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Yucca Mountain Site Characterization Project

An Analysis of a Joint Shear Model for Jointed Media With Orthogonal Joint Sets

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Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550
for the United States Department of Energy
under Contract DE-AC04-76DP00789

SF2900Q(8-81)

"Prepared by Yucca Mountain Site Characterization Project (YMSCP) participants as part of the Civilian Radioactive Waste Management Program (CRWM). The YMSCP is managed by the Yucca Mountain Project Office of the U.S. Department of Energy, DOE Field Office, Nevada (DOE/NV). YMSCP work is sponsored by the Office of Geologic Repositories (OGR) of the DOE Office of Civilian Radioactive Waste Management (OCRWM)."

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Distribution
Category UC-814

An Analysis of a Joint Shear Model for Jointed Media with Orthogonal Joint Sets

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Abstract

This report describes a joint shear model used in conjunction with a computational model for jointed media with orthogonal joint sets. The joint shear model allows nonlinear behavior for both joint sets. Because nonlinear behavior is allowed for both joint sets, a great many cases must be considered to fully describe the joint shear behavior of the jointed medium. An extensive set of equations is required to describe the joint shear stress and slip displacements that can occur for all the various cases. This report examines possible methods for simplifying this set of equations so that the model can be implemented efficiently from a computational standpoint. The shear model must be examined carefully to obtain a computationally efficient implementation that does not lead to numerical problems.

This work was completed under WBS 1.2.4.2.3.1, but was worked on under the earlier WBS 1.2.4.6.1. This work is considered to be preliminary and scoping in nature and will not be directly used in the development or modification of scientific and engineering software used in the Yucca Mountain Site Characterization Project. However, the results presented here may assist in formulating a basis for design of new software or modification of exisiting software after the SNL Software Quality Assurance Plan and its implementing procedures are formally approved.

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1. A BILINEAR SHEAR STRESS VERSUS SLIP DISPLACEMENT MODEL

1.1 Introduction

The effects of fractures in rocks can be modeled with a continuum approach. This continuum model captures the gross response of jointed rock by distributing the individual responses of the joints throughout the rock structure. A continuum model for rock with a single joint set was first proposed by Morland (1974) and then examined by Thomas (1982). The model was extended to orthogonal joint sets by Chen (1986).

A computational model for a two-dimensional continuum model for jointed media with orthogonal joint sets was first implemented by Chen (1987). This implementation assumes that joints occur in sets that are more or less parallel and regularly spaced. The deformation response normal to the joint is nonlinear elastic and based on a rational polynomial. Joint shear stress is treated as linear elastic in the shear stress versus slip displacement before attaining a critical stress level governed by the Coulomb friction criterion. Beyond the critical stress value, a linear relation analogous to strain-hardening plasticity governs the rest of the shear stress versus slip displacement relation.

The original shear model described by Chen (1987) has undergone several modifications. These modifications are documented in two Sandia National Laboratory (SNL) memorandums: Chen to Costin and Bauer, April 18, 1988, and Chen to Costin and Bauer, December 8, 1988. The algorithm implemented in these two modifications is based on the simultaneous satisfaction of the strain rate decomposition and the shear stress versus slip displacement relation. The code used to implement this algorithm is listed in the second of the above two memos. There is no formal documentation of the conventions and equations implementing this algorithm.

The shear model allows nonlinear behavior for both of the joint sets. The nonlinear behavior can be either elastic-plastic or elastic-perfectly plastic. Allowing nonlinear shear behavior to occur for both of the joint sets in an orthogonal joint set model leads to a great number of cases that must be considered. If the model is to be fully implemented, all cases that can arise must be recognized, and the equations governing each case must be determined. This report lists the equations required to describe all cases and documents the conventions used to derive these equations.

If the shear model is to be efficiently implemented in a computational model, the governing equations must be carefully examined. This report includes a detailed analysis of the equations describing the shear model that reveals two important characterisitics

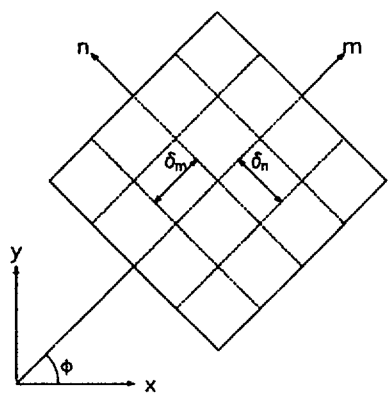


Figure 1.1. Section of Jointed Rock Medium

of the system of equations. First, the system of equations governing the shear model can be greatly simplified while allowing essentially full generality. Second, it is possible to predict the type of nonlinear shear behavior pattern for a given strain increment based solely on parameters at the beginning of the increment. The prediction is based on two inequalities that are derived in this report. Because it is possible to predict the pattern of nonlinear behavior for a strain increment, the equations can be programmed in a fully structured manner.

1.2 Notation Conventions

Figure 1.1 shows a section of jointed medium with orthogonal joint sets. The coordinate system xy is a global coordinate system. The joint planes with a normal in the m direction are referred to as joint set m; the joint planes with a normal in the n direction are referred to as joint set n. These two joint sets establish a coordinate system mn. The angle θ is referred to as the joint set angle, and it measures the angle between the $x-$ axis and the reference joint set.

The normal stress in the direction of the $m-$ axis is T_{mm}, the normal stress in the direction of the $n-$ axis is T_{nn}, and the shear stress is T_{mn}. The sign convention used for the shear stress follows the convention used by Fung (1965). Suppose that the outward normal for the face of some differential element is in the positive m direction. If the shear stress component T_{mn} on this face is in the positive n direction, then the shear stress for the differential element is positive. If the shear stress component T_{mn} on this face is in the negative n direction, the shear stress for the differential element is negative. The same rule applies if the outward normal for the face of some differential element is in the positive n direction. If the shear stress component T_{mn} on this face is in the positive m direction, then the shear stress for the differential element is positive.

To transform the stresses from the xy to the mn coordinate system as shown in Figure 1.1, the following equations are used:

$$T_{mm} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta, \quad (1.1)$$
$$T_{nn} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta, \quad (1.2)$$
$$T_{mn} = (-\sigma_{xx} + \sigma_{yy})\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta). \quad (1.3)$$

The stress components in the xy coordinate system are σ_{xx}, σ_{yy}, and τ_{xy}. To transform the stresses from the mn to the xy coordinate system, substitute $-\theta$ for θ in the above equations. The resulting set of equations for the inverse transformation follows:

$$\sigma_{xx} = T_{mm}\cos^2\theta + T_{nn}\sin^2\theta - 2T_{mn}\cos\theta\sin\theta, \quad (1.4)$$
$$\sigma_{yy} = T_{mm}\sin^2\theta + T_{nn}\cos^2\theta + 2T_{mn}\cos\theta\sin\theta, \quad (1.5)$$
$$\tau_{xy} = (T_{mm} - T_{nn})\sin\theta\cos\theta + T_{mn}(\cos^2\theta - \sin^2\theta). \quad (1.6)$$

The spacing between the joint planes for joint set m is δ_m and for joint set n is δ_n. The properties for joint set m are all denoted with a subscript m. The maximum joint closure in the m direction is denoted as $(u^d_{max})_m$, and the half-closure stress for joint set m is A_m. These two parameters define behavior normal to joint set m, i.e., in the m direction. The parameters defining the shear behavior for joint set m are the joint shear stiffness G_{sm}, the joint shear hardening G'_{sm}, the joint cohesion C_{0m}, and the coefficient of friction μ_m. Similar notation is used for joint set n.

