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INTERPRETATION OF TWO-DIMENSIONAL CHARACTERISTICS OF STOCHASTIC, FRACTAL, HETEROGENEOUS, POROUS MEDIA MODELS FROM ONE-DIMENSIONAL SAMPLING

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Abstract

Two-dimensional (2D) stochastic fractal models of heterogeneous porous media were created to provide complex representations of porous media which yield velocity distributions that approach the advective variability observed in field systems. These models were used to investigate the application of sparse one-dimensional (1D) sampling to characterize the two-dimensional hydraulic conductivity (K) distributions. Steady-state flow was simulated in the binary K fields of the models, and yielded 2D velocity fields for analysis.

Texture of the generated porous media (distribution of K) was varied by creating two models with different lacunarities, but the same fractal dimension. Lacunarity and fractal dimension of the generated porous media models were calculated. Two methods of analyzing lacunarity of the porous media models were used (one method follows Mandelbrot (1983), the other was developed in this research).

The properties of 1D profiles of hydraulic conductivity and velocity (as could be obtained from boreholes) were calculated and examined to determine if they could differentiate the 2D properties of the porous medium models. Fractal dimension of the 1D hydraulic conductivity profiles was determined by the box-flex counting method. Four methods of fractal analysis of velocity profiles were used (box-flex counting, rescaled range, spectral density, and threshold value), and one method of non-fractal analysis (coefficient of variation) was employed.

The two textures were best differentiated by the lacunarity measurement developed in this research. Lacunarity of 1D samples through the K field calculated by the method developed in this research differentiated the 2D models. Measurement of fractal dimension of the velocity profiles yielded inconsistent results in differentiating texture (two methods were successful in differentiating texture, one needs further development, and the final method was not sensitive to differences in texture).

Integration of the hydraulic conductivity and velocity profiles will provide the most information about the nature of the 2D hydraulic conductivity fields. The K profiles indicate location and size of the heterogeneities in the porous medium models, while the velocity profiles reveal the connectedness of high K zones. This information is valuable for generation of representative stochastic porous medium models from sparse data samples.

Introduction

The discontinuity or connectedness of high hydraulic conductivity zones is perhaps the single most influential feature that affects contaminant transport; however, it is extremely difficult to map from data gathered from sparse boreholes. While thorough geologic mapping may identify connected zones of high K at the large scale, much of the heterogeneity critical to contaminant transport at the site scale is unmappable with currently available techniques. Most of the small-scale heterogeneity is due to micro-structures and almost imperceptible variations in packing, grain-size distribution, clay content, or degree of cementation (Freeze and Cherry, 1979).

The purpose of this project was to develop a method for characterizing porous medium heterogeneity from sparse data. Such a method will reduce the cost of data required to model an porous medium by improving our knowledge of the nature of media between boreholes based on data gathered in boreholes. The study presumes that vertical profiles of horizontal velocities are available for analysis. Currently such profiles cannot be measured at the desired level of detail in the field. This study explores the potential usefulness of such profiles before effort is invested in developing technology to measure velocity profiles.

A Monte Carlo procedure was used to create two-dimensional (2D) stochastic fractal porous mediums with binary K fields. These models were intended to provide complex representations of K fields whose properties (degree of porous medium heterogeneity, connectedness of high K zones), would be similar to the variability of sediments in a field setting, thus facilitating the extension of this research to field situations. The use of stochastic fractal porous medium models also creates the opportunity for future research into modeling of solute transport via differential advection rather than with the classical advection-dispersion equation.

This entire work is discussed in greater detail by Zlatev (1991). For the benefit of those who have not worked with fractals, the following brief descriptions are provided.

