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AGENCY FOR NUCLEAR PROJECTS/ NUCLEAR WASTE PROJECT OFFICE

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PRELIMINARY

RISK ASSESSMENT FOR THE YUCCA MOUNTAIN HIGH-LEVEL NUCLEAR WASTE REPOSITORY SITE: ESTIMATION OF VOLCANIC DISRUPTION

Chih-Hsiang Ho

Department of Mathematical Sciences

University of Nevada, Las Vegas

Las Vegas, NV 89154 (U.S.A.)

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INTRODUCTION

The annual report for the research in the area of "Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: Estimation of Volcanic Disruption" includes the following contributions:

A. **Articles**

(1) Ho, C.-H. 1992. Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: Estimation of Volcanic Disruption. Mathematical Geology, 24: 347-364.

(2) Ho, C.-H. 1992. Statistical Control Chart for Regime Identification in Volcanic Time Series, Mathematical Geology, 24: (in press).

B. **Abstracts and papers presented**

(1) " Prediction of Explosive Eruptions at Volcan de Colima, Mexico." invited speaker at the 2nd International Reunion of Volcanology held in Colima, Mexico, January 20-24, 1992.

(2) "Volcanic Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site." presented at the 29th International Geological Congress held in Kyoto, Japan, August 24, 1992 - September 3, 1992.

(3) "Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: Estimation of Volcanic Disruption," presented at the Panel on Structural Geology & Geoengineering Meeting on Volcan-

ism held in Las Vegas, Nevada, September 14-16, 1992 (copy of the overheads of the presentation is included).

FUTURE WORK: Sensitivity Analysis

Future work will concentrate on the following:

(1) Sensitivity analysis in risk assessment for the proposed repository with respect to (a) models for the recurrence rate (b) models for the site disruption parameter (c) definition of a single event, and (d) dates of the defined events.

(2) Development of models for stochastic phenomena, which have general application worldwide, will continue.

(3) Several major papers will be prepared and submitted for publication.

Mathematical Geology, Vol. 24, No. 4, 1992

Risk Assessment for the Yucca Mountain High-Level Nuclear Waste Repository Site: Estimation of Volcanic Disruption[1]

Chih-Hsiang Ho[2]

In this article, we model the volcanism near the proposed nuclear waste repository at Yucca Mountain, Nevada, U.S.A. by estimating the instantaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and by using a homogeneous Poisson process to predict future eruptions. We then quantify the probability that any single eruption is disruptive in terms of a (prior) probability distribution, since not every eruption would result in disruption of the repository. Bayesian analysis is performed to evaluate the volcanic risk. Based on the Quaternary data, a 90% confidence interval for the instantaneous recurrence rate near the Yucca Mountain site is (1.85×10^{-6}/yr, 1.26×10^{-5}/yr). Also, using these confidence bounds, the corresponding 90% confidence interval for the risk (probability of at least one disruptive eruption) for an isolation time of 10^{4} years is (1.0×10^{-4}, 6.7×10^{-4}), if it is assumed that the intensity remains constant during the projected time frame.

KEY WORDS: Bayesian analysis, nonhomogeneous Poisson process, prior distribution, volcanic risk, Weibull distribution.

INTRODUCTION

In the United States, spent fuel and high-level radioactive waste will be permanently disposed of in a geologic repository. Disposal of the spent fuel and high-level waste is scheduled to begin in the year 2010. The candidate site for the first U.S. geologic repository is located at Yucca Mountain, Nevada, approximately 100 miles, or about 160 kilometers, northwest of Las Vegas, Nevada. Comprehensive studies are underway on the potential host rock formation. These studies are called site characterization. An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site

[1] Received 29 May 1991; accepted 11 October 1991.

[2] Department of Mathematical Sciences, University of Nevada-Las Vegas, 4505 Maryland Parkway, Las Vegas, Nevada 89154.

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(Department of Energy, 1986) because of the presence of multiple basalt centers of Quaternary age.

Yucca Mountain is located in the southcentral part of the Southwestern Nevada Volcanic Field (SNVF), a major volcanic province of the southern Great Basin first defined by Christiansen et al. (1977) and extended by Byers et al. (1989). Interested readers are referred to the papers of Byers et al. (1989) for the location of geographic features of the SNVF, and Crowe (1990) for the basaltic volcanic episodes of the Yucca Mountain region. Crowe and Perry (1989, Fig. 1) divide the Cenozoic volcanism of the Yucca Mountain region into three episodes that include (1) an older episode of large volume basaltic volcanism (12 to 8.5 Ma [million years]) that coincides in time with the termination of silicic volcanic activity, (2) the formation of five clusters of small volume basalt scoria cones and lava flows (9 to 6.5 Ma), all located north and east of the Yucca Mountain site, and (3) the formation of three clusters of small volume basalt centers (3.7 to .01 Ma), all located south and west of the Yucca Mountain site. The two youngest episodes form northwest-trending zones that parallel the trend of structures in the Spotted Range-Mine Mountain section of the Walker Lane belt. Crowe and Perry (1989), and Crowe (1990) suggest a southwest migration of basaltic volcanism in the Yucca Mountain area based on this structural parallelism, a pattern that may reflect an earlier southwest migration of silicic volcanism in the Great Basin. Smith et al. (1990a) provide a different point of view of the migration trends of volcanism in the Yucca Mountain region. Specifically, they conclude that future volcanic events in the Yucca Mountain area will be associated with Quaternary centers in Crater Flat, at Sleeping Butte, or at the Lathrop Wells cone (see Fig. 1). Based on their assumption, a future eruption may occur either to the north-northeast or south-southwest of an existing cone or group of cones. A more detailed discussion will be provided in later sections.

Concern that future volcanism might disrupt the proposed Yucca Mountain repository site motivated the assessment of the volcanic risk to the Yucca Mountain area, straddling the southern corner of the Nevada Test Site (NTS), where nuclear materials have been handled for more than three decades. Crowe and Carr (1980) calculate the probability of volcanic disruption of a repository at Yucca Mountain, Nevada using a method developed largely by Crowe (1980). Crowe et al. (1982) refine the volcanic probability calculations for the Yucca Mountain area using a simple Poisson model:

$$Pr\,[\text{no disruptive events before time } t] = \exp(-\lambda t p),$$

where λ is the recurrence rate of volcanic events and p is the probability of a repository disruption, given an event (a volcanic eruption). Theoretically, the

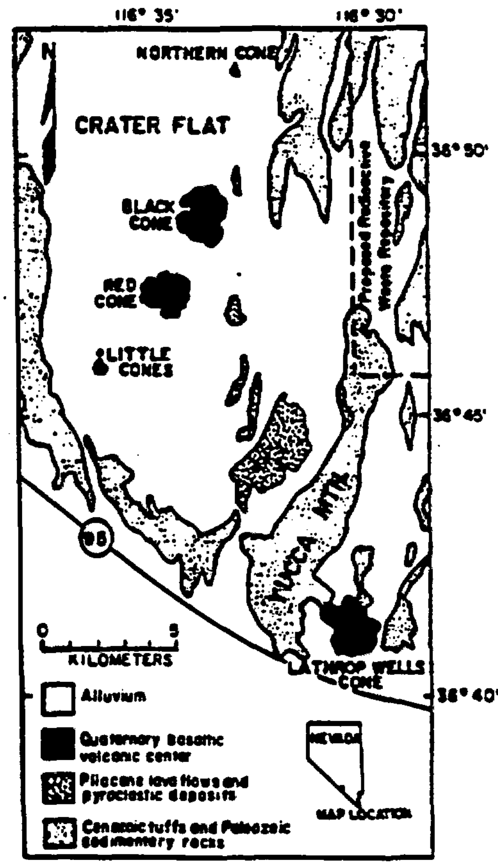

Fig. 1. Generalized geologic map of Crater Flat volcanic field area and boundary of proposed radioactive waste repository; inset map shows locations of the Crater Flat volcanic field (Source: Wells et al., 1990, Fig. 1).

probability formula (Crowe et al., 1982) is derived from the following assumptions:

1. Volcanic eruptions in successive time periods of length t for each period are independent and should follow a Poisson distribution with a constant mean (average rate) λt, i.e., a simple Poissonian volcano (see Wickman, 1966).

2. Every eruption has the same probability of repository disruption p. That is, there is no heterogeneity with respect to disruptiveness.

3. The disruptive events are independent of one another.

The parameter p is estimated as a/A, where a is the area of the repository and A is some minimal area that encloses the repository and the area of the volcanic events. Crowe et al. (1982) develop a computer program to find either the minimum area circle or minimum area ellipse (defined as A) that contains the volcanic centers of interest and the repository site. A is defined to accommodate tectonic controls for the localization of volcanic centers and to constrain λ to be uniform within the area of either the circle or ellipse. The rate of volcanic activity is calculated by determination of the annual rate of magma production for the NTS region using refined age data (Crowe et al., 1982). Resulting probability values using the refined mathematical model are calculated for periods of 1 year and 10^5 years. As calculated by Crowe et al. (1982), the probability of volcanic disruption of a waste repository located at Yucca Mountain falls in the range 3.3×10^{-10} to 4.7×10^{-8} during the first year, and increases approximately linearly with isolation time. Note that this model and the resultant values are used in all subsequent analyses by Crowe (e.g., Crowe 1986, 1990; Crowe and Perry, 1989; Crowe et al., 1988, 1989).

ISSUES THAT ARISE IN CONNECTION WITH A SIMPLE POISSON MODEL

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risks, and probabilities. Reliable historical data make possible the construction of activity patterns for several volcanoes (Wickman, 1966, 1976; Klein, 1982, 1984; Mulargia et al., 1985, 1987). Unfortunately, there is no historical record of volcanism near Yucca Mountain. The eruptive history of basaltic centers (dates of volcanic eruptions) at NTS must therefore be developed by detailed volcanologic studies (field mapping, petrology, geochemistry, geochronology, including magnetic polarity determinations, tectonic setting, and geophysical studies).

As mentioned earlier, there is a large and growing body of literature on probabilistic modeling for volcanism. Much of the debate in the literature is centered on the choice of distribution models (principally homogeneous Poisson vs. nonhomogeneous Poisson models). Although the simple Poisson model has proved successful in some comparisons of its predictions with observations (e.g., Gardner and Knopoff, 1974; McGuire and Barnhard, 1981), it might be inadequate to model the volcanism at NTS for the following reasons:

(a) A simple Poisson model does not allow for the possibility of a waning (or developing) volcanic time trend, which is one of the major concerns in quantifying the volcanism at the Yucca Mountain region. It should be obvious that the chronological order in which the volcanic eruptions occur is an extremely important aspect of a historical eruptive data set. We have written about this elsewhere (Ho, 1991a), but will review the basic arguments to illustrate this point. In Fig. 2, we use the pseudo-data provided by Ascher (1983). For example, even an eyeball analysis of Fig. 2 is adequate to strongly suggest that volcanic activities are "waning," "random," and "developing," since as time increases, the eruptions occur less frequently, about as frequently, and more frequently, respectively. The simple Poisson model, however, assumes that the average recurrence rate (λ) is constant throughout the entire life of the volcanic activity. Once this assumption is made, the model would treat these data sets as equivalent and, therefore, would take the average of the five numbers (14, 34, 42, 72, and 244) as the estimated repose time and its reciprocal as the estimated recurrence rate (λ). It is therefore of interest to explore alternative model(s) derived from less restrictive model assumptions allowing for the incorporation of the time trend of volcanism at the NTS area.

(b) As no historical record is available for the Yucca Mountain region, identifying the number of eruptions depends on clear understanding of eruptive processes and reliable dating techniques. Crowe et al. (1982), and Crowe and Perry (1989) determine the rate of magma production for the NTS region by fitting a linear regression line to a data set of four points collected from four volcanic centers. Each value thus represents magma volume of a single eruption at a corresponding volcanic center. The mean magma volume during the last 4 million years is calculated by taking the average of these four values. The ratio (rate/mean) is then calculated as an estimate ($\hat{\lambda}$) for the constant mean of the assumed simple Poisson model. Ho et al. (1991) criticize the statistical work of Crowe et al. (1982), and Crowe and Perry (1989) as seriously flawed. Specifically, the probabilistic results of Crowe et al. are based on idealized model assumptions, a premature database, and inadequate estimates of the required parameters, which lead to questionable conclusions about volcanic stability of the proposed Yucca Mountain repository.

For the reasons discussed, a formal structure needs to be developed to ensure that volcanic risk assessment is based on an adequate model. In this paper, we model the volcanism near the proposed nuclear waste repository at Yucca Mountain by estimating the instantaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and by using a homogeneous Poisson process to predict future eruptions. We then quantify the prob-

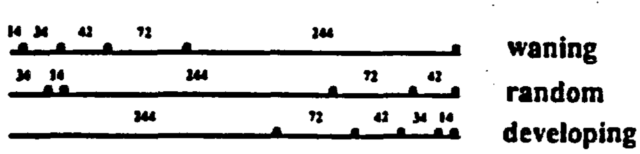

Fig. 2. Dot Diagrams of volcanic time series of three volcanoes in their original chronological orders.

ability that any single eruption is disruptive in terms of a (prior) probability distribution, because not every eruption would result in disruption of the repository. Bayesian analysis is performed to model the disruptive frequency.

