgated to ensure the safety of the public, require that radiation will not be released above some established safe limit, as determined by the Environmental Protection Agency (EPA), for at least 10,000 years after the repository is permanently sealed.

An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site (DOE, 1988) because the late Tertiary geologic history of southwestern Nevada has been dominated by volcanism and the consequent deposition of volcanic flows and tuffaceous rocks. Yucca Mountain, similar to most surrounding ranges, is composed dominantly of a series of Miocene ashflow tuff units and silicic volcanic rocks.

RELATED ISSUES

Yucca Mountain is located in the south-central part of the Southwestern Nevada Volcanic Field (SNVF), a major volcanic province of the southern Great Basin first defined by Christiansen and others (1977) and extended by Byers, Carr, and Orkild (1989). Interested readers are referred to the papers of Byers, Carr, and Orkild (1989) for the location of geographic features of the SNVF, and Crowe (1990) for the basaltic volcanic episodes of the Yucca Mountain region. Before developing formal results, it is useful to review briefly the controversy about some issues related to the volcanological studies at the Yucca Mountain region, straddling the southwestern corner of the Nevada Test Site (NST) where nuclear materials have been handled for more than three decades.

Modeling Assumptions for the Recurrence Rate

Present understanding of eruptive mechanisms of basaltic volcanism is not advanced enough yet to allow deterministic predictions of future activity. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, hazards, risks, and probabilities. There is a large and growing body of literature on probabilistic modeling for volcanism. Much of the debate in the literature is centered on the selection of distribution models (principally homogeneous Poisson vs. nonhomogeneous Poisson models).

Several probabilistic assessments of volcanic hazard at Yucca Mountain are available (Crowe, Johnson, and Beckman, 1982; Crowe and Perry, 1989; Ho and others, 1991; Ho, 1991a, 1992). All rely on dividing the probabilistic hazard assessment into two phases of estimating: (1) λ, the recurrence rate of volcanic activities near the Yucca Mountain region, and (2) p, the conditional probability of site disruption given a volcanic event. Crowe and his coworkers (Crowe, Johnson, and Beckman, 1982; Crowe and others, 1989) have estimated λ using a simple Poisson model. Ho (1990, 1991b) examines the applicability of the simple Poisson model for volcanic eruption forecasting. He notes that whereas the simple Poisson model can be used for modeling volcanic events from some volcanoes, it may not be appropriate in all situations. Therefore, Ho (1991a, 1991b, 1992) proposes a Weibull model which allows for decreasing or increasing trends in volcanism through time. A brief description of the technique is reviewed in the next section.

The Eruptive History of the Basaltic Volcanism

An area of difficulty in reconstructing the eruptive history of the Quaternary basalt near the Yucca Mountain site is in determining whether each center formed in a single eruption (monogenetic) or multiple time-separate eruptions (polycyclic). Additionally, it is difficult to establish an age of eruptive activity for each center with a reasonable degree of accuracy. Small volume basalt centers traditionally have been assumed to be monogenetic centers (Wood, 1980). However, detailed studies of the Lathrop Wells and Sleeping Butte centers (Wells and others, 1990; Crowe and Perry, 1991) have raised the possibility that some basalt centers may form episodically (polycyclic volcanism).

The Lathrop Wells volcanic center is located 20 km south of the potential Yucca Mountain site, at the south end of the Yucca Mountain range. It has long been recognized as the youngest basalt center in the region. However, determination of the age and eruptive history of the center remains the subject of considerable debate. Isotopic ages between 3.8 and 0.3 Ma have been obtained from the cinder cone centers in Crater Flat by Turrin and Champion (1991). However, Wells and others (1990) have argued that the Lathrop Wells cone may be as young as 20 ka based on geomorphic and pedogenic characteristics as well as on the scatter of isotopic ages. Wells and others (1990) further suggest that this center contains at least three discrete and temporally separate eruptive events that may have occurred over time spans of 1–10 ka, based on mapping of stratigraphic relations of tephra (volcanic debris) units here and elsewhere in the Basin and Range (Crowe and others, 1989). In contrast, $^{40}Ar/^{39}Ar$ age dating of two separate flow units in the Lathrop Wells volcanic center yields arithmetic means of ages of 183 ± 21 and 144 ± 35 ka (Turrin, Champion, and Fleck, 1991). On the basis of this dating and as yet unpublished K/Ar dates, Turrin, Chapman, and Fleck (1991) conclude that there were two eruptive events at Lathrop Wells, dated at 136 ± 8 ka and 141 ± 9 ka. They speculate that the time interval between flows may be less than 100 years because field mapping and paleomagnetic data indicate remanent magnetization directions only a few degrees apart for the two flow units. Differences in remanent magnetization directions can be accounted for by secular (temporal) variation of the Earth's magnetic field, the rates of which have been calibrated in other volcanic fields at approximately 4° per 100 years. Thus, they interpret the nearly identical remanent directions in the two Lathrop Wells flow units to imply a short duration (<100 years) of eruptive activity. However, this interpretation of the paleomagnetic data is controversial. Because both remanent directions are similar to the time-averaged geomagnetic field in the study area, these directions could represent equally well eruptions separated by 100 years, 10 ka, 100 ka, or 1 Ma. Therefore, it is not surprising that data are inconsistent at this early stage of site characterization studies.

Structural Controls of Basaltic Volcanic Activity

Crowe and Perry (1989) describe the distribution of volcanic centers, emphasizing a southwest stepping of volcanism between 6.5 and 3.7 Ma. They describe a recurrence pattern of basaltic events where new eruptive sites are marked by probable coeval clusters of centers. These clusters seem to be of similar age within the limits of K–Ar age determinations. They note that all basalt centers of the youngest episode of volcanism, except the basalt of Buckboard Mesa, occur in a narrow northwest trending zone. They named this zone the Crater Flat volcanic zone (CFVZ; see Fig. 1). Crowe and Perry (1989), and Crowe (1990) suggest a southwest migration of basaltic volcanism in the Yucca Mountain area based on this structural parallelism, a pattern that may reflect an earlier southwest migration of silicic volcanism in the Great Basin. Smith and others (1990) examine the spatial and temporal patterns of post-6 Ma volcanism in the southern Great Basin. They describe the area of most recent volcanism (AMRV; see Fig. 1) near Yucca Mountain as an area enclosing all known post-6 Ma volcanic centers in the region and examine the implications of the infor-

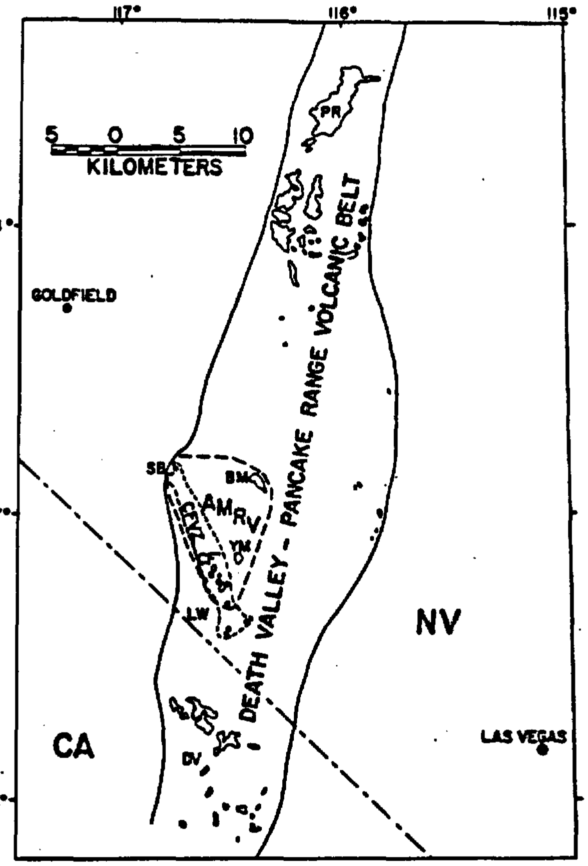

Figure 1. Proposed area of most recent volcanism (AMRV) is outlined by a heavy dashed line and includes Lathrop Wells cone (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), and volcanic centers within Crater Flat (CF). For comparison Crater Flat Volcanic Zone (dashed line) and Death Valley-Pancake Range Volcanic Belt (solid line) are shown. PR = Pancake Range. YM = proposed drift perimeter at Yucca Mountain. DV = Death Valley (Source: Smith and others, 1990, fig. 2).

mation for an assessment of volcanic hazard. Smith and others (1990) and Smith, Bradshaw, and Mills (1993) provide a different point of view of the migration trends of volcanism in the Yucca Mountain region. They suggest that the structural control of basaltic volcanism should be evaluated at two scales. First, the control of large-scale regional structures (strike-flip faults, detachments) and volcano alignments related to these structures should be evaluated. Second, control of structures on and adjacent to Yucca Mountain and volcano alignments related to these structures should be evaluated. Models for structural control because of the different scales of geological structures may be different. For example, northwest-striking structures may result in a regional alignment of Pliocene and Quaternary cones in a northwest direction. But, at the scale of Yucca Mountain, northeast striking structures control the alignment of volcanoes (Smith and others, 1990). Although both models may be supported by the data, a judgment must be made as to which model is most appropriate for volcanic hazard studies at Yucca Mountain. In contrast to the work of Crowe, Johnson, and Beckman (1982), Ho (1992) recently has incorporated numerically the possibility that the sites of future volcanism may be controlled by specific segments of structures developed by Smith and others (1990) into the site disruption parameter, **p**. Details of point estimation and prior determination of **p** will be provided later.

Counts of Volcanic Events

In order to estimate the recurrence rate of the volcanism and the volcanic hazard to the repository, the definition of a single event has to be addressed. An accurate count of the number of eruptions is possible for volcanoes with a complete historical record. As no historical record is available for the Yucca Mountain region, identifying the number of eruptions depends on a clear understanding of eruptive processes and a reliable dating technique. Crowe and others (1983) indicate that a main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone. Therefore, Ho (1991a, 1992) attributes a single date to the cluster and creates a separate event with that date for each main cone in the cluster, using this definition of a single eruption from Crowe and others (1983). An alternative definition of a single event would be a single cluster of volcanic centers, because one may argue that all main cones in a cluster could arise from the same eruption.

Rationale

It is useful to perform sensitivity study of volcanic hazard assessment for the Yucca Mountain site because the controversy over the important issues we have briefly reviewed. The following development is to account for some significant geological factors raised by experts. Specifically, we will concentrate on the treatment of the model and the data.

MODELS FOR VOLCANIC ACTIVITY

Simple Poisson Process

The application of statistical methods to volcanic eruptions is put onto a sound analytical footing by Wickman (1965, 1976) in a series of papers that discuss the applicability of the methods and the evaluation of recurrence rates for a number of volcanoes. Wickman observes that, for some volcanoes, the recurrence rates are independent of time. Volcanoes of this type are termed "Simple Poissonian Volcanoes." Theoretically, the probability model for simple Poissonian volcanoes is derived from the following assumptions:

> Volcanic eruptions in successive time periods of length t for each period are independent and should follow a Poisson distribution with a constant mean (average rate) $\mu = \lambda t$, where λ is the recurrence rate in unit time and is assumed to be constant throughout the entire life of the volcanic activity.

If λ is assumed constant over t, the process is referred to as a homogeneous Poisson process (HPP). Because λ is constant and the increments are independent, it turns out that one does not need to be concerned about the location of the observation time interval, and an HPP is applicable for any interval of length t, $[s, s + t]$, $\mu = \lambda t$. That is, regardless of the interval selected, the variable remains Poisson with the appropriate mean. If events occur according to a Poisson process with parameter λ, then the waiting time until the first occurrence, T_1, follows an exponential distribution, $T_1 \sim \text{Exp}(\theta)$ with $\theta = 1/\lambda$. Furthermore, the times between consecutive occurrences are independent exponential variables with the same mean time between occurrences, $1/\lambda$. The assumption of a constant recurrence rate λ suggests that the volcanism, which depends on the availability of magma and a functioning triggering mechanism, as well as on their mutual interaction, is relatively uniform and does not get "exhausted" by loss of gases or for other reasons.

Suppose we assume that the successive volcanic eruptions at the Yucca Mountain region follow a simple Poisson process. Let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$. The following theoretical results are useful for this study:

(1) The maximum likelihood estimator for the recurrence rate λ is (Ho and others, 1991):

$$\hat{\lambda} = n/t$$

(2) The number of future eruptions, N, during $[t, t + t_0]$ would be distributed as a homogeneous Poisson random variable with constant rate λt_0,

$$P(N = k) = \exp[-\lambda t_0][\lambda t_0]^k/k!,\ k = 0, 1, \cdots$$

Weibull Process

If the volcanic trend is decreasing or increasing, the model should be generalized to allow λ to be, respectively, a decreasing or increasing function of t. More generally, one might want to allow the recurrence rate to be an arbitrary nonnegative function of t. Specifically, for volcanism, Ho (1991a, 1991b) considers a nonhomogeneous Poisson process (NHPP) with intensity function $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ for $\beta, \theta > 0$. The parameters β and θ may be referred to as shape and scale parameters, respectively. Because $\lambda(t)$ is the failure rate for the

Weibull distribution, the corresponding process has been termed the Weibull Process (WP). Goodness-of-fit, maximum likelihood (ML) estimates of β and θ, confidence intervals, and inference procedures for this process are presented in Bain and Engelhardt (1980), Bassin (1969), Crow (1974, 1982), Finkelstein (1976), and Lee and Lee (1978). A WP is appropriate for three types of volcanoes: increasing-recurrence-rate ($\beta > 1$), decreasing-recurrence-rate ($\beta < 1$), and constant-recurrence-rate ($\beta = 1$). This generalized model can be considered a goodness-of-fit test for an exponential model ($\beta = 1$) of the volcanic interevent times, which is equivalent to a homogeneous Poisson model of the events. In a simulation study, Bain, Engelhardt, and Wright (1985) conclude that the test which is derived as an optimal test for the WP also is powerful as a test of trend for general NHPP's. In other words, the test is "robust" against other model assumptions. This is the rationale of our choice of a WP to amend a simple Poisson model which neglects the time trend of the volcanic activities. Again, suppose we assume that the successive volcanic eruptions at the Yucca Mountain region follow a WP. For a time-truncated WP, let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at time $0 < t_1 < t_2 < \cdots < t_n$. Some useful theoretical results to be used later are summarized as follows:

(1) The maximum likelihood estimates (MLE) of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n \Big/ \sum_{i=1}^{n} \ln (t/t_i)$$

$$\hat{\theta} = t/n^{1/\hat{\beta}}$$

(2) If a WP is assumed during the observation time period $[0, t]$, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ at time t. In the application of the WP to volcanic eruptive forecasting, the estimate of $\lambda(t)$ is of considerable practical interest because $\lambda(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. Crow (1982) derives the MLE for $\lambda(t)$ as

$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\theta})(t/\hat{\theta})^{\hat{\beta}-1} = n\hat{\beta}/t$$

(3) Using the same WP, the number of occurrences, N, in time $[t, t + t_0]$, is a Poisson random variable,

$$P(N = k) = \exp[-m(t_0)][m(t_0)]^k/k!; \quad k = 0, 1, \cdots$$

where

$$m(t_0) = \int_t^{t+t_0} \lambda(s)\, ds$$

$$= [(t + t_0)^{\beta} - t^{\beta}]/\theta^{\beta}$$

(m obviously depends on t but our notation suppresses t because t is the known observation period.)

MODELING OF VOLCANIC DISRUPTION

Classical Approach

If we consider the fact that not every eruption would result in disruption of the repository, and let p be the probability that any single eruption is disruptive, then the number of occurrences of such a disruptive event $X(t_0)$ in $[0, t_0]$ also follows a homogeneous Poisson random variable with constant rate (Meyer, 1965, p. 156). Notice that this fact applies to the WP as well (see result 3 of the previous section). An important element in assessing the suitability of the site is an assessment of the potential for future volcanic disruption of the repository. Therefore, the probability of at least one disruptive event during the next t_0 years is of considerable practical interest and is quoted as "hazard" (see e.g., UNESCO, 1972). In a classical statistical analysis, we would use the Poisson probability distribution formula,

$$\text{hazard} = \Pr(\text{at least one disruptive event before time } t_0)$$
$$= 1 - \exp\{-\lambda \mathrm{p} t_0\}$$

for an HPP. And

$$\text{hazard} = 1 - \exp\{-m(t_0)\mathrm{p}\}$$

for a WP. Point or interval estimates for the hazard can be obtained based on those of p, λ, and $m(t_0)$.