Fractal Geometry

Fractal geometry is a branch of mathematics that is well suited to quantifying the patterns found in nature (Mandelbrot, 1967, 1983 and many others; Feder, 1988). Fractal shapes are self-similar; that is, when magnified, a small portion of the shape is identical to the larger piece. For natural objects (e.g. porous media), this self-similarity is a statistical self-similarity. Fractal objects have dimensions that are greater than or equal to their topological dimension (D_T), less than or equal to their embedding Euclidean dimension (E), and are not necessarily an integer (e.g. a jagged line has a fractal dimension, D_F, between 1 and 2; a rougher line will "fill" more 2D space and will have a D_F closer to 2). Intersections through stochastic fractal objects produce integer reductions in the value of D_F; for example, a volumetric object with $D_F = 2.6$ embedded in three-dimensional space, will have $D_F = 1.6$ when sectioned by a plane, and $D_F = 0.6$ when intersected by a line.

Lacunarity

Lacunarity is an indication of the texture of a fractal object. Lacunarity of an object is related to the size of zones within the object; an object of high lacunarity has larger zones, while in an object of low lacunarity, zones are more disseminated. Two objects with very different texture (lacunarity) can have identical fractal dimensions. Similarly, two different porous media (e.g. sandstone and siltstone) may have the same fractal dimension but different lacunarities. Although measurement of D_F does not seem to differentiate materials of differing hydraulic conductivity (Barton et al., in press), we expect that lacunarity will be a better measure for such differentiation

Generation of Hydraulic Conductivity and Velocity Fields

Stochastic fractal porous medium models were created by calculating the size-frequency distribution of the low K squares for each generation, and randomizing their location in the initiator square (Figure 1). Two fractal models (one of high and one of low lacunarity, Figure 1) were chosen to investigate the ability to make predictive statements about the fractal character of the 2D model from interpretations of the 1D profile data. Five realizations of each model were examined in this study, their characters are similar but the location of heterogeneities vary as determined by the seed which is used to begin the Monte Carlo process. In keeping with the recursive algorithms of fractal geometry, the larger squares (first generation) were removed at a random location first, then squares of successive generations were randomly removed to create the third-generation fractal models that were used as binary K fields. In order to maintain a constant value of fractal dimension, squares were not allowed to overlap.

These hydraulic conductivity distributions were used as input to a 2D steady-state ground water flow model with fixed heads at each side to generate the hydraulic head distribution. The hydraulic conductivity and head data were used to calculate the velocity field using Darcy's Law. Some examples of hydraulic conductivity and velocity profiles, taken from models of different lacunarity, are illustrated in Figure 2.

The velocity profiles sampled from the high and low lacunarity models exhibit different character (Figure 2). The velocity profiles from the high lacunarity models typically had large plateaus of high and low velocity, with abrupt, but fewer changes from high to low velocity. The size of the lenses were larger, but there were fewer lenses, so the velocity profiles reflected a model that had larger zones of constant hydraulic conductivity. Large changes