MODELING OF VOLCANISM

The Weibull Process

A simple Poisson process is more specifically known as a homogeneous Poisson process (HPP) since the rate λ is assumed to be independent of time t. The homogeneous Poisson model generally gives a good fit to many volcanoes for forecasting volcanic eruptions. If eruptions occur according to a homogeneous Poisson process, the repose times between consecutive eruptions are independent exponential variables with mean $\theta = 1/\lambda$. The exponential distribution is applicable when the eruptions occur "at random" and are not due to aging, etc. If we replace the constant λ with a function of t, denoted by $\lambda(t)$, then another type of Poisson process is derived, known as a nonhomogeneous Poisson process (NHPP). If $X(t)$ denotes the number of occurrences in a specified interval $[0, t]$ for an NHPP, then it can be shown that $X(t)$ is distributed as a nonhomogeneous Poisson random variable (Parzen, 1962, p. 138) with parameter $\mu(t)$, where

$$\mu(t) = \int_0^t \lambda(s)\, ds$$

The choice for the nonhomogeneous intensity function, $\lambda(t)$, is important in modeling the volcanism at the Yucca Mountain area. In this paper, our choice of $\lambda(t)$ is

$$\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$$

which gives

$$\mu(t) = (t/\theta)^{\beta}$$

In this case, the time to first occurrence follows a Weibull distribution, $WEI(\theta, \beta)$. This parameter (μ) is an increasing function of t if $\beta > 1$ and a decreasing function of t if $\beta < 1$. Of course, the Weibull process is a generalization of the exponential case ($\beta = 1$, which assumes a no-memory property), so it is useful for situations which entail waning, growth, etc. (Ho, 1991b). For example, the birth process (new volcanoes) and the death process (extinction) of volcanoes are included. Clearly the Weibull model does include the simple Poisson model, since when $\beta = 1$ the Weibull reduces to the exponential (a simple Poisson model). The Weibull model has been frequently applied in a variety of ways (e.g., Brillinger, 1982; Kiremidjian and Anagnos, 1984), and we shall show that it appears best suited to meet the requirements discussed in the previous section.

In a Weibull process, the time to first occurrence, say T_1, follows a Weibull distribution, $WEI(\theta,\beta)$. The time to second occurrence, or the time between occurrences, does not follow a Weibull distribution. This is in contrast to the exponential case in which the times between occurrences are also exponentially distributed. The Weibull process will be referred to as failure-truncated, in reliability terminology, if it is observed until the first n failure times, $t_1, \ldots, t_n$, have occurred, and it will be referred to as time-truncated if it is observed for a fixed time t. For volcanic eruptive forecasting near the Yucca Mountain region, the time-truncated case makes more sense, since t can be extended to the present date to include the repose time following the last eruption.

Suppose we assume that the successive volcanic eruptions at the Yucca Mountain region follow a simple Weibull process. For a time-truncated Weibull process, let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at times $0 < t_1 < t_2 < \ldots < t_n$. The maximum likelihood estimates (MLE) of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n \Big/ \sum_{i=1}^{n} \ln (t/t_i)$$

$$\hat{\theta} = t/n^{1/\hat{\beta}}$$

These are similar to the failure-truncated case if t is replaced by t_n. Simple calculations yield the following estimates for the data sets in Fig. 2:

Volcano	$\hat{\beta}$
Waning	0.63
Random	0.99
Developing	5.40

The β estimated for the simple Poissonian volcano (random) clearly is consistent with $\beta = 1$, that is with a homogeneous Poisson process. Since the recurrence rate is proportional to $t^{\beta-1}$, the β's estimated for the waning and developing volcanoes imply decreasing and increasing recurrence rates at which eruptions are occurring, respectively. These results are in complete agreement with an eyeball analysis of Fig. 2. In sharp contrast, if we fitted the simple Poisson model to these data sets, we would obtain exactly the same parameter estimates for all volcanoes. This demonstrates the rationale of our choice of a Weibull intensity to model the volcanism at the Yucca Mountain region.

If a Weibull model is assumed during the observation time period $[0, t]$, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ at time t. Furthermore, assuming that the intensity, $\lambda(t)$, remains constant thereafter, then the subsequent inter-event times are independent exponential variables with recurrence rate $\lambda(t)$ and the mean time to the next eruption at cumulative observation time t is $1/\lambda(t)$. In the application of the Weibull process model to volcanic eruptive forecasting, the estimate of $\lambda(t)$ is of considerable practical interest since $\lambda(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. Crow (1982) derives the MLE for $\lambda(t)$ as

$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\theta})(t/\hat{\theta})^{\hat{\beta}-1}$$

Since the number of eruptions during some specified length of time t_0 would be distributed as a homogeneous Poisson random variable with constant rate $\lambda(t)t_0$, estimates of probability of future eruptions are readily available from $\hat{\lambda}(t)$ and the Poisson probability distribution function. Note that while we use historical eruptive data during $[0, t]$ to estimate the instantaneous recurrence rate $\lambda(t)$ at time t based on an NHPP with Weibull intensity, we then use an HPP to predict future eruptions based on a recurrence rate $\lambda(t)t_0$ for future time, $[t, t + t_0]$. In other words, we incorporate the time trend (developing or waning) into our estimate of the instantaneous recurrence rate and description of the general trend, but we take a neutral position, i.e., constant rate for future events, when predicting future eruptions. The rationale for this procedure is that, although eruptions are caused by specific physical events or processes, there might be many causal factors with random influences on the sequence of eruptions, e.g., regimes with various occurrence rates were identified in the eruptive history of Mount Etna by Mulargia et al., 1987. As a result, the future time trend is assumed to be described by an HPP for forecasting purposes.

Estimation of Recurrence Rate

According to Crowe and Perry (1989), the youngest zone of basaltic activity in the vicinity of Yucca Mountain is characterized by basaltic centers occurring as clusters of scoria cones and lava flows. These clusters include the 3.7-Ma basalts in southeastern Crater Flat, the 2.8-Ma basalt of Buckboard Mesa, the sequence of four 1.2-Ma centers in Central Crater Flat, two centers of the 0.28-Ma Sleeping Butte site, and the Lathrop Wells center. The age of the Lathrop Wells center has been refined from the original 0.27 Ma (Crowe et al., 1982) to 0.01 Ma (Crowe and Perry, 1989). This date (0.01 Ma) is in the range of 0 to 0.02 Ma, period of the most recent volcanic activity of the Lathrop Wells Cone as reported by Wells et al. (1990).

In order to estimate the recurrence rate of the volcanism, some other relevant issues have to be addressed. An accurate count of the number of eruptions is possible for volcanoes with a complete historical record. As no historical record is available for the Yucca Mountain region, identifying the number of eruptions depends on clear understanding of eruptive processes and reliable dating techniques. Crowe et al. (1983) indicate that a main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone. Therefore, we count each widely recognized main cone as a major event, but do not require that the main cones in each center (or cluster of centers) be of separate ages, since traditional K-Ar dating commonly produces large errors in the age ranges recorded by the volcanoes near Yucca Mountain which would mask the differences of dates and would lead to an underestimation of the recurrence rate. For instance, the 3.7-Ma basalts include at least four volcanic centers. According to Smith et al. (1990b), basalts of the 3.7-Ma cycle are the most voluminous in Crater Flat. Outcrops cover at least 20 km^2 (see Smith et al. 1990b, Fig. 5). To be consistent with our discussion, we count four main cones in the 3.7-Ma units. The sequence of four 1.2-Ma centers in central Crater Flat includes Red Cone, Northern Cone, Black Cone, and two Little Cones (Fig. 1). Jointly with two Sleeping Butte Cones, one Lathrop Wells Cone, and the basalt of Buckboard Mesa, we form a slightly more detailed set of data for the statistical analysis. Notice that, although we count each main cone as a major event, every counted Quaternary main cone (Red Cone, Black Cone, etc.) is a well-known volcanic center (and a possible cluster of volcanic centers). Smith et al. (1990a) concentrate on the group of five cinder cone complexes in the central part of Crater Flat in Fig. 1. Based on their discussion, the cones form a 12-km-long arcuate chain. Details of vent alignment are best observed on Black Cone and Red Cone in the central part of the chain. In the Black Cone complex, the cinder cone is the most prominent topographic feature (about 100 m high and 500 m in diameter), but it may only account for a small volume of flows. A larger volume of basalt erupted from at least ten vents located north, south, and east of Black Cone. These vents are commonly represented by scoria mounds composed of cinder, ash, and large bombs. Vents are aligned along two sub-parallel zones that strike approximately N35E. One zone includes Black Cone and four scoria mounds; the other zone lies 300 m to the southeast of Black Cone and contains at least seven mounds. Dikes exposed in eroded mounds strike northeast and parallel the trend of the vent zones. The Red Cone complex contains three vent zones, two trend approximately N45E, and a third zone strikes N50W (see Smith et al., 1990a, Fig. 3). This provides substantive justification of our treatment of the data set.

Another key issue in the site characterization studies is the disagreement over age-dating of the rocks. For example, the K-Ar dates for Red Cone presented by Smith et al. (1990b, Table 4) are: 0.98 $\pm$ 0.10 Ma for dike, 1.01 $\pm$ 0.06 Ma for amphibole bearing unit, and 0.95 $\pm$ 0.08 for basalt on top of Red Cone. Until more reliable dating techniques are available, we have no way to distinguish the ages of the cones within each cluster but to assign the respective cycle age to each cone. The dates then are: 3.7, 3.7, 3.7, 3.7, 2.8, 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01. This may slightly affect the estimation of β

since, in contrast to the exponential model, the Weibull model is sensitive to the locations, numbers, and relative sizes (to t) of the ordered t_i's. Also, specifying t is important in modeling the volcanism at NTS. Most of the volcanic risk assessment studies in the Yucca Mountain area are centered around the post-6-Ma (Pliocene and younger) and Quaternary (< 1.6 Ma) volcanism (Crowe et al., 1988, 1989, Smith et al., 1990a; Wells et al., 1990). We shall use the above dates to estimate the recurrence rate of volcanism during the following two observation periods: Pliocene and younger (<6.0 Ma), and Quaternary (<1.6 Ma).

Let the beginning of the Pliocene period ($\pm$ 6.0 Ma) be time zero, so t = 6.0 Ma. The estimated instantaneous recurrence rate ($\hat{\lambda}$) is about 5.0×10^{-6}/yr ($\hat{\beta} = 2.29$). For the study on Quaternary volcanism, t = 1.6 Ma, and the dates are: 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01. The estimated instantaneous recurrence rate is about 5.5×10^{-6}/yr ($\hat{\beta} = 1.09$). Volcanism during these two observation periods yield similar recurrence rates. The estimated recurrence rate, $\hat{\lambda}(t)$ (= 5.5×10^{-6}/yr), based on Quaternary volcanism, is a point estimate with no assessment of uncertainty. It is not an example of the sort of statistical practice statisticians try to encourage. It is emphasized that interval estimates are more informative than point estimates. A 90% confidence interval for $\lambda(t)$ has the form $\pi_2^{-1}\hat{\lambda}(t) < \lambda(t) < \pi_1^{-1}\hat{\lambda}(t)$, where π_1 and π_2 are the values given by Crow (1982, Table 2). In this case the corresponding 90% confidence interval for $\lambda(t)$ is: $5.5 \times 10^{-6}/2.981 < \lambda(t) < 5.5 \times 10^{-6}/0.436$, or ($1.85 \times 10^{-6}$, 1.26×10^{-5}). These confidence bounds based on the period of the most recent volcanic activity (Quaternary volcanism) are denoted as $\hat{\lambda}_1$ and $\hat{\lambda}_2$, respectively, and will be used for further analysis.

MODELING OF VOLCANIC DISRUPTION

In the previous section, we use historical eruptive data during [0, t] to estimate the instantaneous recurrence rate $\lambda(t)$ at time t based on an NHPP with Weibull intensity. Furthermore, assuming that the intensity remains constant thereafter, then the number of eruptions during some specified length of time t_0 would be distributed as a homogeneous Poisson random variable with constant rate $\lambda(t)t_0$. If we consider the fact that not every eruption would result in disruption of the repository, and let p be the probability that any single eruption is disruptive, then the number of occurrences of such a disruptive event $X(t_0)$ in [0, t_0] also follows a homogeneous Poisson random variable with constant rate $\lambda(t)pt_0$ (Meyer, 1965, p. 156). An important element in assessing the suitability of the site is an assessment of the potential for future volcanic disruption of the repository. Since the phenomenon is stochastic, the answer is necessarily probabilistic (e.g., Dalal et al., 1989). Therefore, the probability of at least one disruptive event during the next t_0 years is of considerable practical interest and is quoted as "risk." In a classical statistical analysis, we would use the Poisson probability distribution formula,

$$\begin{aligned} \text{risk} &= \Pr\,(\text{at least one disruptive event before time } t_0) \\ &= 1 - \exp\{-\lambda(t)\mathrm{p}t_0\} \end{aligned} \tag{1}$$

and would find point or interval estimates for $\lambda(t)$ and p and act accordingly. In the following development, we shall use a Bayesian approach, in which we permit prior distribution for p, and shall consider $\lambda(t)$ to be fixed. The prior distribution, $\pi(p)$, of p expresses our beliefs regarding the numerical values of p. This would incorporate uncertainty about the probability of repository disruption p that are eventually averaged out as shown in the following equation. In this case, using the model of constant $\lambda(t)(=\hat{\lambda}_1$, or $\hat{\lambda}_2)$,

$$\text{risk} = 1 - \int_p \exp\{-\lambda(t)pt_0\}\pi(p)\,dp \tag{2}$$

Crowe et al. (1982) assume that every eruption has the same probability of repository disruption p, and provide a point estimate for $\mathrm{p}(= a/A)$. Their estimated values of p range from 10^{-4} to 10^{-3}. The calculations are based on a fixed value of a (= area of the repository $\pm$ 8 km^2), and several choices of A. (An area, ranging from 1953 km^2 to 69,466 km^2, corresponds closely to a defined volcanic province and satisfies the requirement of a uniform value of λ.) This approach offers computational simplicity. However, the existing data base is inadequate to reasonably constrain A. The technical machinery (Bayesian approach) involved in Eq. (2) would support much more informative answers if the prior distribution $\pi(p)$ is adequately chosen.