Bayesian Approach

For the Bayesian approach, we consider λ and $m(t_0)$ to be fixed for both the HPP and the WP, and we permit prior distribution for p. The prior distribution, $\pi(p)$, of p expresses our beliefs regarding the numerical values of p. This would incorporate uncertainties about the probability of repository disruption p that are averaged eventually as shown in the following equations. In this situation, using the model of constant λ

$$\text{hazard} = 1 - \int_p \exp\{-\lambda p t_0\}\pi(p)\,dp$$

for the HPP. And,

$$\text{hazard} = 1 - \int_p \exp\{-m(t_0)p\}\,\pi(p)\,dp$$

for the WP. The technical machinery (Bayesian approach) involved in the given equations would support more informative answers if the prior distribution $\pi(p)$ is adequately selected.

Point Estimation and Prior Determination of p

Crowe, Johnson, and Beckman (1982) assume that every eruption has the same probability of repository disruption p, and provide a point estimate for p($= a/A$). The calculations are based on a fixed value of a ($=$ area of the repository estimated at 6–8 km^2), and several choices of A (an area, ranging from 1953 km^2 to 69,466 km^2, that corresponds closely to a defined volcanic province and satisfies the requirement of a uniform value of λ). Additional results of point estimates of p proposed by several experts are listed in table 7.1 of Crowe, Perry, and Valentine (1993). The values range from 1.1×10^{-3} to 8×10^{-2}. We shall use these two bounds of p for the classical approach in the sensitivity data analysis.

We now turn to the description of the prior density for the Bayesian approach (Ho, 1992). Because the permissible range of p is $0 \le p \le 1$, without use of expert opinions regarding the geological factors at NTS, a natural choice for $\pi(p)$ is a noninformative prior. For instance, $U(0, 1)$ (uniform 0, 1) assumes an average of 50% "direct hit," which is unrealistically conservative (overestimation). Ho (1992) settles on one particular prior based on the geological structure of the volcanic centers at NTS.

According to Smith and others (1990), the area of most recent volcanism (AMRV) includes all known post-6-Ma volcanic complexes in the Yucca Mountain area and encompasses the four Quaternary volcanic centers in Crater Flat, the Lathrop Wells cone, several centers in southeastern Crater Flat, two centers at Sleeping Butte, and a center at Buckboard Mesa within the moat of the Timber Mountain caldera. They conclude that future volcanic events in the Yucca Mountain area will be associated with Quaternary centers in Crater Flat, at Sleeping Butte, or at the Lathrop Wells cone (see Fig. 2). Based on their assumption, a future eruption may occur either to the north-northeast or south-southwest of an existing cone or group of cones. They show high risk zones within the AMRV in Figure 3 by placing two rectangles on each group of Quaternary cones. The proposed high-level nuclear waste repository at Yucca Mountain falls within the larger high-risk Lathrop Wells rectangle and just to the east of the high-risk zones constructed for the Crater Flat chain as described in Figure 3. The dimensions of the larger Lathrop Wells rectangle are 50 km long and 3 km wide as determined by analog studies of Pliocene volcanic centers in the Fortification Hill field (Lake Mead area, Arizona and Nevada) and the Reveille Range (south-central Nevada). The lower half of this rectangle is outside the AMRV.

Now, using the idea of Crowe, Johnson, and Beckman (1982), assume

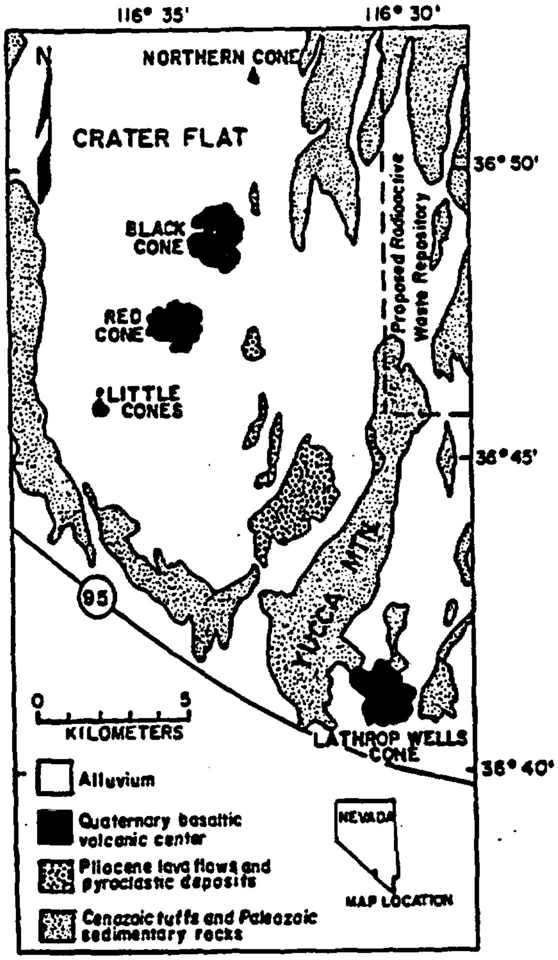

Figure 2. Generalized geologic map of Crater Flat volcanic field area and boundary of proposed radioactive waste repository; inset map shows location of Crater Flat volcanic field (source: Wells and others, 1990, fig. 1).

there is no heterogeneity with respect to disruptiveness in the upper-half of the rectangle that encloses the repository (the eruptions to the south-southwest of the Lathrop Wells cone are outside the AMRV, and have near zero probability of disrupting the site). So, given $A = 75$ km^2 ($=$ half of the area of the rectangle), $a = 8$ km^2 (area of the repository), we obtain $p = a/A = 8/75$. Therefore, a more informative prior, $U(0, 8/75)$, which assumes 8/75 as the

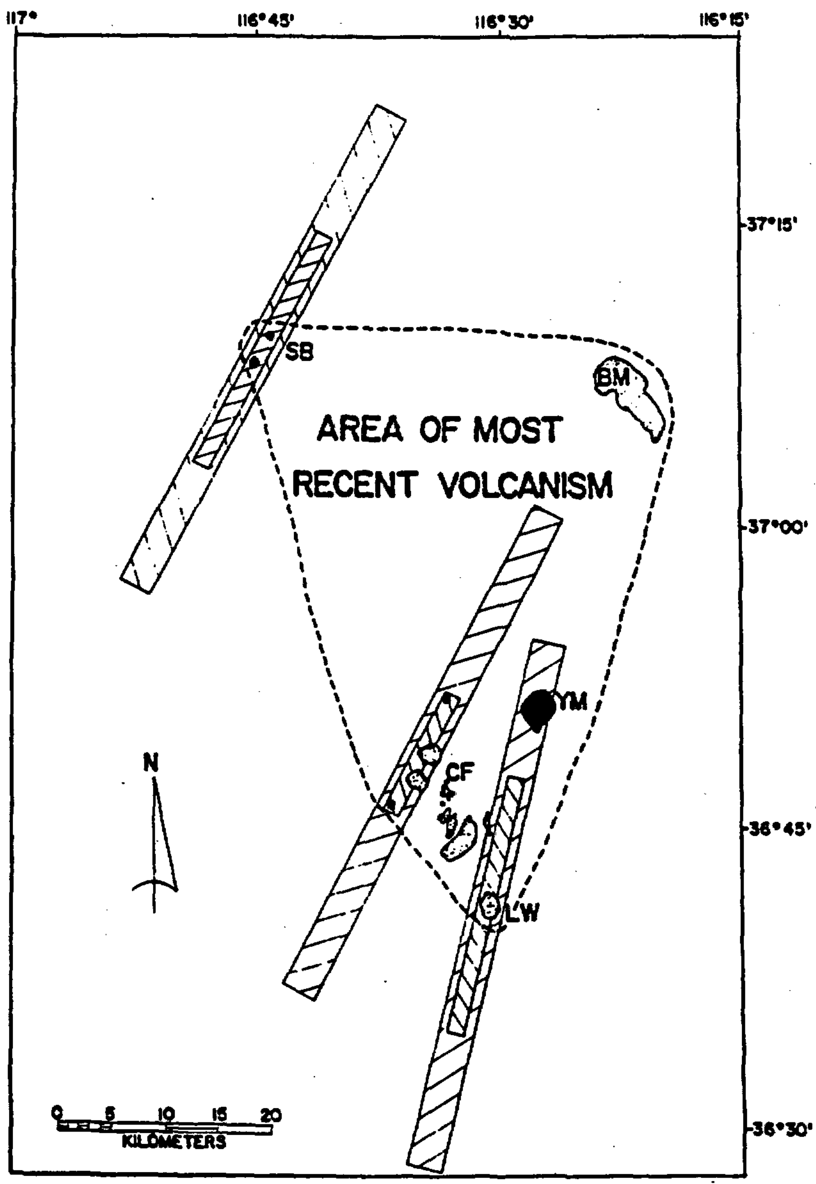

Figure 3. Map outlining ARMV (dashed line) and high-risk zones (rectangles) in Yucca Mountain (YM) area that include Lathrop Wells (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), volcanic centers within Crater Flat (CF) (source: Smith and others, 1990, fig. 7).

upper limit for p seems to be more suitable. We shall conduct all Bayesian analysis based on this prior which is developed from the geological structure of the volcanic centers at NTS.

DATA

The following is a summary of field and chronology data using the most current information from site characterization studies (Crowe, Perry, and Valentine 1993). Pliocene volcanic events in the Yucca Mountain region include:

(1) 4.6 Ma Centers: **Basalt of the Thirsty Mesa.** This is a lava mesa formed from three coalesced vents. It is treated as one or three events with an age of 4.6 Ma.

(2) 4.4 Ma Center: **Basalt of the Amargosa Valley.** This volcanic event is represented by the aeromagnetic anomaly located a few kilometers south of the town of Amargosa Valley.

(3) 3.7 Ma Centers: **Basalt of southeast Crater Flat.** This Pliocene unit consists of five centers representing one to five events. The age of the centers is assumed to be well dated at 3.7 Ma.

(4) 2.9 Ma Centers: **Basalt of Buckboard Mesa.** This consists of one center or event forming a lava mesa and small cone in the moat zone of the Timber Mountain caldera.

Quaternary events in the Yucca Mountain region include:

(1) 1.1 Ma Centers: **Quaternary basalt of Crater Flat.** These are treated by Crowe, Perry, and Valentine (1993) as four individual centers. Smith, Bradshaw, and Mills (1993) suggest that at least six centers now must be considered in the calculation from the Quaternary basalt of Crater Flat: NE Little Cone, SW Little Cone, Black Cone, Northern Cone, Red Cone 1, Red Cone 2.

(2) 0.38 Ma Centers: **Basalt of Sleeping Butte.** These are treated as two individual centers clustered on a northeast-trend 45 km northwest of the Yucca Mountain site.

(3) 0.1 Ma Centers: **Lathrop Wells Center.** This is treated as a single event center formed in two pulses of activity, one at about 100 to 140 ka, the other at >40 ka. The existence of a potential young volcanic event (10 ka) at the center remains controversial (Crowe, Perry, and Valentine 1993).

Another key issue in the sensitivity analysis is to specify the observation period, $[0, t]$, in modeling the volcanic history at NTS. Most of the volcanic hazard assessment studies in the Yucca Mountain area are centered around the post-6-Ma (Pliocene and younger) and Quaternary (< 1.6 Ma) volcanism (Crowe and others, 1982; Smith and others, 1990; Wells and others, 1990; Connor and Hill, 1993). We shall use the given dates to estimate the recurrence rate of volcanism during the following two observation periods: Pliocene and younger (<6.0 Ma), and Quaternary (< 1.6 Ma). Therefore, let the beginning of the Pliocene period (± 6.0 Ma) be time zero, so $t = 6.0$ Ma. For the study on Quaternary volcanism, $t = 1.6$ Ma. Prediction of future volcanic activities (volcanic eruption and site disruption) will focus on the entire life of the repository (A 10^4 year period is recommended as the required isolation period during which radioactive waste may decay to an acceptable level). Thus, we shall evaluate the hazard with $t_0 = 10{,}000$.

DATA ANALYSIS

Three types of models are considered in the following sensitivity analysis. The first model (HPP) assumes that both past and future volcanic activities follow an HPP. The second model (WP-HPP) uses a WP to estimate the instantaneous recurrence rate based on the historical data at NTS. The model then switches from a WP of past events to a predictive HPP (a constant rate for future events). The third model (WP) assumes that the prior historical trend based on a WP would continue for future activities. Hazards (at least one disruptive event before time t_0) using both classical and Bayesian approaches are evaluated based on the data for the following two observation periods: Pliocene and younger, and Quaternary.

As we have mentioned, another key issue in the site characterization studies is the disagreement about age-dating of the rocks and counts of volcanic events. The following treatment of the data is to account for some significant differences raised by experts. The dates (in Ma) summarized from the previous section are: 4.6 (1 to 3 events), 4.4, 3.7 (1 to 5 events), 2.9, 1.1 (4 to 6 events), 0.38 (2 events), 0.1, 0.01 (this remains controversial but possible). Combinations of various counts at volcanic centers of controversy and inclusion (or exclusion) of the youngest date (= 0.01) generate 90 ($= 3 \times 5 \times 3 \times 2$) different data sets (Pliocene and younger volcanism). Sensitivity analysis is performed for each data set and only the minimum and the maximum hazards for each model are summarized in Table 1 (Quarternary volcanism) and Table 2 (Pliocene volcanism).

CONCLUDING REMARKS

In characterizing the Yucca Mountain site, scientists must study geology, hydrology, volcanoes, earthquakes, and variability in climate. A geologic repository has never been attempted and it presents a number of challenges. The

Table 1. Results of Sensitivity Analysis for Proposed Yucca Mountain Repository Site Based on Data of Quaternary Volcanism

Model	Recurrence rate (min, max)	Hazard: Classical $p = 1.1 \times 10^{-3}$	Hazard: Classical $p = 8 \times 10^{-2}$	Hazard: Bayesian
HPP	$(4.38 \times 10^{-6}, 6.25 \times 10^{-6})$	$(4.81 \times 10^{-5}, 6.87 \times 10^{-5})$	$(3.49 \times 10^{-3}; 4.99 \times 10^{-3})$	$(2.33 \times 10^{-1}, 3.33 \times 10^{-1})$
WP-HPP	$(5.83 \times 10^{-6}, 8.23 \times 10^{-6})$	$(6.40 \times 10^{-5}, 9.06 \times 10^{-5})$	$(4.65 \times 10^{-3}, 6.56 \times 10^{-3})$	$(3.10 \times 10^{-1}, 4.38 \times 10^{-1})$
WP	$(5.83 \times 10^{-6}, 8.23 \times 10^{-6})$	$(6.41 \times 10^{-5}, 9.06 \times 10^{-5})$	$(4.65 \times 10^{-3}, 6.57 \times 10^{-3})$	$(3.10 \times 10^{-1}, 4.38 \times 10^{-1})$

Table 2. Results of Sensitivity Analysis for Proposed Yucca Mountain Repository Site Based on Data of Pliocene and Younger Volcanism

Model	Recurrence rate (min, max)	Hazard: Classical $p = 1.1 \times 10^{-3}$	Hazard: Classical $p = 8 \times 10^{-2}$	Hazard: Bayesian
HPP	$(1.83 \times 10^{-6}, 3.33 \times 10^{-6})$	$(2.02 \times 10^{-5}, 3.67 \times 10^{-5})$	$(1.47 \times 10^{-3}, 2.66 \times 10^{-3})$	$(9.77 \times 10^{-4}, 1.78 \times 10^{-3})$
WP-HPP	$(3.41 \times 10^{-6}, 5.67 \times 10^{-6})$	$(3.75 \times 10^{-5}, 6.24 \times 10^{-5})$	$(2.72 \times 10^{-3}, 4.53 \times 10^{-3})$	$(1.82 \times 10^{-3}, 3.02 \times 10^{-3})$
WP	$(3.41 \times 10^{-6}, 5.67 \times 10^{-6})$	$(3.75 \times 10^{-5}, 6.24 \times 10^{-5})$	$(2.72 \times 10^{-3}, 4.53 \times 10^{-3})$	$(1.82 \times 10^{-3}, 3.02 \times 10^{-3})$

sensitivity analysis performed in this paper is an assessment of the potential for future volcanic activity at NTS. The numbers presented are subject to change should the volcanic history be modified pending the final results of the site characterization study. We now summarize this study with a few key points and comments.

