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MODELLING THE BENDING/BOWING OF COMPOSITE BEAMS SUCH AS NUCLEAR FUEL: THE BOW CODE

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Arrays of tubes are used in many engineered structures, such as in nuclear fuel bundles and in steam generators. The tubes can bend (bow) due to in-service temperatures and loads. Assessments of bowing of nuclear fuel elements can help demonstrate the integrity of fuel and of surrounding components, as a function of operating conditions such as channel power.

The BOW code calculates the bending of composite beams such as fuel elements, due to gradients of temperature and due to hydraulic forces. The deflections and rotations are calculated in both lateral directions, for given conditions of temperatures. Wet and dry operation of the sheath can be simulated.

BOW accounts for the following physical phenomena: circumferential and axial variations in the temperatures of the sheath and of the pellet; cracking of pellets; grip and slip between the pellets and the sheath; hydraulic drag; restraints from endplates, from neighbouring elements, and from the pressure-tube; gravity; concentric or eccentric welds between endcaps and endplate; neutron flux gradients; and variations of material properties with temperature.

The code is based on fundamental principles of mechanics. The governing equations are solved numerically using the finite element method. Several comparisons with closed-form equations show that the solutions of BOW are accurate. BOW's predictions for initial in-reactor bow are also consistent with two post-irradiation measurements.

1. Introduction

Arrays of tubes and beams sometimes operate at high temperatures and loads, causing deflections, bending, creep, and stress-relaxation. The operating conditions, and deflections, may change with time.

Fig. 1 shows some complex arrays of tubes/beams used in the power industry, e.g. in steam generators and in nuclear fuel. Nuclear reactors impose unique loading conditions on core structures:

- New isotopes (and even materials) are produced by the fission reaction, and some isotopes decay. This changes the distribution of temperature.
- Varying temperatures change the material properties significantly.
- High temperatures can change the phase ($\alpha \rightarrow \beta$) of the material, resulting in a change of volume.
- Some structural materials embrittle due to irradiation (neutrons), and due to absorption/diffusion of hydrogen/deuterium leading to hydrides/deutrides.
- Some corrosion is caused by the coolant and by fission products like iodine/cesium.
- Small clearances among some neighbouring surfaces mean that potential contacts/slips need to be addressed.

Deformations and structural integrity of the critical components are usually determined by combining ex-

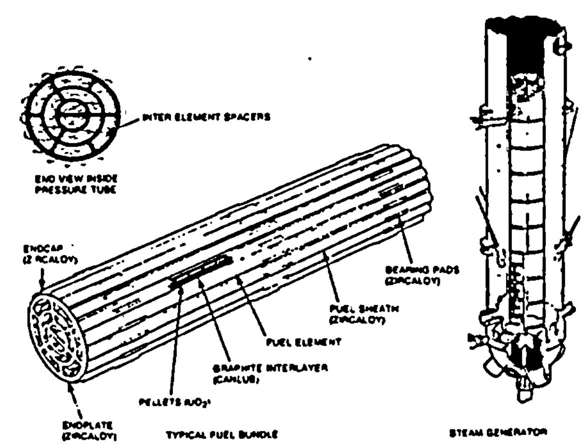

Fig. 1. Some arrays of tubes in the power industry.

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periments with design calculations. Commercial computer-codes for stress-analysis sometimes do not permit studying economically all the above features of nuclear reactors, hence new codes and models are being developed by the nuclear industry [1,2,3] *. This paper describes one such Canadian code, BOW. It calculates the bending of composite beams arranged in simple or in complex arrays, and subjected to high temperatures (possibility of phase change), to temperature-gradients, and to mechanical/hydraulic loads. To extend its applicability to nuclear fuel, BOW also allows for complex two-dimensional contacts/slips between neighbouring surfaces. Thus, BOW can be applied to bending (bowing) of nuclear fuel elements (fig. 1) if the fuel elements become partially dry during accidents, for example during a loss of regulation, or during a loss of pump, or during a loss of coolant. BOW can also be used to calculate the bending during normal operating conditions, i.e. without the drypatches.

This paper describes the features, theory, and accuracy of the BOW code. But first, a brief discussion is given of the specific reasons that lead to the bending of nuclear fuel, as well as of the evolution of bowing models in Canada.

2. Background

Bowing is defined as the lateral deflection of a fuel element from the axial centreline. Fig. 1 identifies the components of a typical fuel bundle.

During operation, coolant temperature varies among the subchannels between the elements. This causes variations in coolant temperature around the circumference of a fuel element (ΔT_1). Non-uniform heat transfer coefficient between the coolant and the sheath and radial gradients of neutron flux also cause temperature gradients, labelled ΔT_2 and ΔT_3 respectively.