Determination of the Prior

We now turn to the determination of the prior density. Since the permissible range of p is $0 < p < 1$, without use of expert opinions regarding the geological factors at NTS, a natural choice for $\pi(p)$ is a noninformative prior. For instance, $U(0, 1)$ (uniform (0, 1)) assumes an average of 50% "direct hit," which is unrealistically conservative (overestimation). We shall settle on one particular prior based on the geological structure of the volcanic centers at NTS and conduct all further analysis in relation to Eq. (2).

According to Smith et al. (1990a), the area of most recent volcanism (AMRV) includes all known post-6-Ma volcanic complexes in the Yucca Mountain area and encompasses the four volcanic centers in Crater Flat, the Lathrop Wells cone, several centers in southeast Crater Flat, two centers at Sleeping Butte, and a center at Buckboard Mesa within the moat of the Timber Mountain Caldera (Fig. 3). They conclude that future volcanic events in the Yucca Moun-

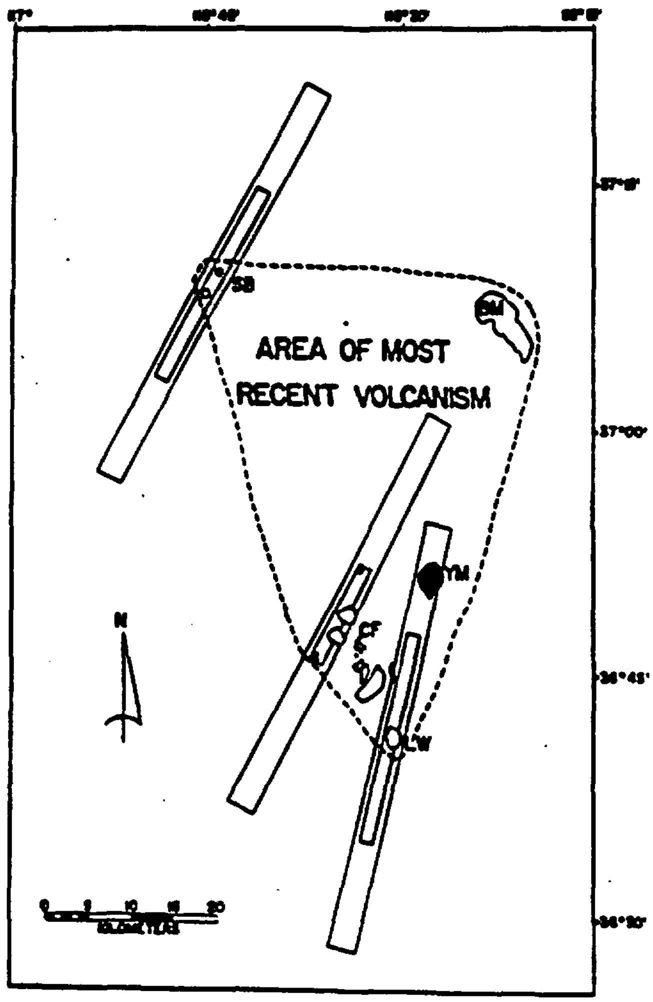

Fig. 3. Map outlining the AMRV (dashed line) and high-risk zones (rectangles) in the Yucca Mountain (YM) area that include Lathrop Wells (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), volcanic centers within Crater Flat (CF) (Source: Smith et al., 1990a, Fig. 7).

tain area will be associated with Quaternary centers in Crater Flat, at Sleeping Butte, or at the Lathrop Wells cone. Based on their assumption, a future eruption may occur either to the north-northeast or south-southwest of an existing cone or group of cones. They show high risk zones within the AMRV in Fig. 3 by placing two rectangles on each group of Quaternary cones. The proposed high-level nuclear waste repository at Yucca Mountain falls within the larger high-risk Lathrop Wells rectangle and just to the east of the high-risk zones constructed for the Crater Flat chain as described in Fig. 3. The dimensions of the larger Lathrop Wells rectangle are 50 km long and 3 km wide as determined by analog studies of Pliocene volcanic centers in the Fortification Hill field (Lake Mead area, Arizona and Nevada) and the Reveille Range (south-central Nevada). The lower half of this rectangle is outside the AMRV.

Now, using the idea of Crowe et al. (1982), assume there is no heterogeneity with respect to disruptiveness in the upper-half of the rectangle that encloses the repository (the eruptions to the south-southwest of the Lathrop Wells cone are outside the AMRV, and have near zero probability of disrupting the site). So, given $A = 75$ km^2 (= half of the area of the rectangle), $a = 8$ km^2 (area of the repository), we obtain $p = a/A = 8/75$. Therefore, a more informative prior, $U(0, 8/75)$, which assumes $8/75$ as the upper limit for p seems to be more suitable.

Risk Calculations

10^4 years is recommended as the required isolation period during which radioactive waste may decay to an acceptable level (see Crowe, 1986). The principal question we must answer: Can Yucca Mountain safely isolate for 10^4 years the radioactive waste? Thus, this period is the minimum length of time for which future volcanic hazards must be forecasted. The interval estimation of the risk (Eq. 2) for the chosen prior is based on this time frame, $\hat{\lambda}_1$, and $\hat{\lambda}_2$. We evaluate the risk for cases with $t_0 = 1$, and 10^4. Notice that Eq. (2) is integrated analytically, and the risk is a function of $\lambda(t)$ (given t_0) because the uncertainty of the probability of repository disruption p has been averaged out. Thus, from confidence bounds $\hat{\lambda}_1$ and $\hat{\lambda}_2$ on $\lambda(t)$, the corresponding confidence bounds on the risk for $t_0 = 1$ are 1.0×10^{-7} and 6.7×10^{-7}, increasing approximately linearly with isolation time t_0. As a result, a 90% confidence interval for the probability of site disruption for an isolation time of 10^4 years is $(1.0 \times 10^{-3}, 6.7 \times 10^{-3})$.

We have attempted to develop a formal structure that will have broad applicability to the common problem of estimating the instantaneous recurrence rate of volcanic activity based on the inter-event times. Within this framework we have specifically calculated the probability of site disruption based on the geological structure of the volcanic centers near the proposed Yucca Mountain site. Because the paper deals with questions of great importance, we clarify the differences between our work and that of Crowe et al.'s by including Table 1 which lists both approaches side-by-side.

Table 1. Summary of the Risk Assessment Methodologies for the Proposed Yucca Mountain Repository Site

Major questions	Answers: Crowe *et al.*	Answers: Ho
1. What is the assumed model?	A simple Poisson model	An NHPP for past events and an HPP for future events
2. What is a single event?	A volcanic center or cluster of centers	A main cone
3. How many events are counted? (Total, Quaternary volcanism)	(4, 2) (Crowe et al., 1982, Fig. 3; Crowe and Perry, 1989, Fig. 3)	(13, 8)
4. What is the variable of interest?	Magma volume	Inter-event time
5. What is the estimated recurrence rate (λ)?	$\approx 10^{-6}$/yr (Crowe, 1986)	(1.85×10^{-6}/yr, 1.26×10^{-5}/yr), a 90% C.I. for the instantaneous recurrence rate
6. What is the estimated disruptive parameter (p)?	$a/A = 1.1 \times 10^{-4}$–4.1×10^{-3} (Crowe et al., 1982)	Quantified by a prior, $U(0, 8/75)$
7. What is the estimated probability of site disruption for the projected time frame, i.e., 10^4 years?	3.3×10^{-6}–4.7×10^{-4} (Crowe et al., 1982)	(1.0×10^{-1}, 6.7×10^{-3}), a 90% C.I.

CONCLUDING REMARKS

In characterizing the Yucca Mountain site, scientists will study geology, hydrology, volcanoes, earthquakes, and climate. Such a geologic repository has never been attempted and it presents a number of challenges. The probability model developed in this paper deals with only some of the purely statistical studies which are based on past performance of the volcanoes at NTS. We now conclude this section with a few comments and point to some further work.

1. In modeling the recurrence rate, in general, it is unnecessary to treat it as constant for future events as it is reasonably assumed that the prior historical trend would continue. However, for the Yucca Mountain study, the projected time frame (10^4 years) is only a small fraction ($10^4/1.6 \times 10^6 = 6.25 \times 10^{-3}$) of the Quaternary period and is about 5% of the average repose time ($= 1.6 \times 10^6/8 = 2 \times 10^5$). This relatively short time scale suggests switching from an NHPP model of past events to a predictive HPP model. An HPP model is further justified on the basis of mathematical simplicity (e.g., Eq. 1), objectivity (given the uncertainty of future geophysical phenomena), and a slight increasing trend ($\hat{\beta} = 1.09$ for the Quaternary volcanism). Of course, if future advances

in volcanology suggest a continuous trend, the model can easily be updated to incorporate this requirement (e.g., Ho, 1991b).

2. We attribute a single date to the cluster and create a separate event with that date for each main cone in the cluster, using the definition of a single eruption from Crowe et al. (1983). Although this may appear to overlook the possibility that all main cones in a cluster could arise from the same eruption, Ho (1991a) points out that Crowe et al. (1989) and Wells et al. (1990) classify the Lathrop Wells volcanic center as a polygenetic volcano so some cones may have erupted more than once, leading to an underestimation of the recurrence rate. For example, the estimated recurrence rate would be doubled, provided the Lathrop Wells volcano (the youngest volcanic center) has erupted four times (Ho, 1991a). Furthermore, there are about 13 vents at Red Cone volcanic center (see Smith et al., 1990a, Fig. 3), so the recurrence rate would also be underestimated if these nearby vents have distinguishable ages. After all, every counted Quaternary main cone is a well-known volcanic center. All the above considerations are valuable. Further developments are necessary to complete and document those points previously mentioned for each Quaternary center to ensure that the probabilistic risk assessment is based on an adequate characterization of the volcanic record of the Yucca Mountain region.

3. Finally, if we are asked to deal with a method (Bayesian approach) that requires considerable use of subjective judgment, members of the licensing agency (Nuclear Regulatory Commission) of the repository might be left with the feeling that the whole exercise lacks scientific rigor, being unaccustomed to such mixing of "objective" facts with "subjective" judgments. This is hardly a new problem for scientists and engineers dealing with the mathematically intractable problems of the real world (Apostolakis, 1990). However, the beauty of the Bayesian approach is that it has no theoretical difficulty incorporating all kinds of information into predictive distributions (e.g., Ho, 1990). One could construct priors for the recurrence rate λ or the Weibull parameters and construct a predictive distribution for the number of eruptions in a given time span. Unfortunately, the quality of the current data set makes it impractical to use a Bayesian approach because of too many zero inter-event times (being unable to separate the ages of the cones) and too few data points. As the site characterization studies are more developed, a more informative data set allowing for the possibilities of polygenetic and polycyclic volcanism and based on reliable dating techniques may become available.

The task of quantifying volcanism at Yucca Mountain is as complicated as trying to predict the time of the next catch only based on a few piles of dead fish. (People would debate on the unknown fishing technique(s) used (fishing net, a single hook, etc.) to define a single event. They would also disagree on the freshness of each fish measured.) The issue of the high-level nuclear waste repository at Yucca Mountain has many geological and political considerations.

Some proponents of the repository will denounce opposition based on volcanic considerations as farfetched, while others will insist that any risk of site disruption poses an unacceptable threat to population. As a decision must be made, one using available information and educated estimates based on an adequate model is preferable to one decided in ignorance. We believe that our use of the Weibull model is the simplest approach that captures the basic elements (trend, objectivity, predictability, and mathematical simplicity) of the site characterization studies. It also accounts for all significant geological factors and can be easily amended to incorporate future advances in volcanology.

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STATISTICAL CONTROL CHART FOR REGIME IDENTIFICATION IN VOLCANIC TIME SERIES

Chih-Hsiang Ho

Department of Mathematical Sciences

University of Nevada, Las Vegas

4505 Maryland Parkway

Las Vegas, NV 89154 (U.S.A.)

ABSTRACT

In an important paper, Mulargia et al. (1987) address the importance of quantitative and objective identification of different regimes of a volcano. They develop a procedure based on the two-sample Kolmogorov-Smirnov (K-S) statistic. The K-S test is a general-purpose test that discriminates between two data sets as belonging to two different regimes based on their empirical distribution functions. The empirical distribution function is designed to describe the aggregate behavior of the volcanic activity, and it is constructed from the orders of the length of the collected repose times in each data set.