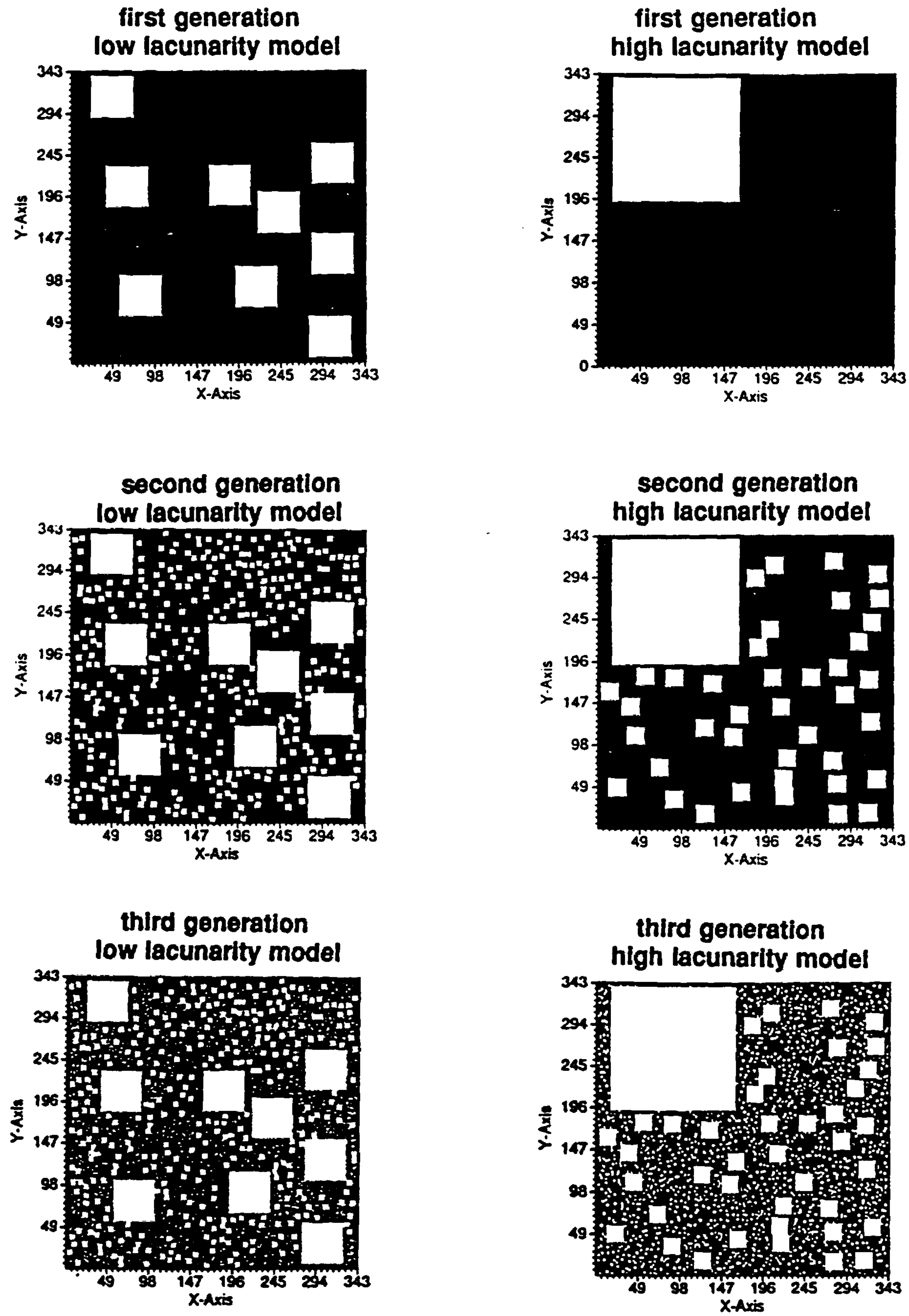

Figure 1 Generation of stochastic fractal porous media models

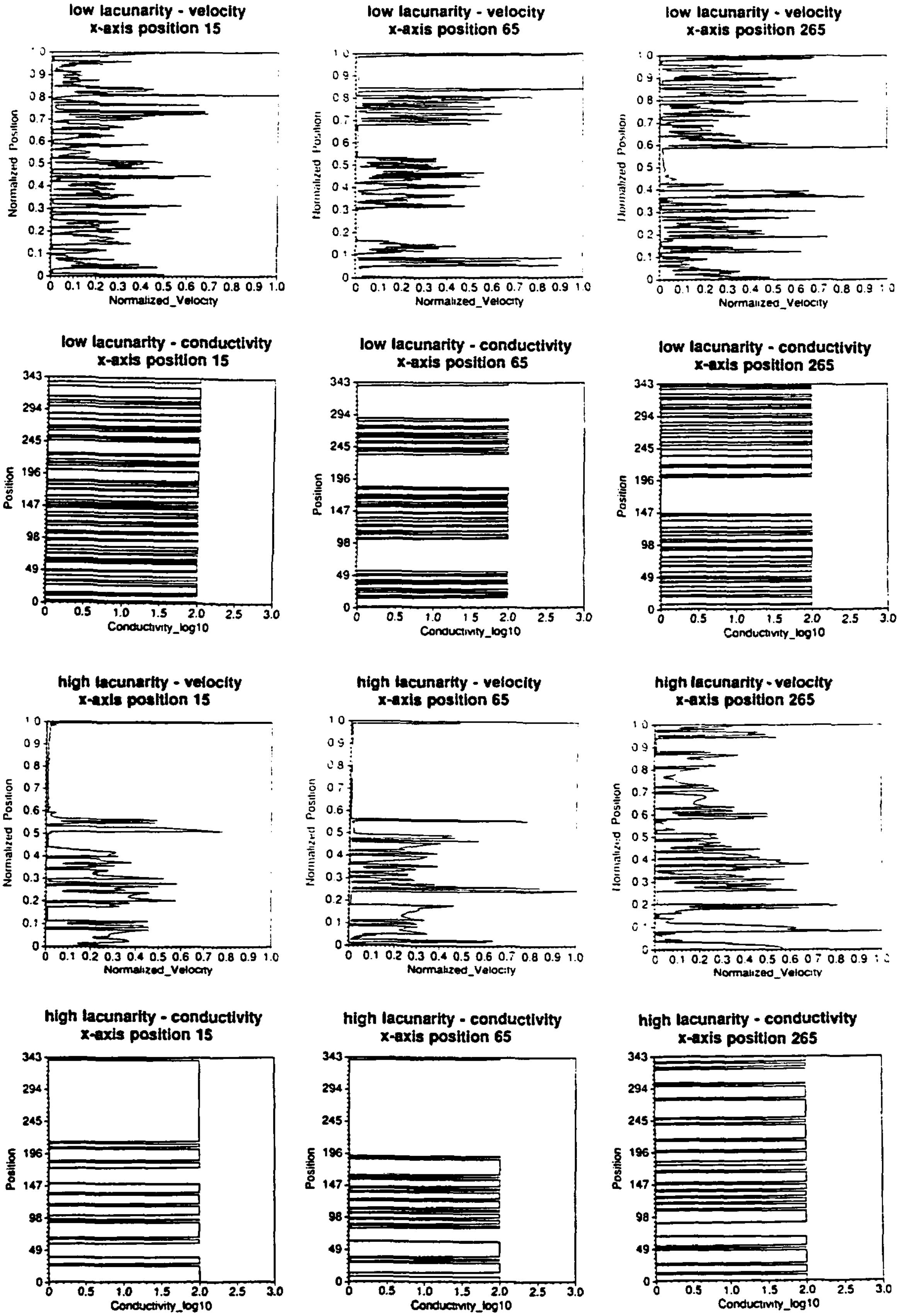

Figure 2 Profiles of hydraulic conductivity and velocity from high and low lacunarity models

in the velocity profiles were commonly due to a feature that was directly intersected by the profile itself; however, if the profile was taken proximal to a low K lens, the velocity reflected that condition. This phenomenon is the key to using velocity profiles as clues to the connectivity of units between holes. The effects of larger lenses were more easily recognized at greater distances than the effects of the many smaller lenses.

Velocity profiles from the low lacunarity models were more irregular; the zones of high and low velocity were thinner, and the changes between them were more gradual, reflecting the presence of small lenses that were not intersected by the profile. The size of the lenses were smaller, relative to the lenses of the high lacunarity model, but there were more lenses.

Fractal Dimension of the 2D Hydraulic Conductivity Fields

The similarity dimension is defined as:

$$D_s = \log(N(r) / \log)(1/r) \qquad (1)$$

where:

N(r) = number of boxes of size r
r = size of boxes

D_s is an approximation to the fractal dimension that considers the self-similarity that define fractals. While typically used for estimating the fractal dimension of mathematically exact fractals (Feder, 1988, Falconer, 1990), similarity dimension was used to estimate the fractal dimension of the stochastic fractal models. For all two-dimensional fractal models in this research, $D_s = 1.896$.