(1) The estimated values of recurrence rate and hazard (Table 1) based on the Quaternary data are higher than those obtained from the data of post-6-Ma volcanism (Table 2). However, they are of the same order of magnitude across-the-board. The most obvious reason is that the length of the Pliocene period (t = 6.0 Ma) outweighs the greater number of events.

(2) The instantaneous recurrence rates produced by the WP generally are higher than rates obtained from the HPP, which shows the volcanic trend at the Yucca Mountain region is increasing ($\beta > 1$). Because the WP incorporates the time trend, evidence of additional events will not increase necessarily the instantaneous recurrence rate as it would for the HPP recurrence rate. Additional older events would serve to decrease β, whereas evidence of more recent events (< 1 Ma) would most likely increase β, that is, the rate is increasing with time.

(3) The classical approach using $p = 1.1 \times 10^{-3}$ and $p = 8 \times 10^{-2}$ yields the lowest and the highest values respectively for the hazard. Because the minimum value for p is almost two orders of magnitude smaller than the maximum value (indicating considerable uncertainty exists about the estimation of p), it is not unexpected that the highest hazard is also two orders of magnitude greater than the lowest hazard. Note that the Bayesian approach yields hazards that are of the same order of magnitude as those calculated using the higher value of p. The Bayesian approach used an informed prior obtained from the geological information of the AMRV. Using $U(0, 1)$ as a noninformative prior for p would have increased the volcanic hazard. This may argue that the low value of p used in the classical approach is too small and that a more realistic lower bound for p is at least one order of magnitude greater.

(4) Results of both WP and WP-HPP models are almost identical, because the projected time frame (10,000 years) is only a small fraction ($10^4/1.6 \times 10^6 = 6.25 \times 10^{-3}$) of the Quaternary period, about 5% of the average repose time. This relatively short time scale suggests that switching from a WP model of past events to a predictive HPP model is justified (i.e. the difference is negligible). The predictive HPP model has the advantages of greater simplicity, parameters and confidence intervals are easier to calculate, and hypothesis testing is more developed.

(5) The Lathrop Wells volcanic center is located 20 km south of the potential Yucca Mountain site, at the south end of the Yucca Mountain range. It has long been recognized as the youngest basalt center in the region. However, determination of the age and eruptive history (monogenetic vs. polycyclic volcanism) of the center remain the subject of considerable debate. My results show

(not shown in the tables) that the inclusion of the potential young volcanic event (10 ka) at Lathrop Wells increases the values of the hazard across-the-board, a result not unexpected (see conclusion 2). Should further young events be determined at Lathrop Wells or other sites in the AMRV, all hazard values would increase, but those from the WP and WP-HPP models could change proportionally more than those from the HPP as the evidence of increasing trend is strengthened.

(6) As expected, data with the least (most) count of events yield the lowest (highest) values of both the recurrence rate and hazard using the model of HPP. In contrast, the WP model is sensitive to the numbers, and relative sizes (to t) of the ordered t_is. If early sparse t_is (cumulative observation times) were accompanied later by greater number of t_js toward the end of the observation period, then $\hat{\beta}$ would be large, showing an increasing trend of eruption through time, and vice versa. For example, the data set which produces the lowest hazard in Table 2 (WP and WP-HPP models only) is: 4.6, 4.6, 4.6, 4.4, 3.7, 2.9, 1.1, 1.1, 1.1, 1.1, 0.38, 0.38, 0.1 ($\hat{\beta}$ = 1.57). The hazard actually is higher if we only count one event for the Basalt of the Thirsty Mesa (= 4.6 Ma) and keep the same counts for the others (in this situation, $\hat{\beta}$ = 2.05). Along the same line of argument, the data set which produces the highest hazard in Table 2 is: 4.6, 4.4, 3.7, 2.9, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 0.38, 0.38, 0.1, 0.01 ($\hat{\beta}$ = 2.43).

In summary, sensitivity analysis based on the data of Quaternary volcanism predicts that the hazard is between 4.81×10^{-5} and 6.57×10^{-3}, whereas the corresponding hazard based on the data of Pliocene and younger is between 2.02×10^{-5} and 4.53×10^{-3}. Therefore, the estimated overall probability of at least one disruption of a repository at the Yucca Mountain site by basaltic volcanism for the next 10,000 years should be bounded between 2.02×10^{-5} and 6.57×10^{-3}.

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APPENDIX 2

Article (2): Volcanic Hazard Assessment Incorporating Expert Knowledge: Application to the Yucca Mountain Region, Nevada, U.S.A.

Volcanic Hazard Assessment Incorporating Expert Knowledge:
Application to the Yucca Mountain Region, Nevada, U.S.A.

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ABSTRACT

Multiple-expert hazard/risk assessments have considerable precedent, particularly in the Yucca Mountain site characterization studies. A certain amount of expert knowledge is needed to interpret the geological data used in a probabilistic data analysis. As is often the case in science, experts disagreed on crucial points. Consequently, lack of consensus in some studies is a sure outcome. In this paper, we present a Bayesian approach to statistical modeling in volcanic hazard assessment for the Yucca Mountain site. Specifically, we show that the expert opinion on the site disruption parameter **p** is elicited on the prior distribution, $\pi(p)$, based on geological information that is available. Moreover, $\pi(p)$ can combine all available geological information motivated by conflicting but realistic arguments (e.g., simulation, cluster analysis, structural control, $\cdots$, etc.). The incorporated uncertainties about the probability of repository disruption **p** will eventually be averaged out by taking the expectation over $\pi(p)$. We use the following priors in the analysis: (1) priors chosen for mathematical convenience: Beta (r, s) for $(r, s) = (2,2)$, $(3,3)$, $(5,5)$, $(2,1)$, $(2,8)$, $(8,2)$, and $(1,1)$; and (2) three priors motivated by expert knowledge. Sensitivity analysis is performed for each prior distribution. Our study concludes that estimated values of hazard based on the priors chosen for mathematical simplicity are uniformly higher than those obtained based on the priors motivated by expert knowledge. And, the model using the prior, Beta (8,2), yields the highest hazard ($= 2.97 \times 10^{-2}$). The minimum hazard is produced by the "three-expert prior" (i.e., values of $\mathbf{p}$ are equally likely at 10^{-3}, 10^{-2}, and 10^{-1}). The estimate of the hazard is 1.39×10^{-3}, which is only about one order of magnitude smaller than the maximum value. The term, "hazard", is defined as the probability of at least one disruption of a repository at the Yucca Mountain site by basaltic volcanism for the next 10,000 years.

KEY WORDS: Expert knowledge, Nonhomogeneous Poisson process, Power-law process, Prior distribution, Volcanic hazard.

INTRODUCTION

In the ongoing national debate on nuclear power as a source of electricity, a key issue is the disposition of the high-level radioactive wastes produced in the process. At an earlier stage in this debate, Congress, aware of the importance of the waste issue, passed the Nuclear Waste Policy Act of 1982. This legislation required the federal government to develop a geologic repository for the permanent disposal of high-level radioactive waste from civilian nuclear power plants. This waste consists primarily of spent nuclear fuel. A brief summary of key events in the high-level nuclear waste program follows. In 1982, the Nuclear Waste Policy Act directed the Department of Energy (DOE) to site, design, construct, and operate a geologic repository for spent nuclear fuel from civilian reactors and other high-level radioactive waste. DOE established the Office of Civilian Radioactive Waste Management (OCRWM) in 1983 in response to the legislation and started site characterization at three candidate sites in 1986. In 1987, Congress directed that site characterization continue only at the Yucca Mountain site in southern Nevada. In 1988, OCRWM issued a Site Characterization Plan for the Yucca Mountain site, which identified plans for scientific and engineering work needed to determine if the site was suitable. The approach to site characterization outlined in the 1988 Site Characterization Plan called for extensive testing to obtain a comprehensive understanding of the Yucca Mountain site to allow decisions to be made simultaneously on site suitability, licensing, and repository design issues.

An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site (DOE, 1988) because the late Tertiary geologic history of southwestern Nevada has been dominated by volcanism and the consequent deposition of volcanic flows and tuffaceous rocks. Yucca Mountain, similar to most surrounding ranges, is composed dominantly of a series of Miocene ashflow tuff units and silicic volcanic rocks.

Crowe et al. (1982) use a homogeneous Poisson process (HPP) (see e.g., Wickman, 1965) to evaluate the volcanic hazard at the Yucca Mountain region (YMR). The main component of the mathematical calculation is $\exp\{-t_0 \lambda \mathbf{p}\}$, where $t_0 = 10,000$ years (the projected time period), $\lambda =$ the recurrence rate in unit time and is assumed to be constant throughout the entire life of the volcanic activity, and $\mathbf{p}$ is estimated as a/A (a is the area of the repository, and A is the area of a chosen sample region). Once A is chosen, $\mathbf{p}$ is constant and is a fixed probability of site disruption for any given eruptions that occur in the future. Connor and Hill (1995) provide 2-D probability maps for the YMR using nonparametric techniques. The corresponding component of Connor and Hill (1995) is $\exp\left\{-t_0 \iint_a \lambda_r(x,y)\,dxdy\right\}$, where λ_r is a function of location (x,y). For a small region a, the estimated $\lambda_r(x,y)$ is a constant for all (x,y) in a. Therefore, $\iint_a \lambda_r(x,y)\,dxdy = a\lambda_r$, which implies that λ_r is equivalent to λ/A. In other words, λ (in Crowe et al., 1982) represents the number of eruptions in unit time, and λ_r (in Connor and Hill, 1995) is the number of eruptions in unit time and unit area; λ and λ_r are estimated accordingly, but are both assumed to be constant in time for the probability calculations.

In this paper, our basic equation for the volcanic hazard is $\underset{p}{E}\{\exp[-m(t_0)p]\}$. Where $m(t_0) = \mu(t+t_0) - \mu(t)$, and $\mu(s) = \mu(s|\theta,\beta) = (s/\theta)^{\beta}$ represents the expected number of events to time s. Once the functional form of μ is specified, a nonhomogeneous Poisson process (NHPP) is fully characterized. Hence, we use β to report the time-trend: increasing ($\beta > 1$), decreasing ($\beta < 1$), or random ($\beta = 1$, as has been assumed to be so in the previous two models), and let the prior historical time-trend continue for future volcanic activities. We then permit prior distribution, $\pi(p)$, for **p**. Notice that, Bayesian approaches with meaningful chosen priors would bring improvements to the analysis, because of their ability to incorporate expert knowledge into the inferential mechanism. A noteworthy feature of our approach is that the expert opinion is elicited on the prior distribution, $\pi(p)$, based on geological information that is available. Sensitivity based on two classes of priors will be discussed: (a) priors chosen for mathematical convenience, and (b) priors motivated by expert knowledge.

A MAJOR DILEMMA

Although we have sophisticated models for calculating the probabilities of geological events (e.g., see Cox and Lewis, 1966; Cressie, 1993; Daley and Vere-Jones, 1988; Hunter and Mann, 1992; Ripley, 1981; Rodionov, 1987), the criteria for selecting data and interpreting results are still relatively primitive. In the case of the Yucca Mountain Project, it is very difficult to judge the validity of the data and the statistical models for a particular purpose as described in the previous section.

As a first requirement, a statistical model must be based on a sound geological interpretation of the tectonic, structural, and magmatic processes governing the distribution of volcanism in space and time. With this information, we can draw on the records of volcanoes with similar settings and styles of eruptive activity to estimate the possible repose intervals, frequencies of eruption, and volcanic hazards. Therefore, a certain amount of expert judgement (knowledge) is needed to interpret the geological information collected (and used) in a probabilistic data analysis. Consequently, lack of consensus in volcanic hazard/risk assessment is a sure outcome, because experts disagreed on crucial points as is often the case in science. In this paper, we demonstrate a Bayesian approach in volcanic hazard assessment to incorporate multiple-expert knowledge.

THE UNDERLYING MODEL

Power-Law Process

Volcanism in the Yucca Mountain region, Nevada, has been the topic of numerous studies focusing on the probability of disruption of a proposed high-level radioactive waste repository by volcanic activity (Connor and Hill, 1995; Crowe et al., 1982; Ho, 1991, 1992, 1995; Ho et al., 1991; Sheridan, 1992). These studies are pursued largely because the proposed waste repository is located within 10 to 20 km of at least five Quarternary cinder cones and the high-level radioactive waste must be isolated from the surrounding environment for a period of at least 10,000 years. The area of the actual repository is currently estimated to be 6-8 km^2. Our focus of interest is to estimate the probability of at least one disruptive event at the Yucca Mountain site during the next 10,000 years. This probability is termed as "volcanic hazard" or "hazard" (see e.g., UNESCO, 1972) throughout the remaining of the paper.

The use of the nonhomogeneous Poisson process has recently gained popularity in volcanic data analysis as a simple and versatile tool to assess the waxing or waning time-trends of a volcano and to assess its volcanic hazards (see e.g., Ho 1991, 1995). In this paper, we present an NHPP with a mean value function denoted by $\mu(t|\Theta)$, where Θ is a vector of parameters. The nondecreasing function $\mu(t|\Theta)$ represents the expected number of events to present time, t. Once the functional form of $\mu(t|\Theta)$ is specified, the NHPP is fully characterized. An alternate characterization of the NHPP is through its intensity function $\lambda(t|\Theta)$, where

$$\lambda(t|\Theta) = \frac{d}{dt}\mu(t|\Theta).$$

Along the same line of arguments in Ho (1991, 1995), we let $\Theta = (\theta, \beta)$ and write

$$\mu(t|\theta) = (t/\theta)^{\beta},$$

so that

$$\lambda(t|\theta,\beta) = (\beta/\theta)(t/\theta)^{\beta-1}.$$

This form, termed the *power law*, has found applications in reliability due to its flexibility (in the sense that the intensity function can be constant, decreasing, or increasing) and the fact that the time to first arrival in the process is a Weibull distribution. It is because of this latter property that the underlying Poisson process has sometimes been referred to as the "Weibull process" (Crow, 1974). A noteworthy feature of our approach by replacing the expected number of events in an HPP, λt, with $\mu(t) = (t/\theta)^{\beta}$ is that we use β to report the time-trend: increasing ($\beta > 1$), decreasing ($\beta < 1$), or random ($\beta = 1$, as has been assumed to be so in the models of Crowe et al., 1982, and Connor and Hill, 1995), and let the prior historical time-trend continue for future volcanic activities. Another noteworthy feature of our approach is that we also let the experts be in control of the estimation of the site disruption parameter, **p**. The remaining of this paper is designed to address the methods of incorporating the available expert knowledge to the probability calculations.

Classical Approach Versus Bayesian Approach

Crowe et al. (1982) assume that every eruption has the same probability of repository disruption **p**, and provide a point estimate for **p** $(= a/A)$. The calculations are based on a fixed value of a (= area of the repository estimated at 6-8 km^2), and several choices of A (an area, ranging from 1,953 km^2 to 69,466 km^2, that corresponds closely to a defined volcanic province and satisfies the requirement of a uniform value of λ). In contrast to the above classical approach, Ho (1992, 1995) permits prior distribution, $\pi(p)$, for **p**. The focus of this paper is to demonstrate that Bayesian approaches with meaningful chosen priors would bring improvements to the analysis, because of their ability to incorporate expert knowledge into the inferential mechanism. Specifically, we shall extend the ideas of Ho's model (1992, 1995) to show that the expert opinion on the site disruption parameter **p** is elicited on the prior distribution, $\pi(p)$, based on geological information that is available. Moreover, $\pi(p)$ can combine all available geological information motivated by conflicting but realistic arguments (e.g., simulation, cluster analysis, structural control, $\cdots$, etc.). The incorporated uncertainties about the probability of repository disruption $\mathbf{p}$ will eventually be averaged out by taking the expectation over $\pi(p)$. Thus our approach should prove superior to both the sample theory method that does not use expert information and the existing Bayesian approaches that do not better exploit the knowledge that is available. To achieve this goal, we first review the mathematical groundwork (most of which has been addressed in Ho, 1992, 1995) in the next section. We then concentrate on the determination of the prior distributions which either are chosen for mathematical convenience or are motivated by expert knowledge.