In this article, we use the idea of statistical process control to distinguish between the variation inherent in the observed repose times and the extraordinary variation that signals a real change in the regimes. We construct a table of control limits, and we demonstrate the procedure of regime identification based on a simple control chart. It shows a point outside the control limits almost as soon as the process enters a new regime. The basis of the statistical process control mechanism is a simple Poisson process, which is state of the art. The proposed control charting procedure is an eruption by eruption procedure. which follows the original chronological order of the eruptions. This procedure is applied to the eruptive history of the Mount Etna volcano. The application shows schematically that the procedure presents a visual interpretation of the identified regimes and can be practically translated for tabular or manual use.

KEY WORDS: Exponential distribution, process control, Weibull Poisson process.

INTRODUCTION

Identification of different regimes in time series data is essential to modeling the system and understanding the underlying processes. In the case of volcanoes, Mulargia et al. (1987) analyze the cumulative distributions of eruptions and volume output of the Etna volcano for the period 1600-1980 and detect several points of change of regime. They develop an algorithm for regime identification. A brief description follows:

1. They scan a data set of N sequential events and apply the Kolmogorov-Smirnov (K-S) two-sample test to all pairs of sets (of sizes m and n) which can be generated by the partition:

   $$m = i$$

   $$n = N - m$$

   $$i = 3, \cdots, N - 3.$$

   The partition point. which provides the most significant K-S test statistic subject to a specified significance level, is identified as the first or principal change-point.

2. Repeating the procedure on each of the subsets partitioned by the first change-point. a second (relative to segment 1) and a third (relative to segment 2) change-point are determined: applying the procedure successively to each of the subsets obtained. all significant change-points can be identified

following a sequential tree structure.

3. Each of the identified regimes is then tested by a K-S one-sample goodness-of-fit test to determine if the regime belongs to a standard distribution such as normal, exponential, etc.

4. Once the distribution of each regime is determined, confidence intervals for the points of change are determined through Monte Carlo simulation.

The Kolmogorov-Smirnov statistics have been frequently applied in volcanological studies (e.g., Klein, 1982; Mulargia et al., 1985) and are described in texts on statistics (e.g., Berry and Lindgren, 1990). In the case of volcanoes, the aggregate behavior of the volcanic activity is described by the empirical distribution function (the cumulative relative frequency) based on the orders of the length of the collected repose times in each data set. Therefore, any random permutation of the same data set of repose times yields the same result if the K-S test is applied to determine the distribution in each of the data sets.

It is obvious that the chronological order in which the volcanic eruptions occur is an extremely important aspect of a historical eruptive data set. We have written about this elsewhere (Ho, 1991a,b), but we will review the basic arguments to illustrate this point. We use the pseudo-data of five numbers (14, 34, 42, 72, and 244) provided by Asher (1983) to construct dot diagrams (Figure 1). These graphs strongly display that volcanic activities are "waning," "random," and "developing," since as time increases, the eruptions occur less frequently, about as frequently, and more frequently, respectively. The K-S one-sample test, however, indicates that the exponential distribution provides the same results of near-perfect fit (p-value $\doteq 1$) to all three data sets. The one-sample K-S statistic (e.g., Berry and Lindgren, p. 567) is based on the largest absolute difference between cumulative distribution functions. This same measure of distance, applied to two sample cumulative distribution functions, leads to the two-sample K-S statistic (e.g., Berry and Lindgren, p. 569) for testing the hypothesis that two populations are identical. For the pairwise comparisons based on the present data sets (random vs. developing, waning vs. developing, and random vs. waning), the K-S two-sample test also provides the same degree of evidence (p-value $\doteq 1$) for the null hypothesis (there is no difference between two distributions), since the orderings based on the length of repose times are exactly the same for all three data sets. Moreover, the sample size, N, in each data set must be specified in advance when the K-S test is applied. It implies that in the process of regime identification using the algorithm of Mulargia et al. (1987), the incorporation of any additional new eruption(s) in the future requires a complete new search from scratch. Consequently, the regimes previously identified could change even at the same level of significance, which is frustrating and intuitively unacceptable. Since the data occur naturally in a sequential fashion, it will be useful to have alternative procedures allowing for repeated significance tests on the accumulating data.

The change-point problem can be considered one of the central problems of statistical inference, linking together statistical control theory, theory of estimation and testing hypotheses, classical and Bayesian approaches, fixed sample and sequential procedures. Extensive references are given by Shaben (1980), Zacks (1983), and Wolfe and Schechtman (1984). None of these sources contain any references to a procedure which has the advantages of both simplicity and speed in detecting the change-points in a stochastic process. For the following development, we use the idea of statistical process control to distinguish between the variation inherent in the observed repose times and the extraordinary variation that signals a real change in the regimes. We also design a control chart as a tool for ease of use.

STATISTICAL PROCESS CONTROL

Statistical control is a sophisticated concept because it recognizes that variability will be present and requires only that the pattern of variability remain the same. A variable that continues to be described by the same distribution when observed over time is said to be in statistical control, or simply in control. We are already quite advanced in the art of thinking statistically when we describe a variable as stable or in control if its distribution does not change with time. Books by Montgomery (1985) and Ryan (1989) review much of the work in this area. We wish to distinguish between the variation inherent in the repose times observed and the extraordinary variation that signals a real change in the eruptive time-history of a volcano. This objective raises the following question: What is the distribution (or process of interest) under investigation?

There is a large, growing body of literature on probabilistic modeling of volcanic eruption time-series (Wickman, 1966, 1976; Klein, 1982, 1984; Mulargia et al., 1985; Mulargia et al., 1987; Ho, 1990, 1991ab). Much of the debate in the literature is centered on the choice of distribution models (principally, homogeneous Poisson versus nonhomogeneous Poisson models). For instance, Wickman (1966) observes that, for some volcanoes, the recurrence rates are independent of time. These volcanoes are called "simple Poissonian volcanoes." Wickman also uses a sequence of activity states (Markov chains), with the duration of the states being random variables distributed according to an exponential probability density function, for several volcanoes other than the simple Poissonian volcanoes. As mentioned earlier, Mulargia et al. (1987) discuss the random nature of the eruptive activity and also conclude that the eruptions of the Etna volcano occur at different regimes along the sampled period, each according to a simple Poisson process (the repose times between consecutive eruptions in each regime follow an exponential distribution). Therefore, we can rephrase our objective as follows: to produce a diagnostic technique for regime identification using a simple Poisson process as the basis of the statistical process control mechanism.

Control charts, which were first developed in the 1920s and 1930s, provide a mechanism for recognizing whether the process is in control. A control chart will be effective if it shows a point outside the control limits almost as soon as the process goes out of control. The point to be identified is the boundary value of two different regimes for the volcanological studies. This point goes under the name of change-point or scan-point (e.g., see Mulargia et al., 1987). A basic element of control charting is that data have been collected from the process of interest at a sequence of time points. Depending on the aspect of the process under investigation, some statistic (a number calculated from the observations in a sample) is chosen. The value of this statistic is then calculated for each sample in turn. A traditional control chart then results from plotting these calculated values over time. If the points on the chart all lie between the two control limits, the process is deemed to be in control. That is, the process is believed to be operating in a stable fashion reflecting only natural random variations. An out-of-control "signal" occurs whenever a plotted point falls outside the limits. This is assumed to be attributable to a new regime, and a search for another change-point commences. We shall design the control limits so that an in-control process generates very few false alarms, whereas a process not in control quickly gives rise to a point outside the limits.

CONTROL CHART FOR A POISSON PROCESS

There is a strong analogy between the logic of control charting and hypothesis testing. The null hypothesis (H_0) here is that the process is in control. When an in-control process yields a point outside the control limits (an out-of-control signal), a type I error (rejecting H_0 when H_0 is true) has occurred. Appropriate choice of control limits (corresponding to specifying a rejection region in hypothesis testing) will make this error probability suitably small.

For volcanism, Ho (1991a,b) considers a nonhomogeneous Poisson process (NHPP) with intensity function, $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ for $\beta, \theta > 0$. The parameters β and θ are sometimes referred to as shape and scale parameters, respectively. Because $\lambda(t)$ is the failure rate for the Weibull distribution, the corresponding process has been called the Weibull Poisson process (WPP). Goodness-of-fit, maximum likelihood (ML) estimates of β and θ, confidence intervals, and inference procedures for this process are presented in Bain and Engelhardt (1980), Bassin (1969), Crow (1974, 1982), Finkelstein (1976), and Lee and Lee (1978). A WPP is appropriate for three types of volcanoes: increasing-recurrence-rate ($\beta > 1$), decreasing-recurrence-rate ($\beta < 1$), and constant-recurrence-rate ($\beta = 1$). This generalized model can be considered a goodness-of-fit test for an exponential model ($\beta = 1$) of the volcanic inter-event times, which is equivalent to a homogeneous Poisson model of the events. In a simulation study, Bain et al. (1985) conclude that the test which is derived as an optimal test for the WPP also is rather powerful as a test of trend for general NHPP's. In other words, the test is "robust" against other model assumptions. This is the rationale of our choice of a WPP and the optimal test to be described below.

Suppose we assume that the successive volcanic eruptions of a specific volcano follow a WPP. Let $t_1, \cdots, t_n$ be the first n successive times of eruptions of a volcano. These times are measured from the beginning of the observation period (cumulative length of time over which the eruptions occur), so $t_1 < t_2 < \cdots < t_n$. The following theoretical results (for proof see Bain and Engelhardt, 1991, Ch. 9) are useful for constructing the control limits:

1) The maximum likelihood estimators for β and θ at t_n are

$$\hat{\beta}_n = n / \sum_{i=1}^{n-1} \ln(t_n/t_i) \tag{1}$$

and

$$\hat{\theta}_n = t_n / n^{1/\hat{\beta}_n} \tag{2}$$

2) A size α test of $H_0 : \beta_n = \beta_0$ against $H_A : \beta_n \neq \beta_0$ is to reject H_0 if $2n\beta_0/\hat{\beta}_n \leq \chi^2_{\alpha/2}(2n-2)$ or $2n\beta_0/\hat{\beta}_n \geq \chi^2_{1-\alpha/2}(2n-2)$, where $\chi^2_{\alpha/2}(2n-2)$ is the $100\alpha/2$ percentile of a chi-square distribution with 2n-2 degrees of freedom.

First, the parameters estimated from Equations (1) and (2) provide us with a quantitative value to characterize the volcanic activity at the nth eruption, which is the first step toward the construction of the statistic required for plotting over time. Second, suppose we wish to decide whether an exponential distribution seems appropriate (in-control signal) for the data up to the nth eruption. This suggests a test of $H_0 : \beta_n = 1$ against $H_A : \beta_n \neq 1$. Result 2 indicates that a chi-square test is appropriate, and the control limits are readily available from a table of the chi-square distribution.

Designing a CSLR procedure

If the eruption process is stable over time, the observed test statistic, $2n/\hat{\beta}_n$, should continue to be described by a chi-square distribution with 2n-2 degrees of freedom. We use this idea by drawing the $(1-\alpha)100\%$ control limits at

$$LCL_\alpha = \text{lower control limit} = \chi^2_{\alpha/2}(2n-2)$$

$$UCL_\alpha = \text{upper control limit} = \chi^2_{1-\alpha/2}(2n-2).$$

Table 1 provides the control limits for $\alpha = .01$, .05, and .1 (corresponding to 99%, 95%, and 90% control limits).

The next step in examining the eruptive process is to plot the statistic [$= 2n/\hat{\beta}_n$ or $2\sum_{i=1}^{n-1} \ln(t_n/t_i)$] against the time order in which the measurements were recorded. Since it requires at least two repose times for the statistical process control at each stage, cumulative sums of log ratios (CSLR) can be defined by

$$S_2 = 2ln(t_2/t_1)$$

$$S_3 = 2[ln(t_3/t_1) + ln(t_3/t_2)] = 2\sum_{i=1}^{2} ln(t_3/t_i)$$

$$\vdots$$

$$S_\ell = 2[ln(t_\ell/t_1) + \cdots + ln(t_\ell/t_{\ell-1})] = 2\sum_{i=1}^{\ell-1} ln(t_\ell/t_i)$$

$$= S_{\ell-1} + 2(\ell-1)ln(t_\ell/t_{\ell-1})$$

These cumulative sums are plotted over time. That is, at time ℓ of the ith stage, we plot a point at height S_ℓ. At the current time point r in the current stage i, the plotted points are $(2, S_2)_i, (3, S_3)_i, \cdots, (r, S_r)_i$.

If at current time r, either $S_r \leq \chi^2_{\alpha/2}(2r-2)$ or $S_r \geq \chi^2_{1-\alpha/2}(2r-2)$, the process is judged to be out of control. The first inequality suggests the process has shifted to an increasing time trend and thus a different regime has started at time $r-1$. Similarly, the second inequality suggests the process has shifted to a decreasing time trend. In either case, the (r-1)th eruption is identified as a change-point, which is the boundary point of two different regimes. Therefore, the (r-1)th time point is regarded as time zero for the search of the next change-point. The control charting procedure continues until no more significant points can be found or until the size of the data set becomes too small (minimum = 2). Each of the identified regimes then belongs to a simple Poisson process without need for further goodness-of-fit testing.

EMPIRICAL EXAMPLE: THE CASE OF MOUNT ETNA

For comparison purposes, the dates of eruptions in Table 2 reproduce the time series of flank eruptions of the Etna volcano in Table 1 of Mulargia et al. (1987). Figure 2 (line 4) shows the dot diagram of these eruptions in their original chronological order (in days). The purpose of the following analysis is to investigate the sensitivity of the proposed control chart.

