FEAST: A Two-Dimensional Non-Linear Finite Element Code for Calculating Stresses

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Abstract

The computer code FEAST calculates stresses, strains, and displacements. The code is two-dimensional. That is, either plane or axisymmetric calculations can be done. The code models elastic, plastic, creep, and thermal strains and stresses. Cracking can also be simulated. The finite element method is used to solve equations describing the following fundamental laws of mechanics: equilibrium; compatibility; constitutive relations; yield criterion; and flow rule. FEAST combines several unique features that permit large time-steps in even severely non-linear situations. The features include a special formulation for permitting many finite elements to simultaneously cross the boundary from elastic to plastic behaviour; accomodation of large drops in yield-strength due to changes in local temperature;and a three-step predictor-corrector method for plastic analyses. These features reduce computing costs. Comparisons against twenty analytical solutions and against experimental measurements show that predictions of FEAST are generally accurate to ± 5%.

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FEAST: Code non linéaire à deux dimensions utilisant la méthode des éléments finis pour le calcul des contraintes

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Résumé

Le code d'ordinateur FEAST calcule les efforts, contraintes et déplacements. Il s'agit d'un code à deux dimensions permettant d'effectuer des calculs plans ou axisymétriques. Le code élabore des modèles d'efforts et contraintes thermiques, de fluage et élastiques et plastiques. Il peut élagement simuler la fissuration. La méthode des éléments finis est utilisée pour résoudre les équations représentant les lois fondamentales de la mécanique suivantes : l'équilibre; la compatibilité; les relations constitutives; le critère de limite d'élasticité; et la loi de l'écoulement. Plusieurs caractéristiques uniques sont réunies dans le code FEAST en vue de permettre de grands intervalles de temps même dans des situations manifestant une non-linéarité sévère. Les caractéristiques en question comprennent une disposition spéciale par laquelle de nombreux éléments finis peuvent franchir simultanément la limite du comportement élastique au comportement plastique; l'adaptation aux grandes baisses de la limite conventionnelle d'élasticité entraînées par les variations de la température locale; et une méthode de prévision et correction à trois étapes pour les analyses plastiques. Les caractéristiques ci-dessus réduisent le coût du calcul. Les comparaisons du code FEAST avec vingt solutions analytiques et avec des mesures expérimentales indiquent que les prévisions FEAST sont généralement justes à ± 5 %.

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INTRODUCTION

FEAST* is a general-purpose finite-element code for calculating static stresses, strains, and displacements in two-dimensional, non-linear systems. It has been used for several analyses of stresses in CANDU** fuel, including pellets, sheaths, endcaps, endplates and plenums. Reference 1 described some recent applications of FEAST in design/analyses of CANDU fuel. These included: hourglassing of UO_2 pellets; fatigue of Zircaloy sheaths due to power-cycling; elastic-plastic stress-concentrations at sheath/endcap welds; creep collapse of sheaths; cracking of plenums due to thermal stresses; and stresses in endplates due to gravity.

The present paper focusses on those theoretical aspects of the finite element method that are unique to FEAST. The paper first summarizes the features and capabilities of FEAST, then describes their theoretical details. The paper also compares FEAST predictions to some analytical solutions and to experimental measurements. Two illustrative examples are given.

There are several other stress-analysis codes (Refs. 2, 3), developed independently, that share common features with FEAST. The purpose of the present paper is not to make a detailed comparison of the features of these codes, but to report a portion of the Canadian modelling effort.

The incentive to develop FEAST came from intended applications to nuclear fuel. Therefore, the numerical scheme in FEAST was chosen to minimize computing time in applications involving the following: high temperatures (300-2000°C); extensive brittle cracking (up to half the volume in the mesh); significant plastic flow (local strain approaching 4%); high creep (e.g. 0.3% per hour); and load-histories lasting many years.

FEAST contains no correlations specific to nuclear fuel. Hence it can also be used to analyze other components like pressure tubes, calandria tubes, and even non-nuclear systems.

FEATURES

FEAST accounts for the effects of elastic, plastic, creep, and thermal strains and stresses. Cracking can also be simulated. Since the code is two-dimensional, either plane or axisymmetric calculations can be done. Because the finite element method (Ref. 4) is used, complex geometries, curved surfaces, and unusual boundary-conditions can be accomodated easily and accurately (Refs. 1, 5). Material properties like Young's modulus and plastic modulus can differ in different parts of the structure, depending on local temperature, on prior heat-treatment, and on local strain.

The user can specify arbitrary distributions of forces, pressures, and temperatures. As well, zero, non-zero, and/or limiting values of displacement can be imposed on parts of the body. These can all be specified as functions of time.

FEAST solves the classical equations of equilibrium (Ref. 6). This ensures that internal stresses balance each other and applied loads at each point. FEAST also ensures compatibility (Ref. 6) among neighbouring fibres. This means that the calculated strains do not create artificial holes, nor do they assign two different points of the material to occupy the same physical space. In addition, FEAST ensures that at each point of the structure, the stress/strain law is consistent with local conditions like temperature and plasticity. The von-Mises equation (Ref. 7) is used to determine if local stresses are elastic or plastic. For elastic elements, the constitutive relations are given by the multidimensional form of the Hooke's law (Ref. 6). For plastic elements, the von-Mises yield surface is combined with the Levy-Mises flow rule (Ref. 7). This ensures that the principal axes of increments in plastic strains, coincide with the principal axes of total stresses. In this paper, plastic strains refer to instantaneous permanent strains. Permanent strains that develop with time, are called creep. The formulations for creep (Ref. 8) are similar to those for plasticity and, in addition, allow for dependence on time.

A three-step predictor-corrector method (Ref. 9) contributes to large time-steps, thus to low computing cost, while maintaining good accuracy for linear and for non-linear problems. FEAST also contains a special formulation (Ref. 10) for elastic-plastic elements. This removes the usual requirement that the time-step go through the knee of the elastic-plastic curve.

For axisymmetric analysis, FEAST prints the following results:

- radial and axial displacements of nodes,
- radial, circumferential, axial, shear, and effective stresses of elements,
- yield strengths of elements,
- radial, circumferential, axial, shear, and effective strains of elements,
- principal stresses and their directions,
- minimum and maximum values of the above components of stresses and strains, and their locations.

Similar parameters are also printed for plane-stress calculations. In addition, FEAST saves the strains and stresses on a magnetic tape, for post-processing and for plotting of contours.