The jointed rock model uses strain rate equations to calculate stress increments. Transforming the strain increments from the xy to the mn coordinate system is similar to the transformation of stresses shown in Equations 1.1 through 1.3.

$$e_{mm} = \epsilon_{xx}\cos^2\theta + \epsilon_{yy}\sin^2\theta + 2\epsilon_{xy}\cos\theta\sin\theta, \quad (1.7)$$
$$e_{nn} = \epsilon_{xx}\sin^2\theta + \epsilon_{yy}\cos^2\theta - 2\epsilon_{xy}\cos\theta\sin\theta, \quad (1.8)$$
$$e_{mn} = (-\epsilon_{xx} + \epsilon_{yy})\sin\theta\cos\theta + \epsilon_{xy}(\cos^2\theta - \sin^2\theta). \quad (1.9)$$

The strain components in the xy coordinate system are $\epsilon_{xx}, \epsilon_{yy}$, and ϵ_{xy}.

The joint displacement in the m direction is u^d_m, and the joint displacement in the n direction is u^d_n. The joint slip displacement along joint set m is denoted as u_{sm}. Note that this slip is parallel to the joint planes for joint set m and is in the $\pm n$ direction. The joint slip displacement along joint set n is denoted by u_{sn}. This slip is parallel to the joint planes for joint set n and is in the $\pm m$ direction.

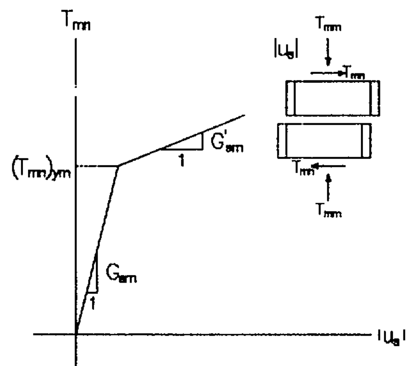


Figure 1.2. Nonlinear Shear Behavior for Joints

1.3 The Bilinear Shear Model

The bilinear shear stress versus slip displacement response for the slip behavior of the joints is shown in Figure 1.2. The onset of the nonlinear response is assumed to be governed by a linear Mohr-Coulomb criterion. A scalar slip function is given as

$$F = \mid T_{mn} \mid + \mu_m T_{mm} - C_{0m}. \tag{1.10}$$

In Equation 1.10, T_{mm} is the normal stress across a joint in joint set m; T_{mn} is the shear stress across the joint. The coefficient of friction across the joint is μ_m, and C_{0m} is the joint cohesion. The joint behavior is elastic if $F < 0$ and inelastic if $F > 0$. The joint shear stiffness is G_{sm} in the elastic range and G'_{sm} in the inelastic range for joint set m. This behavior is shown in Figure 1.2.

If the value for G'_{sm} is nonzero, the curve describing the shear stress versus slip displacement behavior will be referred to as being elastic-plastic. If G'_{sm} is zero, the behavior will be referred to as elastic-perfectly plastic. The point where the curve changes slope corresponds to a shear stress value designated as $(T_{mn})_{ym}$. This is the yield stress for the curve. The nomenclature is similar for joint set n.

2. LOADING AND UNLOADING CASES FOR THE BILINEAR SHEAR MODEL

2.1 Shear Stress-Shear Strain Relations

Because of the bilinear curve for the shear stress versus slip displacement, a variety of cases must be considered in order to fully characterize the shear behavior. This section discusses those various cases and the manner in which the equations are derived to characterize these various cases.

The computational joint model implemented by Chen uses an incremental solution process. The strain-stress relations are written in a rate form, and it is assumed the strain rate is constant over time increment Δt. If the stresses and strain rates are known at time t, it becomes possible to calculate the stresses at time $t + \Delta t$.

The total shear strain increment from time t to time $t + \Delta t$ can be expressed as

$$\Delta e_{mn} = (\Delta e_{mn})_{matrix} + (\Delta e)_m + (\Delta e)_n, \tag{2.1}$$

where $(\Delta e_{mn})_{matrix}$ is the shear strain in the rock matrix and $(\Delta e)_m$ and $(\Delta e)_n$ are the contributions to the total shear strain Δe_{mn} from joint sets m and n, respectively. If small angle changes are assumed, the strain contribution arising from joint set m relates to the slip displacement increment Δu_{sm} by

$$(\Delta e)_m = \Delta u_{sm}/(2\delta_m), \tag{2.2}$$

and the strain contribution arising from joint set n relates to the slip displacement increment Δu_{sn} by

$$(\Delta e)_n = \Delta u_{sn}/(2\delta_n). \tag{2.3}$$

If the shear stress behavior is strictly in the elastic range, the shear slip for joint set m relates to the shear stress by

$$u_{sm} = T_{mn}/G_{sm}, \tag{2.4}$$

and the shear slip for joint set n relates to the shear stress by

$$u_{sn} = T_{mn}/G_{sn}. \tag{2.5}$$

The sign of the shear slip displacements is determined by the sign of the shear stress. If the shear stress is positive, then the slip displacements are also positive. The physical

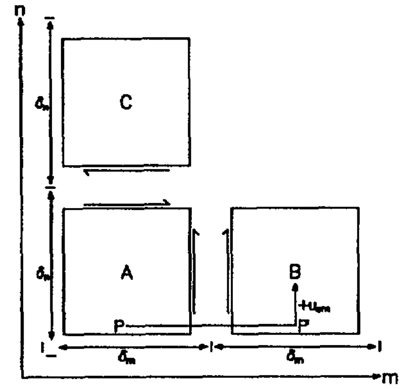


Figure 2.1. Sign convention for joint shear stresses and slip displacement

meaning of a positive slip displacement is shown in Figure 2.1. This figure shows three blocks of material (A, B, and C) in a jointed material. The shear forces acting on block A are positive by the convention presented in Section 1. First consider the undeformed configuration for the three blocks. Moving from any point P in block A by a distance δ_m in the positive m direction generates some corresponding point P' in block B. If block A is held fixed and block B is allowed to slip in the n direction according to the elastic constitutive relation for slip, point P' will move in the positive n direction. The shear forces acting on blocks A and B determine the relative motion of the two blocks. The positive shear stress indicates that, if block A is held fixed, the relative motion of block B to block A will be in the positive n direction for a positive shear stress. A similar situation holds for block C in relation to block A. If block A is held fixed, the relative motion of block C to block A will be in the positive m direction for a positive shear stress.

If the shear stress behavior is strictly in the elastic range, the equations relating the slip displacements to the shear strain contributions from the joint sets and the equations relating the slip displacements to the shear stress can be combined so that Equation 2.1 can be written as

$$\Delta e_{mn} = \left[\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}}\right] \Delta T_{mn}, \tag{2.6}$$

where G is the shear modulus for the rock matrix. Equation 2.6 can be rewritten as

$$\Delta T_{mn} = \frac{2G\Delta e_{mn}}{1 + G/(\delta_m G_{sm}) + G/(\delta_n G_{sn})}. \tag{2.7}$$

For each time step in the computational model, the above value for ΔT_{mn} is calculated and used to determine a value for T_{mn} at time $t + \Delta t$. Because the above increment is based only on elastic constants, it will be denoted as $(\Delta T_{mn})_{elastic}$; a trial value for T_{mn} at time $t + \Delta t$ calculated from this increment will be denoted by $(T_{mn})^{t+\Delta t}_{elastic}$.

$$(T_{mn})^{t+\Delta t}_{elastic} = T^{t}_{mn} + (\Delta T_{mn})_{elastic} \tag{2.8}$$

The value for $(T_{mn})^{t+\Delta t}_{elastic}$ is the primary parameter used to determine which sets of equations are used to calculate the shear stress increment and slip displacements for a given time step. It is not, however, the only parameter used to determine which equations will be used for calculating shear stress increments and slip displacements. The elastic shear stress increment is useful for delineating some major categories of shear behavior for the joint sets. Within some of the major categories, several other parameters and relations among these parameters must be used before selecting the proper set of equations.

In addition to the value for $(T_{mn})^{t+\Delta t}_{elastic}$, it is necessary to calculate yield points for the bilinear curves defining the shear stress versus slip displacement relations at each time step. The yield points at time t are denoted as $(T_{mn})^{t}_{ym}$ and $(T_{mn})^{t}_{yn}$ and at time $t + \Delta t$ as $(T_{mn})^{t+\Delta t}_{ym}$ and $(T_{mn})^{t+\Delta t}_{yn}$.