Temperature gradients result in differential thermal expansions or strains. To accommodate them, the fuel element bows in the direction of the longer side. This deflection is magnified by the bending moment provided by hydraulic drag loads. Fig. 2 shows a schematic of the driving forces.

If the operating power of a fuel element is increased, drypatches will eventually form on the sheath. This would change the magnitude of temperature gradients (ΔT_2), affecting the amount of bow. Fig. 3 shows the bowed shape of a fuel element after an experiment in a

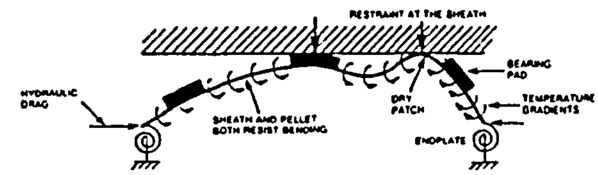

Fig. 2. Driving forces for bowing.

test reactor using superheated steam, during which the element experienced localized overheating.

The eventual objective of the model reported here is to predict the extent to which an element will bow for given temperature conditions, and hence to help establish limiting conditions for which element distortion is low enough that the thermohydraulic conditions in the fuel channel are acceptable. The aim is to develop a tool that can be used to assess fuel element bow under conditions of nominal system pressure, high element power, and mild dryout that lasts a few seconds. Due to the short duration of dryout, time-dependent processes like creep and irradiation-swelling are not expected to be significant and hence are not modelled in the code. The focus is on predicting the initial (before creep) deflections of the fuel element. This paper describes the current status of the code.

Fig. 4 shows the evolution of bowing models in Canada. A model was proposed by Veeder and Schankula [1] for bowing due to thermal effects. This was followed by a model by Gerrard and Clendening for bowing due to hydraulic effects. A numerical model, ELDEMO, is also available. All three models assume that the sheath alone resists the external bending moments, without assistance from the pellets. Tayal proposed a closed-form model combining thermal and hydraulic effects, and he also introduced the strengthening effect of fuel pellets as they affect the resistance to bowing.

All of the above models assume that fuel deflection is symmetric about the longitudinal midpoint of the element, called 'midlength' for brevity. Measurements using electrically heated tubes in a test reactor show [4]

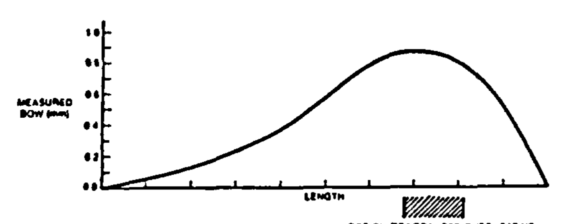

Fig. 3. Bow measured after irradiation.

* [ ] denotes references

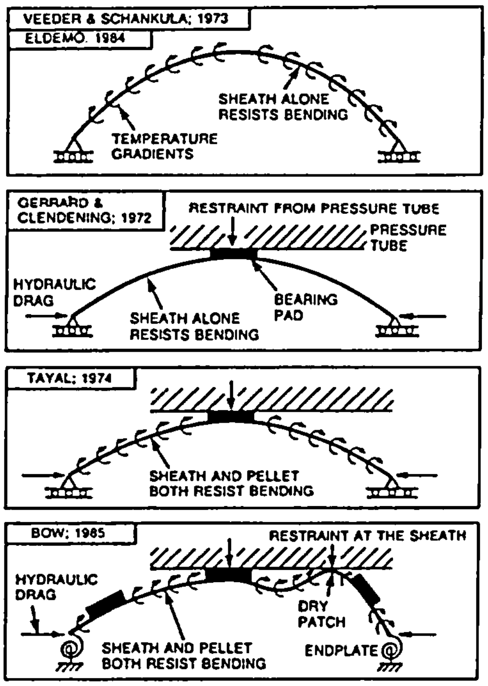

Fig. 4. Canadian models for bowing.

that during mild dryout, the temperatures can be asymmetric about the element midlength as well as around the element circumference, see fig. 5. This can lead to unsymmetric bowing with respect to the midlength, see for example fig. 3. Hence the previous models are not applicable to mild dryout and a new code, BOW, was developed.

Although the incentive to develop BOW came from intended applications to nuclear fuel, the code addresses the more general situation of bending of composite tubes/rods/beams, in nuclear and in non-nuclear environments. The highlights of the BOW code have been briefly described earlier [5]. The present paper provides a more detailed description of the features, theory, and accuracy of BOW.