Lacunarity of the 2D Hydraulic Conductivity Fields

Cantor dust is a fractal, broken line, with segments of the line being of variable length. A line formed by segments of the length of high hydraulic conductivity segments along vertical profiles in the porous media models forms a Cantor dust model. Mandelbrot (1983) proposed that a ratio of the measured mass to the expected (average) mass be used as a method to define the lacunarity of a Cantor dust:

$$L = E\left[\frac{W}{E(W)} - 1\right]^2 \qquad (2)$$

where:

L = value of the lacunarity
W = mass (e.g. length of a segment)
E(W) = expected (average) mass of a segment.

To calculate lacunarity of the 2D hydraulic conductivity fields, first, the 2D model was read as a series of vertical hydraulic conductivity profiles. Along these profiles, the size-frequency distribution of the mass "segments" (lengths of connected high K) was recorded. For each profile, the summation of the mass segment values (sum of the mass segment size multiplied by its frequency of occurrence) was divided by the total number of mass segments in each profile to yield the expected mass (E(W)) of segments for that profile. The lacunarity of each profile was calculated using equation 2. Two-dimensional lacunarity was the average of lacunarity for all the 1D profiles. Lacunarity values calculated using this method did not show significant differences between high and low lacunarity 2D models. The average lacunarity calculated for the high lacunarity models was 0.8 (ranging from 0.76 to 0.83), while average lacunarity for low lacunarity models was 0.75 (ranging from 0.74 to 0.76). Although the lacunarity calculated by equation 2 does differentiate the models, a more sensitive measure was desired.

Therefore, another method, which parallels Mandelbrot's equation, was developed to calculate lacunarity. Mandelbrot (1983) explains that a less lacunar (lower lacunarity) object should appear more homogeneous (i.e. a smaller area can be taken for a representative sample). It follows that the mass segments of a less lacunar object would have smaller variance than a higher lacunarity object. Consequently the variance of the mass segments along a 1D profile was calculated, and these 1D lacunarities were averaged to estimate lacunarity of the 2D model. The equations used are:

$$L1 = E[\,W - E(W)\,]^2 \quad \text{(one-dimensional)} \qquad (3)$$

and

$$L2 = E(L1) \quad \text{(two-dimensional)} \qquad (4)$$

where:

L1 = lacunarity of one-dimensional profiles
W = mass of segment along profile
E(W) = expected (average) mass of segments along profile
L2 = expected (average) lacunarity of profiles for two-dimensional model.

The results of these calculations distinctly separate the high and low lacunarity models. Average lacunarity for the high lacunarity models was 133 (ranging from 122 to 152) and average lacunarity calculated for the low lacunarity models was 16.7 (ranging from 16.5 to 16.9). A comparison of the average lacunarity values calculated by this method for the 2D models shows approximately an 8:1 ratio. The difference between the two approaches is that Mandelbrot's equation is normalized by the expected mass of the profile, while the latter approach does not involve normalization. The absolute differences in the profiles are important, therefore normalization is not advantageous in this situation. Defining the lacunarity calculation for 1D and 2D objects as the variance of the size of the mass segments yields a simple, sensitive equation for examining texture.

Analysis of One-dimensional Profiles

The purpose of these analyses was to make predictive statements about the 2D distribution of heterogeneities and the lacunarity of the hydraulic conductivity field from 1D data samples. One-dimensional profiles of velocity and hydraulic conductivity were taken from the hypothetical fractal models to simulate the information that could be gathered from boreholes in the field. It was hypothesized that the velocity field generated by steady-state flow through a random, fractally distributed K field may be fractal, consequently the 1D velocity profiles were analyzed for their fractal characteristics and compared with the properties of the 2D hydraulic conductivity fields. The coefficient of variation of the 1D velocity profiles (a non-fractal method) was also analyzed as a possible tool for differentiating the models.