FRAMEWORK

The finite element method (Refs. 4, 5, 9) divides the analyzed region into a number of smaller, idealized, subregions called finite elements. The elements are connected to each other at the corner-points, called nodes. Figure 1 shows some triangular elements and nodes. Equations relating forces to displacements are formed for each finite element. These equations account for the shape, size, and location of each element, for

* FEAST - Finite Element Analyses for STresses

** CANDU - CANada Deuterium Uranium - is a registered trademark of Atomic Energy of Canada Limited the type of load (e.g. axisymmetric expansion), and for the physical processes (e.g. plasticity, creep). Then these equations are assembled to describe the entire system. The system equations are modified to account for boundary conditions. The equations are then solved to obtain displacements, strains, and stresses.

The following paragraphs describe the theory behind FEAST. 'Plane stress' calculations represent a special case of 'axisymmetric' calculations. Hence, the more general 'axisymmetric' option is used here to illustrate the principles. 'Plane stress' calculations use similar principles, and are not discussed in this paper for brevity. Tensor notation usually simplifies the description of the theory of the finite element method. However, the matrix notation (Ref. 4), though cumbersome, is more widely understood, hence is used in this paper. The symbols are defined towards the end of the paper in the section 'Nomenclature'. For axisymmetric structures, the matrices contain four rows in the following order: radial, circumferential, axial, and shear. Shear refers to the r-z plane.

In the displacement approach given by Zienkiewicz (Ref. 4), force-balance provides the following equation for a finite element:

$$\{dF\}^e = [K]^e \{d\delta\}^e + \{dF\}^e_{\epsilon o} + \{dF\}^e_{\sigma o} \quad (1)$$

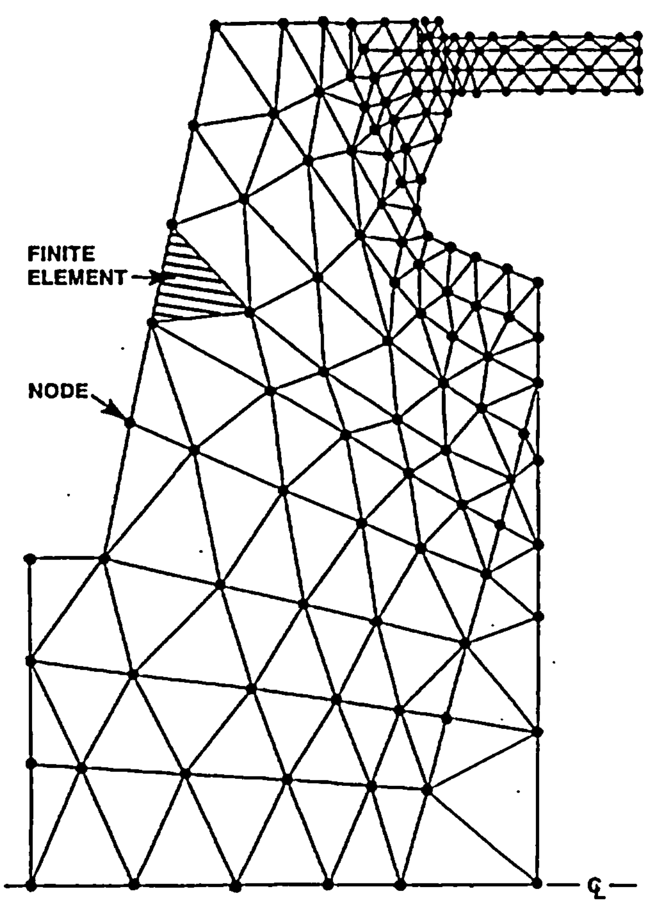

FIGURE 1 TRIANGULAR FINITE ELEMENTS IN A HYPOTHETICAL ENDCAP

By using the principle of virtual work, the following equations can be obtained (Ref. 4):

$$\{d\epsilon\}^e = [B] \{d\delta\}^e \quad (2)$$

$$\{d\sigma\}^e = [D] \{d\epsilon\}^e \quad (3)$$

$$[K]^e = \iiint_V [B]^T [D] [B] \, dV \quad (4)$$

$$\{dF\}^e_{\epsilon o} = - \iiint_V [B]^T [D] \{d\epsilon_o\} \, dV \quad (5)$$

$$\{dF\}^e_{\sigma o} = \iiint_V [B]^T \{d\sigma_o\} \, dV \quad (6)$$

Zienkienics (Ref. 4) gives the explicit equations for [B]. The equations for the slope matrix ([D]), and for the force vector {dF}, depend on the physical process considered, e.g. elasticity, plasticity, creep, cracking. Their derivations are discussed later in this paper.

By repeated application of equation 4, the stiffnesses ($[K]^e$) of all finite elements are obtained. They are then assembled into a global stiffness matrix, which describes the stiffness of the entire system.

This process gives a set of simultaneous linear equations relating known external loads ({dR}), to unknown nodal displacements ({dδ}), via the known global stiffness ([K]), as follows:

$$\{dR\} = [K] \{d\delta\} + \{dF\}_{\epsilon o} + \{dF\}_{\sigma o} \quad (7)$$

Solution of equation 7 provides displacements, strains, and stresses.

The above equations are equally valid for linear and for nonlinear problems. Plasticity and creep make the problem nonlinear. Hence FEAST solves equation 7 incrementally. The total load is divided into a series of smaller loads. The total values of displacements, strains, and stresses are the sums of previous total plus the current increment.

Each increment of load is kept reasonably large by using a three-step predictor-corrector method (Ref. 9). It uses three iterations per load-step (also called time-step). Figure 2 shows this schematically. Point 0 represents the solution at the end of the previous time-step. To discuss Figure 2, let us assume that the temperature is higher during the current time-step, giving additional thermal strain and lower yield strength. Points 1, 2 and 3 represent the solutions during the current time-step, after iterations 1, 2, and 3 respectively.

The first iteration accounts for the drop in yield strength due to increase in local temperature. It also corrects for residual errors from the previous solution.

The second iteration applies half the load-increment. This provides the average stiffness of the system during the time-step.

The final iteration applies the full increment in load. It uses the average stiffness calculated in second iteration, and thus provides a reasonable calculation of final displacements and stresses for the time-step.

To solve equation 7, we now need to derive the slope matrices, $[D]$, and the force vectors, $\{dF\}$, for the modelled processes: elasticity, thermal strain, cracking, plasticity and creep. These are discussed in the following five sections. Table 1 compiles the resulting equations for the slope matrices ($[D]$).