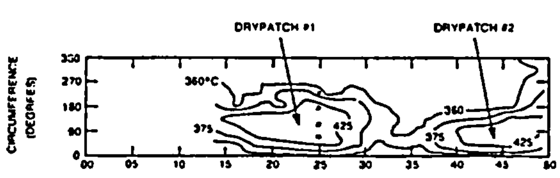

Fig. 5. Sheath temperatures during an experiment.

3. Features of BOW

- Similar to the model by Veeder and Schankula [1], BOW accounts for *circumferential* gradients in fuel temperature due to: incomplete mixing of the coolant; non-uniform heat transfer coefficient between the sheath and the coolant; and variation of neutron flux around the circumference of the element.
- BOW permits the temperature profile to be *axially* unsymmetric, thus the effects of localized drypatches can be studied.
- BOW includes the additional bending provided by hydraulic drag.
- BOW accounts for the effect of gravity (pellet weight).
- BOW accounts for the change in metallurgical phase due to increase in temperature.
- BOW solves for deflections in two lateral directions.
- *Within* the fuel element, relative slip and/or grip is allowed between the pellet and the sheath, as in [1].
- BOW permits the fuel element to be laterally restrained by neighbouring *external* surfaces like other fuel elements and pressure tubes. The restraints can be applied at many points, e.g. at bearing pads, at spacer pads, and at sheaths. The clearances are allowed to vary circumferentially and axially, see fig. 1. After contact, slip is permitted along the surface of contact.
- BOW accounts for restraints provided by the endplate.
- BOW accounts for the flexural rigidity of the pellets, which can help resist bowing.
- Cracking of the pellets is considered.
- BOW permits either concentric or eccentric welds between the endplate and the endcap.
- BOW accounts for variations in material properties as functions of local temperatures.
- The output of BOW includes lateral deflections and rotations in horizontal and vertical directions. The net displacement is also printed.

4. Solving the bending equation

The classical equation governing bending of beams is [6]:

$$\frac{1}{\rho} = \frac{M}{EI}. \qquad (1)$$

The symbols are explained in the section 'Nomenclature' at the end of the text.

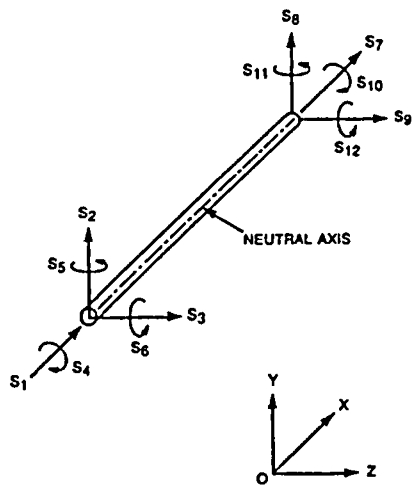

Fig. 6. Beam element.

BOW solves the above equation, for both lateral directions. The finite-element-method [7] is employed because of its high accuracy and low cost. Beam elements are used; fig. 6 shows the basic beam element [7] used in BOW. Each finite element represents a short length of the fuel element.

Each finite element is subdivided into a number of radial and circumferential segments. At each cross-section, the net moment due to temperature gradients and the net flexural rigidity (EI), are obtained by numerical integration over all the radial and circumferential segments of the section, using the following equations:

$$M_{T_y} = \int_A \alpha ETz \, dA, \quad \text{and}$$

$$M_{T_z} = \int_A \alpha ETy \, dA. \tag{2}$$

Simpson's rule [8] is used for the numerical integration.

In horizontal reactors such as CANDUs (*CAN*ada *D*euterium *U*ranium), pellet weight also contributes to the bowing/sag of the fuel element. This is accommodated in BOW by applying distributed loads on each finite element [7]. The magnitude of the load depends on the weight per unit length of the fuel element, and its distribution in the two lateral directions depends on the orientation of the fuel element. Thus fuel elements of arbitrary diameters and densities can be analyzed, as well as fuel elements in horizontal or in vertical reactors.

The net deflection due to temperature gradients and due to gravity provides a moment-arm to axial forces such as hydraulic drag. This causes additional bending, and is sometimes called the 'P-δ' effect. Although it is possible to build this effect into the stiffness matrix of each finite element, doing so gives off-diagonal terms [2] that significantly increase the bandwidth of the system stiffness matrix. This increases the computing cost in proportion to the square of the bandwidth. To keep the computing cost low, the effect of hydraulic drag is calculated iteratively in BOW.

The above procedure is sufficient if the hydraulic drag is concentric to the axial centerline of the fuel element. However, eccentric axial loads can be applied on the fuel element by latches and sidestops during loading and unloading of the fuel bundle. Moreover, eccentric welds between the endplate and the endcap can cause eccentric drag loads on the fuel element. In such cases, an additional bending moment is applied on the fuel element, equal to the axial load times the eccentricity.