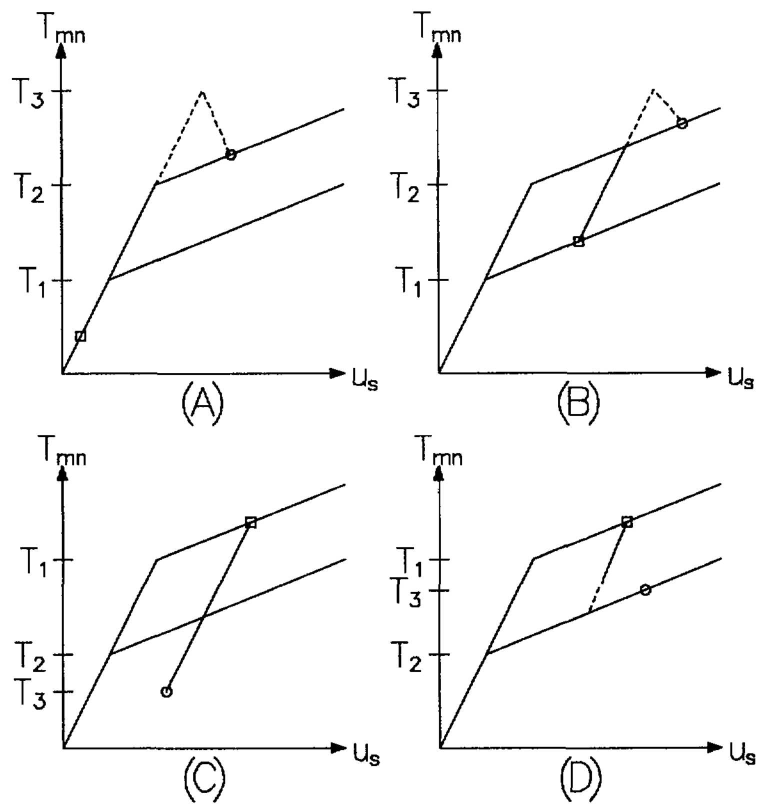
2.2 Inelastic Behavior for a Single Joint Set

Several cases need to be considered if inelastic behavior occurs for a load step, and these are depicted in Figure 2.2. Joint set m is used for the examples shown in Figure 2.2; analogous cases exist for joint set n.

Consider the behavior shown in Figure 2.2A. The shear stress at time t is on the elastic portion of the shear stress versus slip displacement curve. Suppose a load increment occurs, and the value for $(T_{mn})^{t+\Delta t}_{elastic}$ is greater than $(T_{mn})^{t+\Delta t}_{ym}$. The value for T_{mn} cannot lie above the present shear stress versus slip displacement curve. The shear stress increment must track the current shear stress versus slip displacement curve, which is a bilinear curve. The equation for this bilinear curve is used to determine the value for T_{mn} at time $t + \Delta t$.

Another case to consider is one where the T_{mn} is on the inelastic portion of a shear stress versus slip displacement curve at time t and a load increment occurs. This situation is shown in Figure 2.2B. The shear stress increases along an elastic curve until it reaches a point on the current shear stress versus slip displacement curve known as the effective yield, $(T_{mn})_{eym}$. The shear stress is still constrained to lie below the current shear stress versus slip displacement curve; therefore, any increment in the shear stress above the effective yield point must occur along the inelastic portion of the current shear stress versus slip displacement curve.

The effective yield for the case shown in Figure 2.2B is the intersection of two curves. One of these curves is an elastic curve that passes through point T^{t}_{mn} and that has a slope



T_1 – yield stress at time t
T_2 – yield stress at time t + Δ t
T_3 – shear stress at time t + Δ t
based on elastic calculations
□ – shear stress at time t
○ – shear stress at time t + Δ t

Figure 2.2. Cases for Loading and Unloading

G_{sm}. Associated with this curve is an offset slip displacement denoted as u_{sm}^{offset}. This offset value changes from time step to time step. The other curve is the inelastic portion of the current shear stress versus slip displacement curve. The offset slip displacement is defined as

$$u_{sm}^{offset} = u_{sm}^{t} - T_{mn}^{t}/G_{sm}. \tag{2.9}$$

The effective yield stress in terms of the offset slip displacement is

$$(T_{mn})_{eym} = (T_{mn})_{ym}^{t+\Delta t} + \frac{G'_{sm} u_{sm}^{offset}}{1 - G'_{sm}/G_{sm}}. \tag{2.10}$$

The effective yield stress is defined by Equation 2.10, except when a sign change occurs for T_{mn} for a load increment. When the sign for T_{mn}^{t} differs from that of $(T_{mn})_{elastic}$, then the effective yield stress is the regular yield stress $(T_{mn})_{ym}^{t+\Delta t}$ at time $t + \Delta t$.

The last two cases of inelastic behavior that need to be considered are shown in Figures 2.2C and 2.2D. Both cases involve unloading situations. For the case shown in Figure 2.2C, the shear stress is originally on the inelastic portion of the shear stress versus slip displacement curve. Unloading takes place along an elastic curve with slope G_{sm}, and the value for $T_{mn}^{t+\Delta t}$ is less than the effective yield stress at the current time. The final value for T_{mn} lies below the current shear stress versus slip displacement curve.

The situation shown in Figure 2.2D is similar to that in Figure 2.2C except that the value for T_{mn} at time $t + \Delta t$ is greater than the effective yield stress. The value for T_{mn}, therefore, lies above the current shear stress versus slip displacement curve. Since this is not a valid value for T_{mn} at the current time, unloading occurs along an elastic curve with slope G_{sm} until the effective yield stress is reached. The value for T_{mn} at time $t + \Delta t$ lies on the inelastic portion of the current shear stress versus slip displacement curve.

2.3 Shear Model for Orthogonal Joint Sets

The preceding section describes some key concepts required to correctly model shear behavior for a single joint. These concepts are also important for the characterization of orthogonal joint sets. This section discusses the equations required to describe shear behavior for a material with orthogonal joint sets. The value for $(T_{mn})_{elastic}^{t+\Delta t}$ is useful for determining the proper set of equations to describe joint shear behavior. Because of this, the following sections are grouped on the basis of the relation of $(T_{mn})_{elastic}^{t+\Delta t}$ to the effective yield stresses. The value for $(T_{mn})_{elastic}^{t+\Delta t}$ is not the only deciding factor in selecting the correct equations to describe shear behavior for the joints, but it does provide a useful parameter to define major divisions characterizing shear behavior for the joints. The divisions based on $(T_{mn})_{elastic}^{t+\Delta t}$ will be referred to as cases, and there are five cases that must be considered. The cases can be further divided into what will be called variations.

2.3.1 Case 1: Elastic Shear Behavior for Both Joint Sets

The simplest case for the shear behavior of the joint sets is one where both joint sets exhibit elastic shear behavior. If $(T_{mn})^{t+\Delta t}_{elastic} \leq (T_{mn})_{eym}$ and $(T_{mn})^{t+\Delta t}_{elastic} \leq (T_{mn})_{eyn}$, then the behavior for both joint sets is elastic, and the value for $T^{t+\Delta t}_{mn}$ is $(T_{mn})^{t+\Delta t}_{elastic}$. The equations for this case can be summarized as follows:

- The stress increment is

$$\Delta T_{mn} = \frac{2G\Delta e_{mn}}{1 + G/(\delta_m G_{sm}) + G/(\delta_n G_{sn})}. \tag{2.11}$$