Mathematical Layout

Let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at time $t_i \{i = 1, \cdots, n\}$, $0 < t_1 \leq t_2 \leq \cdots \leq t_n \leq t$. Consider an NHPP with a mean value function denoted by

$$\mu(t|\theta, \beta) = (t/\theta)^{\beta},$$

so that

$$\lambda(t|\theta,\beta) = (\beta/\theta)(t/\theta)^{\beta-1}.$$

Some useful theoretical results to be used later are summarized as follows:

(1) The number of new eruptions, N, in a future time interval $[t, t+t_0]$ follows

$$\Pr(N=k) = \exp[-m(t_0)][m(t_0)]^k \Big/ k!, \quad k = 0, 1, \cdots$$

[This is the probability of obtaining k events out of N.]

where

$$\begin{aligned} m(t_0) &= \int_t^{t+t_0} \lambda(s)\, ds \\ &= \mu(t+t_0) - \mu(t) \\ &= \left[(t+t_0)^\beta - t^\beta\right] \Big/ \theta^\beta. \end{aligned}$$

(2) The maximum likelihood estimates (MLEs) of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n \Big/ \sum_{i=1}^{n} \ln(t/t_i)$$
$$\hat{\theta} = t \Big/ n^{1/\hat{\beta}}.$$

(3) In a classical analysis, we would use the Poisson probability distribution

formula,

$$\begin{aligned}\text{Hazard} &= \Pr(\text{at least one disruptive event before time } t_0) \\ &= 1 - \exp\{-\lambda \mathrm{p} t_0\}\end{aligned}$$

for an HPP model with a fixed value of **p**. For the Bayesian approach, the prior distribution, $\pi(p)$, of **p** expresses our beliefs regarding the numerical value of **p**. This would incorporate expert knowledge/uncertainties about the probability of repository disruption **p** that are averaged (by taking the expectation of the hazard over the distribution of **p**) eventually as shown below for an NHPP model:

$$\text{Hazard} = \underset{p}{E}\{1 - \exp[-m(t_0)\,p]\},$$

where

$$m(t_0) = \mu(t + t_0) - \mu(t),$$

and can be estimated using the equations summarized in (1) and (2).

ASSESSING PRIOR PROBABILITIES

A probability distribution for an unknown parameter is intended to embody one's uncertainty, based on beliefs, convictions, experience, hunches, and past data. It is subjective or personal because the probability distribution involves beliefs and opinions that are peculiar to an individual. For the Yucca Mountain case, it is reasonable to assume that the site disruption parameter varies for different geological interpretations, and it may be proper to treat **p** as a random variable. The introduction of the probability distribution function (pdf) $\pi(p)$, which usually is called a prior distribution for the parameter **p**, constitutes an additional assumption that may be inevitable. Also, averaging the volcanic hazard relative to a pdf $\pi(p)$ is a procedure that provides a possible way to combine several estimators when all of their validity can be justified to certain degree based on some geological theories and/or methods.

A whole class of estimators can be produced by considering different pdf's $\pi(p)$. It is useful to have a class of estimators in a problem, although if there is some physical reason to justify choosing a particular $\pi(p)$, then the estimator associated with that $\pi(p)$ would presumably be the best one to use in this problem. There are different philosophies involved with choosing prior distributions $\pi(p)$. Specifically, for the site disruption parameter, we shall concentrate on two classes of prior distributions and the sensitivity analysis will be performed based on the Yucca Mountain volcanic data.

PRIORS CHOSEN FOR MATHEMATICAL CONVENIENCE

Bayesian approaches with meaningful chosen prior would bring improvements to the analysis, because their ability to incorporate expert knowledge into the inferential mechanism. However, most of the prior distributions used are chosen more for their mathematical convenience (e.g., see Berger, 1985) than due to a realistically motivated argument. For example, because the permissible range of p is $0 \leq p \leq 1$, without use of expert opinions regarding the geological factors at NTS, a natural choice for $\pi(p)$ is a noninformative prior. For instance, $U(0,1)$ (uniform $0, 1$) assumes an average of 50% "direct hit." Also, the family of beta distributions provides a versatile and useful set of models for distributions on the unit interval $0 < p < 1$. The family of *beta distributions* (denoted as Beta (r, s)) is defined by the density

$$f(p) = \frac{1}{B(r,s)} p^{r-1} (1-p)^{s-1}, \quad \text{for } 0 < p < 1,$$

where r and s are positive parameters, and

$$B(r,s) = 2\int_0^{\pi/2} \cos^{2r-1}\theta \sin^{2s-1}\theta d\theta = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)}.$$

For $r > 1$ and $s > 1$ they are unimodal, with maximum density at the value $p = (r-1)/(r+s-2)$, having various degrees of peakedness depending on the size of $r+s$, and various degrees of skewness depending on the ratio of r to s. (For $r = s$ they are symmetric.) For $r = s = 1$ the density is uniform on $(0,1)$. Several members of the beta family are shown in Figure 1, and will be used for the data analysis. A discussion on the sensitivity will also be available.

PRIORS MOTIVATED BY EXPERT KNOWLEDGE

The idea of probabilistic natural hazard assessment is not new (e.g., Cornell, 1988; Cressie, 1993; Hunter and Mann, 1992; Rodionov, 1987, and many references therein). But what is new here is a consideration of this idea to a dense collection of priors as our mechanism for combining expert opinions. Potentially useful estimators (priors) can be developed through the following structure which illustrate the train of thought that is involved in the proposed model.

Priors Summarizing the Aggregate Opinion of a Single Expert

The literature on normative approaches for the formation of aggregate opinion on the Yucca Mountain volcanic study is now available; see, for instance, the articles of Ho (1992), and Ho (1995). We now turn to the description of the prior density developed by Ho (1992, 1995) as an illustrative example of how the $\pi(p)$, which incorporates expert knowledge and geological information, is chosen for the purpose of volcanic hazard assessment at the Yucca Mountain site.

According to Smith et al. (1990), the area of most recent volcanism (AMRV) includes all known post-6-Ma volcanic complexes in the Yucca Mountain area and encompasses the four Quarternary volcanic centers in Crater Flat, at Sleeping Butte, and a center at Buckboard Mesa within the moat of the Timber Mountain caldera. Smith et al. (1990) conclude that future volcanic events in the Yucca Mountain area will be associated with Quarternary centers in Crater Flat, the Lathrop Wells cone, several centers in southeastern Crater Flat, two centers at Sleeping Butte, or at the Lathrop Wells cone (see figure 7, Smith et al., 1990). Based on this assumption, a future eruption may occur either to the north-northeast or south-southwest of an existing cone or group of cones. Smith et al. (1990) show high risk zones within the AMRV by placing two rectangles on each group of Quaternary cones. The proposed high-level nuclear waste repository at Yucca Mountain falls within the larger high-risk Lathrop Wells rectangle and just to the east of the high-risk zones constructed for the Crater Flat chain as described in figure 7, Smith et al. (1990). The dimensions of the larger Lathrop Wells rectangle are 50 km long and 3 km wide as determined by analog studies of

Pliocene volcanic centers in the Fortification Hill field (Lake Mead area, Arizona and Nevada) and the Reveille Range (south-central Nevada). The lower half of this rectangle is outside the AMRV.

Now, using the idea of Crowe, et al. (1982), assume there is no heterogeneity with respect to disruptiveness in the upper-half of the rectangle that encloses the repository (the eruptions to the south-southwest of the Lathrop Wells cone are out the AMRV, and have near zero probability of disrupting the site). So, given $A = 75$ km^2 (= half of the area of the rectangle), $a = 8$ km^2 (area of the repository), then $\mathbf{p} = a/A = 8/75$. Therefore, a more informative prior $U(0, 8/75)$, which assumes $\mathbf{p} = 8/75$ as the worst-case scenario, and $\mathbf{p} = 0$ as the best-case scenario for $\mathbf{p}$ respectively seems to be more suitable than $U(0,1)$. That is, any value between 0 and 8/75 is considered equally likely as the true value of the unknown parameter $\mathbf{p}$ without additional geological information. This prior is quoted as a "one-expert prior" for further references. Notice that, alternatively, one could use the following discrete $\pi(p)$: $\pi_1 = \Pr[\mathbf{p} = 0]$, and $\pi_2 = \Pr[\mathbf{p} = 8/75]$, where $\pi_1 + \pi_2 = 1$. The weight $\pi_i\,(0 < \pi_i < 1)$ needs to be carefully evaluated. In general, a discrete prior summarizing the estimation results (presented by p_i's) based on different geological (or statistical) methods has the following structure:

$$\pi_i = \Pr\left[\mathbf{p} = p_i\right], \quad i = 1, \cdots, k;$$

where

$$\pi_i \geq 0, \text{ and } \sum_{i=1}^{k} \pi_i = 1.$$

Priors Combining Opinions of Multiple Experts

Multiple-expert hazard/risk assessments have considerable precedent, particularly in the Yucca Mountain site characterization studies. Our approach provides the flexibility of combining multiple-expert knowledge into a single prior distribution. Typically, in the elicitation of opinion more than one source is tapped. Suppose that N opinions are collected, and in each case it is assumed probabilistic in nature. For instance, results of point estimates of $\mathbf{p}$ proposed by several experts are listed in table 7.1 of Crowe et al. (1993). The values range from 1.1×10^{-3} to 8×10^{-2}. Ho (1995) has used these two bounds of $\mathbf{p}$ for the classical approach in the sensitivity data analysis. The listed results (table 7.1 of Crowe et al., 1993) represent three major groups of thought: (1) classical approach based on a simple Poisson process from Crowe et al. (1982), $p \approx 10^{-3}$; (2) Monte Carlo simulation results from Sheridan (1992), $p \approx 10^{-2}$; and (3) AMRV risk rectangles proposed by Smith et al. (1990), $p \approx 10^{-1}$. A discrete prior for our illustrative example is ready: $\pi_1 = \Pr[\mathrm{p} = 10^{-1}]$, $\pi_2 = \Pr[\mathrm{p} = 10^{-2}]$, and $\pi_3 = \Pr[\mathrm{p} = 10^{-3}]$; where $\pi_i \geq 0$, and $\pi_1 + \pi_2 + \pi_3 = 1$. The probabilities/weights can be further determined by members of a different expert-panel through a survey or other methods. A straightforward solution is to let $\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$. This prior is quoted as a "three-expert prior" for further analysis.

SENSITIVITY ANALYSIS

In this study, only the sensitivity on the choice of the priors will be evaluated. We note that the sensitivity based on several other factors has been addressed in a paper of Ho (1995). We list the basic assumptions required to conduct this sensitivity analysis on the priors.

(1) We let the beginning of the Pliocene period ($\doteq$ 6.0 Ma) be time zero, so $t = 6.0$ Ma.

(2) Prediction of future volcanic activities (volcanic eruption and site disruption) will focus on the entire life of the repository (A 10^4 year period is recommended as the required isolation period during which radioactive waste may decay to an acceptable level). Thus, we shall evaluate the volcanic hazard with

$t_0 = 10,000$ years.

(3) A key issue in the site characterization studies is the disagreement about age-dating of the rocks and counts of volcanic events (e.g., see Ho, 1995), but we will not be too concerned with the dates in this work. The only set of dates adopted for this study is: Basalt of the Thirsty Mesa, 4.6 Ma (1 event); Basalt of the Amargosa Valley, 4.4 Ma (1 event); Basalt of Southeast Crater Flat, 3.7 Ma (1 event); Basalt of Buckboard Mesa, 2.9 Ma (1 event); Quaternary basalt of Crater Flat, 1.1 Ma (4 events); Basalt of Sleeping Butte, 0.38 Ma (2 events); Lathrop Wells Center, 0.1 Ma (1 event).

Returning to the prior distributions now, recall that $\pi(p)$ denotes the pdf of $\mathbf{p}$. The following priors are included in the analysis: (1) Priors chosen for mathematical convenience: Beta (r, s) for $(r, s) = (2,2)$, $(3,3)$, $(5,5)$, $(2,1)$, $(2,8)$, $(8,2)$, and $(1,1)$; and (2) Priors motivated by expert knowledge: (a) one-expert prior, $U(0, 8/75)$; (b) three-expert prior, $\pi_1 = \Pr[\mathbf{p} = 10^{-1}]$, $\pi_2 = \Pr[\mathbf{p} = 10^{-2}]$, and $\pi_3 = \Pr[\mathbf{p} = 10^{-3}]$, where $\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$; and (c) two-expert prior, $\pi_1 = \Pr[\mathbf{p} = 10^{-1}]$, and $\pi_2 = \Pr[\mathbf{p} = 10^{-3}]$, where $\pi_1 = \pi_2 = \frac{1}{2}$. Sensitivity analysis is performed for each prior distribution and the results of hazard are summarized in Figure 2.

CONCLUSIONS

In this article we present a volcanic hazard model that incorporates the expert knowledge into the inferential mechanism. We have provided several examples illustrating how this model can be applied to the volcanic hazard assessment for the Yucca Mountain site. It is our hope that this method can be applied in the future to other geological studies. We now summarize this study with a few key points and comments:

(1) For this study, the estimated overall probability of at least one disruption of a repository at the Yucca Mountain site by basaltic volcanism for the next 10,000 years is bounded between 1.39×10^{-3} and 2.97×10^{-2} (Figure 2). The numbers reported in Figure 2 were rounded to the same format.

(2) The model using the prior, Beta (8,2) (see, Figures 1 and 2), yields the highest hazard ($= 2.97 \times 10^{-2}$). The minimum hazard is produced by the "three-expert prior" (i.e., values are equally likely at 10^{-3}, 10^{-2}, and 10^{-1}). The value of the hazard is 1.39×10^{-3}, which is only about one order of magnitude smaller than the maximum value.

(3) The estimated values of hazard based on the priors chosen for mathematical simplicity are uniformly higher than those obtained based on the priors motivated by expert knowledge (these include $U(0, 8/75)$, which is used in Ho, 1992, 1995).

(4) While there is some variation (after the fourth digit to the right of the decimal point but is not shown in Figure 2 due to rounding), we can see that the estimates using Beta $(1,1)$, Beta $(2,2)$, Beta $(3,3)$, and Beta $(5,5)$ are very comparable. If we do not want to use a highly informative prior, any of these four seems to be suitable, though if we wish to be somewhat less (more) conservative we might opt for Beta $(2,8)$ (Beta $(8,2)$) which favors lower (higher) probability but not extremely so.