The deflections in y and z directions are independent of each other: the stiffness matrix does not contain terms linking the two directions [7]. Hence BOW saves further on computing costs by decomposing the stiffness matrix and obtaining independent solutions in the two directions. The two displacements are then added, as vectors, to obtain the net displacement.

5. Neutron flux gradient

The following major materials are used in the fuel channel of a CANDU nuclear reactor: heavy water, Zircaloy, and UO_2. Each of these has a different cross-section for capturing thermal neutrons. Hence the neutron flux differs at different radial locations in the channel, see fig. 7 [9]. The neutron flux is the highest in the outer elements. The flux is usually the highest at the pellet surface that faces the pressure tube, and lowest near the axial centerline of the fuel element. In typical UO_2 pellets, the neutron flux can change by about 30% at different radial and circumferential locations within an element. This causes a circumferentially asymmetric distribution of temperature, and is a major driving force for bowing.

The effect of flux gradients is accommodated in BOW by following the approach of Veeder and Schankula [1]. They assumed that the neutron-flux distribution through the bundle is given by $I_0(\kappa R)$, where κ is the inverse diffusion length for thermal neutrons in the homogenized fuel channel (pellet, coolant, sheath). For this distribution of neutron flux, Veeder-Schankula

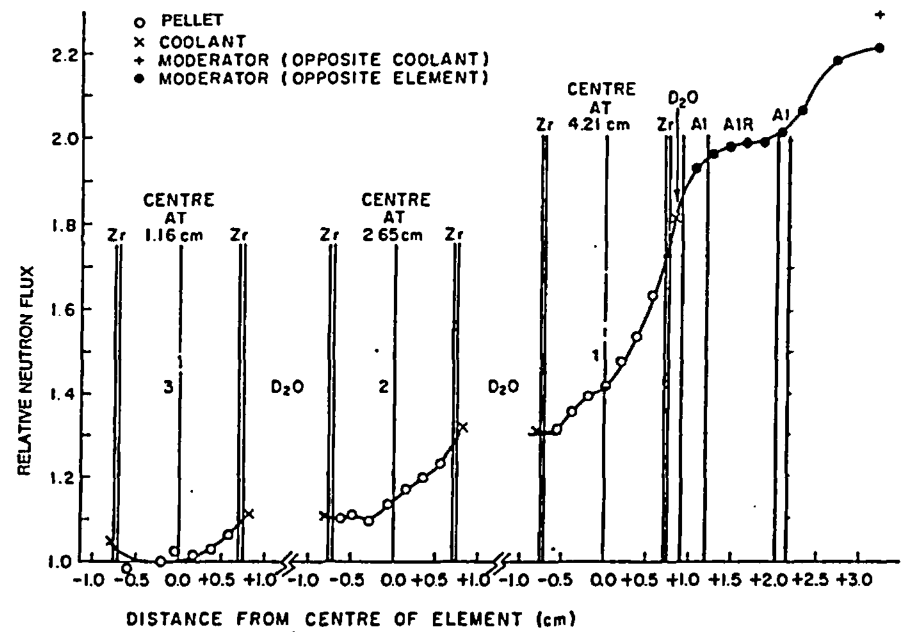

Fig. 7. Neutron flux profile.

obtained the following Flux Gradient Factor, D:

$$D = \frac{I_1(\kappa R_i) I_2(\kappa r_1)}{I_0(\kappa R_i) I_1(\kappa r_1)}. \tag{3}$$

Thus, the Flux Gradient Factor is a dimensionless number, and is a function of enrichment, of the diameter of the fuel bundle, and of the diameter of the fuel element.

The temperature-gradient due to the above D is calculated from the expressions given in ref. [1], and fed into eq. (2) to calculate the bending moments in the pellet. The resulting curvature of the pellet, however, is not transmitted fully to the sheath, as explained below.

If the fuel stack were one solid rod of UO_2, the sheath would likely assume the curvature of that rod. However, when the fuel stack consists of many individual pellets, neighbouring pellets are separated by interfaces across which tensile stresses cannot exist. Hence, as shown in fig. 8, it is possible for individual pellets to bend locally, i.e., without combining their curvatures. In this case, the net curvature of the sheath can be less than that of individual pellets. The 'Curvature Transfer Factor' describes the extent to which a pellet can impose its curvature on the sheath. This factor varies between zero and one [1], and BOW will calculate bending for input values of this factor.