Regarding the first eruption idicated in Table 2, June 28, 1607, as time point 0, Figure 3a schematically illustrates that the upper 90% control limit is first crossed at $(9, S_9)_1$ [S_9 (= 26.43) is larger than the UCL (= 26.30) corresponding to $\alpha = .1$ and $r = 9$ in Table 1]. It suggests the process has shifted to a decreasing time trend and thus a different regime has started at time point 8. As a result, the first change-point is identified as the March 3, 1702, eruption (corresponding to the 8th time point, see Column 1, Table 2) based on 90% control limits. This concludes the first stage. We then shift time point 0 to the first change-point and repeat the same procedure for the second stage of the control charting procedure. The resulting control chart is shown in Figure 3b, which shows that the lower control limit is crossed at $(3, S_3)_2$. A significantly increasing time trend (relative to regime 2) has started at time point 2 of stage 2. Therefore, the second change-point is identified at $(2, S_2)_2$, which corresponds to the May 1, 1759, eruption (see Column 2, Table 2). Also, this eruption is treated as time point 0 for the next search. At stage three. all plotted points are between the limits (Figure 3c), indicating an in-control process as far as variation is concerned. Therefore, based on 90% control limits, three regimes are identified in the period 1600-1980, with change-points at March 3, 1702, and May 1, 1759. The dot diagrams in Figure 2 (lines 1-3) display these three regimes. Reading these graphs in Figure 2, we are convinced that a long repose ($= 19,364$ days or $\doteq 53$ years) after the eruption of March 3, 1702, contributes significantly to the breakdown of these regimes based on the 90% control limits. Interestingly enough, although the points plotted in Figure 4 demonstrate that the overall time trend has the same pattern as that described in the previous three regimes. all points are within the 95% control limits. In other words, using 95% control limits. the conclusion of Mulargia et al. (1987) that Etna (flank eruptions only) behaves as a nonstationary Poisson volcano in the period 1600-1980 is not substantiated by the present approach based on the original chronological order of the eruptions. It demonstrates that the evidence of an out-of-control signal is moderate for this data set.

Mulargia et al. (1987), using the technique based on the orders of the length of repose times, draw the following conclusion: The eruption of May 30, 1865 splits the time series of flank eruptions of Mount Etna (1600 - 1980) into two different regimes at the 0.05 significance level. The dot diagram (line 4) in Figure 2 reveals nothing interesting about the May 30, 1865, eruption, which is also supported by the technique proposed in the present study. However, a histogram for the data in each regime obtained by Mulargia et al. (1987) might show different aggregate behavior of the volcanic activity, because the K-S test is based on the orders of the length of the collected repose times in each data set. Of course, there is a possibility for a "false alarm" in either technique.

Finally, what are we to conclude from the fact that Mulargia et al. (1987) split the time series into two regimes while this study concludes three regimes based on the 90% control limit and only one regime if the 95% control limit is used? In this article, we present a new approach for regime identification based on the original chronological order of the eruptions. In so doing we strive for neither generality nor consistency (with the results of Mulargia et al., 1987). In the spirit of data analysis, it seems sensible to examine different aspects of the data by a variety of tests to help illuminate the nature of the data. In particular, our results for Mount Etna provide an integrated way of addressing the various aspects of the regimes of a volcano. Perhaps the above question should be rephrased as: Is Mount Etna a simple Poissonian volcano?

CONCLUSIONS

We conclude this section by stating a few characteristics of the proposed technique.

(1) The computation is extremely easy (a hand calculator will do the job).

(2) The technique is based on the original chronological order of the eruptions. The results (regimes) can then be easily displayed and interpreted by dot diagrams.

(3) Regimes previously identified won't be affected by updating of future eruptions, if the same level of control limit is used.

(4) A simple Poisson process, currently regarded as the underlying distribution, is assumed as the basis of the statistical process control mechanism. This assumption is engineered to kill two birds with one stone, as each identified regime belongs to a simple Poisson process and requires no further goodness-of-fit tests.

(5) The control chart is a useful tool designed for manual use. It also provides a mechanism for monitoring volcanic activities to identify instability and unusual circumstances, and prompt action can follow.

(6) When faced with the formality of significance testing, we recommend adjusting signficance levels (control limits) to account for multiple tests. (This

may be viewed by some volcanologists as strange.) Our efforts for future studies will be devoted to this goal and to some quality assessments of the procedure.

ACKNOWLEDGEMENTS

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Table 2. Time Series of Flank Eruptions of Etna Volcano According to Mulargia et al. (1987)

Time point (r) for stage 1	2	3	Date	
0			1607-06-28	
1			1610-02-06	
2			1614-07-01	
3			1634-12-19	
4			1646-11-20	
5			1651-01-16	
6			1669-03-11	
7			1689-03-14	
8	0		1702-03-03	*
9	1		1755-03-09	
	2	0	1759-05-01	*
	3	1	1763-02-05	
		2	1763-06-20	
		3	1766-05-27	
		4	1780-04-20	
		5	1792-05-11	
		6	1792-06-01	
		7	1802-11-15	
		8	1809-03-28	
		9	1811-10-28	
		10	1819-05-27	
		11	1832-11-01	
		12	1843-11-17	
		13	1852-08-20	
		14	1865-05-30	
		15	1874-08-29	
		16	1879-05-26	
		17	1879-05-27	
		18	1886-05-18	

Table 2. continued

19	1892-07-11
20	1908-05-29
21	1910-03-23
22	1911-09-09
23	1918-11-29
24	1923-06-16
25	1928-11-03
26	1928-11-04
27	1942-06-30
28	1947-02-21
29	1949-12-02
30	1950-11-25
31	1971-05-07
32	1971-05-11
33	1974-03-11
34	1978-04-29
35	1978-08-24
36	1978-11-18

* Change point based on 90% control limits

Table 1. Control Limits for Regime Identification

Time point (r)	90%		95%		99%	
	$LCL_{.1}$	$UCL_{.1}$	$LCL_{.05}$	$UCL_{.05}$	$LCL_{.01}$	$UCL_{.01}$
2	.10	5.99	.05	7.38	.01	10.60
3	.71	9.49	.48	11.14	.21	14.86
4	1.64	12.59	1.24	14.45	.68	18.55
5	2.73	15.51	2.18	17.53	1.34	21.96
6	3.94	18.31	3.25	20.48	2.16	25.19
7	5.23	21.03	4.40	23.34	3.07	28.30
8	6.57	23.68	5.63	26.12	4.07	31.32
9	7.96	26.30	6.91	28.85	5.14	34.27
10	9.39	28.87	8.23	31.53	6.26	37.16
11	10.85	31.41	9.59	34.17	7.43	40.00
12	12.33	33.92	10.98	36.78	8.64	42.80
13	13.84	36.41	12.40	39.37	9.89	45.56
14	15.38	38.88	13.84	41.93	11.16	48.29
15	16.92	41.33	15.31	44.46	12.46	50.99
16	18.49	43.77	16.79	46.98	13.79	53.67
17	20.07	46.19	18.28	49.48	15.09	56.37
18	21.66	48.60	19.79	51.97	16.46	59.01
19	23.27	50.99	21.32	54.44	17.85	61.62
20	24.88	53.38	22.87	56.90	19.25	64.22
21	26.51	55.76	24.42	59.35	20.67	66.80
22	28.14	58.12	25.99	61.78	22.10	69.37
23	29.79	60.48	27.56	64.20	23.55	71.93
24	31.44	62.83	29.15	66.62	25.00	74.47
25	33.10	65.17	30.75	69.03	26.47	77.00
26	34.76	67.50	32.35	71.42	27.96	79.53
27	36.44	69.83	33.96	73.81	29.45	82.04
28	38.11	72.15	35.58	76.20	30.95	84.54
29	39.80	74.47	37.20	78.57	32.46	87.03
30	41.49	76.78	38.84	80.94	33.98	89.51

(1-α)100% level

Table 1. continued

31	43.19	79.08	40.47	83.30	35.50	91.98
32	44.89	81.38	42.12	85.66	37.04	94.45
33	46.59	83.67	43.77	88.01	38.58	96.91
34	48.30	85.96	45.42	90.35	40.13	99.36
35	50.02	88.25	47.08	92.69	41.68	101.81
36	51.74	90.53	48.75	95.03	43.24	104.25
37	53.46	92.81	50.42	97.36	44.81	106.68
38	55.19	95.08	52.10	99.68	46.39	109.11
39	56.92	97.35	53.77	102.00	47.97	111.53
40	58.65	99.62	55.46	104.32	49.55	113.94
41	60.39	101.88	57.15	106.63	51.14	116.35
42	62.13	104.14	58.84	108.94	52.74	118.75
43	63.87	106.39	60.53	111.25	54.34	121.16
44	65.62	108.65	62.23	113.55	55.94	123.55
45	67.37	110.90	63.93	115.84	57.55	125.94
46	69.12	113.14	65.64	118.14	59.17	128.33
47	70.88	115.39	67.35	120.43	60.79	130.71
48	72.64	117.63	69.06	122.72	62.41	133.09
49	74.40	119.87	70.78	125.00	64.04	135.46
50	76.16	122.11	72.49	127.28	65.67	137.83

For $r > 50$, $$LCL_\alpha = (2r-2)\left(1 - \frac{2}{9(2r-2)} + z_\alpha\sqrt{\frac{2}{9(2r-2)}}\right)^3$$

$$UCL_\alpha = (2r-2)\left(1 - \frac{2}{9(2r-2)} - z_\alpha\sqrt{\frac{2}{9(2r-2)}}\right)^3$$