ELASTIC AND THERMAL STRAINS

If the stresses are in the elastic range, FEAST uses the equations given by Zienkiewics (Ref. 4) for slope $[D]$, see Table 1. Similarly, thermal expansion is treated as an initial strain. This is also calculated by the equations given by Zienkiewicz (Ref. 4):

$$\{d\epsilon_o\}^t = \alpha\ (d\omega)\ \{1,\ 1,\ 1,\ 0\}^T \qquad (8)$$

Equation 8 is used in equation 5.

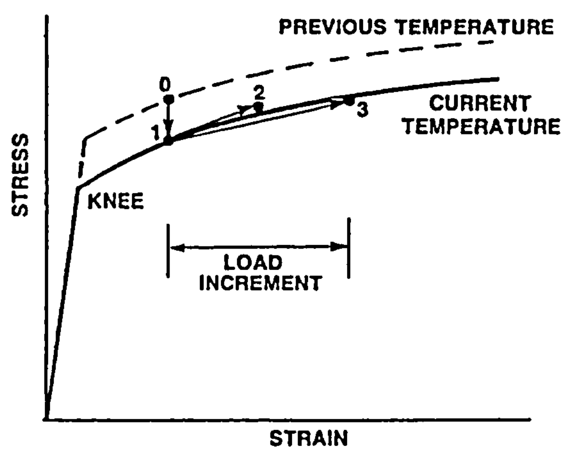

FIGURE 2 THE THREE ITERATIONS FOR THE PLASTIC SOLUTION

CRACKING

FEAST permits 'radial' cracks to develop in the system, see Figure 3, if the hoop stress exceeds the fracture stress. 'Plane stress' calcualtions are done for cracked finite elements. The $[D]$ matrix for cracked elements is modified to give zero hoop stress, see Table 1. In addition, one must account for the redistribution of the hoop stress that was present in the element prior to the formation of the crack. The correction takes the form of an initial stress. The resulting incremental forces are given by (Ref. 11):

$$\{dF\}^e_{\sigma_o} = \iiint_V [B]^T\ \{0,\ -\sigma_\theta,\ 0,\ 0\}^T\ dV \qquad (9)$$

These nodal forces are used in equation 6.

PLASTICITY

For plasticity calculations, FEAST considers permanent strains that are instantaneous, independent of strain-rate, and independent of thermal activation.

FEAST accounts for the following six features (Ref. 7) of incompressible plastic flow in ductile materials:

1. The components of stresses combine in such a manner that the effective stress lies on the yield surface;

2. The size of the yield surface depends on local temperature;

3. Due to work-hardening and/or strain-softening, the slope of the stress-strain curve is a function of accumulated strain;

4. The principal axes for increments in plastic strain coincide with the principal axes for total stresses;

5. Plastic flow does not change the volume of the material; and

6. Work done is positive during plastic flow, i.e., plastic strains are not recovered by removing loads.

The above features require changes in the slope matrix $[D]$ of equation 3, and in the initial load vector $\{dF\}^e_{\sigma_o}$ of equation 6. This section describes the pertinent equations used in FEAST for $[D]$ and $\{dF\}^e_{\sigma_o}$ of plastic material.

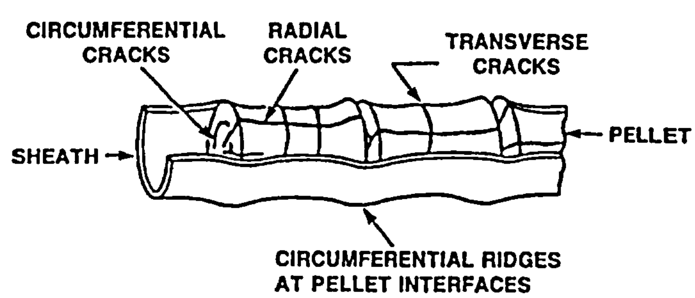

FIGURE 3 EXAMPLES OF POSSIBLE CRACKS IN A NUCLEAR FUEL ELEMENT

The incremental theory of plasticity (Ref. 7) shows good agreement with experiments. In the incremental theory, the above physical principles are expressed by the following three equations:

Total plastic strain:

$$\{d\epsilon\} = \{d\epsilon\}^{elas} + \{d\epsilon\}^p + \{d\epsilon\}^t \qquad (10)$$

Levy-Mises flow rule (Ref. 7):

$$d\lambda = \frac{(d\epsilon_r)^p}{\sigma_r'} = \frac{(d\epsilon_\theta)^p}{\sigma_\theta'} = \frac{(d\epsilon_z)^p}{\sigma_z'} = \frac{(d\gamma)^p}{2\tau} \qquad (11)$$

von-Mises yield function (Ref. 7):

$$2\bar{\sigma}^2 = (\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + 6\ \tau^2 \qquad (12)$$

It is convenient to express the yield function (equation 12) in its incremental form:

$$\frac{2}{3}\ \bar{\sigma}\ (d\bar{\sigma}) = \sigma_r'\ (d\sigma_r) + \sigma_\theta'\ (d\sigma_\theta) + \sigma_z'\ (d\sigma_z) + 2\tau\ (d\tau) \qquad (13)$$