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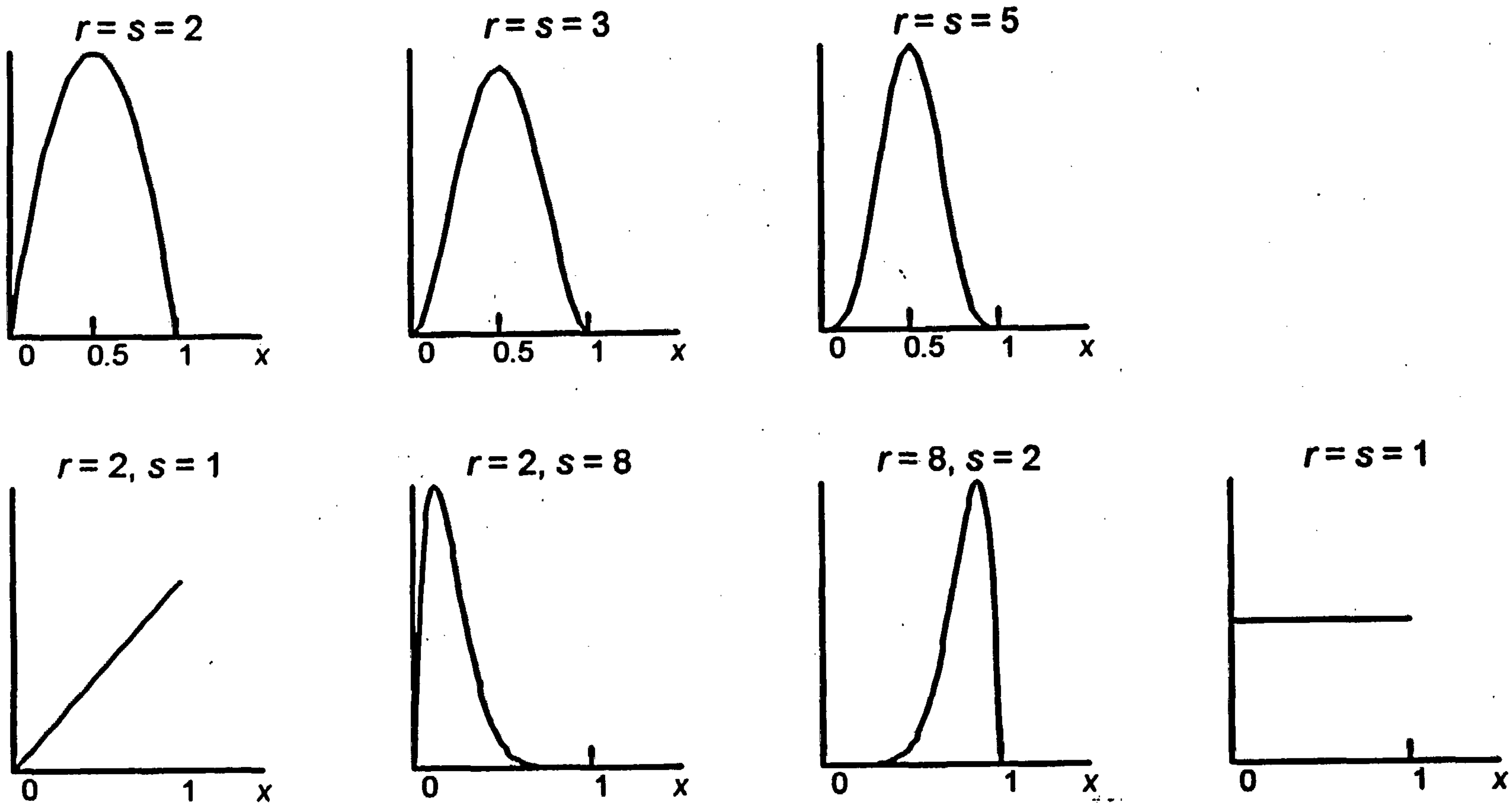

Figure 1 Priors chosen for mathematical convenience: $\pi(p) \sim \mathrm{Beta}(r,s)$

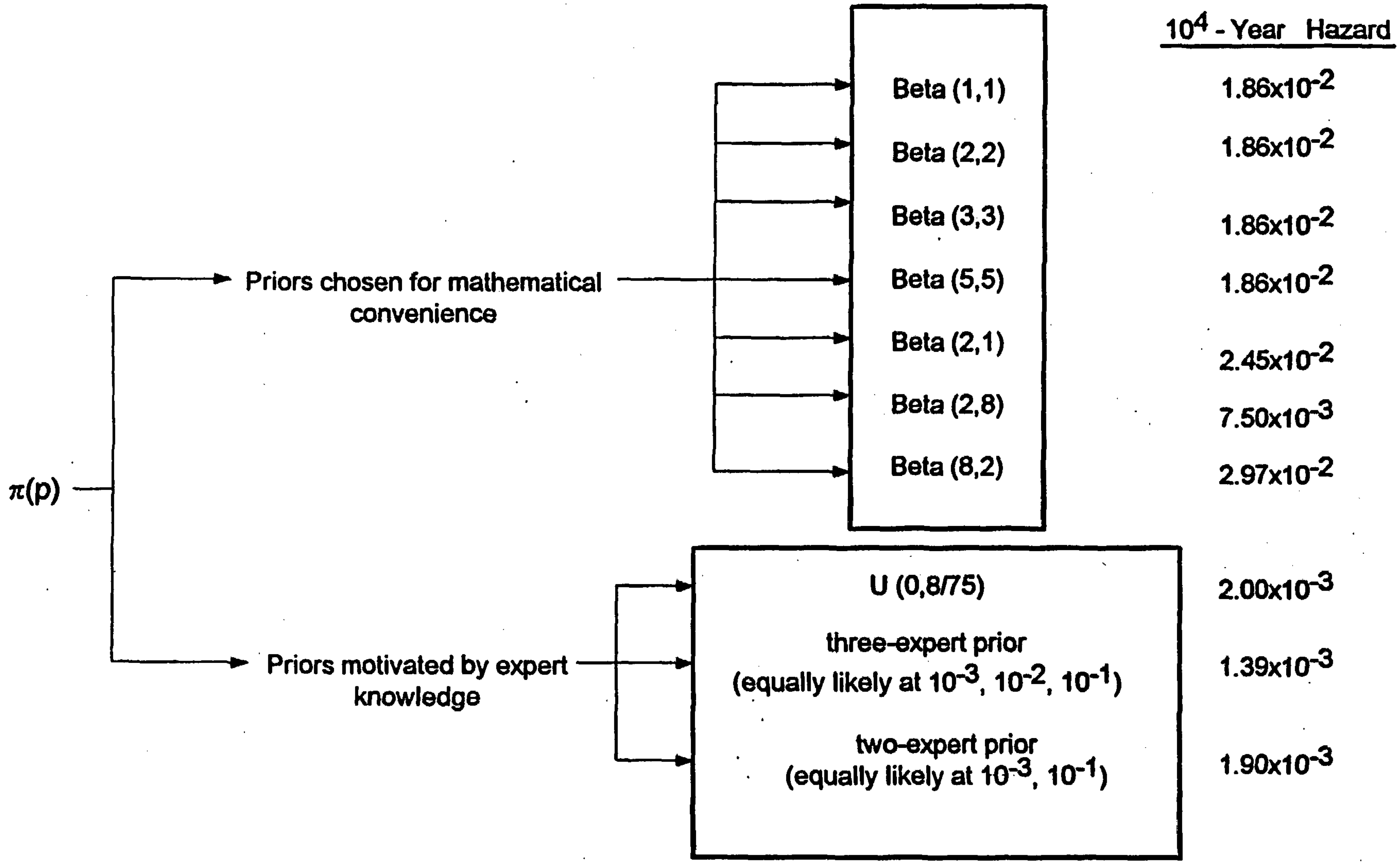

Figure 2 Summary of sensitivity analysis for proposed Yucca Mountain Repository site based on two classes of prior distributions

APPENDIX 3

Article (3): Volcanic Time Trend Analysis

VOLCANIC TIME TREND ANALYSIS

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ABSTRACT

I propose test statistics to quantitatively test whether the time-trends of two or more sets of volcanic events are similar or dissimilar. Specifically, the results of the tests provide the following information: (1) Are the volcanic events Poissonian or non-Poissonian? (2) Do the volcanic events show a significantly increasing (or decreasing) time-trend during the observation period? (3) Is the difference between two sets of volcanic recurrence intervals statistically significant? (4) Can we test whether a group of volcanoes (≥ 3) show the same time-trend (increasing, decreasing, or random)? (5) Is there a clear-cut guideline for volcanic model selection process? Furthermore, an empirical example using volcanic data provides efficient computation algorithms for producing informative time-trend analyses.

KEY WORDS: Nonhomogeneous Poisson process, Power-law process, Test statistics, Volcanic time-trend.

INTRODUCTION

Volcanoes and their processes cover an enormous spectrum: from inconspicuous fissures to majestic peaks and from mild steaming to terrifying paroxysms. To understand volcanism - an essential step towards either combating its dangers or utilizing its resources - we must gauge its full breadth and attempt to wrestle its elements into some kind of framework (Simkin and Siebert, 1994). This paper is one of many efforts toward that end.

The subject of volcanic hazards has received increased attention in the past decade. Because of widespread interest in the subject, the use of the nonhomogeneous Poisson process (NHPP) has recently gained popularity in volcanic data analysis as a simple and versatile tool to assess the waxing or waning time-trends of a volcano and to assess its volcanic hazards (see e.g., Ho 1990, 1995). As I argued in an earlier work (Ho, 1991), a general population of volcanoes can be related to an NHPP. I proposed a new method for time-trend analysis and demonstrated its usefulness on some real data. The method was designed to model a single volcano (or a volcanic system treated as one point process). The main purpose of this article is to propose statistical tests for quantitative comparison of time-trends of several volcanic processes.

The article is organized as follows: (1) The first major section gives notation and reviews of an NHPP to model the time-trend of a single volcano; (2) The second major section introduces an F-test for testing the similarity of two volcanoes; (3) The third major section provides computation algorithms for comparison of more than two volcanoes; (4) The fourth major section illustrates the train of analyses and the numerical computations that are involved in the proposed methods using an empirical example; and (5) the last section presents summary remarks and generalizations.

TREND ANALYSIS FOR ONE VOLCANO

A homogeneous Poisson process (HPP) assumes a constant recurrence rate, λ, for the volcanic events. If the volcanism is waning or developing, the model should be generalized to allow λ to be, respectively, a decreasing or increasing function of t. If one replaces the constant λ with a function of t, denoted by $\lambda(t)$, then another type of Poisson process can be derived, known as an NHPP. An NHPP has a mean value function denoted by $\mu(t|\Theta)$, where Θ is a vector of parameters. The nondecreasing function $\mu(t|\Theta)$ represents the expected number of events to time, t. Once the functional form of $\mu(t|\Theta)$ is specified, the NHPP is fully characterized. An alternate characterization of the NHPP is through its intensity function $\lambda(t|\Theta)$, where

$$\lambda(t|\Theta) = \frac{d}{dt}\mu(t|\Theta).$$

For volcanism, I (see Ho, 1991) let $\Theta = (\theta, \beta)$ and write

$$\mu(t|\Theta) = (t|\theta)^{\beta},$$

so that

$$\lambda(t|\theta, \beta) = (\beta/\theta)(t/\theta)^{\beta-1}.$$

This form, termed the *power law*, has found applications in reliability due to its flexibility (in the sense that the intensity function can be constant, decreasing, or increasing) and the fact that the distribution of the time to first arrival in the process is a Weibull. It is because of this latter property that the underlying Poisson process has sometimes been referred to as the "Weibull process" (Crow, 1974). A noteworthy feature of my approach by replacing the expected number of events in an HPP, λt, with $\mu(t) = (t/\theta)^{\beta}$ is that I let the volcanic data speak the time-trend for themselves: increasing ($\beta > 1$), decreasing ($\beta < 1$), or random ($\beta = 1$ which assumes a no-memory property). To model the volcanic time-trend using a power-law process (PLP), let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at time $0 < t_1 \leq t_2 \leq \cdots \leq t_n \leq t$. Some useful theoretical results to be used later are summarized as follows:

(1) Let $S = \sum_{i=1}^{n} \ln(t/t_i)$, then the maximum likelihood estimators (MLEs) of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n/S$$
$$\hat{\theta} = t/n^{1/\hat{\beta}}.$$

(2) Under the null hypothesis $H_0 : \beta = 1, 2S \sim \chi^2(2n)$. Therefore, a size α test of $H_0 : \beta = 1$ against $H_A : \beta \neq 1$ is to reject H_0 if $2S \leq \chi^2_{\alpha/2}(2n)$ or $2S \geq \chi^2_{1-\alpha/2}(2n)$,

where $\chi^2_{\alpha/2}(2n)$ is the $100\alpha/2$ percentile of a chi-square distribution with $2n$ degrees of freedom.

(3) If PLP is assumed during the observation time period $[0, t]$, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ at time t. In the application of the PLP to volcanic eruptive forecasting, the estimate of $\lambda(t)$ is of considerable practical interest because $\lambda(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. Crow (1982) derives the MLE for $\lambda(t)$ as

$$\hat{\lambda}(t) = \left(\hat{\beta}/\hat{\theta}\right)\left(t/\hat{\theta}\right)^{\hat{\beta}-1} = n\hat{\beta}/t.$$

Clearly, the PLP generalizes the HPP, because when $\beta = 1$ the PLP reduces to an HPP. The chi-square test defined in (2) provides us a quantitative method to objectively evaluate whether the time-trend of the volcanic activities during the observation period (a) remains approximately Poissonian? or (b) shows a significantly increasing (or decreasing) time-trend? I note that in a simulation study, Bain et al. (1985) conclude that the chi-square test which is derived as an optimal test for the PLP also is rather powerful as a test of trend for general NHPP's. In other words, the test is "robust" against other model assumptions. This is the rationale of choosing a PLP to model the volcanic eruptions.

F-TEST FOR TESTING SIMILARITY OF TWO VOLCANOES

If data are obtained from a single volcano and inferences are made only for that volcano, then a PLP with fixed values of the parameters is an appropriate model. However, there are many situations in which more than one volcano are involved in a simple exploratory analysis. For example, Klein (1982) compares repose times for differences between large and small, summit and flank, and Kilauea and Mauna Loa eruptions.

Engineers are able to compare several repairable systems based on statistical methods. Volcanic eruptions are individually unique, but volcanism as a whole is a nonunique process in which repeated combinations of rate balances give rise to categorically similar patterns worldwide. Given sufficiently redundant information, pattern recognition and comparisons with the observed patterns become automatic. I expect to also demonstrate a generalized method of quantitative description and comparisons of the volcanic processes.

Suppose now that independent volcanic repose time series of sizes n_1 and n_2 are observed, and two PLPs with shape parameters β_1 and β_2 are assumed respectively for each process. Let S_1 and S_2 be the corresponding statistics as described in (1), then the overall time-trends of these two volcanic processes can be quantitatively compared using the following test.

(4) Let $F = n_2S_1/n_1S_2$, then under the null hypothesis $H_0 : \beta_1 = \beta_2$, $F \sim F(2n_1, 2n_2)$. And, a size α test of $H_0 : \beta_1 = \beta_2$ against $H_A : \beta_1 \neq \beta_2$ is to reject H_0 if $F \leq F_{\alpha/2}(2n_1, 2n_2)$ or $F \geq F_{1-\alpha/2}(2n_1, 2n_2)$, where $F_{\alpha/2}(2n_1, 2n_2)$ is the $100\alpha/2$ percentile of an F-distribution with $2n_1$ and $2n_2$ degrees of freedom. Dot plots showing the visible time-trends of the volcanoes in the empirical studies section will demonstrate the usefulness of the F-test.

TEST STATISTIC FOR MORE THAN TWO VOLCANOES

Suppose $k\,(> 2)$ volcanoes are observed for a fixed length of time, t, and volcano i has n_i eruptions at successive time $0 < t_{i1} \leq t_{i2} \leq \cdots \leq t_{in_i} \leq t$. Again, some useful theoretical results to be used later are summarized as follows:

(5) Let $S_i = \sum_{j=1}^{n_i} \ln(t/t_{ij})$, then the maximum likelihood estimator (MLE) of β_i derived by Engelhardt and Bain (1987) is

$$\hat{\beta}_i = n_i/S_i,$$

which has the same form as the MLE of the parameter β, as described in (1), for a single PLP model. Also, the useful relationship to the chi-square distribution for $\hat{\beta}_i$, in the PLP case carries over to the case with more than one PLP. Namely, the chi-square test described in (2) is also applicable for testing $H_0 : \beta_i = 1$ against $H_A : \beta_i \neq 1$ for any $i = 1, 2, \cdots, k$.

(6) A test of equality of shape parameters, $H_0 : \beta_1 = \beta_2 = \cdots = \beta_k$ rejects this hypothesis at the approximate level α if

$$M \geq c\chi^2_{1-\alpha}(k-1)$$

where

$$c = 1 + \frac{1}{3(k-1)}\left(\sum_{i=1}^{k}\frac{1}{2n_i}\right) - \frac{1}{2N}$$

$$M = 2N\left[\ln\left(\sum_{i=1}^{k} n_i/\hat{\beta}_i\right) - \ln N\right] + 2\sum_{i=1}^{k} n_i \ln\left(\hat{\beta}_i\right)$$

and

$$N = \sum_{i=1}^{k} n_i, \text{ the total number of eruptions for all } k \text{ volcanoes.}$$

Interested readers are referred to the article of Engelhardt and Bain (1987) for theoretical development and further references. In the next section, I apply these computation algorithms to volcanic data to produce informative time-trend analyses.

EMPIRICAL EXAMPLE

Data

Simkin et al. (1981) constructed a chronology of known volcanic events over the past 8,000 years. The record has been updated through December 31, 1993 (Simkin and Siebert, 1994). The eruption records (adopted from Volcanoes of the World, 2nd edition, Simkin and Siebert, 1994) of the following three volcanoes in New Zealand are studied for time-trend analyses: White Island, Tongariro, and Ruapehu. (New Zealand contains the world's strongest concentration of youthful rhyolitic volcanoes, and voluminous ignimbrite sheets blanket much of North Island.)