where z_α is the 100α percentile from $N(0,1)$

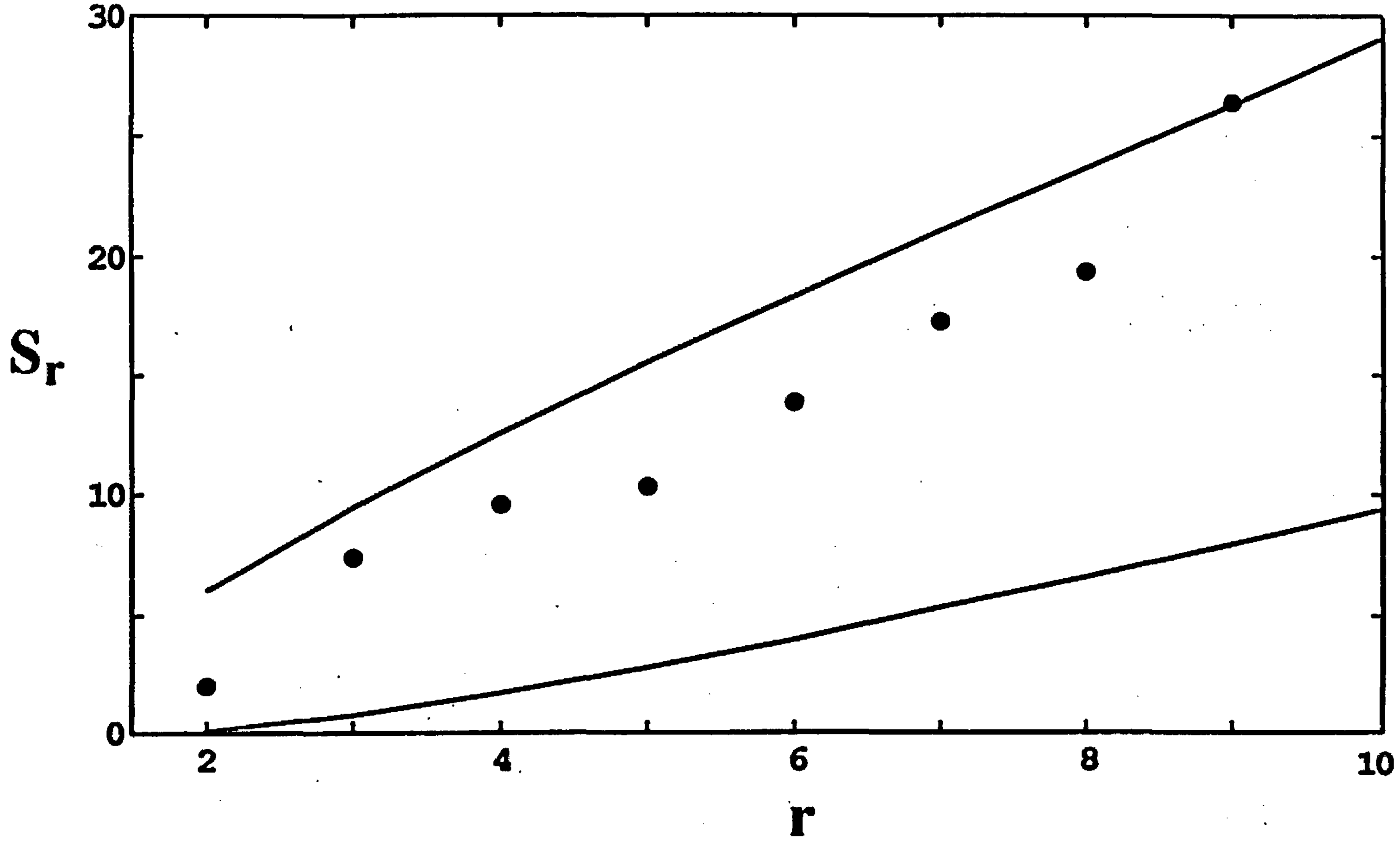


Figure 3a. Stage 1 control chart for Mount Etna based on 90% control limits

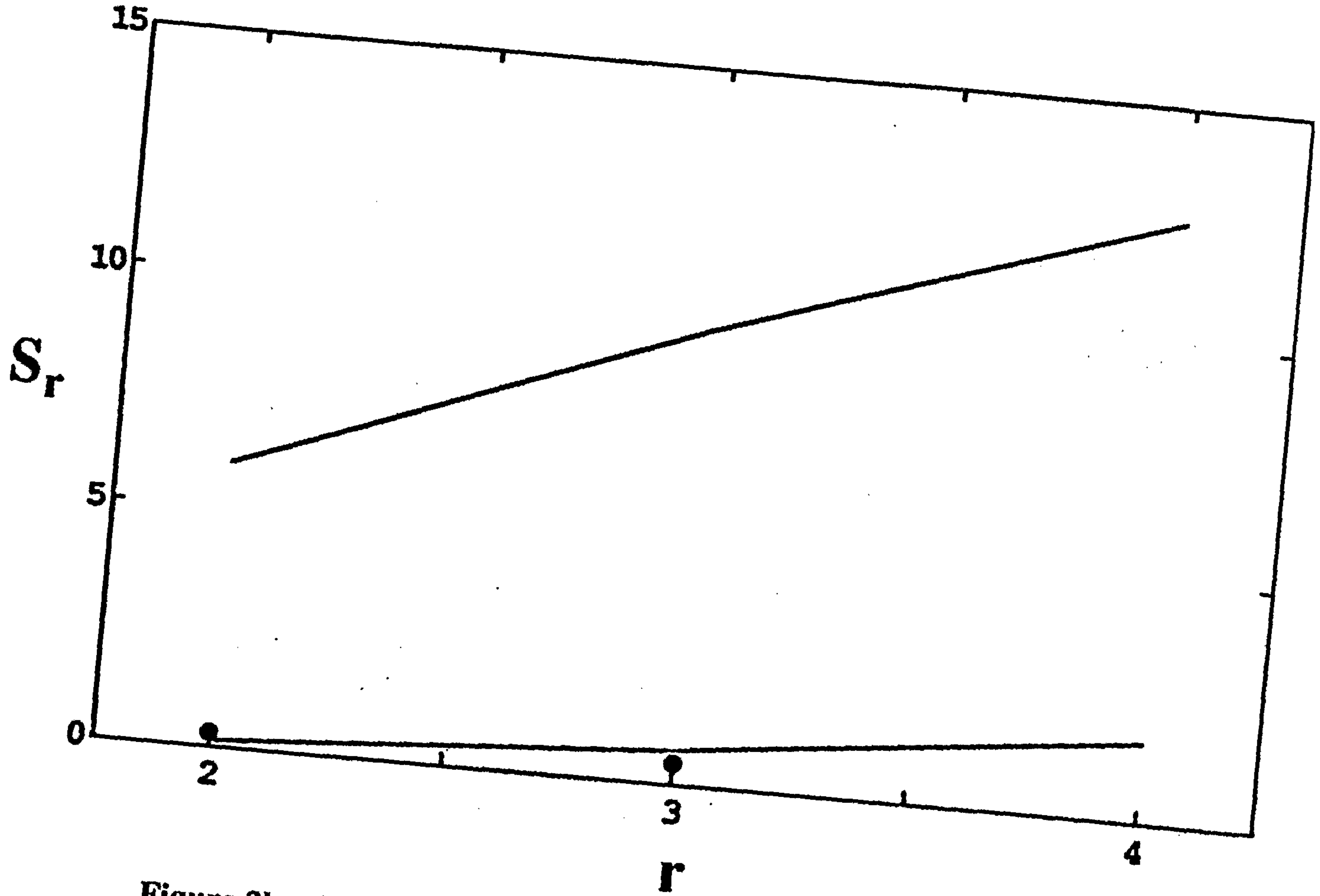


Figure 3b. **Stage 2 control chart for Mount Etna based on 90% control limits**

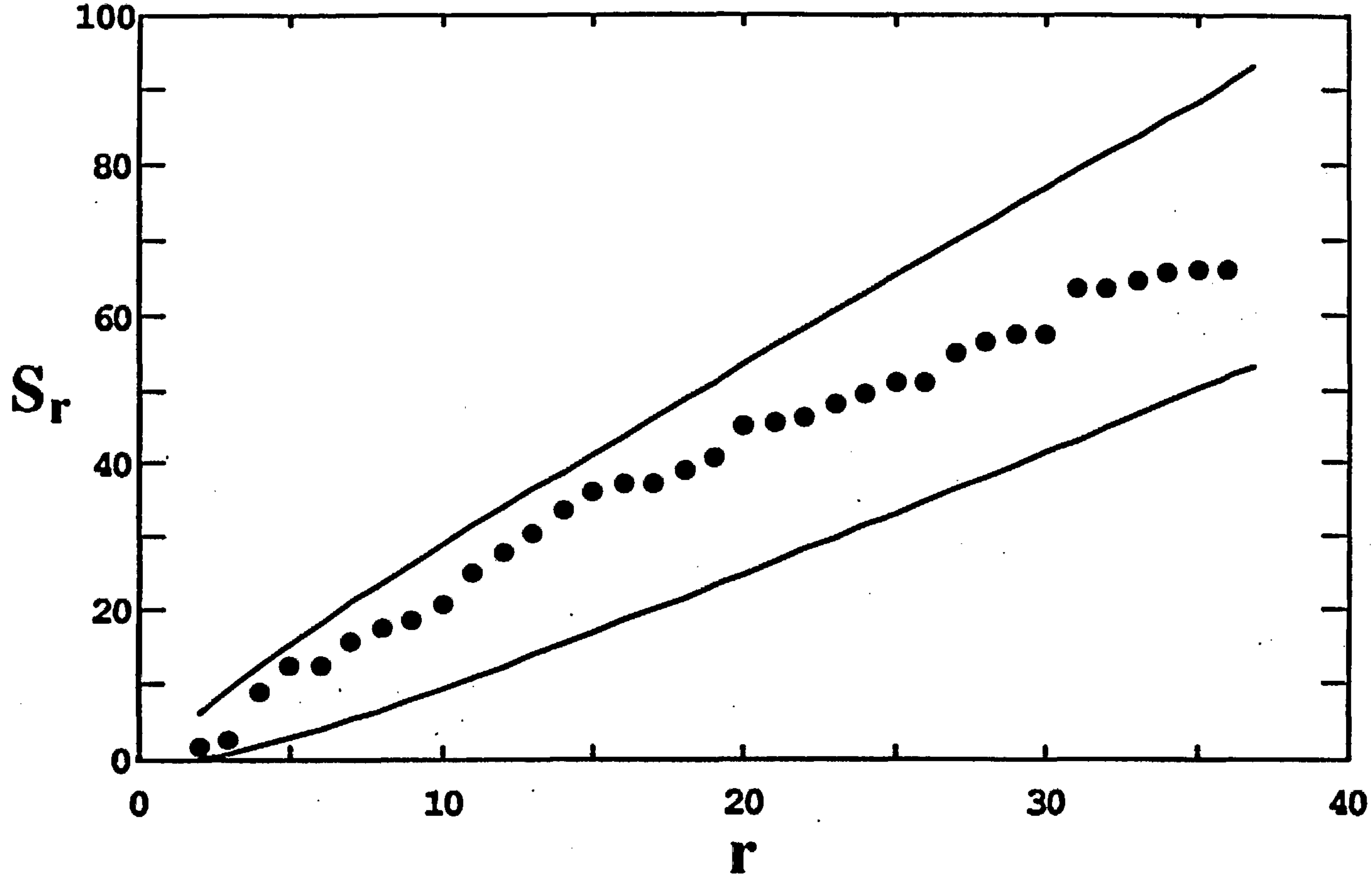


Figure 3c. Stage 3 control chart for Mount Etna based on 90% control limits

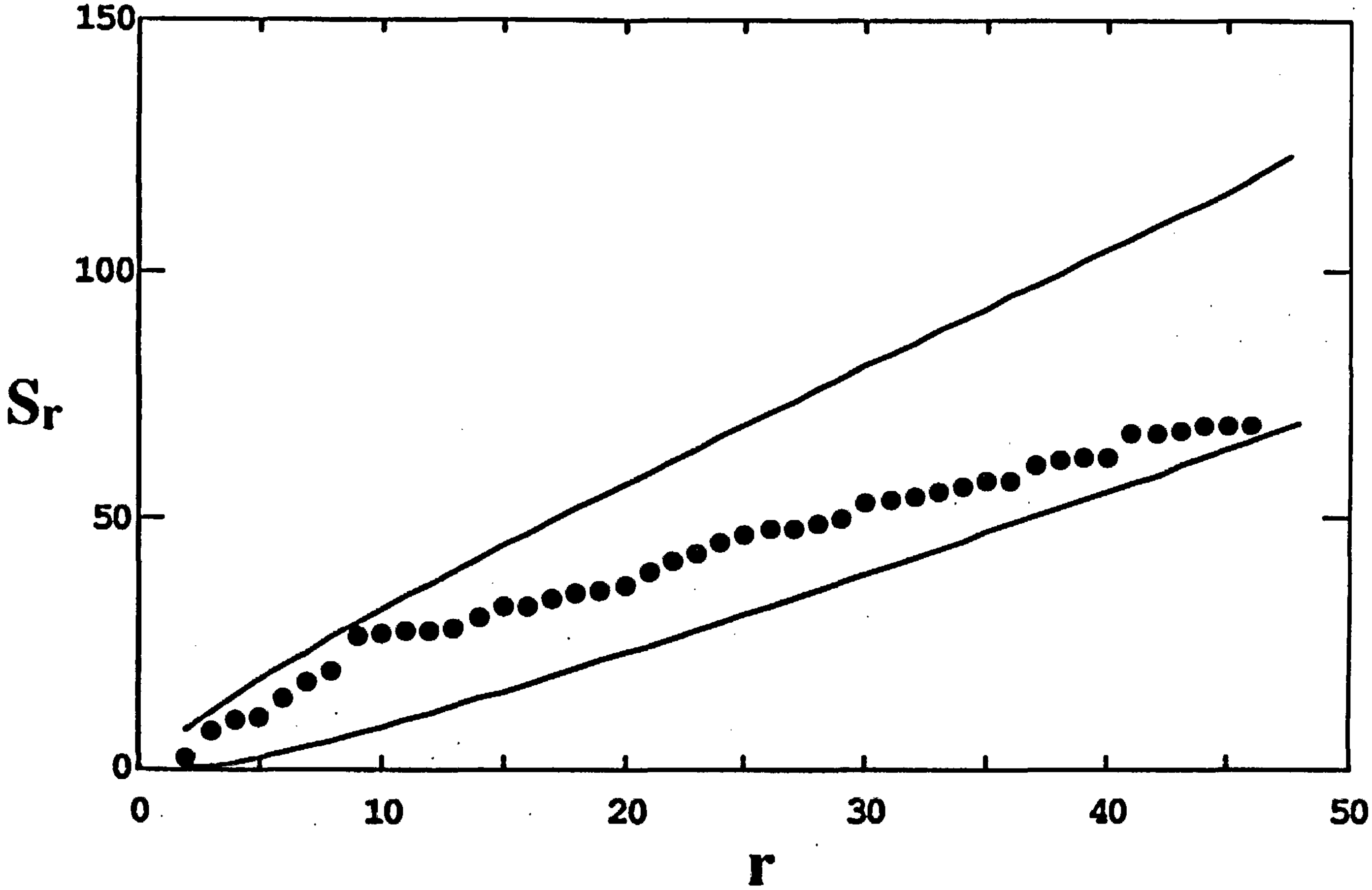


Figure 4. Control chart for Mount Etna based on 95% control limits

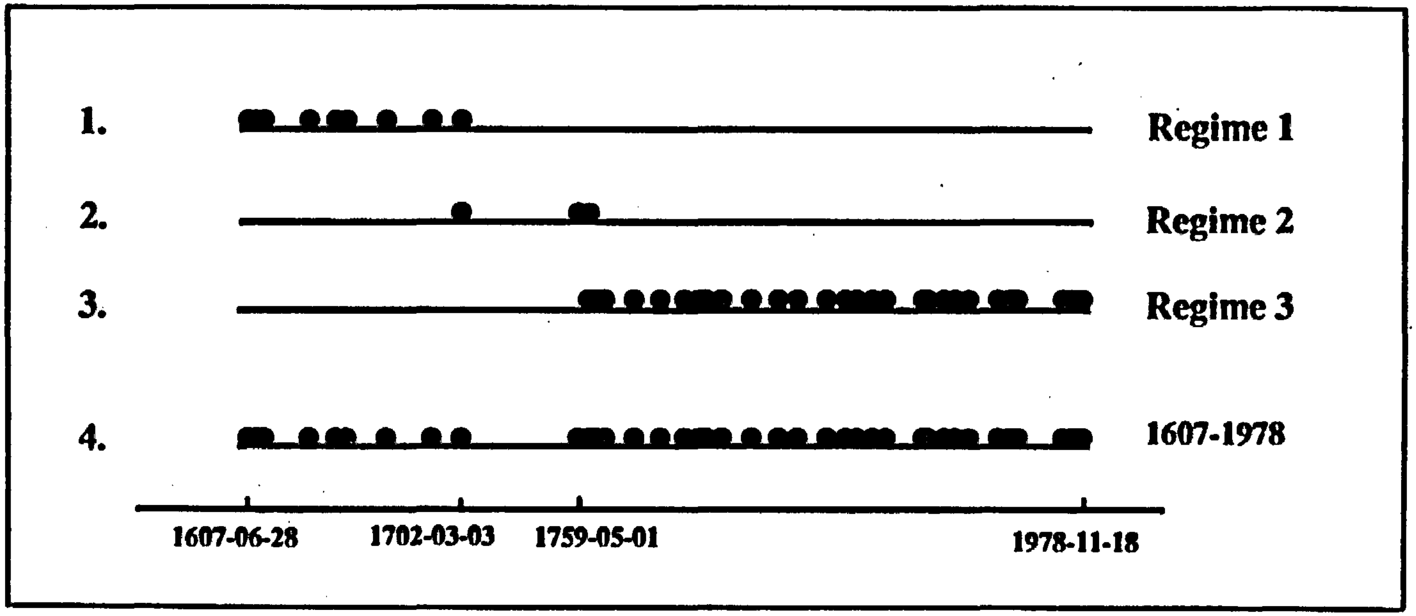


Figure 2. Dot diagrams of three Etna regimes based on 90% control limits and the original time series data.