TABLE 1

SUMMARY OF THE DIFFERENT [D] MATRICES CORRESPONDING

TO THE VARIOUS MATERIAL BEHAVIOUR THAT AN ELEMENT MAY EXHIBIT

	AXISYMMETRIC	PLANE STRESS, OR RADIALLY CRACKED
Elastic behaviour	$[D]^e = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$	$[D]^{e,ck} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & 0 & \nu & 0 \\ 0 & 0 & 0 & 0 \\ \nu & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$
Instantaneous plastic flow behaviour	$[D]^p = [D]^e - \frac{3}{2m\bar{\sigma}^2}\left(\frac{E}{1+\nu}\right)\{\sigma'\}\{\sigma'\}^T$, where $\bar{\sigma}$ = effective stress, σ' = deviatoric stresses, $m = 1 + \frac{2}{3}\left(\frac{1+\nu}{E}\right)E^P$	$[D]^{p,ck} = [D]^{e,ck} - S\{S'\}\{S'\}^T$, where $S = \sigma_r' S_1 + \sigma_z' S_3 + 2\tau S_4 + \frac{4}{9}\bar{\sigma}^2 E^P$; $\{S'\}^T = \frac{S_1}{S}, 0, \frac{S_3}{S}, \frac{S_4}{S}$; $S_1 = \frac{E}{1-\nu^2}(\sigma_r' + \nu\sigma_z')$; $S_3 = \frac{E}{1-\nu^2}(\sigma_r' + \nu\sigma_z')$; $S_4 = \frac{E\tau}{1+\nu}$
Time-dependent creep behaviour	$[D]^{creep} = \frac{1}{f} \begin{bmatrix} a^2-b^2 & b(b-a) & b(b-a) & 0 \\ b(b-a) & a^2-b^2 & b(b-a) & 0 \\ b(b-a) & b(b-a) & a^2-b^2 & 0 \\ 0 & 0 & 0 & \frac{f}{c} \end{bmatrix}$	$[D]^{creep,ck} = \frac{1}{(b^2-a^2)} \begin{bmatrix} -a & 0 & b & 0 \\ 0 & 0 & 0 & 0 \\ b & 0 & -a & 0 \\ 0 & 0 & 0 & \frac{b^2-a^2}{c} \end{bmatrix}$
	where $a = \frac{1}{E} + \frac{(d\bar{\varepsilon}^c)}{2\bar{\sigma}}$; $b = \frac{\nu}{E} - \frac{(d\bar{\varepsilon}^c)}{4\bar{\sigma}}$; $c = \frac{2(1+\nu)}{E} + \frac{3(d\bar{\varepsilon}^c)}{2\bar{\sigma}}$; $f = a(a^2-b^2) + 2b^2(b-a)$	
Elastic and plastic behaviour	$[D]^{ep} = W[D]^e + (1-W)[D]^p$, where W is an estimate of the fraction of stress increment during which the element behaves elastically.	$[D]^{ep,ck} = W[D]^{e,ck} + (1-W)[D]^{p,ck}$, where W is an estimate of the fraction of stress increment during which the element behaves elastically.

Equations 10, 11, and 13 provide three linear equations in three unknowns: $\{d\epsilon\}^P$, $d\lambda$, and $d\bar{\sigma}$. By following the procedure given by Yamada (Ref. 12), the three unknowns are eliminated by simultaneously solving the three equations. Then, equation 3 is used to express the elastic strains in terms of stresses. This yields the following expression:

$$\{d\sigma\} = [D]^P\{d\epsilon\} + \{d\sigma_o\} - \frac{\alpha E\,(d\omega)}{1-2\nu}\{1, 1, 1, 0\}^T \tag{14}$$

Equation 14 provides an explicit relation between stresses and strains. Compared with some other formulations (Ref. 13) that require inversion of matrices, equation 14 results in low computing time.

The slope matrix, $[D]^P$, is given in Table 1. The last term in equation 14 represents thermal stresses, and is similar to equation 8.

The vector $\{d\sigma_o\}$ simulates the reduction in flow stress due to an increase in local temperature. This feature was especially formulated for FEAST, and aids in large time-steps. For axisymmetric solids, $\{d\sigma_o\}$ is given by:

$$\{d\sigma_o\} = \frac{d\bar{\sigma}^t}{m\bar{\sigma}}\{\sigma'\} \tag{15}$$

$$\text{where } m = 1 + \frac{2}{3}\,\frac{(1+\nu)E^P}{E} \tag{16}$$

For radially cracked elements, $\{d\sigma_o\}$ is given by:

$$\{d\sigma_o\} = \frac{2}{3}\,\bar{\sigma}\,d\bar{\sigma}^t\,\{S'\} \tag{17}$$

where:

$$\{S'\}^T = \{\frac{S_1}{S}, 0, \frac{S_2}{S}, \frac{S_4}{S}\}$$

$$S = \frac{4}{9}\bar{\sigma}^2 E^P + \sigma_r' S_1 + \sigma_z' S_3 + 2\tau S_4$$

$$S_1 = \frac{E}{1-\nu^2}(\sigma_r' + \nu\,\sigma_z')$$

$$S_3 = \frac{E}{1-\nu^2}(\nu\,\sigma_r' + \sigma_z')$$

$$S_4 = \frac{E}{1+\nu}\,\tau$$

For plastic materials, $[D]^P$ is used instead of $[D]$ in equation 3. The $\{d\sigma_o\}$ term of equation 6 is obtained from equations 15 and 17.

ELASTIC AND PLASTIC BEHAVIOUR

Many non-linear codes for stress analysis require that all finite elements that start out elastic at the beginning of a time-step, must also stay elastic during that entire time-step. This restraint simplifies the calculation of slope matrices ([D]) for individual finite elements. But it also means that each finite element must go through the knee of the stress-strain curve, see Figure 2. For applications typical of nuclear fuel, this frequently results in a large number of very small time-steps, thus large computing cost.

A special formulation in FEAST avoids this cost. Large time-steps are used. In each increment, many elements are allowed to exhibit different combinations of elastic and plastic behaviour. Weighted stress-strain relations are used to calculate the slope matrices ([D]) of finite elements that go from elastic to plastic state during a time-step. The resulting slope matrix is given in Table 1. It is important to accurately calculate the weighting function W. FEAST uses the method described briefly in Reference 10 and in more detail in Reference 14. The method consists of first defining the load-path in the six-dimensional stress-space. Then, the intersection is found between the load-path and the yield-surface. For the von-Mises yield function, this results in a quadratic equation in the six-dimensional space. Its solution is obtained by the normal methods of algebra. Further details are available from References 10 and 14. This is a major feature of FEAST and permits large calculation-steps.

CREEP

For creep calculations, FEAST considers permanent strains that develop with time. The model incorporates the following physical features (Ref. 8): (i) creep does not change the volume of the material; (ii) the principal axes for increments in creep strains coincide with the principal axes for total stresses; and (iii) positive work is done by external forces. These features are similar to those for plasticity, and their mathematical description (Ref. 8) is provided by equations 10, 11, and 13.

The creep rate of the material, $\mathring{\epsilon}^c$, is assumed to be known as a function of local stress, temperature and strain. That is,

$$\mathring{\bar{\epsilon}}^c = \mathring{\bar{\epsilon}}^c\,(\bar{\sigma},\ T,\ \bar{\epsilon}) \qquad (18)$$

Above equation is combined with equations 10, 11 and 13. Then, mathematical manipulations are done similar to plasticity, and an equation similar to equation 14 is obtained. The resulting slope matrix ([D]) for creep is given in Table 1.