The slip displacement increment u_{sm} for joint set m is

$$\Delta u_{sm} = \frac{(\Delta T_{mn})_{elastic}}{G_{sm}}, \tag{2.12}$$

and the slip displacement increment u_{sn} for joint set n is

$$\Delta u_{sn} = \frac{(\Delta T_{mn})_{elastic}}{G_{sn}}. \tag{2.13}$$

2.3.2 Case 2: Inelastic Shear Behavior for Joint Set m and Elastic Shear Behavior for Joint Set n

Now consider the case where $(T_{mn})^{t+\Delta t}_{elastic} > (T_{mn})_{eym}$ and $(T_{mn})^{t+\Delta t}_{elastic} \leq (T_{mn})_{eyn}$. The value for $(T_{mn})^{t+\Delta t}_{elastic}$ indicates that the behavior for joint set m is nonlinear, but the behavior for joint set n is linear. Two different variations must be considered when this particular situation arises. The first variation involves elastic-perfectly plastic behavior for joint set m. If joint set m is elastic-perfectly plastic, the value for T_{mn} at time $t + \Delta t$ cannot exceed $(T_{mn})_{eym}$. The value for $T^{t+\Delta t}_{mn}$ is set to $(T_{mn})_{eym}$, and the stress increment becomes

$$\Delta T_{mn} = (T_{mn})_{eym} - T^{t}_{mn}. \tag{2.14}$$

The slip displacement increment for joint set n is

$$\Delta u_{sn} = \Delta T_{mn}/G_{sn}. \tag{2.15}$$

By using Equation 2.1 and the relation

$$\Delta u_{sm} = 2\delta_m(\Delta e)_m \tag{2.16}$$

and the relation

$$\Delta u_{sn} = 2\delta_n(\Delta e)_n, \tag{2.17}$$

it is possible to write the slip displacement increment for joint set m as

$$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_n G_{sn}}\right). \tag{2.18}$$

The second case to consider is elastic-plastic behavior for joint set m. To calculate the stress increment for this case, write Equation 2.1 as

$$\Delta e_{mn} = \frac{\Delta T_{mn}}{2G} + \frac{\Delta u_{sm}}{2\delta_m} + \frac{\Delta u_{sn}}{2\delta_n}. \tag{2.19}$$

Equation 2.19 can be rewritten as

$$\Delta e_{mn} = \frac{\Delta T_{mn}}{2G} + \frac{1}{2\delta_m}\left(\frac{(T_{mn})_{eym} - T^t_{mn}}{G_{sm}} + \frac{T^{t+\Delta t}_{mn} - (T_{mn})_{eym}}{G'_{sm}}\right) + \frac{\Delta T_{mn}}{2\delta_n G_{sn}}. \tag{2.20}$$

Eliminating $T^{t+\Delta t}_{mn}$ from Equation 2.20 yields

$$\Delta e_{mn} = \frac{\Delta T_{mn}}{2G} + \frac{1}{2\delta_m}\left(\frac{(T_{mn})_{eym} - T^t_{mn}}{G_{sm}} - \frac{(T_{mn})_{eym} - T^t_{mn}}{G'_{sm}}\right) + \frac{\Delta T_{mn}}{2\delta_m G'_{sm}} + \frac{\Delta T_{mn}}{2\delta_n G_{sn}}. \tag{2.21}$$

Solving for ΔT_{mn} from Equation 2.21 yields

$$\Delta T_{mn} = \frac{2\Delta e_{mn} + ((T_{mn})_{eym} - T^t_{mn})(\frac{1}{\delta_m G'_{sm}} - \frac{1}{\delta_m G_{sm}})}{\frac{1}{G} + \frac{1}{\delta_m G'_{sm}} + \frac{1}{\delta_n G_{sn}}}. \tag{2.22}$$

Equations 2.15 and 2.18, which describe the joint slip displacements for the case when joint set m is elastic-perfectly plastic, also describe the joint slip displacements when joint set m is elastic-plastic.

The two variations just described can be summarized as follows:

- When

  $$G'_{sm} = 0, \tag{2.23}$$

  then

  $$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}. \tag{2.24}$$

- When

  $$G'_{sm} \neq 0, \tag{2.25}$$

  then

  $$\Delta T_{mn} = \frac{2\Delta e_{mn} + ((T_{mn})_{eym} - T^t_{mn})(\frac{1}{\delta_m G'_{sm}} - \frac{1}{\delta_m G_{sm}})}{\frac{1}{G} + \frac{1}{\delta_m G'_{sm}} + \frac{1}{\delta_n G_{sn}}}. \tag{2.26}$$

The slip displacement for joint set n is

$$\Delta u_{sn} = \Delta T_{mn}/G_{sn}, \tag{2.27}$$

and the slip displacement for joint set m is

$$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_n G_{sn}} \right). \tag{2.28}$$

The latter two equations for the slip displacements are valid for both $G'_{sm} = 0$ and $G'_{sm} \neq 0$.

It is possible to write Equation 2.22 as

$$\Delta T_{mn} = \frac{2GR_m \Delta e_{mn} + ((T_{mn})_{eym} - T^t_{mn})(1 - R_m)\frac{G}{\delta_m G_{sm}}}{R_m + \frac{G}{\delta_m G_{sm}} + \frac{R_m G}{\delta_n G_{sn}}}, \tag{2.29}$$

where

$$R_m = G'_{sm}/G_{sm}. \tag{2.30}$$

Equation 2.29 is applicable for the variations $G'_{sm} = 0$ and $G'_{sm} \neq 0$. The ratio R_m is equal to 0 when $G'_{sm} = 0$, which describes elastic-perfectly plastic behavior for joint set m. For $R_m = 0$, Equation 2.29 yields

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}, \tag{2.31}$$

which is the desired result for elastic-perfectly plastic behavior.

2.3.3 Case 3: Elastic Shear Behavior for Joint Set m and Inelastic Shear Behavior for Joint Set n

Now suppose that $(T_{mn})^{t+\Delta t}_{elastic} \leq (T_{mn})_{eym}$ and $(T_{mn})^{t+\Delta t}_{elastic} > (T_{mn})_{eyn}$. For this particular case, the shear behavior for joint set n is nonlinear, but the behavior for joint set m is linear. This is similar to the case just described, except that the joint sets are interchanged in terms of behavior. The equations for the preceding section are applicable if the subscripts associated with joint sets are simply interchanged. The equations for the present case will only be summarized.

- When

$$G'_{sn} = 0, \tag{2.32}$$

then

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.33}$$

- When

$$G'_{sn} \neq 0, \tag{2.34}$$

then

$$\Delta T_{mn} = \frac{2\Delta e_{mn} + ((T_{mn})_{eyn} - T^t_{mn})(\frac{1}{\delta_n G'_{sn}} - \frac{1}{\delta_n G_{sn}})}{\frac{1}{G} + \frac{1}{\delta_n G'_{sn}} + \frac{1}{\delta_m G_{sm}}}. \tag{2.35}$$

The slip displacement for joint set m is

$$\Delta u_{sm} = \Delta T_{mn} / G_{sm}, \tag{2.36}$$

and the slip displacement for joint set n is

$$\Delta u_{sn} = \delta_n \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_m G_{sm}} \right). \tag{2.37}$$

As in the preceding case, it is possible to write one equation for T_{mn} that holds for both $G'_{sn} = 0$ and $G'_{sn} \neq 0$. This equation for ΔT_{mn} is

$$\Delta T_{mn} = \frac{2GR_n\Delta e_{mn} + ((T_{mn})_{eyn} - T^t_{mn})(1 - R_n)\frac{G}{\delta_n G_{sn}}}{R_n + \frac{G}{\delta_n G_{sn}} + \frac{R_n G}{\delta_m G_{sm}}}, \tag{2.38}$$

where

$$R_n = G'_{sn} / G_{sn}. \tag{2.39}$$

For $R_n = 0$, Equation 2.38 yields

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}, \tag{2.40}$$

which is the desired result for elastic-perfectly plastic behavior for joint set n.

2.3.4 Case 4: Elastic Shear Increment Exceeds the Effective Yield for Both Joint Sets and the Effective Yields are not Equal

The greatest number of variations arise when the value for $(T_{mn})^{t+\Delta t}_{elastic}$ is greater than both $(T_{mn})_{eym}$ and $(T_{mn})_{eyn}$. The situation of $(T_{mn})_{eym} = (T_{mn})_{eyn}$ is a special case with fewer variations to consider. From a practical standpoint, it is not very likely to encounter the situation of $(T_{mn})_{eym} = (T_{mn})_{eyn}$ during a computational process. This more specialized case will be examined as the last case.