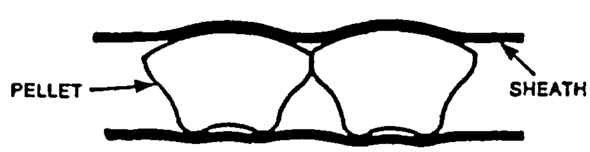

Fig. 8. Partial transfer of curvature from the pellet to the sheath.

The local curvature of the pellet due to the flux gradient is multiplied by the 'Curvature Transfer Factor' and by the local flexural rigidity of the fuel element, to yield the local bending moment on the fuel element due to the flux gradient.

6. Flexural rigidity of composite beams

The sheath bends in response to the temperature gradients and the bending moments discussed above. The bending of the sheath can be resisted by: flexural rigidity of the sheath; mechanical interference between the sheath and the pellets; resistance from the endplates; and lateral restraints from neighbouring surfaces. This section addresses the net flexural rigidity of the fuel element; the next two sections will discuss the influences of the endplates and of the neighbouring surfaces.

The degree to which the pellets help the sheath in resisting bending, is measured by the 'Rigidity En-

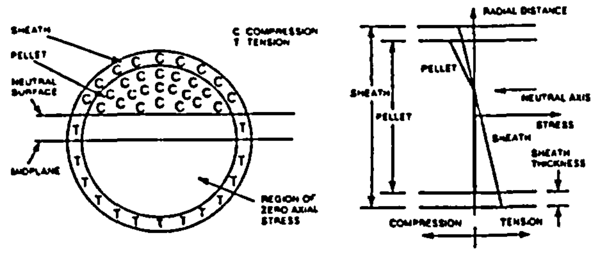

Fig. 9. Stress distribution in the cross section.

hancement Factor'. This factor is a function of: the diametral clearance; the coolant pressure; the gas pressure within the fuel element; and the interfacial pressure between the sheath and the pellets. The factor varies between zero and one, and is an input to the BOW code.

When the Rigidity Enhancement Factor is not zero (pellet helps resist bending), the flexural rigidity (EI) of the finite elements is calculated by modelling the fuel element as a composite beam consisting of the sheath and the pellets [10]. The pellets, which are made from ceramic UO_2, have a low strength against fracture in tension. Hence in calculating the local stiffness, only compressive stresses are considered in the pellet. To ensure equilibrium of force, this requires that the bending moment give a net tensile load in the sheath. This means that the neutral axis shifts away from the geometric centre of the element; see fig. 9. The location of the neutral axis, and the magnitudes of the stresses, are adjusted to satisfy simultaneously the geometric compatibility, the Hooke's law, the force equilibrium, and the magnitude of the applied bending-moment.

The above procedure results in the following transcendental equation [10] for the location of the neutral axis:

$$E_p\left[\tfrac{1}{3}\left(r_2^2-\eta^2\right)^{1/2}\left(2r_2^2+\eta^2\right)-\eta r_2^2\cos^{-1}\frac{\eta}{r_2}\right] - E_s\eta\pi\left(r_1^2-r_2^2\right)=0. \tag{4}$$

Similarly, the following equation can be obtained for the flexural rigidity of the element:

$$J=E_s\left[\frac{\pi}{4}\left(r_1^4-r_2^4\right)\right]+FE_p\left[\frac{r_2^4}{4}\cos^{-1}\frac{\eta}{r_2} + \frac{\eta}{12}\left(r_2^2-\eta^2\right)^{1/2}\left(2\eta^2-5r_2^2\right)\right]. \tag{5}$$

When J is evaluated in plane x–y, its value is equal to EI_y. Similarly, J in plane x–z is equal to EI_z.

When the Rigidity Enhancement Factor F is zero, the pellet does not contribute to the flexural rigidity of the element. The element then bends like an empty sheath. When F is one, the pellet participates fully in resisting external bending moments. From preliminary measurements of the natural frequency of CANDU fuel, an out-reactor value of 0.9 can be deduced for F in irradiated CANDU fuel. The flexural rigidity of fuel elements is higher in-reactor than out-reactor [11], indicating that F is close to 1 inside the reactor.

The composite beam of sheath and pellets resists deflections due to temperature gradients on the sheath and due to hydraulic drag load. Moments on the sheath from curved pellets are resisted by the sheath alone.