BOUNDARY CONDITIONS AND SOLUTION PROCEDURE

We now have all the ingredients for equations relating nodal forces to nodal displacements of the entire system (equation 7). They are assembled using the standard procedure (Ref. 4) of the finite element method. Fresh equations are assembled for each time-step, and for each iteration. The appropriate terms in the stiffness matrix of the system are modified to reflect the boundary conditions, as suggested by Zienkiewics (Ref. 4). This method requires less computing time than the method of altering the size of the stiffness matrix.

Equation 7 is solved by using the method of Gaussian elimination and back-substitution. This provides nodal displacements. Equation 2 then gives strains, and equation 3 gives stresses.

ACCURACY

The predictions of FEAST have been compared against closed-form solutions for about 20 cases. They include combinations of plane-stress, axisymmetry, elasticity, thermal stresses, plasticity, drop in yield strength, and creep. The agreement between FEAST and closed-form solutions is usually within ± 5%.

Reference 1 reported the excellent agreement of FEAST predictions with closed-form solution for creep stresses in a long, thick, closed, internally pressurized cylinder. This paper reports three more comparisons: (i) concentrations of elastic stresses near a circular hole in a rectangular plate subjected to tension; (ii) elastic-plastic stresses in the same plate; and (iii) elastic-plastic stresses in a long, thick, internally pressurized cylinder.

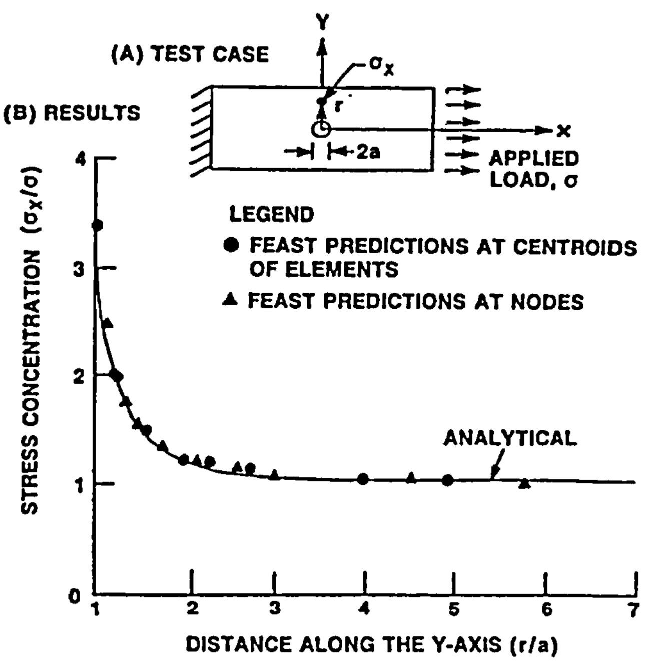

FIGURE 4 ELASTIC STRESSES IN A PLATE : FEAST VS. ANALYTICAL SOLUTION

Case 1: Elastic Stresses in a Plate

Figure 4(a) shows the rectangular plate simulated on FEAST. The plate has a small hole at its center. Uniform uniaxial tension σ is applied in the x direction. This results in non-uniform stresses near the hole (Ref. 15). The stress-concentration is defined as the local value of normal stress in the x direction (σ_x), divided by the applied tension (σ). Figure 4(b) shows the predictions of FEAST for stress concentrations at different points along the y-axis. There is adequate agreement between predictions of FEAST and of closed-form solution (Ref. 15). The agreement can be improved further, if needed, by refining the mesh.

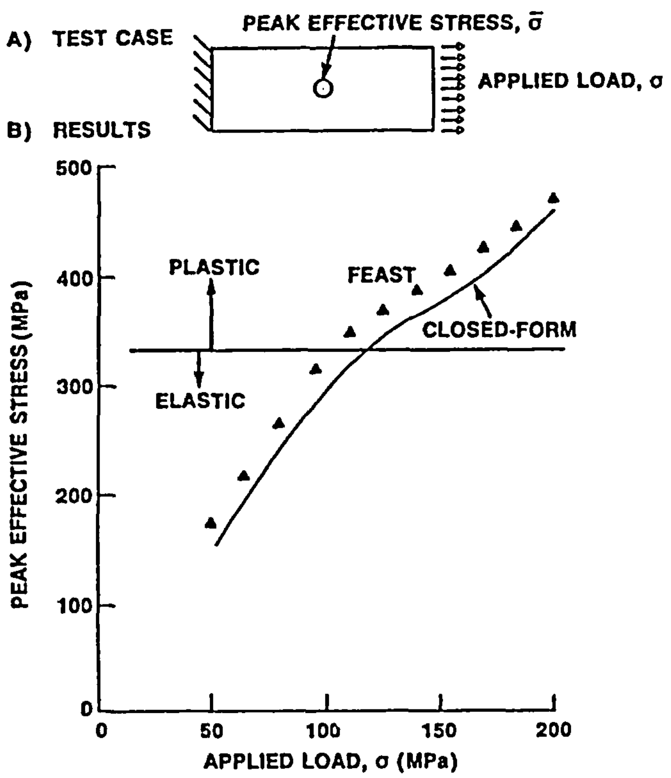

FIGURE 5 PEAK STRESS IN A PLATE FOR ELASTIC-PLASTIC LOAD: FEAST VS. ANALYTICAL SOLUTION

Case 2: Elastic-Plastic Stresses in a Plate

In the problem of Figure 4(a), the peak value of effective (von-Mises) stress occurs at point x=o, y=a. Hence, plastic flow occurs first at that point. We increased the tension applied on the plate, until the peak effective stress was well in the plastic range. Figure 5 shows predictions of FEAST for the peak effective stress, as a function of applied external load, and compares them to the closed-form solution of Tuba (Ref. 16). Plastic flow redistributes stresses, and the agreement between Tuba's solution and FEAST improves as the load increases.

Case 3: Elastic-Plastic Stresses in a Cylinder

Case 3 studied the central region of a long, thick, internally pressurized, axially loaded, cylinder. Axial strain was not allowed, resulting in conditions of plane strain. Figure 6 shows the test case, and the results. Finite element predictions are in reasonable agreement with closed-form solutions (Ref. 17).

VALIDATION

A comparison is available against strain-gauge measurements for compression tests on endcaps of fuel elements. Figure 7 shows that the predictions of the finite element code agree with measured gradients for hoop strains, to within ± 5%.