When $(T_{mn})^{t+\Delta t}_{elastic} > (T_{mn})_{eym}$ and $(T_{mn})^{t+\Delta t}_{elastic} > (T_{mn})_{eyn}$, determining the correct equations to model the shear stress behavior becomes much more complicated. It is necessary not only to distinguish between elastic-plastic and elastic-perfectly plastic situations, but also to examine the relations of the effective yield stresses. Futhermore, if one or both of the joint sets are elastic-plastic, other factors must be examined before applying the equations in this section. These other considerations are noted in the appropriate sections.

The first variation considered is for both joints sets exhibiting elastic-perfectly plastic behavior. If $(T_{mn})_{eym} < (T_{mn})_{eyn}$, then the limiting value for T_{mn} at time $t + \Delta t$ becomes $(T_{mn})_{eym}$; the shear stress cannot exceed the lower effective yield stress. The shear stress for joint set n is limited to elastic behavior even though $(T_{mn})^{t+\Delta t}_{elastic}$ is greater than $(T_{mn})_{eyn}$. The shear stress increment becomes

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}. \tag{2.41}$$

The slip displacement increment for joint set n is

$$\Delta u_{sn} = \Delta T_{mn}/G_{sn}, \tag{2.42}$$

and the slip displacement increment for joint set m is

$$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_n G_{sn}} \right). \tag{2.43}$$

If $(T_{mn})_{eym} > (T_{mn})_{eyn}$ (as opposed to $(T_{mn})_{eym} < (T_{mn})_{eyn}$) and both joint sets have elastic-perfectly plastic shear models, the situation is similar to the one just described. The preceding equations can be used to describe the joint behavior simply by interchanging the subscripts used to denote the two different joint sets. The shear stress increment is given by

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.44}$$

The slip displacement increment for joint set m is

$$\Delta u_{sm} = \Delta T_{mn}/G_{sm}, \tag{2.45}$$

and the slip displacement increment for joint set n is

$$\Delta u_{sn} = \delta_n \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_m G_{sm}} \right). \tag{2.46}$$

The equations for the two preceding situations have appeared previously in the case where $(T_{mn})^{t+\Delta t}_{elastic}$ has indicated nonlinear behavior for one joint set and linear behavior for the other joint set. No new equations have been introduced to describe the shear stress and slip displacement behavior. A different set of conditions have been determined to invoke a previously defined set of equations.

For the next variation, consider an elastic-plastic model for joint set m ($G'_{sm} \neq 0$) and an elastic-perfectly plastic model for joint set n. First consider the case where $(T_{mn})_{eym} < (T_{mn})_{eyn}$. For this variation, the shear stress cannot exceed $(T_{mn})_{eyn}$. The value of $(T_{mn})^{t+\Delta t}_{elastic}$ is not sufficient to determine if this upper limit for the shear stress will be reached. It may be possible that the strain increment is not large enough to produce nonlinear behavior for joint set n. It is possible to determine if the strain increment is large enough to produce nonlinear behavior for joint set n by examining the equation that describes nonlinear behavior for joint set m as an elastic-plastic material and elastic behavior for joint set n. The equation for the stress increment for this particular situation was derived earlier as Equation 2.22.

$$\Delta T_{mn} = \frac{2\Delta e_{mn} + ((T_{mn})_{eym} - T^t_{mn})(\frac{1}{\delta_m G'_{sm}} - \frac{1}{\delta_m G_{sm}})}{\frac{1}{G} + \frac{1}{\delta_m G'_{sm}} + \frac{1}{\delta_n G_{sn}}}. \tag{2.47}$$

Now set the shear stress increment in the above equation equal to $(T_{mn})_{eyn} - T^t_{mn}$. This equality allows the calculation of a shear strain increment (based on quantities at the beginning of the increment) sufficient to produce a value of the shear stress at time $t + \Delta t$ of $(T_{mn})_{eyn}$. The equation for this strain increment is

$$\begin{aligned}\Delta e_{mn} = \; & \tfrac{1}{2}[((T_{mn})_{eyn} - T^t_{mn})(\tfrac{1}{G} + \tfrac{1}{\delta_m G'_{sm}} + \tfrac{1}{\delta_n G_{sn}}) - \\ & ((T_{mn})_{eym} - T^t_{mn})(\tfrac{1}{\delta_m G'_{sm}} - \tfrac{1}{\delta_m G_{sm}})].\end{aligned} \tag{2.48}$$

If the strain increment does not satisfy the inequality

$$\begin{aligned}\Delta e_{mn} > \; & \tfrac{1}{2}[((T_{mn})_{eyn} - T^t_{mn})(\tfrac{1}{G} + \tfrac{1}{\delta_m G'_{sm}} + \tfrac{1}{\delta_n G_{sn}}) - \\ & ((T_{mn})_{eym} - T^t_{mn})(\tfrac{1}{\delta_m G'_{sm}} - \tfrac{1}{\delta_m G_{sm}})],\end{aligned} \tag{2.49}$$

then the equations that describe nonlinear behavior for joint set m and elastic behavior for joint set n must be used to calculate the shear stress increment and slip displacement increments. If, however, the shear strain increment satisfies the above inequality, then both joint sets will behave in a nonlinear manner and the shear stress increment is defined by

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.50}$$

The slip displacement for joint set m must follow the bilinear curve defining the shear stress versus slip displacement. The equation for the slip displacement increment for joint set m is

$$\Delta u_{sm} = \frac{(T_{mn})_{eym} - T^t_{mn}}{G_{sm}} + \frac{(T_{mn})_{eyn} - (T_{mn})_{eym}}{G'_{sm}}. \tag{2.51}$$

The equation for the slip displacement increment for joint set n is

$$\Delta u_{sn} = \delta_n \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta u_{sm}}{\delta_m}\right). \tag{2.52}$$

Now consider the variation where joint set m is elastic-plastic and joint set n is elastic-perfectly plastic, but $(T_{mn})_{eym} > (T_{mn})_{eyn}$ (as opposed to the variation just described where $(T_{mn})_{eym} < (T_{mn})_{eyn}$). The shear stress increment is given by

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.53}$$

The slip displacement increment for joint set m is

$$\Delta u_{sm} = \Delta T_{mn}/G_{sm}, \tag{2.54}$$

and the slip displacement increment for joint set n is

$$\Delta u_{sn} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_m G_{sm}}\right). \tag{2.55}$$

Two more variations are generated if joint set m is elastic-perfectly plastic and joint set n is elastic plastic. This is similar to the two variations just examined. The equations for these two variations are as follows:

- When

$$(T_{mn})_{eym} > (T_{mn})_{eyn} \tag{2.56}$$

then

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}. \tag{2.57}$$

The slip displacement for joint set n is

$$\Delta u_{sn} = \frac{(T_{mn})_{eyn} - T^t_{mn}}{G_{sn}} + \frac{(T_{mn})_{eym} - (T_{mn})_{eyn}}{G'_{sn}}. \tag{2.58}$$

The equation for the slip displacement increment for joint set m is

$$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta u_{sn}}{\delta_n}\right). \tag{2.59}$$

- When

$$(T_{mn})_{eym} < (T_{mn})_{eyn} \tag{2.60}$$

then

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}. \tag{2.61}$$

The slip displacement increment for joint set n is

$$\Delta u_{sn} = \Delta T_{mn}/G_{sn}, \tag{2.62}$$

and the slip displacement increment for joint set m is

$$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_n G_{sn}}\right). \tag{2.63}$$

It must be noted that, in the previous set of equations, the equations for the case of $(T_{mn})_{eym} > (T_{mn})_{eyn}$, are not valid unless the strain increment is large enough to force nonlinear behavior for both joint sets. The strain increment must satisfy the inequality

$$\begin{aligned}\Delta e_{mn} > \; & \tfrac{1}{2}[((T_{mn})_{eym} - T^t_{mn})(\tfrac{1}{G} + \tfrac{1}{\delta_n G'_{sn}} + \tfrac{1}{\delta_m G_{sm}}) - \\ & ((T_{mn})_{eyn} - T^t_{mn})(\tfrac{1}{\delta_n G'_{sn}} - \tfrac{1}{\delta_n G_{sn}})]\end{aligned} \tag{2.64}$$

in order to use the equations specified for the case of $(T_{mn})_{eym} > (T_{mn})_{eyn}$.