7. Resistance from endplates

Residual bow is frequently measured in irradiated fuel bundles. Fig. 10 shows the residual bows [12] in fuel elements after irradiation in a test reactor. In fig. 10, a 'trend line' has been drawn to connect the deflections of individual elements, even though in reality the

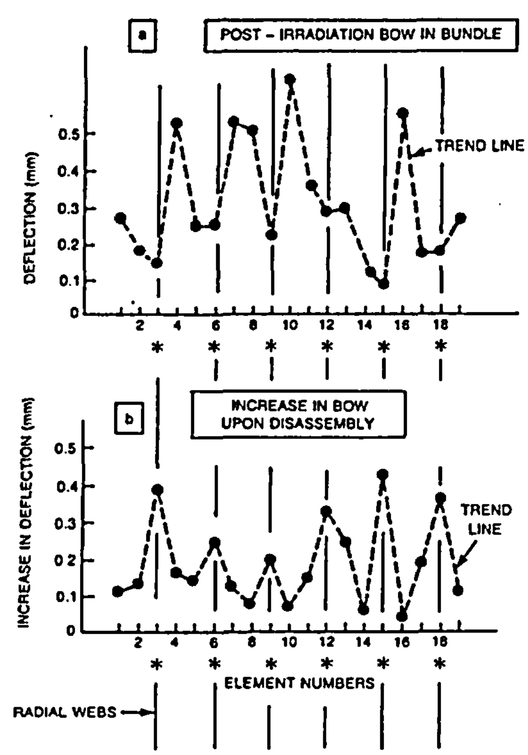

Fig. 10. Effect of endplate on bowing.

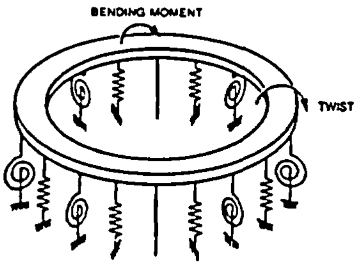

Fig. 11. Endplate model.

deflections are discrete. Fig. 10a shows the deflections when the elements were still in the form of a bundle. In bundle form, elements located at the radial webs of endplates showed significantly less bow than neighbouring elements located away from the radial webs. This indicates that the endplate webs restrain bowing. When the bundle was dissembled into individual elements, all elements increased their bow, see fig. 10b. Bows of elements on the radial webs increased more than those of their immediate neighbours. This also indicates that the endplate provides a significant restraint to bowing. The restraint is larger at the radial webs.

Bow accounts for endplate restraint by modelling endplates as torsional springs [10]. The spring-constant is obtained by modelling the endplate as a circular ring on elastic foundations; see fig. 11. The elastic foundation is provided by the other elements attached to the endplate, as well as by the elements of the neighbouring bundle. The elements behave like springs – both translational and torsional.

The overall approach is as follows: equations of equilibrium are developed for local forces and for local moments in an infinitesimal segment of a circular beam. They are combined with St. Venant's equations [13] relating local curvatures to local displacements and rotations, during bending of a naturally curved beam. The resulting equations are then combined with classical equations relating local moments to local curvatures. This gives two interdependent differential equations for two independent variables: lateral deflection and twist, as functions of angular location along the endplate. These two equations are integrated using Fourier series. From these, the spring-constants of the endplate are determined by differentiation. The mathematical details and the final equations are given in ref. [10]. They are in the forms of Fourier series.

This provides two torsional spring-constants for the endplate. One describes endplate restraint when the element deflects radial to the fuel bundle (K_r), the other for tangential deflection (K_t). For in-reactor conditions of a 37-element bundle, K_r and K_t are 100–200 Nm/rad respectively [10].

For the spring-constant in a direction other than the above two, BOW combines K_r and K_t by using established laws of transformations, see for example Sokolnikoff [14].

8. Contact with neighbours

The usual method of modelling gaps between neighbours is via "gap elements", see for example ref. [15]. Gap elements are versatile, and permit the modelling of contact in a variety of situations. The typical geometry of a CANDU fuel bundle involves many fuel elements, bearing pads, and spacer pads, see fig. 1. To model these all, it was judged that 10–20 gap elements would be needed to specify the circumferential variations in gaps. These would be required at each axial location where significant deflection, and/or contact, can occur. Combined with an attendant increase in bandwidth, the gap elements would increase computing cost significantly and likely would also lead to numerical instabilities. An alternative was therefore sought.

BOW stores in a matrix, the lateral clearances at each node for a number of circumferential directions. The clearances are updated after each calculation of lateral deflection. The calculation is repeated if an overlap is found at any node in any direction. BOW first finds the node with the maximum overlap. In the new calculation, the deflection of this node is set equal to the available clearance, to generate an appropriate restraining force. The process is repeated until overlaps are eliminated at all nodes.

The user can force an overlapping node to stay fixed at the point of contact. This is equivalent to the traditional gap elements. In addition, BOW also provides an option which permits the overlapping node to slip perpendicular to the direction of contact. This allows modelling contact between two neighbouring sheaths.