ILLUSTRATIVE EXAMPLES

FEAST has been used for the following applications in CANDU fuel:

- Estimate the hourglassing of UO_2 pellets. These calculations are related to stress-corrosion-cracking of sheaths.
- Determine stresses in Zircaloy sheaths at circumferential ridges. This is related to assessing the integrity of sheath during fatigue due to power-cycling in a CANDU-600 reactor.
- Calculate stresses at welds between sheaths and endcaps. This study was related to fuel failures (Ref. 18) in Unit 3 of the Bruce reactor, in 1984. The suspected cause of the failures was delayed-hydrogen-cracking (Ref. 18).
- Assess the influence of the location of discontinuity in sheath/endcap weld, on the load-carrying capacity of the bond.
- Check the stability of sheath during creep collapse due to coolant pressure, on pellets of small diameter.
- Assess thermal stresses in graphite plenums, to explain the observed cracking at corners.
- Estimate stresses in endcaps, to check if internal shape can be improved without compromising integrity of fuel in the Pickering reactor.
- Calculate stresses in endplate due to gravity loads on fuel elements, in order to estimate sag of endplates due to creep at high temperatures.

Reference 1 discusses the first five studies above. This paper gives a brief discussion of the last two analyses.

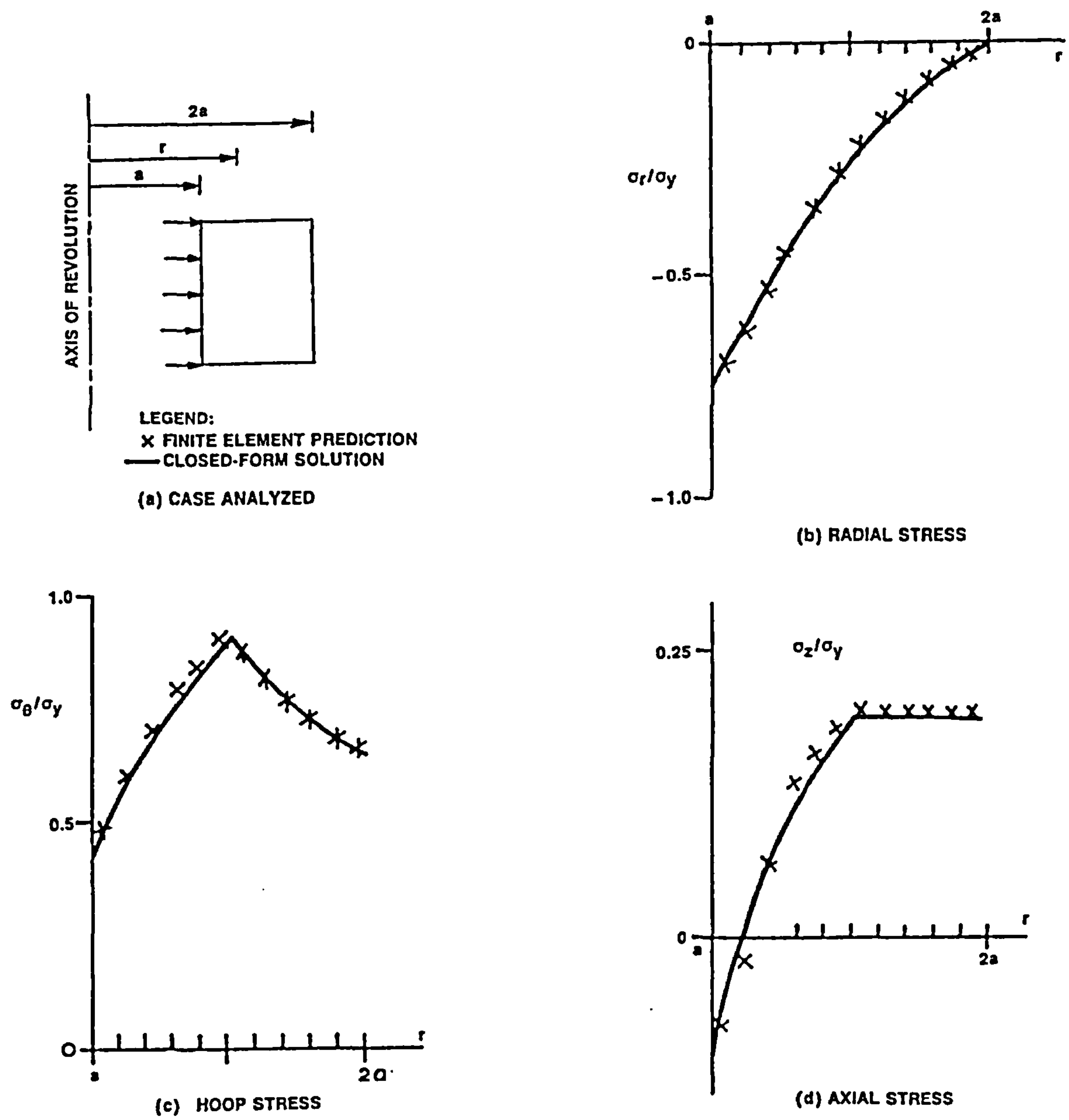

FIGURE 6 ELASTIC-PLASTIC STRESSES IN A CYLINDER: FINITE ELEMENT VS. CLOSED-FORM SOLUTION

Example 1: Endcap Optimization

This axisymmetric analysis investigated the possibility of structural optimization of the internal design of the endcap. It considered the resistance of the endcap against ductile failure due to concentric axial loads during refuelling. Figure 8 shows the effective (von-Mises) stresses in two hypothetical endcaps of Pickering-size fuel. For reasons of commercial proprietary, the geometries shown in the figure do not represent real endcaps; they are used here to illustrate the capabilities of FEAST. Compared to geometry #1, geometry #2 gives a more uniform distribution of effective stress. Geometry #2 also uses less Zircaloy and provides more volume for storing fission gas.

Example 2: Endplate Stresses

This study assessed the stresses in the endplate of a Pickering fuel burdle. The endplate was assumed to carry the gravity loads of the bundle (elements plus endplates). Plane-stress conditions were assumed, ignoring those bending moments which are not in the plane of the endplate. Figure 9 shows the endplate, and the locations of the fuel elements. As expected, the bottom half of the endplate has the largest stresses. Stress concentrations are highest at the corners at the two ends of the bottom spoke. This is consistent with the expected perturbations in stress-flow-lines, at re-entrant corners (Ref. 6). The maximum effective stress is 27 MPa.