RISK ASSESSMENT FOR THE YUCCA MOUNTAIN HIGH-LEVEL NUCLEAR WASTE REPOSITORY SITE: ESTIMATION OF VOLCANIC DISRUPTION

C. -H. HO

Department of Mathematical Sciences

University of Nevada, Las Vegas

(this work is supported by the Nevada Nuclear Waste Project Office)

GOALS

To estimate

1. the recurrence rate

2. the probability of volcanic disruption of the repository during the next 10,000 years

DATA

1. Define a single event
2. Measure each event
3. Count them all

generate a TIME SERIES

A main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone

(Crowe et al. 1983)

Preliminary Data Set

3.7, 3.7, 3.7, 3.7, 2.8, 1. 2, 1. 2, 1. 2, 1. 2, 1. 2, 0. 28, 0. 28, 0. 01

(B) Quaternary

(A) Post-6 Ma

MODEL

MODELING THE VOLCANISM -

RECURRENCE RATE ESTIMATION

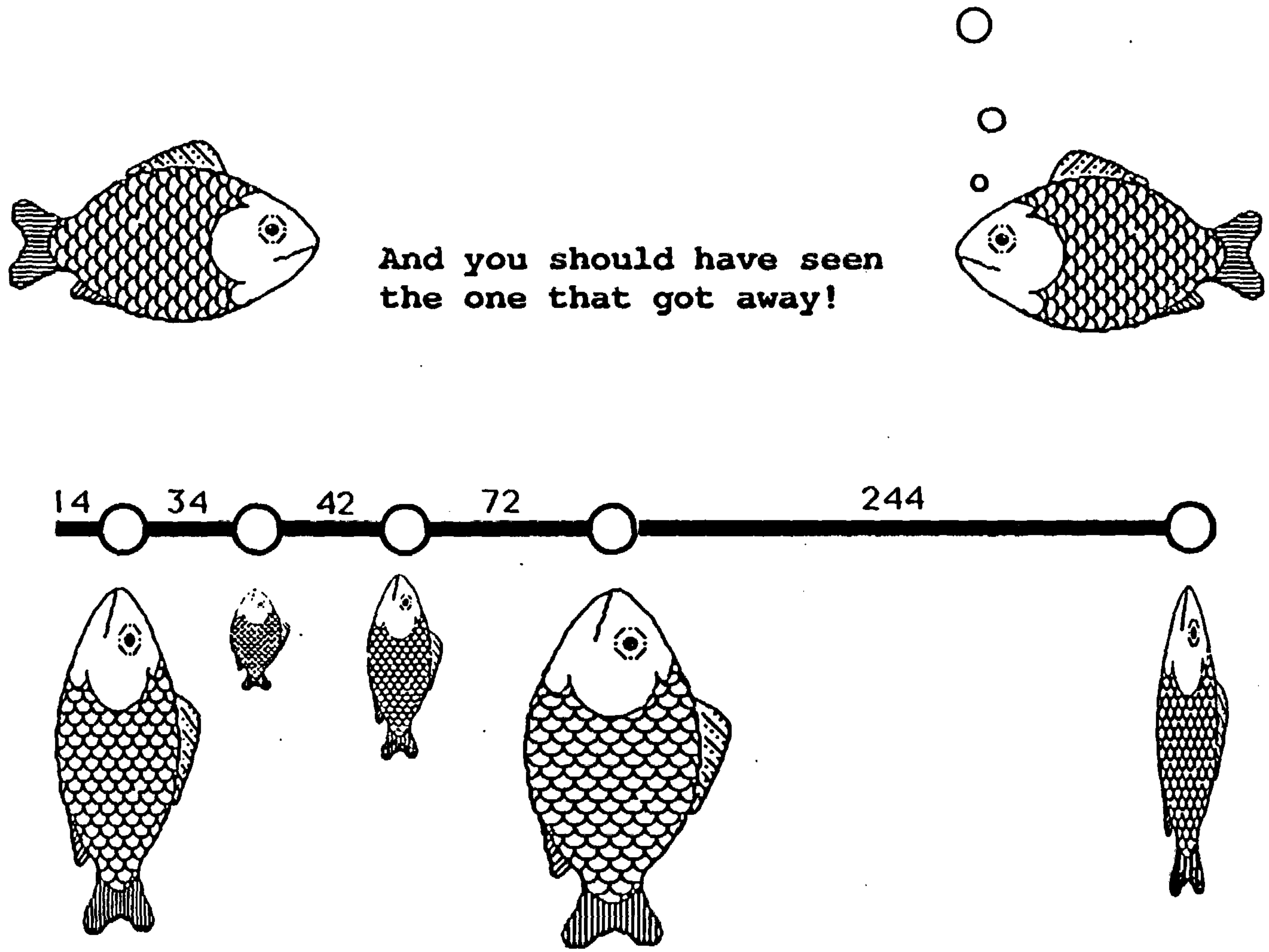
Need a model that captures the basic elements of the study:

1. Time trend

2. Predictability

3. Robust to other model assumptions

4. Mathematical simplicity


And you should have seen
the one that got away!
14
34
42
72
244

1. GENERALIZE a constant λ with $\lambda(t)$, a function of time

2. Model $X(t)$ = number of events in $[0,t]$

   $X(t)$ follows a nonhomogeneous Poisson process (NHPP) with parameter $\mu(t)$

$$\mu(t) = \int_0^t \lambda(s)\, ds$$

(Parzen, 1962, p. 138)

- **Choice of** $\lambda(t) = (\beta/\theta)\,(t/\theta)^{\beta-1}$
- **yields** $\mu(t) = (t/\theta)^{\beta}$
- **implies a Weibull** $(\theta\ ,\ \beta)$

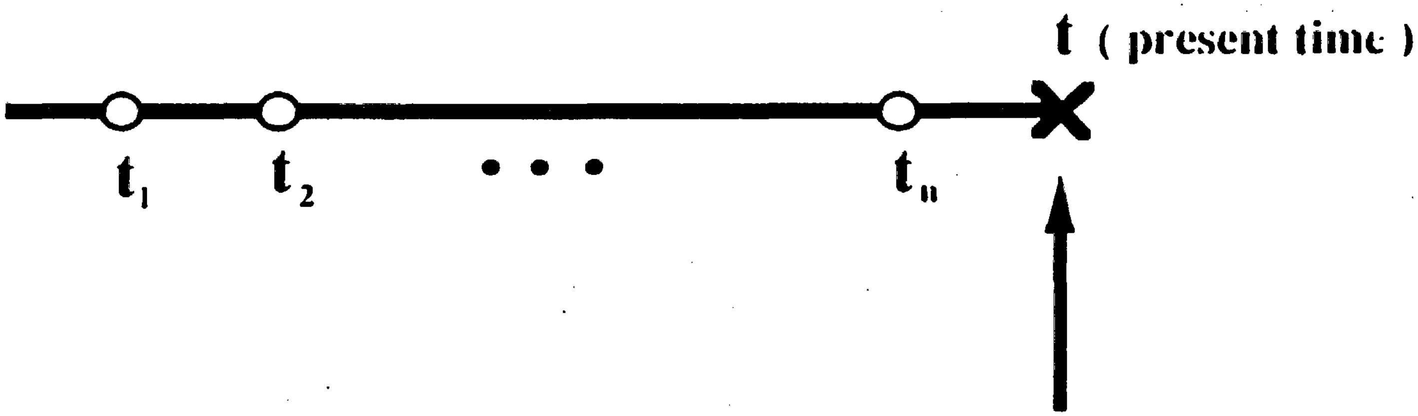
$$\beta \begin{cases} > 1 & \textbf{increasing} \\ = 1 & \textbf{simple Poisson} \\ < 1 & \textbf{decreasing} \end{cases}$$

Let $t_1, t_2, \ldots, t_n$ be the first n successive times of events in $[0,t]$: $t_1 < t_2 < \ldots < t_n$

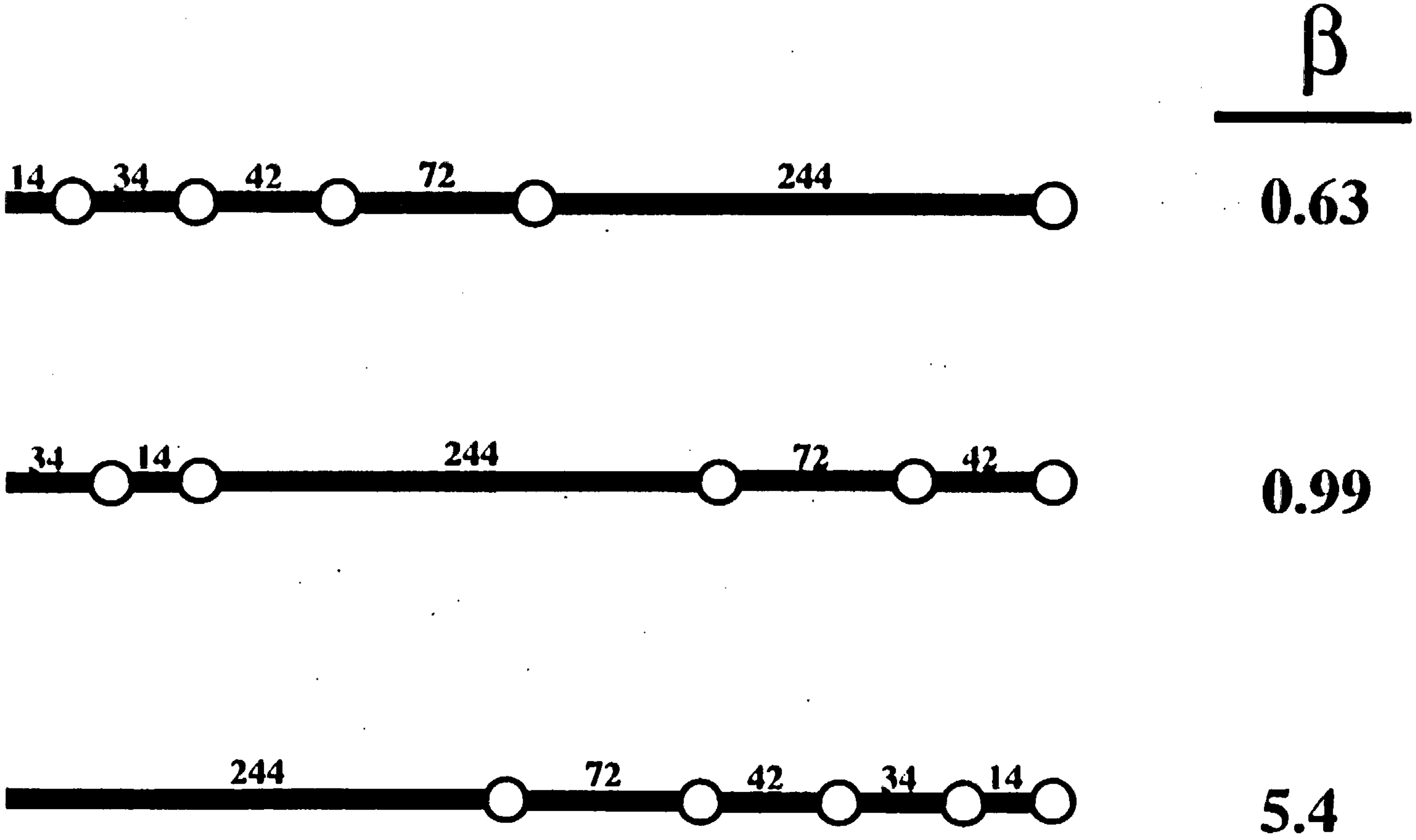
- $\hat{\beta} = n \Big/ \sum_{i=1}^{n} \ln(t/t_i)$
- $\hat{\theta} = t/n^{1/\hat{\beta}}$
- $\lambda = (\hat{\beta}/\hat{\theta})\,(t/\hat{\theta})^{\hat{\beta}-1}$

(Crow 1974, 1982)

Instantaneous Recurrence Rate



$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\Theta})(t/\hat{\Theta})^{\hat{\beta} - 1}$$