The record of volcanic activity analyzed in this article has the form of a point process (i.e., a record of the month and year during which each eruption occurred). Several simplifying assumptions must be made in treating eruptions as a point process in time: (1) Although the onset date of an eruption is generally well-defined by the time when lava first breaks the surface, the duration is harder to determine because of such problems as slowly cooling flows or lava lakes and the gradual decline of activity. I adopt the same definition for repose time as defined by Klein (1982). I, therefore, ignore eruption duration; instead, I take the onset date as most physically meaningful, and measure repose times from one onset date to the next. Thus, my definition of "repose time" differs from the classic one (a noneruptive period). This procedure seems justified because most eruption durations are much shorter than typical repose intervals (Klein, 1982). Each data set of a PLP consists of the cumulative length of time (measured in months) over which the eruptions occur. (2) On several occasions, the months during which eruptions occurred are uncertain and were therefore assigned somewhat arbitrarily. (3) The first eruption on the record of volcano White Island was on the first day of December, 1826. Therefore, this date becomes my choice of the starting point for the observation period for all three volcanoes. And the last day of year 1993 is the end of the observation period, which is the same as that of the listed volcanoes in Simkin and Siebert (1994).

Time-Trend Analyses

To be consistent with the mathematical notations presented in (1) through (6), I label volcanoes White Island, Tongariro, and Ruapehu as volcano no. 1, 2, and 3 respectively for the following analyses and discussions.

(1) During the observation period, December 1, 1826 to December 31, 1993, the data for the number of recurrence intervals are $n_1 = 32$, $n_2 = 70$, and $n_3 = 50$. The estimated shape parameters for the time-trend are $\hat{\beta}_1 = 1.913$, $\hat{\beta}_2 = 1.305$, and $\hat{\beta}_3 = 3.516$ (see Table 1). The result implies that all three volcanoes show an increasing trend (i.e., $\beta > 1$) during the observation period. The two-sided p-values summarized in Table 1 indicate that Tongariro volcano provides only moderate evidence again H_0 ($\beta_2 = 1$) with p-value = 0.037, while the other two volcanoes show strong evidence against H_0. Dot diagrams presented in Figure 1 reconfirm the quantitative results.

(2) Interestingly enough, the instantaneous recurrence rate estimated on December 31, 1993 for Ruapehu volcano $\left(\hat{\lambda}_3 = 0.088/\text{month}\right)$ is higher than that of Tongariro $\left(\hat{\lambda}_2 = 0.046/\text{month}\right)$, although Tongariro volcano produced twenty more eruptions than volcano Ruepehu during the same observation period. It is because that the PLP incorporates the time-trend, evidence of additional events will not increase necessarily the instantaneous recurrence rate as it would for the HPP recurrence rate. This noteworthy feature of the PLP model is of considerable practical interest in volcanic risk/hazard assessment studies (e.g., see Ho, 1995).

(3) For the pairwise comparisons, let's consider volcanoes White Island and Tongariro: $n_1 = 32$, $n_2 = 70$, $S_1 = 16.725$, $S_2 = 53.647$ (see Table 1) and the degrees of freedom for the F distribution are $2n_1 = 64$ and $2n_2 = 140$. The test does not reject $H_0 : \beta_1 = \beta_2$ because $F = 0.682$ and the two-tailed p-value is 0.086 (see Table 2). Thus, we consider that the shape parameter, β, is statistically the same for volcanic activities of White Island and Tongariro volcanoes during the observation period. However, comparisons (see Table 2 and Figure 1) between Tongariro versus Ruapehu, and White Island versus Ruepehu show that the differences are significant with p-values 0.006 and ≈ 0 respectively. (Note that these still stand up nicely to a

Bonferroni corrected alpha of $0.05/3 = .0166\cdots$, if one chooses to do the adjustment of alpha for multiple tests.)

(4) Now, recall from (6) of the previous section that a test of equality of shape parameters, $H_0 : \beta_1 = \beta_2 = \beta_3$ rejects this hypothesis at the approximate level 0.05 if

$$M \geq c\chi^2_{.95}(2).$$

For this study, I get $c = 1.002$, $\chi^2_{.95}(2) = 5.99$, and the critical value $c\chi^2_{.95}(2)$ is approximately 6.002. Because the test statistic M is 26.359, H_0 is rejected at $\alpha = 0.05$ as I have expected from the previous results of pairwise comparisons. Actually, the test is significant at any level since the p-value is ≈ 0. Therefore, I conclude that these volcanoes do not share a common shape parameter, β, which serves as an indicator for the time-trend of the volcanic activities.

(5) Finally, what are the merits of performing the above tests? I shall discuss this issue based on the following scenarios that one might conclude from the trend analyses.

Case 1: $\beta_1 = \beta_2 = \beta_3 = 1$

Case 2: $\beta_1 = \beta_2 = \beta_3 = \beta \neq 1$

Case 3: $\beta_i \neq \beta_j$ for some i, j, where $1 \leq i < j \leq 3$.

For Case 1, a compound homogeneous Poisson process (CHPP) can be used to model the aggregate behavior of these Poissonian volcanoes ($\beta = 1$). In a CHPP model, the recurrence rate for a given volcano or group of volcanoes is described by a gamma distribution (prior) rather than treated as a constant value as in the assumptions of an HPP. I performed Bayesian analysis (Ho, 1990) to link these two distributions together to give the aggregate behavior of the volcanic activity. When the HPP is expanded to accommodate a gamma mixing distribution on λ, a consequence of this mixed (or compound) Poisson model is that the frequency distribution of eruptions in any given time-period of equal length follows the negative binomial distribution

(NBD). Applications of the model and comparisons between this generalized model and an HPP were discussed based on the historical eruptive count data of volcanoes Mauna Loa and Etna (Ho, 1990). Several relevant facts led to the conclusion that the generalized model is preferable for practical use both in space and time. A similar situation can occur with a group of non-Poissonian volcanoes ($\beta \neq 1$). If one replaces the underlying distribution in a CHPP with an NHPP distributed according to a PLP and also let the intensity parameter vary according to a gamma distribution as described in the model of Ho, 1990, then a new model called compound power-law process (CPLP) provides a better fit than a CHPP. Statistical analysis of a CPLP for repairable systems has been presented in an article by Engelhardt and Bain (1987). This model requires several assumptions including the one that I described in Case 2 (i.e., $\beta_1 = \beta_2 = \beta_3 = \beta \neq 1$). My efforts for future studies are to develop the volcanological aspect of a CPLP and to point the potential usefulness of this model in volcanology. For Case 3, to my best knowledge, a single model such as a CHPP for Case 1 and a CPLP for Case 2 is not available.

CONCLUSIONS

Volcanic activity is governed by the complex interaction of several geological, geophysical and geochemical factors. Because of this complexity, even with the present knowledge, eruptions cannot theoretically be predicted. Therefore, the evaluation of eruptive probabilities for a given volcano or a volcanic center remains an open problem in the definition of volcanic risk. There are many unknown areas with respect to geologic understanding of volcanic activity, despite the fact that there are well recognized means of gathering data (field mapping, determinations of the eruptive history of basaltic centers, petrology, geochemistry, geochronology including magnetic polarity determinations, tectonic setting, and geophysical studies) that are well advanced.

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity to be put forward. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risks and probabilities. This paper extends my previous work (Ho, 1991) on testing the significance of increasing or decreasing time-trends of volcanoes. I now add two new test statistics to the geological literature which represents a good cross-application of statistics to geosciences. In summary, the significance of this work is: quantitative comparisons between (or among) volcanoes become possible and a clear-cut guideline for volcanic model selection process evolves.

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Table 1. Summary Statistics for Trend Analysis

	Volcano		
	White Island	Tongariro	Ruapehu
n_i	32	70	50
S_i	16.725	53.647	14.219
$\hat{\beta}_i$	1.913	1.305	3.516
$\hat{\lambda}_i$ (no. of eruptions/month)	0.031	0.046	0.088
Chi-square test statistic (for $H_0: \beta_i = 1$)	33.450	107.293	28.438
p-value (two-tailed)	0.001	0.037	≈ 0

Table 2. Results of F Tests for Pairwise Comparisons

	White Island vs. Tongariro	White Island vs. Ruapehu	Tongariro vs. Ruapehu
F-statistic	0.682	1.838	2.695
p-value (two-tailed)	0.086	0.006	≈ 0

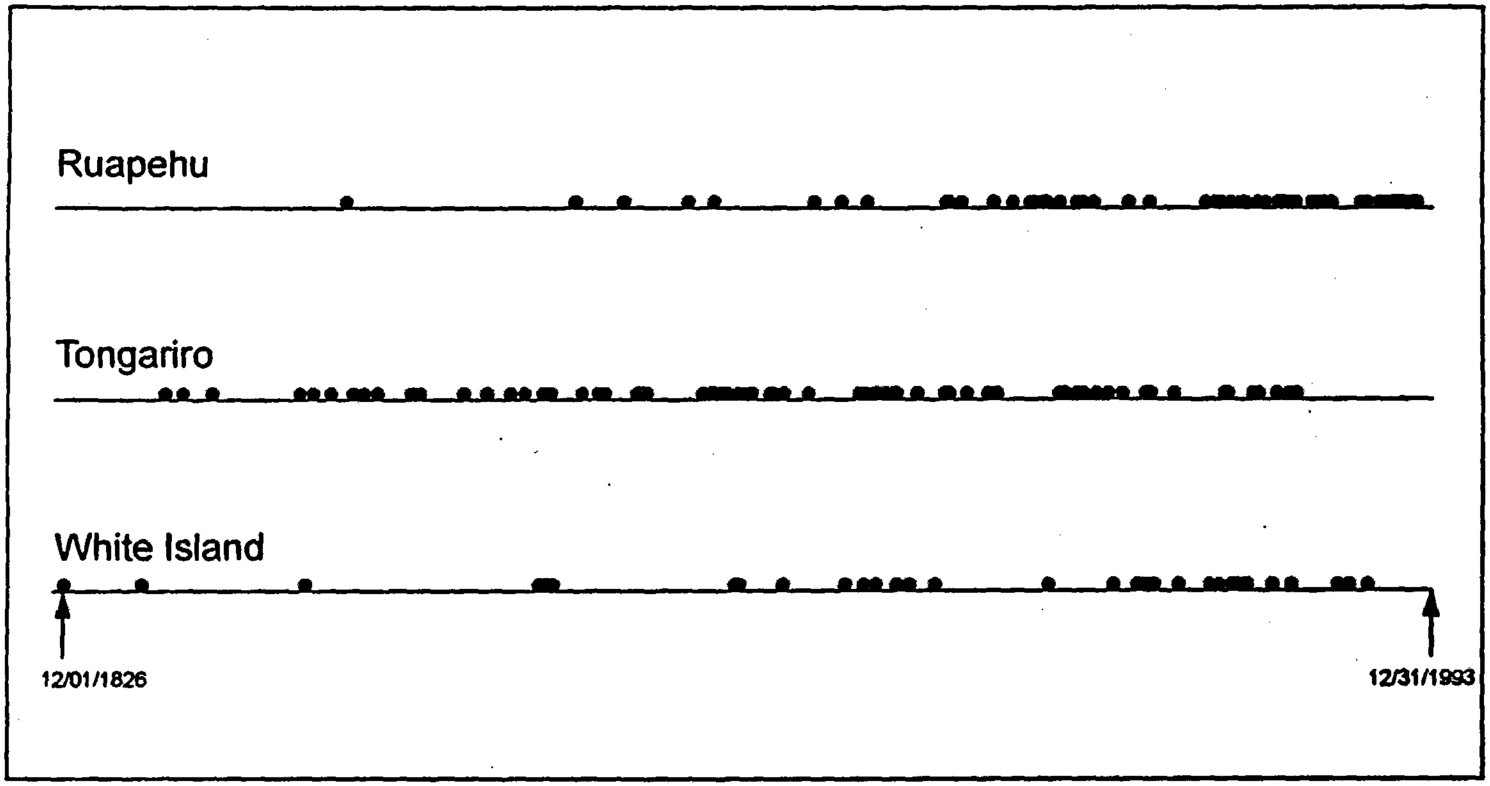

Figure 1. Dot diagrams of recurrence intervals (in months) of volcanoes Ruapehu, Tongariro, and White Island in their original chronological orders observed during December 1, 1826 to December 31, 1993.

APPENDIX 4

Paper Presented

(Volcanic Hazard Analysis at the Yucca Mountain Nuclear Waste Repository Site)

VOLCANIC HAZARD ANALYSIS AT THE YUCCA MOUNTAIN NUCLEAR WASTE REPOSITORY SITE

Presented by:

C.-H. Ho

Department of Mathematical Sciences, University of Nevada, Las Vegas

at

DOE/Geomatrix EXPERT PANEL MEETING ON ALTERNATE HAZARD MODELS, Las Vegas, NV

March 30-31, 1995

Volcanic Hazard Analysis at the Yucca Mountain Nuclear Waste Repository Site

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Summary

Crowe et al. (1982) use a homogeneous Poisson process to evaluate the volcanic hazard at the Yucca Mountain region. The main component of the mathematical calculation is $\exp\{-t_0 \lambda p\}$, where $t_0 = 10{,}000$ years, $\lambda = \text{constant}$, and p is estimated as a/A (a is the area of the repository, and A is the area of a chosen sample region). Once A is chosen, p is constant and is a fixed probability of site disruption for any given eruptions that occur in the future. Connor and Hill (1995) use $\exp\left\{-t_0 \iint_a \lambda_r(x,y)\,dxdy\right\}$ to calculate the hazard, where λ_r is a function of location (x,y). For a small region a, the estimated $\lambda_r(x,y)$ is a constant for all (x,y) in a. Therefore, $\iint_a \lambda_r(x,y)\,dxdy = a\lambda_r$, which implies that λ_r is equivalent to λ / A. In other words, λ (in Crowe et al., 1982) represents the number of eruptions in unit time, and λ_r (in Connor and Hill, 1995) is the number of eruptions in unit time and unit area. They (λ and λ_r) are estimated accordingly, but are both assumed to be constant in time for the probability calculations.

My basic equation for the volcanic hazard is $\underset{p}{E}\{\exp[-\mu(t_0)p]\}$. Where $\mu(t) = \mu(t|\theta,\beta) = (t/\theta)^\beta$ represents the expected number of events to present time, t. Once the functional form of μ is specified, the NHPP is fully characterized. Hence, I let the volcanic data speak for themselves: increasing $(\beta > 1)$, decreasing $(\beta < 1)$, or random ($\beta = 1$, as has been assumed to be so in the previous two models), and let the prior historical time trend continue for future volcanic activities. I then permit prior distribution, $\pi(p)$, for p. Notice that, Bayesian approaches with meaningful chosen prior would bring improvements to the analysis, because of their ability to incorporate expert knowledge into the inferential mechanism. A noteworthy feature of my approach is that the expert (≥ 1) opinion is elicited on the prior distribution, $\pi(p)$, based on geological information that is available. Moreover, $\pi(p)$ can combine all available geological information motivated by conflicting but realistic arguments (e.g., simulation, cluster analysis, structual control, ... , etc.). The incorporated uncertainties about the probability of repository disruption p will eventually be averaged out by taking the expectations of the above equation over $\pi(p)$. Thus my approach should prove superior to both the sample theory method that do not use expert information and the existing Bayesian approaches that do not better exploit the knowledge that is available.