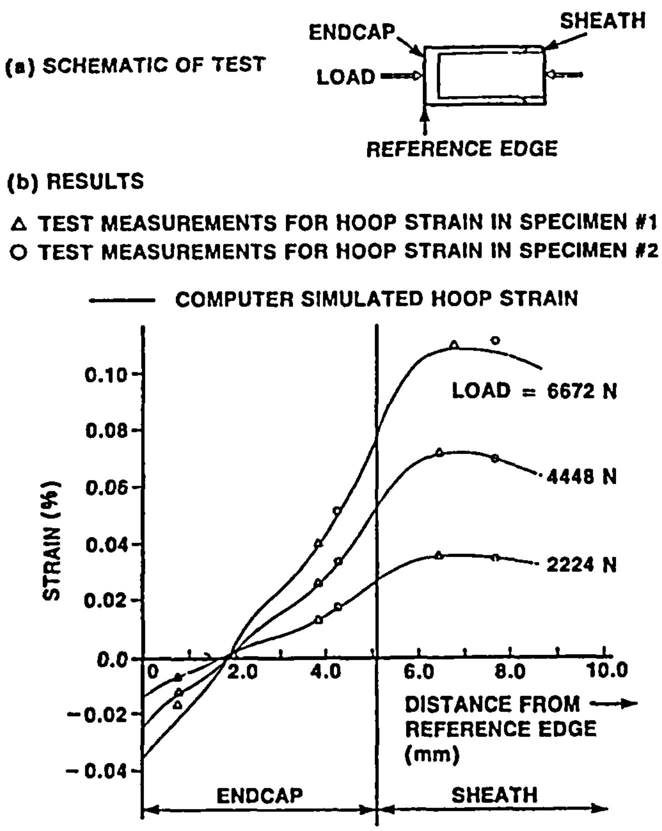

FIGURE 7 HOOP STRAIN IN SHEATH AND IN ENDCAP: FINITE ELEMENT VS. MEASUREMENTS

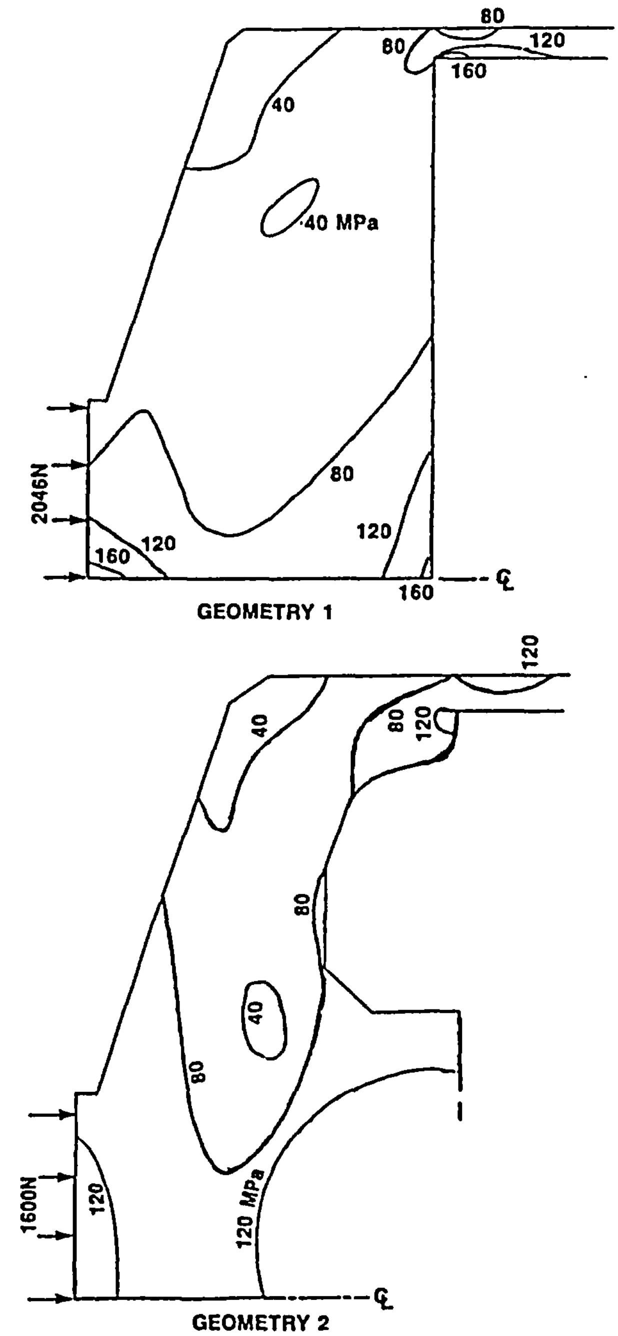

FIGURE 8 CONTOURS OF EFFECTIVE STRESS IN TWO HYPOTHETICAL ENDCAPS

SUMMARY AND CONCLUSIONS

1. A finite element code, FEAST, is available for calculating stresses in non-linear, two-dimensional systems, i.e. for plane-stress or for axisymmetric conditions.

2. It accounts for the following processes: elasticity; thermal stresses; cracking; plasticity; and creep.

3. The mathematical formulations are based on fundamental principles of mechanics: equilibrium; compatibility; Hooke's law; von-Mises yield surface; and Levy-Mises flow rule.

4. FEAST contains the following major features to minimize computing-cost: three-step predictor-corrector scheme for integration; variable-stiffness method to calculate the slope of the stress-strain curve; direct formulation for the slope-matrices of plastic elements; explicit equations for correcting residual errors; and a rigorous estimate of the effective stiffness of elements crossing the elastic/plastic boundary during a time-step.

5. Predictions of FEAST show reasonable agreement with analytical solutions and with experimental data.

6. FEAST has been used for several analyses of stresses in CANDU fuel, including pellets, sheaths, endcaps, sheath/endcap welds, plenums, and endcaps. It can also be used for applications other than CANDU fuel, e.g., pressure tubes and calandria tubes.