Finally, it is possible to consider the variation where both joint sets are elastic-plastic. The shear stress increment is given by

$$\begin{aligned}\Delta T_{mn} = \; & \frac{2\Delta e_{mn}}{\frac{1}{G}+\frac{1}{\delta_m G'_{sm}}+\frac{1}{\delta_n G'_{sn}}} + \\ & \frac{((T_{mn})_{eym}-T^t_{mn})(\frac{1}{\delta_m G'_{sm}}-\frac{1}{\delta_m G_{sm}})+((T_{mn})_{eyn}-T^t_{mn})(\frac{1}{\delta_n G'_{sn}}-\frac{1}{\delta_n G_{sn}})}{\frac{1}{G}+\frac{1}{\delta_m G'_{sm}}+\frac{1}{\delta_n G'_{sn}}}.\end{aligned} \tag{2.65}$$

The slip displacement for joint set m is

$$\Delta u_{sm} = \frac{(T_{mn})_{eym} - T^t_{mn}}{G_{sm}} + \frac{T^{t+\Delta t}_{mn} - (T_{mn})_{eym}}{G'_{sm}}, \tag{2.66}$$

and the slip displacement for joint set n is

$$\Delta u_{sn} = \frac{(T_{mn})_{eyn} - T^t_{mn}}{G_{sn}} + \frac{T^{t+\Delta t}_{mn} - (T_{mn})_{eyn}}{G'_{sn}}. \tag{2.67}$$

As in previous cases, the strain increment must satisfy certain conditions before these equations for the shear stress increment and slip displacement increments can be used. If $(T_{mn})_{eym} < (T_{mn})_{eyn}$, then the strain increment must satisfy the inequality in equation 2.49 in order for equations 2.65, 2.67, and 2.66 to be applicable. If $(T_{mn})_{eyn} < (T_{mn})_{eym}$, then the strain increment must satisfy equation 2.64 in order for equations 2.65, 2.67, and 2.66 to be applicable.

Equation 2.65 can be written in the form

$$\begin{aligned}\Delta T_{mn} = \; & \frac{2G\Delta e_{mn} R_m R_n}{R_m R_n + \frac{GR_n}{\delta_m G_{sm}} + \frac{GR_m}{\delta_n G_{sn}}} + \\ & \frac{\frac{R_n G}{\delta_m G_{sm}}((T_{mn})_{eym}-T^t_{mn})(1-R_m)+\frac{R_m G}{\delta_n G_{sn}}((T_{mn})_{eyn}-T^t_{mn})(1-R_n)}{R_m R_n + \frac{GR_n}{\delta_m G_{sm}} + \frac{GR_m}{\delta_n G_{sn}}}.\end{aligned} \tag{2.68}$$

Note that equation 2.68 gives the result

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn} \tag{2.69}$$

when $R_m = 0$ and $R_n \neq 0$. When $R_m = 0$, joint set m is elastic-perfectly plastic, and equation 2.68 yields the desired result for ΔT_{mn}. If $R_n = 0$ and $R_m \neq 0$, then equation 2.68 yields

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}, \tag{2.70}$$

which is the desired result when joint set n is elastic-perfectly plastic. Equation 2.68 cannot be used when both R_m and R_n are equal to zero. It is not possible to write one equation in a simple form that produces the correct result for ΔT_{mn} for all of the variations that can arise for the present case.

2.3.5 Case 5: Elastic Shear Increment Exceeds the Effective Yield for Both Joint Sets and the Effective Yields are Equal

Now consider the case where the elastic shear stress increment exceeds the effective yield stress for both of the joint sets and the two effective yield stresses are equal. The first variation to consider for this particular case is one where both of the joint sets are elastic-perfectly plastic. For this variation,

$$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.71}$$

The slip displacement for joint set m is

$$\Delta u_{sm} = \delta_m \left(\Delta e_{mn} - \frac{\Delta T_{mn}}{2G} \right), \tag{2.72}$$

and the slip displacement for joint set n is

$$\Delta u_{sn} = \delta_n \left(\Delta e_{mn} - \frac{\Delta T_{mn}}{2G} \right). \tag{2.73}$$

This variation introduces equations for the slip displacement for the joint sets that have not appeared before.

The next variations to consider arise when one joint set is elastic-perfectly plastic and the other is elastic-plastic. These equations have appeared before, and the results for these two variations are only summarized.

- When joint set m is elastic-perfectly plastic ($G'_{sm} = 0$),

  $$\Delta T_{mn} = (T_{mn})_{eym} - T^t_{mn}. \tag{2.74}$$

  The slip displacement increment for joint set n is

  $$\Delta u_{sn} = \Delta T_{mn}/G_{sn}, \tag{2.75}$$

  and the slip displacement increment for joint set m is

  $$\Delta u_{sm} = \delta_m \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_n G_{sn}} \right). \tag{2.76}$$

- When joint set n is elastic-perfectly plastic ($G'_{sn} = 0$),

$$\Delta T_{mn} = (T_{mn})_{eyn} - T^t_{mn}. \tag{2.77}$$

The slip displacement increment for joint set m is

$$\Delta u_{sm} = \Delta T_{mn}/G_{sm}, \tag{2.78}$$

and the slip displacement increment for joint set n is

$$\Delta u_{sn} = \delta_n \left(2\Delta e_{mn} - \frac{\Delta T_{mn}}{G} - \frac{\Delta T_{mn}}{\delta_m G_{sm}} \right). \tag{2.79}$$

The final variation to consider for this case involves elastic-plastic behavior for both joints sets. Equations 2.65, 2.66, and 2.67 are applicable for this variation.

3. ANALYSIS OF PREDICTED SHEAR STRESS VALUES

Suppose that $(T_{mn})^{t+\Delta t}_{elastic}$ predicts that elastic behavior will occur for one joint set and elastic-plastic or elastic-perfectly plastic behavior will occur for the other joint set. When this situation arises, certain prescribed sets of equations are used to calculate the value for $T^{t+\Delta t}_{mn}$ that are significantly different from those used to predict $(T_{mn})^{t+\Delta t}_{elastic}$. The questions arises as to whether or not the equations accounting for nonlinear behavior for one joint set and linear behavior for the other joint set yield the same pattern as that predicted by the linear equation, i.e., do the equations accounting for nonlinear behavior also predict elastic behavior for one joint set and elastic-plastic (elastic-perfectly plastic) behavior for the other joint set? This is important for the computer implementation of joint shear stress model. If it can be shown mathematically that the elastic and nonlinear equations predict the same general behavior, then it is not necessary to add a check after the nonlinear calculations are made. The nonlinear calculations can be made with the knowledge that the pattern predicted by the linear results holds. No check is required to determine if another set of nonlinear equations must be employed, and the elimination of an unnecessary check will make the computer program more efficient.

This section shows that, when $(T_{mn})^{t+\Delta t}_{elastic}$ predicts elastic behavior for one joint set and nonlinear behavior for the other joint set, this pattern does hold when the nonlinear equations are applied. This proof can be carried out only for a particular shear behavior. The results in this section are limited to a particular shear behavior; all results prior to this section are for general shear behavior.