OUTLINE

Definition of the problem

- Layout of a 3-D System

Model

- 1-D Power Law process for time trend
- 2-D Bayesian modeling for disruptive events

The risks of risk assessment

Geological Information

Incorporating Expert Knowledge

Example

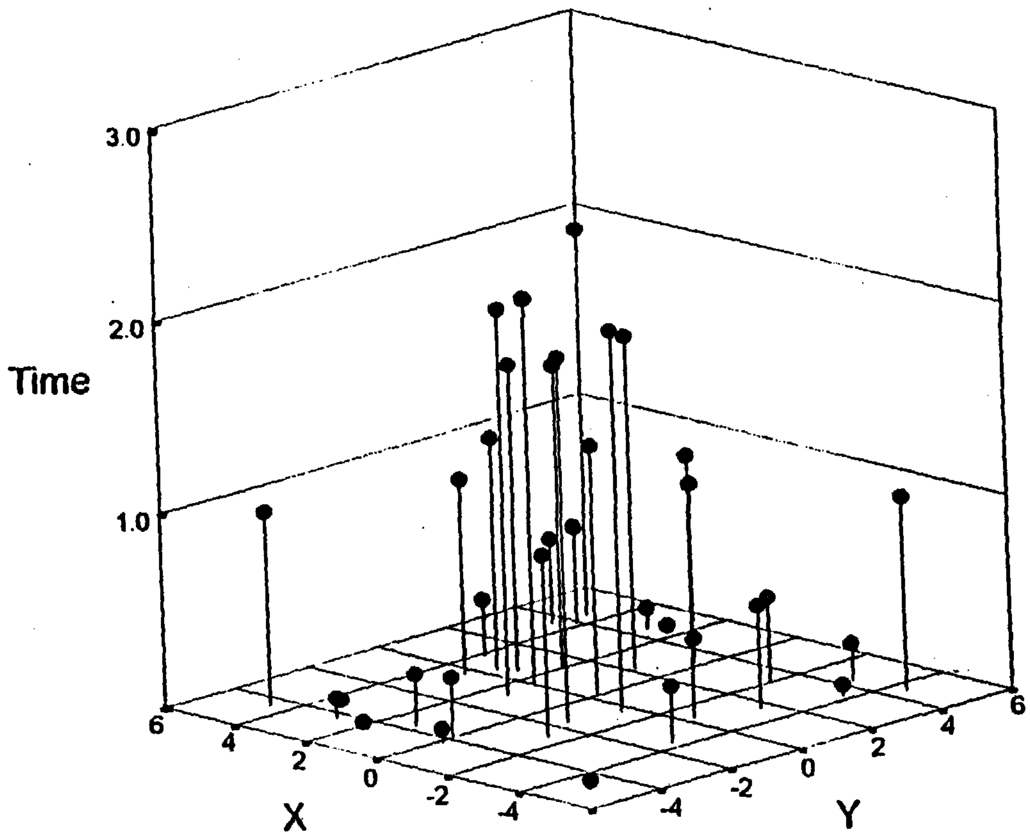
References and Conclusions

Definition

The information on the ith occurrence

$$(x_i, y_i, t_i)$$

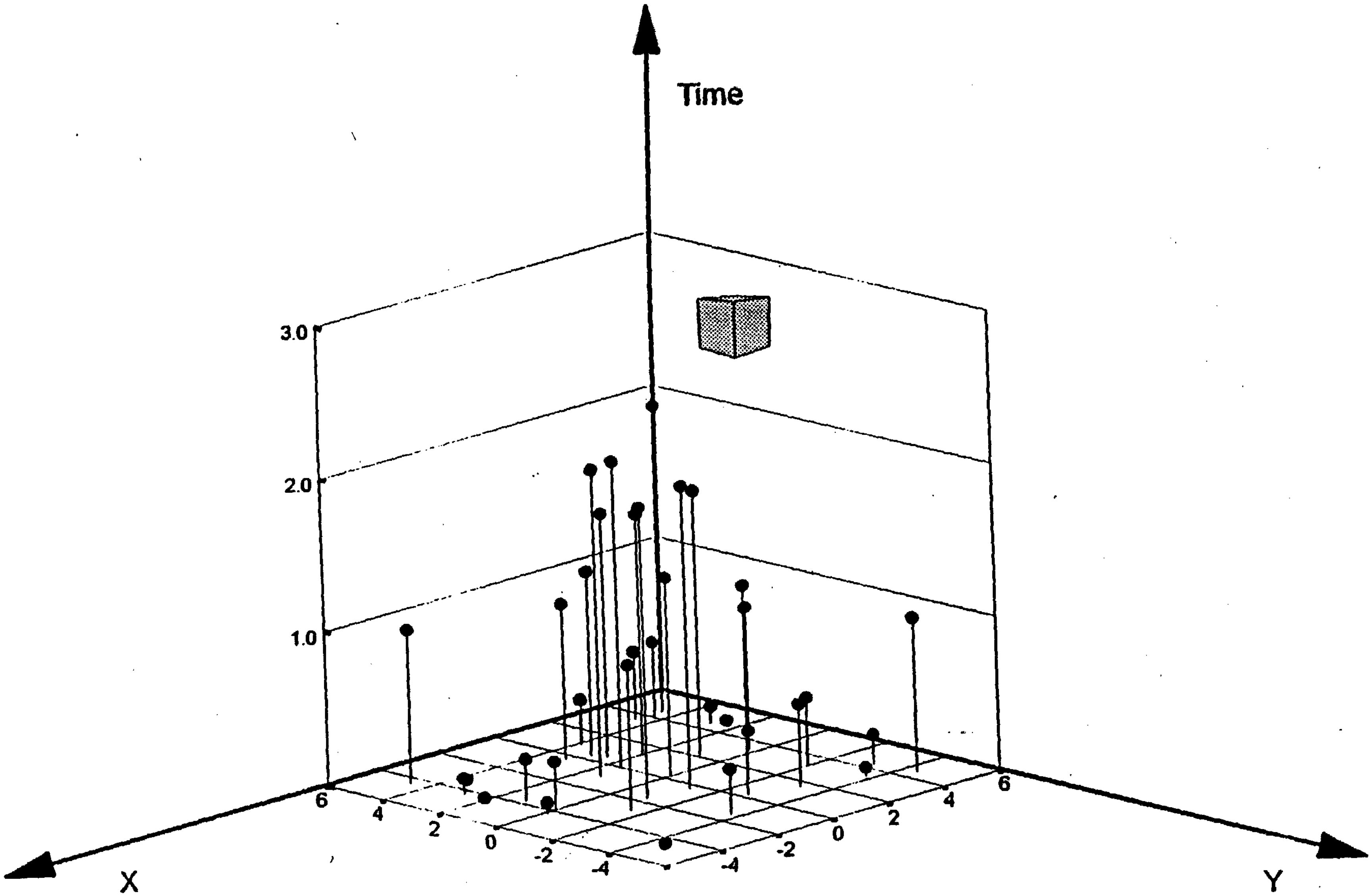
- x_i and y_i are the Euclidean coordinates of the spatical location
- t_i is the time of occurrence relative to the beginning of the study period



3.0
2.0
Time
1.0
6
4
2
0
-2
-4
X
-4
-2
0
2
4
6
Y

A 3-D System

Polycyclic volcanism and polygenetic volcanism would be recorded explicitly in a 3-D plot, and consequently would be handled appropriately by the model.



(Figure 1, Ho et al., 1995a)

1. $\exp\{-t_0\lambda p\} = \exp\{-t_0\lambda(a / A)\}$,
Assuming complete randomness w.r.t. location and time

(Crowe et al., 1982)

2. $\exp\left\{-t_0 \iint_a \lambda_r(x,y)\,dxdy\right\} = \exp\{-t_0 a\lambda_r\}$

(Connor & Hill, 1995)

$$\Rightarrow \qquad \lambda_r \sim \frac{\lambda}{A}$$

where λ = # events/unit time
λ_r = # events/ unit time & unit area

Model Description

$$\exp\{-t_0 \lambda p\} \qquad \text{(Crowe et al., 1982)}$$

$$P(D=0) = \exp\left\{-t_0 \iint_a \lambda(x,y)\,dxdy\right\} \qquad \text{(Connor \& Hill, 1995)}$$

$$\underset{p}{E}\left\{\exp\left[-\underbrace{\mu(t_0)}_{1}\underbrace{p}_{2}\right]\right\} \qquad \text{(Ho, 1992, 1995)}$$

Ideas:

1. $\mu(t) = \mu(t|\theta,\beta) = \left(\dfrac{t}{\theta}\right)^{\beta}$,

   accounted for trend in time

2. $p \sim \pi(p)$,

   incorporating expert knowledge of p

1-D Power Law Process

Consider a NHPP with a mean value function $\mu(t|\Theta)$

- Θ is a vector of parameters
- $\mu(t|\Theta)$ represents the expected number of events to t

Once the functional form of $\mu(t|\Theta)$ is specified, the NHPP is fully characterized.

An alternative characterization of the NHPP is through its intensity function $\lambda(t|\Theta)$, where

$$\lambda(t|\Theta) = \frac{d}{dt}\mu(t|\Theta).$$

For the purposes of this research, we let

$\Theta = (\theta, \beta)$ and write

$$\mu(t|\theta,\beta) = \left(\frac{t}{\theta}\right)^{\beta},$$

so that

$$\lambda(t|\theta,\beta) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}.$$

This form, termed the *power law*, has found applications in reliability due to its flexibility (in the sense that the intensity function can be constant, decreasing, or increasing) and that the distribution of the time to first arrival in the process is a Weibull. The underlying Poisson process has sometimes been referred to as the "Weibull process."

Inferences (MLEs)

Suppose $n > 1$ eruptions are observed during $[0,t]$ at time $0 < t_1 < t_2 < \cdots < t_n$, then

- $\hat{\beta} = n \Big/ \sum_{i=1}^{n} \ln(t/t_i)$
- $\hat{\theta} = t \big/ n^{1/\hat{\beta}}$
- $\hat{\lambda} = n\hat{\beta} \big/ t$ (Crow, 1974, 1982)

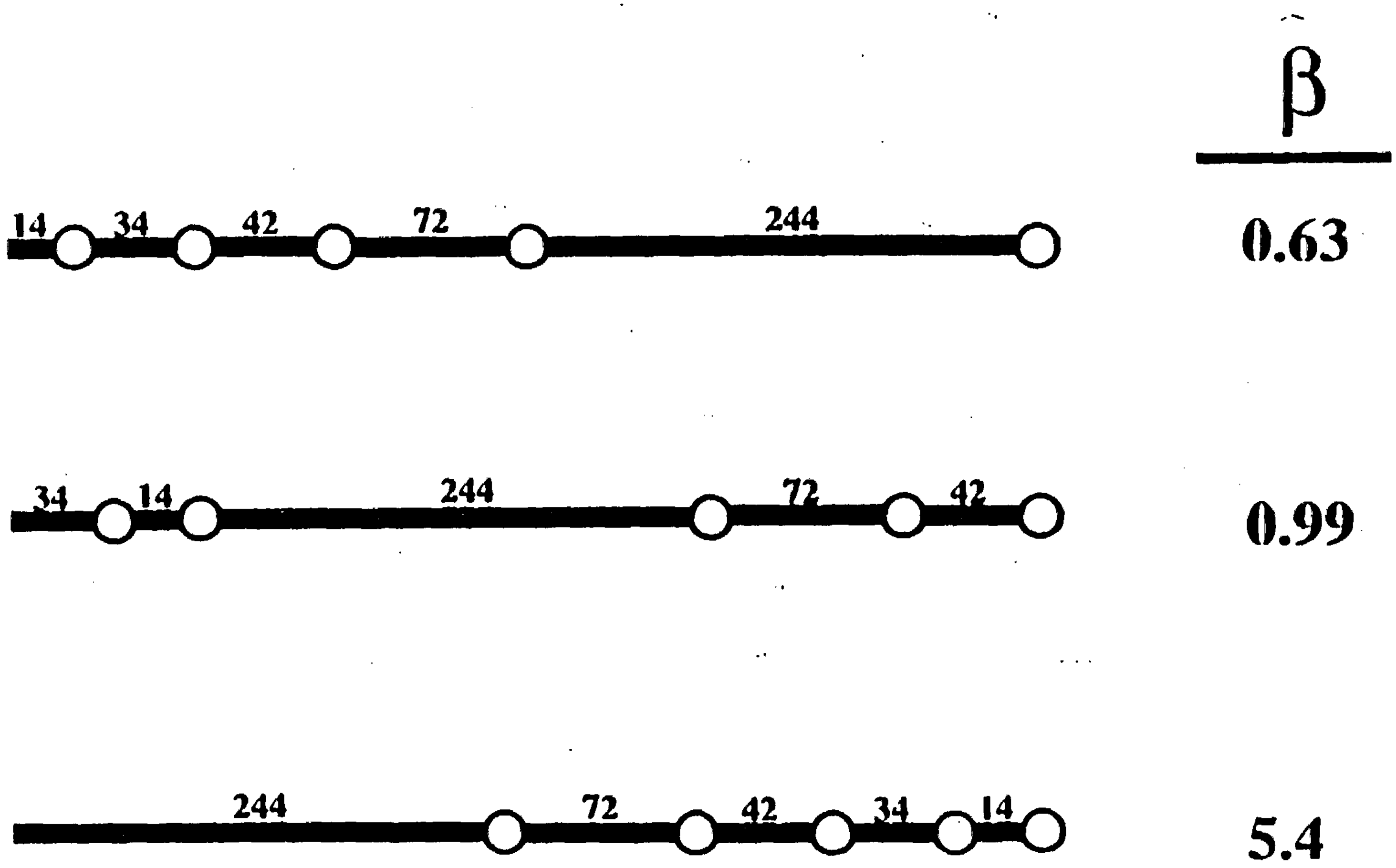
(The instantaneous eruptive status of the volcanism at the end of the observation time t)

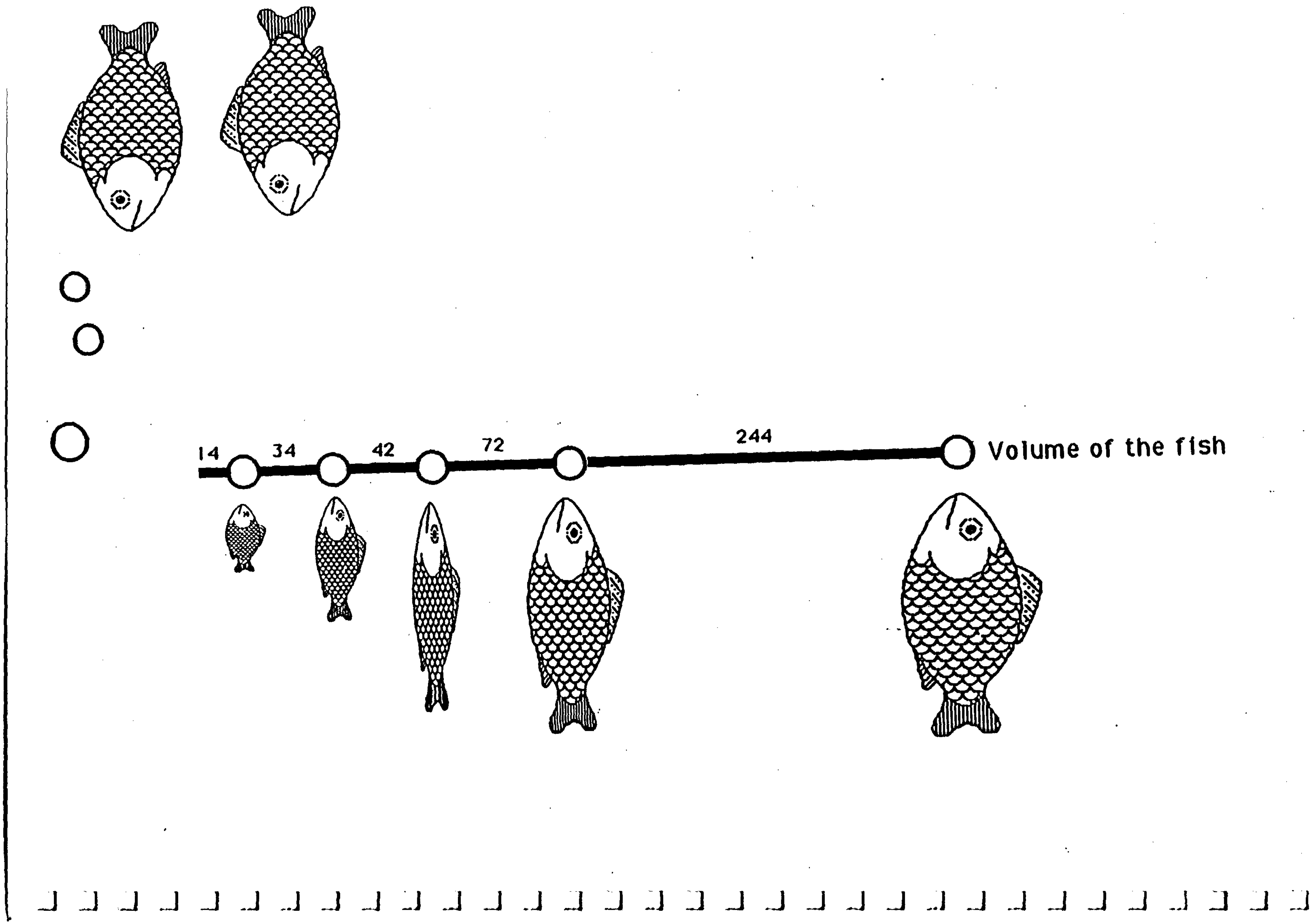
Goodness-of-fit test

$$H_0 : \beta = 1$$

$$\begin{aligned} H_A : \beta &\neq 1 \\ &> 1 \\ &< 1 \end{aligned}$$

$$X^2 = 2n / \hat{\beta} \sim \chi^2(2n)$$


β
14 34 42 72 244
0.63
34 14 244 72 42
0.99
244 72 42 34 14
5.4


14
34
42
72
244
Volume of the fish

Prediction Procedures

The number of occurrences, N, in future time $[t, t+t_0]$ follows:

$$P[N=k] = \exp[-\mu(t_0)][\mu(t_0)]^k / k!, \quad k = 0,1,\cdots$$

where

$$\mu(t_0) = \int_t^{t+t_0} \lambda(s)ds$$

Disruptive Events (2-D Bayesian Modeling)

Let

P = the probability that any single eruption is disruptive

(not every eruption would result in disruption of the repository)

We permit prior distribution, $\pi(p)$, for p.