9. Material properties

BOW uses the following material properties for Zircaloy and for UO_2: Young's modulus; shear modulus; thermal conductivity; and coefficient of thermal expansion. The material properties are obtained from the MATPRO data-base [16], and are functions of the local temperature.

Fig. 12 shows the length-change in Zircaloy due to increase in temperature [16]. Note the decrease in length

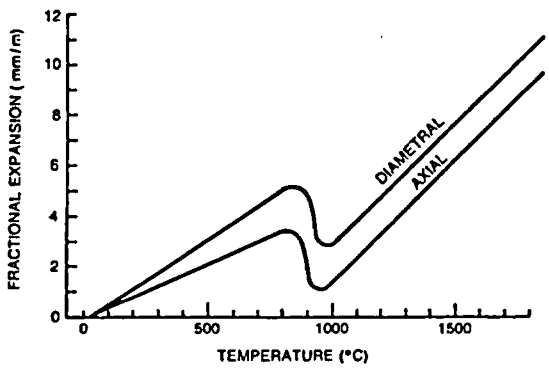

Fig. 12. Thermal expansion of Zircaloy tubing.

between 820 and 970°C. This is likely due to transformation of Zircaloy-4 from alpha to beta phase. This could lead to 'inverse bowing', i.e. bending towards the colder side.

10. Convergence and discretization

Convergence for hydraulic drag and for possible contacts with neighbours usually requires 3 to 5 iterations in BOW. The discretization is controlled by the user. The fuel element is usually divided into 2000 segments: 50 finite elements along the length of the fuel element, times 20 segments around the element circumference, times 2 radial segments across the sheath thickness. For the two-dimensional calculations, including iterations for hydraulic drag and for possible contact between neighbouring surfaces, BOW requires 1.3 seconds of computing time (Central Processor Unit) on a CDC/CYBER 175 computer.

BOW requires 70000 words of central memory on a CDC/CYBER 175 computer. Hence it would fit comfortably in a personal computer like an IBM-PC.

11. Accuracy and validation

The code was tested against eight independent analytical solutions for bending of beams experiencing a variety of loads and restraints. The applied loads included uniformly distributed loads; concentrated transverse and longitudinal forces; and bending moments. The restraints included simple supports; built-in ends;

CASE #	DESCRIPTION OF CASE (SEE NOTE)	MAXIMUM DEFLECTION (mm)					
		HORIZONTAL DIRECTION			VERTICAL DIRECTION		
		THEORY	BOW	% ERROR	THEORY	BOW	% ERROR
1	200 N/m; 400 N/m	7.02	7.02	< 1%	14.04	14.05	< 1%
2	3 Nm 4 Nm	2.07	2.07	< 1%	2.77	2.76	< 1%
3	10 N/m 6 N/m	3.37	3.37	< 1%	2.02	2.02	< 1%
4	3 Nm 4 Nm; 3 Nm 4 Nm	4.04	4.05	< 1%	5.39	5.39	< 1%
5	ΔT = 30°C 60°C; 30°C 60°C	0.310	0.310	< 1%	0.62	0.62	< 1%
6	50 N : 10 N; 0.3 m 0.2 m	5.30	5.30	< 1%	1.06	1.06	< 1%
7	ΔT = 30°C, 60°C; 30°C, 60°C; 565 N; 565 N	0.799	0.799	< 1%	1.60	1.59	< 1%
8	p = 50, 80 N/m; K = 200, 100 Nm/rad; p; K	0.796	0.80	< 1%	1.64	1.64	< 1%

NOTE. THE FIRST LOAD IS APPLIED HORIZONTALLY, THE SECOND VERTICALLY.

Fig. 13. Accuracy of BOW.

and springs. In each case, the agreement was within ±1%, see fig. 13.

Preliminary checks were also made of BOW predictions vs. two Canadian in-reactor experiments, one conducted at the Whiteshell Nuclear Research Establishment (WNRE) and the other at the Chalk River Nuclear Laboratories (CRNL). In both the experiments, CANDU fuel was irradiated under normal operating conditions.

In the WNRE experiment, post-irradiation bows of 0.4 to 1.8 mm were measured, for an average of 1.1 mm. For flux gradient factors in the range 0.032 to 0.7 [1], BOW predicted initial in-reactor bows of 0.56 to 1.2 mm, for an average of 0.9 mm.

In the CRNL experiments, post-irradiation bows of 0.3 to 1.2 mm were measured, for an average of 0.7 mm. BOW predicted initial in-reactor bows of 0.4 to 0.8 mm, for an average of 0.6 mm.