β
14
34
42
72
244
0.63
34
14
244
72
42
0.99
244
72
42
34
14
5.4

Preliminary Data Set

3.7, 3.7, 3.7, 3.7, 2.8, <u>1. 2, 1. 2, 1. 2, 1. 2, 1. 2, 0. 28, 0. 28, 0. 01</u>

(B) Quaternary

(A) Post-6 Ma

(A)
- $\hat{\beta} = 2.29$ (one-sided p-value $\doteq 0.005$)
- $\hat{\lambda} = 5\text{x}10^{-6}$ /yr

(B)
- $\hat{\beta} = 1.09$ (one-sided p-value $\doteq 0.45$)
- $\hat{\lambda} = 5.5\text{x}10^{-6}$ /yr

$$\underline{\hat{\lambda} = 5.5 \times 10^{-6}/\text{yr}}$$

- **The estimated instantaneous recurrrence rate**

- **It represents the instantaneous eruptive status of the volcanism at the end of the observation time t (present)**

Interval estimate of $\lambda(t)$

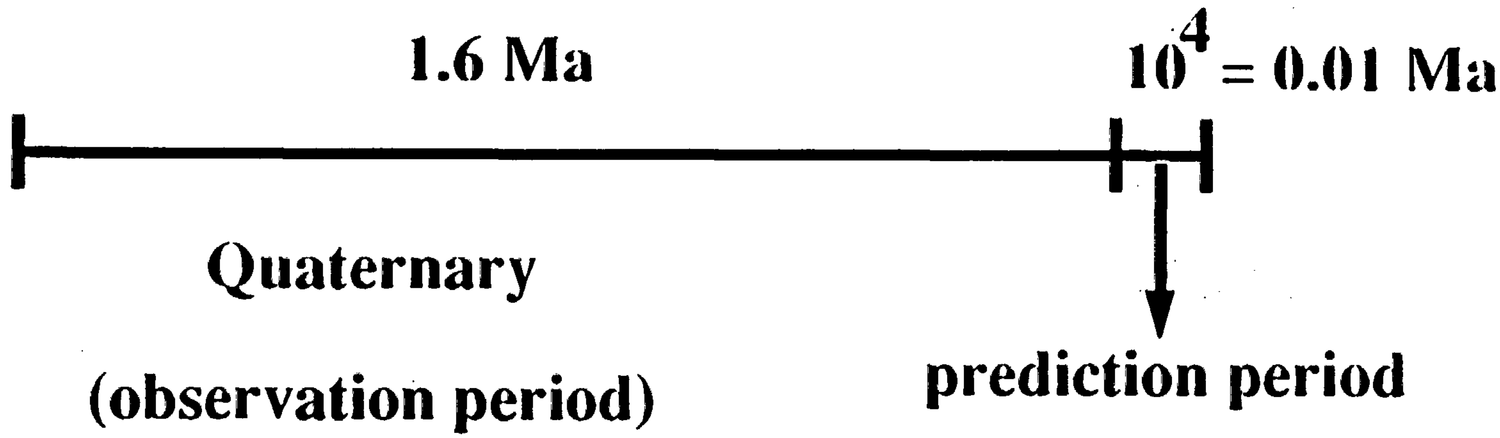
A 90% confidence interval for $\lambda(t)$ is

$\left(\hat{\lambda}_1 , \hat{\lambda}_2\right) = \left(1.85 \times 10^{-6} , 1.26 \times 10^{-5}\right)$, **which**

is more informative than $\hat{\lambda} = 5.5 \times 10^{-6}$ / yr

PREDICTING

FUTURE ERUPTIONS



1. **The projected time frame is about 0.6% of the OP**
2. **It is only 5% of the average repose time**

$\Downarrow$

Suggests switching from a NHPP to a predictive HPP model

MODELING

THE VOLCANIC DISRUPTION

Define

Risk = The probability of at least one disruptive event during the next t_0 years.

$X(t_0)$ = The number of occurrences of such a disruptive event in $[0, t_0]$.

REMARKS

1. In this study, we restrict the risk to bull's-eyed volcanic events which result in the formation of volcanic cones and site disruption.

2. In so doing we neglect the potential impact of all other types of events such as a series of dikes, plugs, and sills, etc.

(What goes on under the surface?)

p = The probability that any single eruption

is disruptive

(not every eruption would result in disruption of the repository)

$$\text{Risk} = 1 - \int_p \exp\left\{-\lambda(t)pt_0\right\}\pi(p)\,dp$$

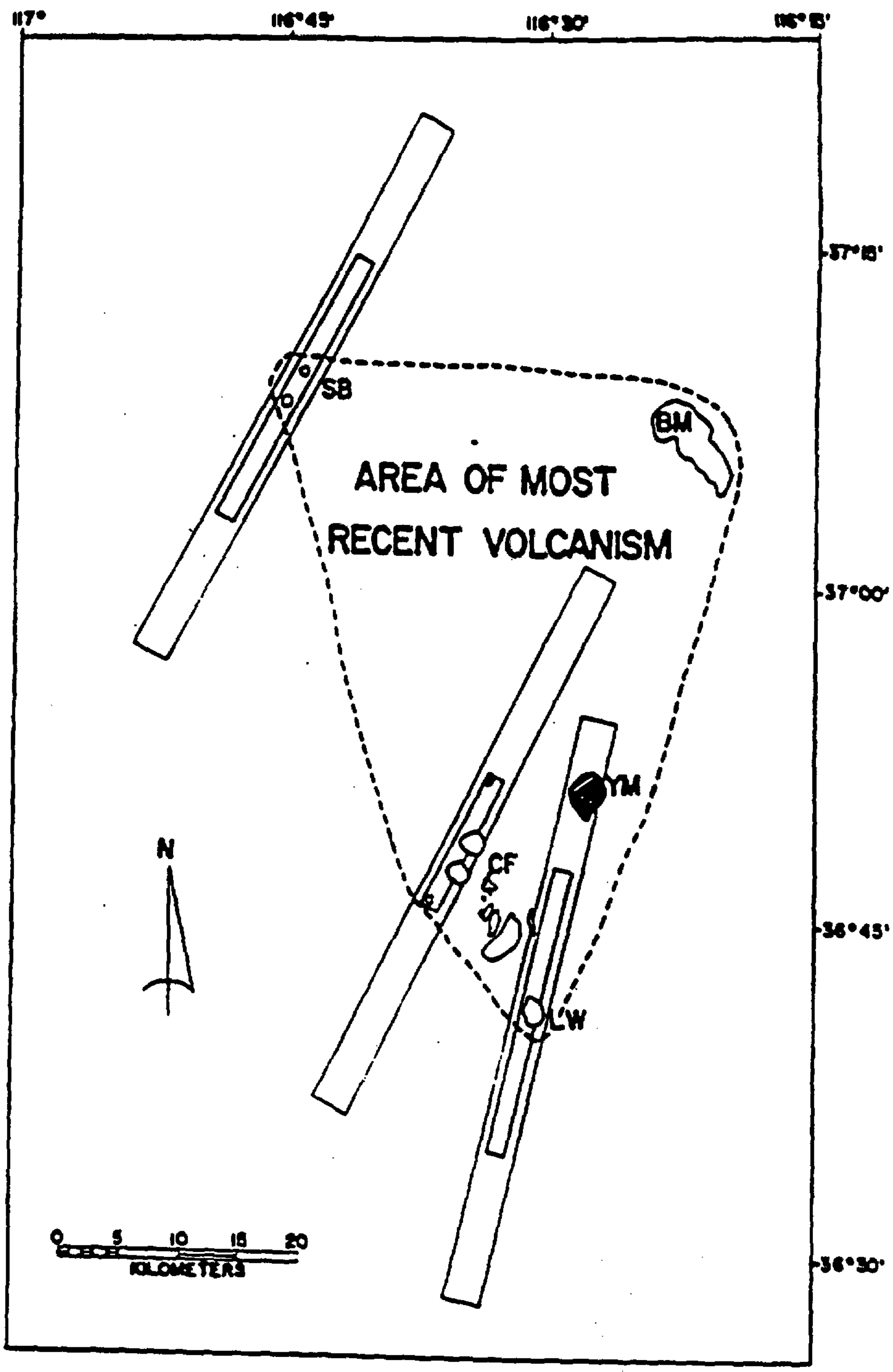
The technical machinery (Bayesian approach) involved in the risk calculation would support much more informative answers if the prior distribution $\pi(p)$ is adequately chosen.

Determination of the Prior

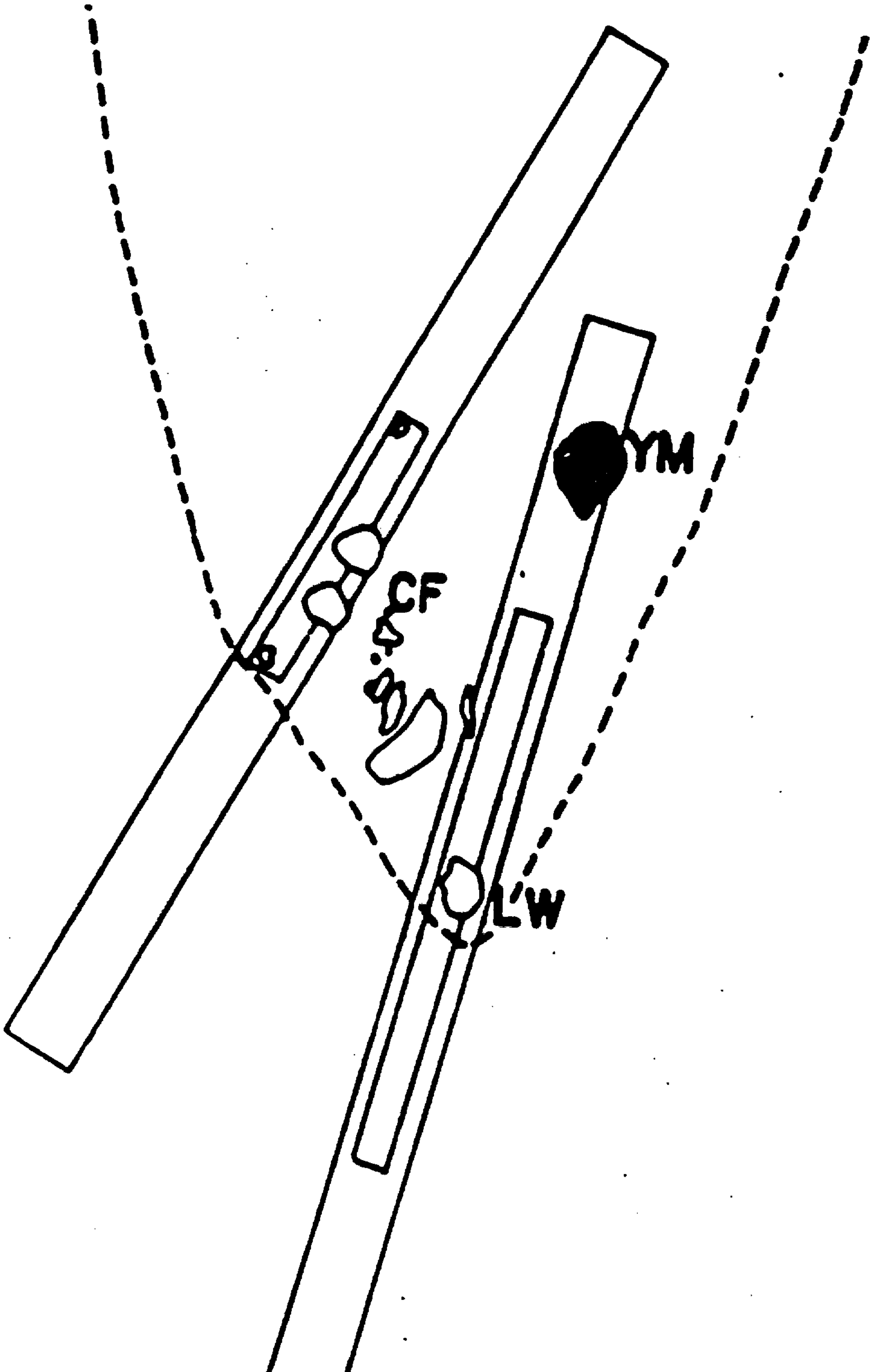
- The permissible range of p is $0 < p < 1$.

- Without use of expert opinions regarding the geological factors at NTS, a natural choice for $\pi(p)$ is a noninformative prior

- For instance, Uniform (0,1) assumes an average of 50% "direct hit" , which is unrealistically conservative (overestimation)



Map outlining the AMRV (dashed line) and high-risk zones (rectangles) in the Yucca Mountain (YM) area that include Lathrop Wells (LW), Sleeping Butte cones (SB), Buckboard Mesa center (BM), volcanic centers within Crater Flat (CF). (Source: Smith et al., 1990a, fig. 7)


YM
CF
LW

We have

1. $A = 75\ km^2$ (= half of the rectangle)

2. $a = 8\ km^2$ (area of the repository, Crowe et al, 1982)

3. $\pi(p) \sim U(0, 8/75)$, which assumes $8/75$ as the upper limit for p

RESULT

A 90% confidence interval for the probability of site disruption for an isolation time of 10^4 years is

$$\left(1.0 \times 10^{-3}\,,\ 6.7 \times 10^{-3}\right)$$