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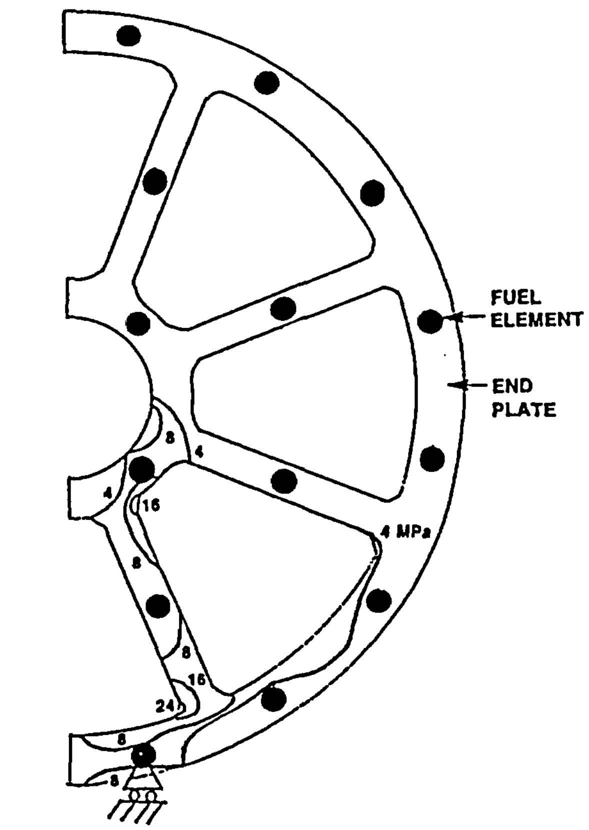

FIGURE 9 CONTOURS OF EFFECTIVE STRESS (MPa) IN AN ENDPLATE, DUE TO IN-PLANE GRAVITY LOADS

NOMENCLATURE

Subscripts

o	:	initial value
r,θ,z	:	polar coordinates
x,y,z	:	cartesian coordinates
ϵ_o	:	initial strain
σ_o	:	initial stress
y	:	yield strength

Superscripts

c	:	creep
ck	:	crack
e	:	element
$elas$	:	elastic
p	:	plastic
t	:	thermal
T	:	transpose
$'$	:	deviatoric
$-$	:	effective

Symbols

$\{\ \}$ represents a column vector, in the following order: radial, circumferential, axial, shear

$[\ ]$ represents a rectangular matrix

$[B]$	:	matrix relating strains to displacements
d	:	infinitesimal increment
$[D]$	:	slope relating strain-matrix to stress-matrix
E	:	Young's modulus
$\{F\}$	:	Force vector
$[K]$	:	Stiffness matrix
m	:	Factor defined by equation 16
$\{R\}$	:	Vector of external forces at nodes
V	:	Volume of the finite element
W	:	Weighting Factor
α	:	Coefficient of linear thermal expansion
$\{\delta\}$	:	Vector of displacements at nodes
$\{\epsilon\}$	:	Strain vector
$\dot{\epsilon}^c$	:	Creep strain rate
$d\lambda$	:	Constant of proportionality in the flow rule
ν	:	Poisson's ratio
$\{\sigma\}$	:	Stress vector
$d\bar{\sigma}^t$	:	Change in yield strength due to increase in local temperature
τ	:	Shear stress
ω	:	Temperature

REFERENCES

1. M. Tayal, D. Lim, "Recent Uses of the Finite Element Method in Design/Analysis of CANDU Fuel", Sixth Annual Conference of the Canadian Nuclear Society, Ottawa, Canada, 1985 June 2-4, pages 1-1 to 1-10. Also, Atomic Energy of Canada Limited, Report AECL-8754.

2. H.D. Hibbitt, 'MARC Series of Programs', Published in: 'Thermal Structural Analysis Programs - A Survey and Evaluation', American Society of Mechanical Engineers, New York, 1972, pages 17-28.

3. H.D. Hibbitt, "ABAQUS/EPGEN - A General Purpose Finite Element Code with Emphasis on Non-Linear Applications", Nuclear Engineering and Design, Volume 77, Number 3, February 1984, Pages 271-297.

4. O.C. Zienkiewicz, "The Finite Element Method in Structural and Continuum Mechanics", McGraw-Hill Book Company (UK) Limited, London, UK, 1967.

5. K.H. Huebner, "The Finite Element Method for Engineers", John Wiley and Sons, New York, 1975.

6. S.P. Timoshenko, J.N. Goodier, "Theory of Elasticity", Third Edition, McGraw-Hill Book Company, New York, 1970.

7. R. Hill, "The Mathematical Theory of Plasticity", Oxford Clarendon Press, UK, 1950.

8. H. Kraus, "Creep Analysis", John Wiley and Sons, New York, 1980.

9. C.S. Desai, J.F. Abel, "Introduction to the Finite Element Method", Van Nostrand Reinhold Company, New York, 1972.

10. M. Tayal, D. Budney, "Load Increment Ratios in the Elastic-Plastic Analysis of Anisotropic Materials by the Variable-Stiffness Finite-Element Method", Journal of Strain Analysis, Volume 11, Number 3, 1976, pages 150-153.

11. O.C. Zienkiewics, S. Valliappan, I.P. King, "Stress Analysis of Rock as a 'No Tension' Material", Geotechnique, Volume 18, 1968, pages 56-66.

12. Y. Yamada, "Recent Developments in Matrix Displacement Method for Elastic-Plastic Problems in Japan", Published in "Recent Advances in Matrix Methods of Structural Analysis and Design", University of Alabama Press, 1971.

13. P.V. Marcal, I.P. King, "Elastic Plastic Analysis of Two-Dimensional Stress Systems by the Finite Element Method", International Journal for Mechanical Science, Volume 9, 1967, pages 142-155.

14. M. Tayal, "A Finite Element Elastic-Plastic Analysis of Orthotropic Bodies Employing Hill's Yield Criterion", M.Sc. Thesis, University of Saskatchewan, Saskatoon, Canada, 1972.

15. R.E. Peterson, "Stress Concentration Design Factors", John Wiley and Sons, 1953.

16. I.A. Tuba, "An Analytic Method for Elastic-Plastic Solutions", Journal for Solid Structures, Volume 3, 1967, pages 543-564.

17. P.G. Hodge, G.N. White, "A Quantitative Comparison of Flow and Deformation Theories of Plasticity", Journal of Applied Mechanics, Volume 27, 1950, pages 180-184.

18. P.T. Truant, "CANDU Reactor Experience: Fuel Performance", Sixth Annual Conference of the Canadian Nuclear Society, Ottawa, Canada, 1985 June 2-4, pages 6.1 to 6.6.