The above measurements are post-irradiation (out-reactor). BOW predicts initial deflections in the reactor. The two can be compared if the thermal recovery (stress relaxation) converts all the initial elastic bow to permanent strain, and if creep/shrinkage/swelling in the reactor add a negligible amount to the permanent (measured) bow. In addition, there are uncertainties in some input data for BOW calculations, e.g. in the flux gradient factor, in the curvature transfer factor, and in the rigidity enhancement factor. Nevertheless, if creep/shrinkage/swelling are assumed to have an insignificant effect on bowing and if the stress relaxation is assumed complete, then the predictions of average bow are within 0.2 mm of the measurements. Also, the range of predicted bow is within the range of measurements.

In these experiments the fuel stayed in the reactor for a few months, so the stresses were substantially relaxed [16]. Some in-reactor creep can be expected, which would add to the predicted bowing and improve the agreement of predictions vs experiments.

12. Illustrative example

The purpose of this example is to illustrate the capability of BOW to model the effect of hypothetical dryouts. The input data was chosen arbitrarily and hence some of it may not be pertinent to actual design or operating conditions.

A natural-UO_2 fuel element producing 66 kW/m was assumed to experience two elliptic drypatches at locations shown in fig. 14. The bulk coolant temperature was assumed to be 300°C at inlet to the bundle, and 305°C at the exit, with a linear variation in between. It was arbitrarily assumed that each drypatch caused a temperature increase of 200°C (i.e. maximum sheath temperature of 500–600°C). The temperature increase was assumed highest at the center of the drypatch, and varied parabolically with distance toward the boundary of the drypatch. Fig. 14 shows the results. When only drypatch #2 is applied, the maximum deflection occurs at about 80% of the element length. When both the drypatches are applied, the maximum deflection of 0.33 mm occurs near the element midlength.

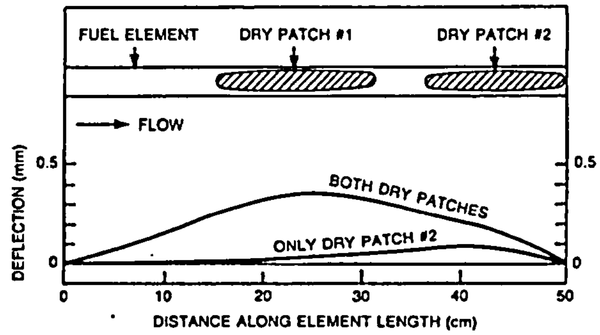

Fig. 14. Predicted bowing for assumed dryout.

13. Summary and conclusions

The computer code BOW predicts deflections of fuel elements in two lateral directions for given conditions of temperatures. It allows for the following physical phenomena: circumferential and axial variations in sheath and pellet temperatures; mechanical interactions between the sheath and the pellets; cracking of pellets; hydraulic drag; restraints from endplates, from neighbouring elements, and from the pressure tube; gravity; concentric or eccentric welds between endcap and endplate; neutron flux gradient; and material property variations.

The code is based on fundamental principles of mechanics. The governing equations are solved numerically using the finite element method. Three aspects of BOW contribute to its numerical stability, to its low computing cost, and to its small storage needs:
- *Iterations*: to account for hydraulic drag; and to model contact/slip between neighbouring surfaces.
- *Analytical formulations*: for restraint from the endplate; for flexural rigidity of the composite beam; and for pellet cracking.
- *Decomposition*: of the stiffness matrix in the two lateral directions.

Several comparisons with closed-form equations show that the solutions of BOW are accurate to $\pm 1\%$. The predictions of BOW for two experimental irradiations are consistent with measurements.

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Nomenclature

A	area of cross-section of the finite element,
D	flux gradient factor,
E, E_s, E_p	Young's modulus; of the sheath, of the pellet,
F	rigidity enhancement factor,
I, I_y, I_z	moment of inertia; about the y axis, about the z axis,
I_0, I_1, I_2	modified Bessel functions of the first kind; of order zero, one, and two respectively,
J	flexural rigidity,
K, K_r, K_t	spring constant; in the radial direction; in the tangential direction,
M, M_{Ty}, M_{Tz}	bending moment; about the y axis due to temperature gradients; about the z axis due to temperature gradients,
r	radial distance from the pellet axis,
r_1, r_2	inner radius of sheath; outer radius of sheath,
R	distance from bundle axis,
R_i	distance of element axis from bundle axis,
$S_1 - S_{12}$	forces and moments,
T	temperature,
ΔT	difference in temperature,
$\Delta T_1 - \Delta T_3$	circumferential difference in sheath temperature,
x, y, z	distance along coordinate axes,
α	coefficient of thermal expansion,
η	distance of neutral axis from the centroid,
κ	inverse diffusion length for thermal neutrons in homogenized bundle (pellet, sheath, coolant),
ρ	radius of curvature.

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