To examine the above question, assume that $(T_{mn})^{t+\Delta t}_{elastic}$ predicts elastic behavior for joint set n and some type of nonlinear behavior for joint set m. For this type of behavior for the two joint sets, it is known that

$$(T_{mn})^{t+\Delta t}_{elastic} \geq (T_{mn})_{eym}, \tag{3.1}$$

$$(T_{mn})^{t+\Delta t}_{elastic} \leq (T_{mn})_{eyn}, \tag{3.2}$$

and

$$(T_{mn})_{eym} \leq (T_{mn})_{eyn}. \tag{3.3}$$

First, consider the case where joint set m exhibits elastic-perfectly plastic behavior. The value for $T^{t+\Delta t}_{mn}$ is set to $(T_{mn})_{eym}$, which means that the value for the stress increment is

$$\Delta T_{mn} = (T_{mn})_{eym} - T^{t}_{mn}. \tag{3.4}$$

Because $T_{mn}^{t+\Delta t}$ is set to $(T_{mn})_{eym}$, the original inequality of $T_{mn}^{t+\Delta t} \le (T_{mn})_{eyn}$ holds. The original prediction of nonlinear behavior for joint set m and linear behavior for joint set n remains the same. This is true for either elastic-plastic or elastic-perfectly plastic behavior for joint set n.

Now consider the case where joint set m exhibits elastic-plastic behavior. It is known that $(T_{mn})_{elastic}^{t+\Delta t}$ is less than the value for $(Tmn)_{eyn}$; therefore it is possible to write the inequality

$$T_{mn}^{t} + \frac{\Delta e_{mn}}{\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}}} < (T_{mn})_{eyn}. \tag{3.5}$$

Equation 3.5 can be rewritten as

$$\Delta e_{mn} < ((T_{mn})_{eyn} - T_{mn}^{t}) \left(\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}} \right). \tag{3.6}$$

The equality used to relate the stress at time $t + \Delta t$ to the strain increment has the form

$$\Delta e_{mn} = \frac{\Delta T_{mn}}{2G} + \frac{T_{mn}^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G'_{sm}} + \frac{(T_{mn})_{eym} - T_{mn}^{t}}{2\delta_m G_{sm}} + \frac{\Delta T_{mn}}{2G_{sn}\delta_n}. \tag{3.7}$$

Equations 3.5 and 3.7 can be combined to produce the inequality

$$\frac{\Delta T_{mn}}{2G} + \frac{T_{mn}^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G'_{sm}} + \frac{(T_{mn})_{eym} - T_{mn}^{t}}{2\delta_m G_{sm}} + \frac{\Delta T_{mn}}{2G_{sn}\delta_n} < ((T_{mn})_{eyn} - T_{mn}^{t}) \left(\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}} \right) \tag{3.8}$$

If $G'_{sm} < G_{sm}$ (the joint shear stiffness model is a hardening model), then it is possible to write

$$\frac{T^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G_{sm}} < \frac{T^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G'_{sm}}. \tag{3.9}$$

This means that

$$\frac{T^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G_{sm}} \tag{3.10}$$

can be substituted for

$$\frac{T^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G'_{sm}} \tag{3.11}$$

in Equation 3.8 and the inequality holds. Equation 3.8 can now be written as

$$\frac{\Delta T_{mn}}{2G} + \frac{T_{mn}^{t+\Delta t} - (T_{mn})_{eym}}{2\delta_m G_{sm}} + \frac{(T_{mn})_{eym} - T_{mn}^{t}}{2\delta_m G_{sm}} + \frac{\Delta T_{mn}}{2G_{sn}\delta_n} < ((T_{mn})_{eyn} - T_{mn}^{t}) \left(\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}} \right), \tag{3.12}$$

which will simplify to

$$\left(T_{mn}^{t+\Delta t} - T_{mn}^{t}\right)\left(\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}}\right) <$$
$$\left((T_{mn})_{eyn} - T_{mn}^{t}\right)\left(\frac{1}{2G} + \frac{1}{2\delta_m G_{sm}} + \frac{1}{2\delta_n G_{sn}}\right). \tag{3.13}$$

Equation 3.13 yields the result

$$T_{mn}^{t+\Delta t} < (T_{mn})_{eyn}. \tag{3.14}$$

This derivation shows that the behavior for joint set n remains in the elastic range as long as $G'_{sm} < G_{sm}$. This is true for an elastic-plastic or an elastic-perfectly plastic model for behavior of joint set n. Once the calculation for ΔT_{mn} has been made, it is not necessary to determine if $T_{mn}^{t+\Delta t}$ is greater than the effective yield for joint set n as long as joint set m is a hardening model.

A similar analysis holds for the case where $(T_{mn}^{t+\Delta t})_{elastic}$ predicts elastic behavior for joint set m and nonlinear behavior for joint set n.

Therefore, it is possible to conclude that, when $(T_{mn}^{t+\Delta t})_{elastic}$ predicts elastic behavior for one joint set and nonlinear behavior for the other joint set for a hardening material, the pattern of joint shear stress behavior predicted by the linear equation will hold after the appropriate nonlinear equations are applied. It is not necessary to examine the general pattern of behavior after the nonlinear equations are applied.

4. COMPUTATIONAL IMPLEMENTATION OF SHEAR MODEL

In the previous section, five different cases were listed for the shear model. Some of these cases have a large number of variations because of the distinction between elastic-plastic behavior versus elastic-perfectly plastic behavior. Although there are a large number of variations, the shear model does lend itself nicely to structured programming. A question naturally arises as to whether or not the set of variations can be simplified in some way so as to decrease the complexity of a computational implementation of the shear model. This section discusses possible methods to simplify the computational implementation of the shear model.

4.1 Generalized Equations

One possible method to simplify the computational implementation of the shear model is by the use of generalized expressions for the shear stress increment. When the elastic shear stress increment $(\Delta T_{mn})_{elastic}$ indicates linear behavior for one joint set and nonlinear behavior for another joint set, it is possible to write one equation for ΔT_{mn} that is valid for both elastic-plastic and elastic-perfectly plastic behavior. Consider a case where the elastic shear stress increment predicts nonlinear behavior for joint set m and linear behavior for joint set n. Equation 2.29 yields valid results for ΔT_{mn} for both $G'_{sm} \neq 0$ and $G'_{sm} = 0$. Furthermore, only one set of equations is needed for the slip displacement increments. The slip displacement equations do not change for elastic-plastic versus elastic-perfectly plastic behavior for this particular case. For the case of linear behavior for joint set m and nonlinear behavior for joint set n, it is possible to write one equation for ΔT_{mn}, one equation for Δu_{sm}, and one equation for Δu_{sn}. A similar situation arises if the elastic stress increment predicts nonlinear behavior for joint set n and linear behavior for joint set m.

When the elastic shear stress increment predicts nonlinear behavior for both joint sets, it is not possible to write one set of equations to handle all the variations. It is possible to write the expression for the shear stress increment in a more general form, but such a form does not hold for all variations. The general expression for the shear stress increment when the elastic shear stress increment predicts nonlinear behavior for both joint sets is Equation 2.68. This equation for shear stress increment will be rewritten here as

$$\begin{aligned} \Delta T_{mn} = \; & \frac{2G\Delta e_{mn} R_m R_n}{R_m R_n + GR_n/(\delta_m G_{sm}) + GR_m/(\delta_n G_{sn})} + \\ & \frac{\frac{R_n G}{\delta_m G_{sm}}((T_{mn})_{eym} - T^t_{mn})(1 - R_m)}{R_m R_n + GR_n/(\delta_m G_{sm}) + GR_m/(\delta_n G_{sn})} + \end{aligned}$$

$$\frac{\frac{R_m G}{\delta_n G_{sn}}((T_{mn})_{eyn} - T^t_{mn})(1 - R_n)}{R_m R_n + G R_n/(\delta_m G_{sm}) + G R_m/(\delta_n G_{sn})}. \tag{4.1}$$

This equation is valid as long as either G'_{sm} or G'_{sn} is not equal to zero. If both of these parameters are zero, the above equation yields three terms with zero divided by zero. Therefore, this equation is not useful if both the joint sets are elastic-perfectly plastic.

Furthermore, for the case of nonlinear behavior for both joint sets, there is no one set of equations for the slip displacement increments that covers all the variations. The equations for the slip displacement increments change significantly depending on a particular variation.