Bayesian approaches with meaningful chosen prior would bring improvements to the analysis, because of their ability to incorporate expert knowledge into the inferential mechanism.

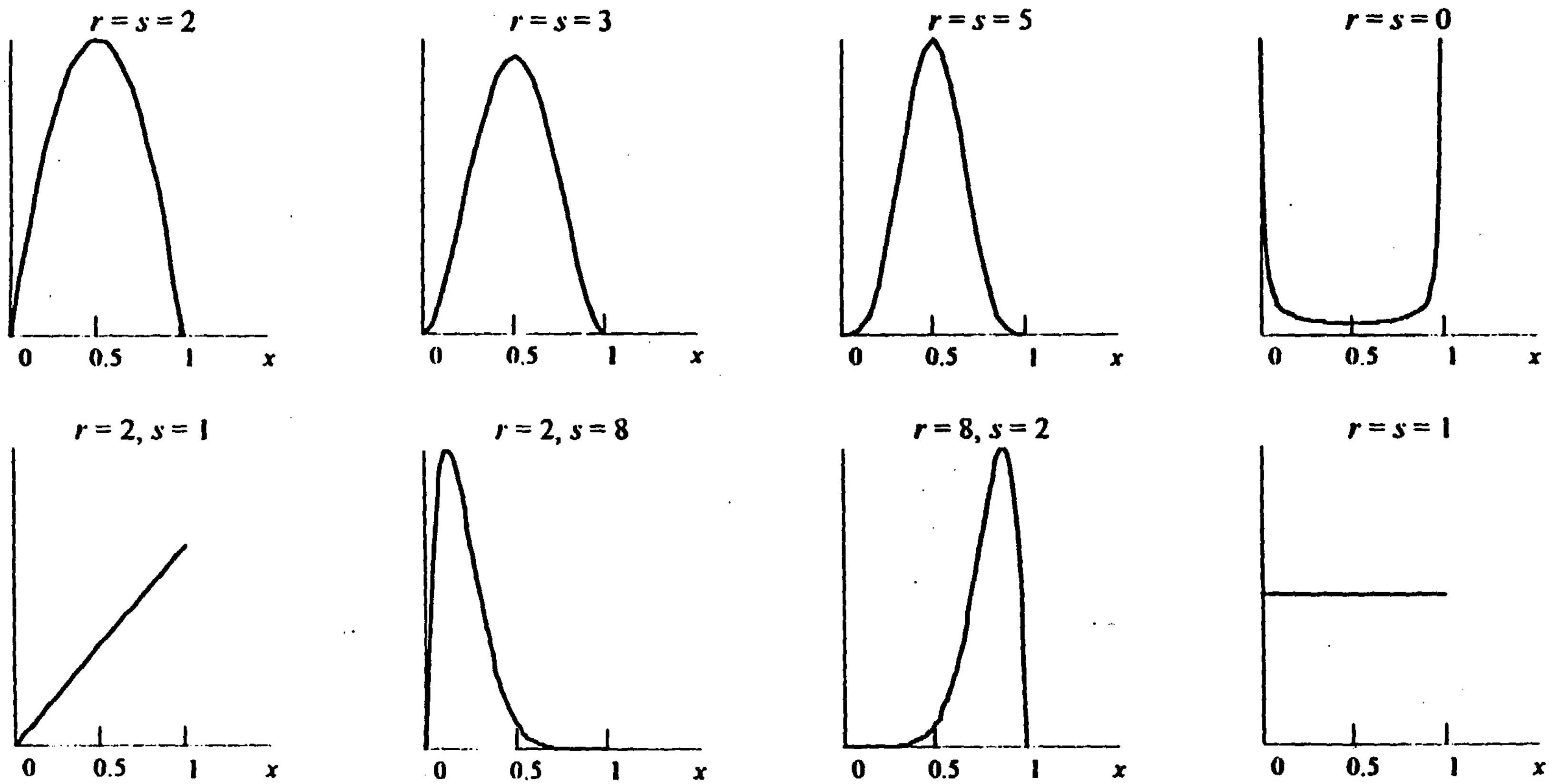
However, most of the prior distributions used are chosen more for their mathematical convenience than due to <u>a realistically motivated argument</u>.

Geological Data and the Prior

- The permissible range of p is $0 < p < 1$.
- Without the input of expert knowledge, the best-case-scenario is $p = 0$; the worst-case-scenario is $p = 1$.

Priors Chosen for Mathematical Convenience

$\pi(p) \sim \text{Beta}\,(r, s)$



(Figure 1, Ho *et al.*, 1995b)

In this research, we describe a Bayesian approach for inference and predictions using a formally developed framework for constructing prior distributions that incorporate expert knowledge and geological information.

The Risks of Risk Assessment

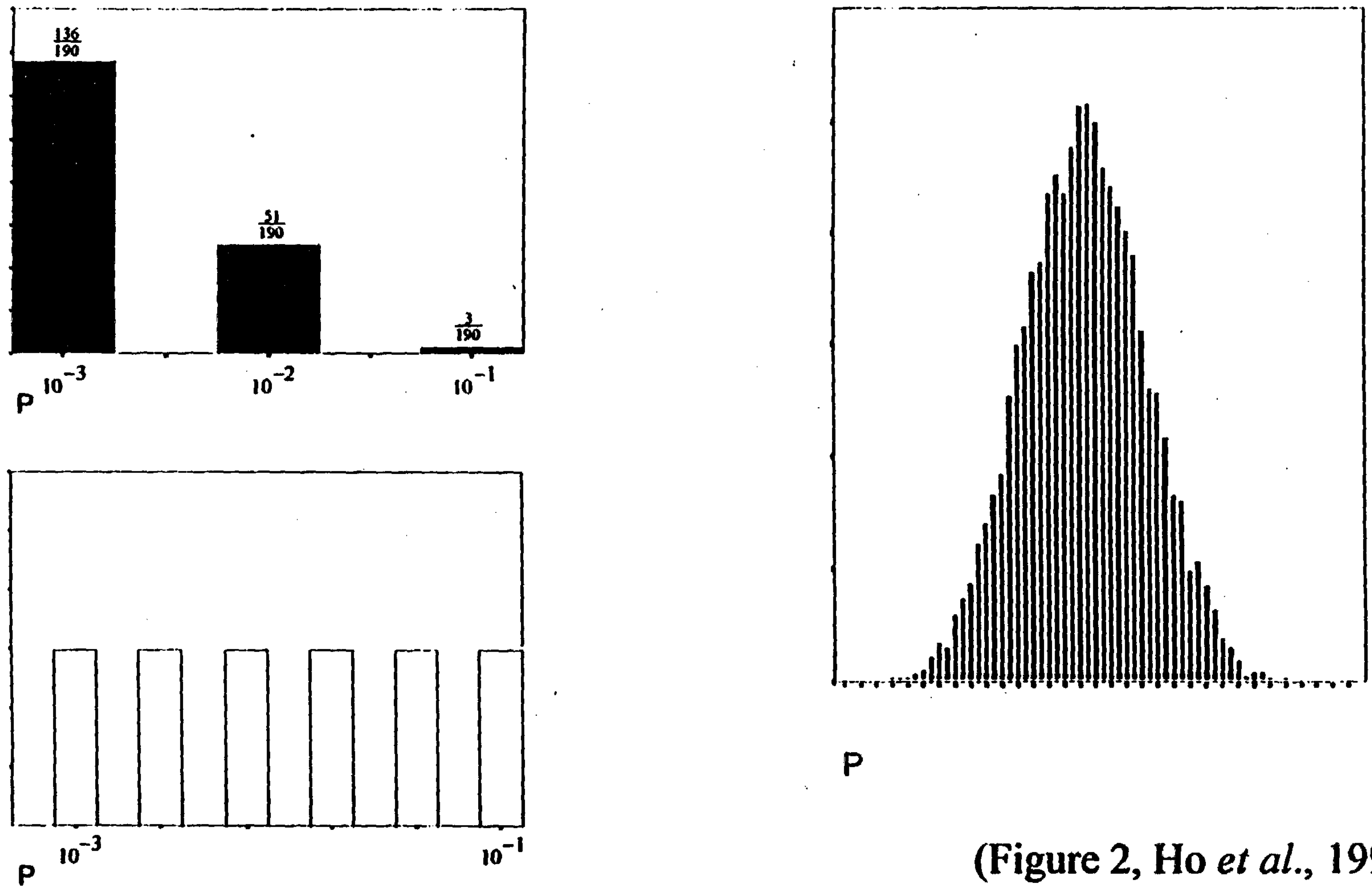
- Incomplete geological data
- Inconsistent statistical methods

Geological Information

- How?
- How many?
- Where?
- When?
- What (to measure)?

Priors Motivated by Expert Knowledge

Discrete $\pi(p)$



(Figure 2, Ho *et al.*, 1995b)

$\pi(p)$

(1)can summarize the results from a single expert
(Ho, 1992, 1995),

and

(2)can combine all available geological information
motivated by (conflicting but) realistic arguments
(e.g., simulation,structural control, ..., etc.).
(Ho et al., 1995b)

A noteworthy feature of our approach is that the expert opinion is elicited on the prior distribution,

$$\pi(p),$$

based on additional geological information that is available.

The prior distribution, $\pi(p)$, of p expresses our beliefs regarding the numerical values of p. This would incorporate uncertainties about the probability of repository disruption p that are averaged eventually as shown below:

$$\text{hazard} = \underset{p}{E}\{1 - \exp[-\mu(t_0)p]\}$$

Thus our proposed approach should prove superior to both the sample theory methods that do not use expert information and the existing Bayesian approaches that do not better exploit the knowledge that is available.

Example

Data: $t = 6.0$ (Pliocene and younger volcanism),

$$4.6, 4.4, 3.7, 2.9, \underbrace{1.1 \times 6}_{H?}, 0.38 \times 2, \underbrace{0.1, 0.01}_{W\&H?}$$

(Crater Flat) (Lathrop Wells)

- $\hat{\beta} = 2.43$
- $\hat{\lambda} = 5.67 \times 10^{-6}$
- hazard $= 3.02 \times 10^{-3}$ $(\pi(p) \sim U(0, 8/75))$
  (Ho, 1995)

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3. Ho et al., 1995a, A 3-D Poisson Process for Volcanic Hazard Assessment (in preparation).
4. Ho et al., 1995b, Volcanic Hazard Assessment Incorporating Expert Knowledge: Application to the Yucca Mountain Region (in preparation).
5. Ho, C.H., 1995, Sensitivity in Volcanic Hazard Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: The Model and the Data. Mathematical Geology, v. 27, p. 239-258 (to appear)

APPENDIX 5

Abstracts submitted for presentations at the 30th Geological Congress Meeting.

Volcanic Hazard Assessment Incorporating Multiple-Expert Knowledge

Chih-Hsiang HO (Dept. of Math., U of Nevada, Las Vegas, NV, U.S.A.)
Eugene I. SMITH (Dept. of Geoscience, U of Nevada, Las Vegas, NV, U.S.A.)
Gene YOGODZINSKI (Dept. of Geol., Dickinson College, Carlisle, PA, U.S.A.)

Multiple expert hazard/risk assessments have considerable precedent, particularly in the Yucca Mountain site characterization studies. (Yucca Mountain has been proposed as the site for permanent underground disposal of high-level radioactive waste from the nation's civilian nuclear power plants and some of the wastes resulting from nuclear weapons production. Scientists have been studying the mountain, located about 100 miles northwest of Las Vegas, Nevada, U.S.A., to gather information that will be used to determine if the site could comply with federal regulations designed to ensure public safety.) A certain amount of expert knowledge is needed to interpret the geological data used in a probabilistic data analysis; Consequently, lack of consensus in some studies is a sure outcome. In this paper, we present a Bayesian approach to statistical modeling in volcanic hazard assessment for the Yucca Mountain site. Specifically, we show that the expert opinion on the site disruption parameter p is elicited on the prior distribution, $\pi(p)$, based on geological information that is available. Moreover, $\pi(p)$ can combine all available geological information motivated by conflicting but realistic arguments (e.g., simulation, cluster analysis, structural control, $\cdots$, etc.). The incorporated uncertainties about the probability of repository disruption p will eventually be averaged out by taking the expectation over $\pi(p)$. We use the following priors in the analysis: (1) priors chosen for mathematical convenience: Beta (r,s) for $(r,s) = (2,2)$, (3,3), (5,5), (2,1), (2,8), (8,2), and (1,1); and (2) three priors motivated by expert knowledge. Sensitivity analysis is performed for each prior distribution.

Speaker's name and mailing address: (Please type)
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Tel: (702) 895-0396 Fax: (702) 895-4343 E-mail: chho@nevada.edu

Session Number and Session Title (First 2 or 3 words. Please type): 19-1 Mathematical characteristics ...

No. of 1st Choice____. Oral____ Poster X Either____ (Check). Title: A 3-D Volcanic Hazard/Risk Assessment Model

No. of 2nd Choice____. Oral____ Poster X Either____ (Check). Title: Volcanic Hazard Assessment Incorporating ...

No session fits my topic____ (Check). I submit another abstract of which I am the first author: Yes X No____ (Check)

A 3-D Volcanic Hazard/Risk Assessment Model: Application to the Yucca Mountain Region, Nevada, U.S.A.

Chih-Hsiang HO (Dept. of Math. Sci., U of Nevada, Las Vegas, NV, U.S.A.)
Eugene I. SMITH (Dept. of Geoscience, U of Nevada, Las Vegas, NV, U.S.A.)

Yucca Mountain has been proposed as the site for permanent underground disposal of high-level radioactive waste from the nation's civilian nuclear power plants and some of the wastes resulting from nuclear weapons production. Scientists have been studying the mountain, located about 100 miles northwest of Las Vegas, Nevada, U.S.A. to gather information that will be used to determine if the site could comply with federal regulations designed to ensure public safety. For instance, volcanism in the Yucca Mountain region (YMR), has been the topic of numerous studies focusing on the probability of disruption of a proposed high-level radioactive waste repository by volcanic activity. These studies are pursued largely because the proposed waste repository is located within 10 to 20 km of at least five Quaternary cinder cones and the high-level radioactive waste must be isolated from the surrounding environment for a period of at least 10,000 years (time). The area of the actual repository is currently estimated to be 6-8 km^2 (space). In this paper, we propose a unified approach for probabilistic volcanic hazard analysis from a 3-D Poisson process using a formally developed framework that incorporate the spatial location and the time of the volcanic events into a single model. The objectives to be carried out in this article are: (1) Define a 3-D Poisson process in volcanological terms; and (2) Perform model fitting techniques based on eruption histories of YMR basaltic volcanism. Specifically, we address the following noteworthy features of the model: (1) the model is volcanologically informative in solving problems of volcanic risk/hazard which depends on the location and time of future events; (2) the computation algorithms of the model fitting procedures are efficient; (3) the model is flexible enough to handle a large class of volcanic risk/hazard studies; (4) the sensitivity on the statistical models, displayed by the experts who have addressed the volcanic hazard/risk assessment problem near the Yucca Mountain region, can be (objectively) evaluated.

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Session Number and Session Title (First 2 or 3 words. Please type): 19-4 New theories and methods ...

No. of 1st Choice_____. Oral_____ Poster X Either_____ (Check). Title: A 3-D Volcanic Hazard/Risk Assessment Model

No. of 2nd Choice_____. Oral_____ Poster X Either_____ (Check). Title: Volcanic Hazard Assessment Incorporatin

No session fits my topic_____ (Check). I submit another abstract of which I am the first author: Yes X No_____ (Check)