In general, then, for the case of nonlinear behavior for both joint sets, it is not possible to write one generalized set of equations in a straightforward manner to cover all of the variations. Some type of branching is required to ensure that the correct equations are used. Otherwise, some very general expressions containing all the variations would need to be written for the slip displacement. These general expressions would also have to include parameters that would ensure that only the correct components would contribute to the slip displacement calculations.

4.2 Adjusting Values of G'_{sm} and G'_{sn} to Approximate Elastic-Perfectly Plastic Behavior

It is possible to eliminate the elastic-perfectly plastic variations by specifying that only nonzero values of G'_{sm} and G'_{sn} can be used for input for the model. To obtain a good approximation to an elastic-perfectly plastic shear stress versus slip displacement behavior for joint set m, for example, it is necessary to specify a "small" value for G'_{sm} as compared to the value of G_{sm}. The definition of "small" varies with the value of G_{sm}. For some given value of G_{sm}, a ratio of G'_{sm} to G_{sm} on the order of 10^{-3} can produce a reasonable approximation to an elastic-perfectly plastic curve for the shear behavior of joint set m. If G_{sm} is larger, it is necessary to adjust the value for G'_{sm} if a close approximation of an elastic-perfectly plastic curve is still desired. Some decision must be made as to how closely an elastic-perfectly plastic curve is to be modeled, and the value for G'_{sm} must be set on the basis of the value of G_{sm}. One fixed ratio between G'_{sm} and G_{sm} will not ensure a good approximation of an elastic-perfectly plastic curve for all values of G_{sm}.

While it is possible to specify small values for the ratios R_m and R_n to closely approximate elastic-perfectly plastic behavior, it may not be desirable in terms of the conditioning of various equations from a computational standpoint. The effects of approximating elastic-perfectly plastic behavior by the use of "small" values for G'_{sm} and G'_{sn} are investigated in the following sections. These studies are based on Sample Problem 2 in the reference by Chen (1987). For this particular problem, a block with orthogonal joint sets undergoes pure shear. The block is 10 in. by 10 in. The rock matrix has a bulk modulus of $K = 667.0$ ksi and a shear stiffness of $G = 400.0$ ksi. The half closure stress A_k is 1.0 ksi, the maximum closure $(u^d_{max})_k$ is -0.003 in., the coefficient of friction μ_k is 0.7, the cohesion C_{0k} is 0.25 ksi, and the joint spacing δ_k is 0.5 in., where $k = m, n$. The joint shear stiffness for both joint sets is 100.0 ksi/in. Set the joint shear hardening for joint set m to 1.0 psi/in. and for joint set n to 1.0 ksi/in. The ratio of the joint shear hardening to the joint shear stiffness for joint set m is 1.0×10^{-5}.

Suppose that the value for the shear stress T_{mn} at the current time is 385.0 psi, the shear yield stress for joint set m is 390.0 psi, and the shear yield stress for joint set n is 400.0 psi. If there is a strain increment of 0.0005 in./in., then the predicted value for the shear stress increment T_{mn} is 23.5294 psi.

Before using the equations for elastic-plastic behavior, calculate the shear stress increment and slip displacement increments by treating joint set m as elastic-perfectly plastic, i.e., use a value of zero for G'_{sm}. For elastic-perfectly plastic behavior, the shear yield stress for joint set m sets the limit for the value for the shear stress at time $t + \Delta t$. The value for $T^{t+\Delta t}_{mn}$ is 390.0 psi and the shear stress increment ΔT_{mn} is 5.0 psi. The slip displacement for joint set m is 4.4375×10^{-4} in., and the slip displacement for joint set n is 5.0×10^{-5} in. The shear strain for the rock matrix is 6.25×10^{-6}. The shear strain for the rock matrix plus the shear strains for both joint sets sum to 0.0005 in./in., which is to be expected.

Calculate the above values using the equations for elastic perfectly plastic behavior. The elastic shear stress increment predicts that both joints should behave in a nonlinear manner. The shear strain increment is not large enough, however, to cause nonlinear behavior for both joint sets. Therefore, it is necessary to use Equation 2.22 to calculate the shear stress increment. This equation is valid for nonlinear behavior for joint set m and linear behavior for joint set n. The stress increment predicted by this equation is 5.0003937 psi, which is very close to the value of 5.0 psi predicted for the elastic-perfectly plastic case. The slip displacement for joint set m for the elastic-plastic case is 5.0003937×10^{-5} in.; the slip displacement for joint set n is 4.4374557×10^{-4} in. These slip displacements are very close to those predicted by the equations for the elastic-perfectly plastic case. This particular example indicates that an elastic-perfectly plastic material can be modeled as an elastic-plastic material with a high degree of accuracy.

4.3 Conclusions

The discussion in Section 2 shows that a great number of variations must be considered when implementing a jointed rock model that allows for nonlinear shear behavior for both joint sets. The shear model can be implemented in a structured manner because it is possible to determine the correct equations to use for the shear calculations based on some simple tests. The first test involves the calculation of $(T_{mn})^{t+\Delta t}_{elastic}$, which is an estimate for the shear stress at time $t + \Delta t$ based solely on elastic properties of the joint sets. If the value for $(T_{mn})^{t+\Delta t}_{elastic}$ is larger than the effective yield stress for joint set m and the effective yield stress for joint set n, then the behavior of both joint sets will be elastic. If the value for $(T_{mn})^{t+\Delta t}_{elastic}$ is larger than the effective yield stress for one joint set and smaller than the effective yield stress for the other joint set, then this pattern will hold when the appropriate set of nonlinear equations is used to calculate $T^{t+\Delta t}_{mn}$. A proof of this is given in Section 3. Finally, if $(T_{mn})^{t+\Delta t}_{elastic}$ is larger than the effective yield stresses for both joint sets, then it may be necessary to make another test to determine the correct set of equations to use. If $(T_{mn})_{eym} \neq (T_{mn})_{eyn}$ and one or both of the joint sets is elastic-plastic, then it is necessary to determine if the strain increment Δe_{mn} is large enough to make both joint sets behave in a nonlinear manner. This check is made by using equation 2.49 or equation 2.64. If the appropriate inequality is satified, then both joint sets will exhibit nonlinear behavior.

This latter section has examined possible ways to simplify the system of equations describing the nonlinear shear behavior for the joint sets. One possible approach to simplification is the use of generalized equations. It is possible to write generalized equations for two of the cases (nonlinear behavior for one joint set and linear behavior for the other joint set), but not for the cases where both joint sets behave in a nonlinear manner.

Another approach is to eliminate the equations for elastic-perfectly plastic behavior by approximating elastic-perfectly plastic behavior through the use of a "small" value for the joint shear hardening coefficient. Based on the sample calculations presented in this report, this appears to be a reasonable approach to simplifying the system of equations. It appears an elastic-perfectly plastic model can be approximated to a high degree of accuracy by an elastic-plastic model with the appropriate choice of the shear hardening modulus. If the joint sets are modeled only as elastic-plastic, the number of variations is significantly reduced and full generality is retained for all practical purposes since elastic-perfectly plastic situations can be closely approximated.

The simplification of the model with the use of only elastic-plastic joint sets and the tests for predicting the nature of the nonlinear behavior of the joint sets opens the possibility of a vectorized computational implementation of the shear model. Because the material model is called repeatedly throughout a computer analysis, the time spent within a material model can be quite significant. Considerations of efficient implementation and vectorization are important for production use of a material model. The information presented in this report is useful for an efficient computational implementation of the jointed rock model and for future consideration of a vectorized version of the jointed rock model. Vectorization of the jointed rock model in general is an important area for future investigation.

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Appendix A
Information from, and Candidate Information for, the SEPDB and the RIB

Information from the Reference Information Base

Used in this Report

This report contains no information from the Reference Information Base. The material properties are assumed values taken in the spirit for which verification problems are performed. However, the values used in this report are representative of average rock mass properties that can be found in the Reference Information Base for the Yucca Mountain site.

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Reference Information Base

This report contains no candidate information for the Reference Information Base.

Candidate Information for the

Site and Engineering Properties Data Base

The report contains no candidate information for the Site and Engineering Properties Data Base.

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