A series of seven profiles (separated by intervals of 50 nodes) were taken through the third generation 2D models. The 1D velocity profiles were taken transverse to the gradient within the 2D steady-state flow field; K profiles were taken at the same node locations as the velocity profiles. The high K zones of each of the seven profiles of the ten realizations were recorded as Cantor dust sets for analysis, while the velocity profiles were line traces.

Fractal dimension of hydraulic conductivity profiles by box-flex counting method

The fractal dimension of the Cantor dust (for each profile) was estimated by the box-counting method using DIMENSIO (Barton, et al, 1988). The average fractal dimension of many 1D Cantor dusts created by intersections taken through a 2D fractal object should be one integer value less than the 2D fractal dimension of the object. The average box-flex fractal dimensions for seven profiles of the high K Cantor dusts was 0.846 for the high lacunarity models and 0.863 for the low lacunarity models. The calculated similarity dimension for the 2D models was 1.896. The difference in fractal dimensions is probably due to the fact that seven profiles are insufficient to obtain a representative average. Analysis of the fractal dimension of every profile in the model is currently being pursued to verify this interpretation. The average D_F of a few sections is likely to be less representative of the 2D D_F in the high lacunarity model because a strong bias can be obtained by having more than a representative portion of the profiles pass through the first generation lens. It is expected that Cantor dusts sets of K from a few boreholes in a field setting will be insufficient to obtain an accurate estimate of the fractal dimension of two- or three-dimensional porous media between boreholes. However the possibility continues to be explored.

Lacunarity of hydraulic conductivity profiles by the variance method

Differences in the calculated lacunarity between profiles within a model are small when the non-normalized method is utilized. Lacunarity of the seven profiles averaged 142 (ranged from 114 to 172) for the high lacunarity model and 13.5 (ranging from 8 to 25) for the low lacunarity model. Because these values are so similar to those computed by evaluating the entire 2D model (133 (122-152) for high lacunarity and 16.7 (16.5-16.9), it can be expected that the lacunarity of Cantor dusts sets of K from a few boreholes in a field setting will be sufficient to obtain a reasonable estimate of the lacunarity of two- or three-dimensional porous media between boreholes.

Fractal dimension of velocity profiles by box-flex counting method

The box-flex counting method in DIMENSIO (Barton et al, 1989) was used to analyze the velocity profiles. Because the velocity profiles are self-affine curves, they had to be normalized (scaled) to fit in a unit square before being analyzed. Two methods were chosen to normalize the velocity profiles: (1) normalize the velocity values to the maximum velocity of each profile, and (2) normalize the velocity values to an arbitrary constant equal to or greater than the maximum velocity expected for any profile. In both cases, the node positions were normalized to the maximum node value.

Although differences between the average values of D_P are small, velocity profiles taken from the lower lacunarity 2D models exhibit a higher average calculated fractal dimension for both normalization methods. This was expected because the lower K zones are more dispersed throughout the low lacunarity models; so, the obstructions to flow create more zones of alternating high and low flow velocities, resulting in a "rougher" plot (i.e. higher fractal dimension) of the velocity profile. Normalizing the velocity profiles to a large constant value (e.g. maximum value at a site) yields greater separation between the high and low lacunarity models (velocity profiles from high lacunarity models had an average fractal dimension of 1.47 {range from 1.44 to 1.49} and the low lacunarity model velocity profiles had an average fractal dimension of 1.51 {range from 1.50 to 1.52} as compared with normalizing to the largest value in each profile where the high lacunarity models had an average of 1.54 {range from 1.53 to 1.55} and the low lacunarity had an average of 1.56 {range from 1.55 to 1.59}. Use of the constant normalizing value allows differentiation of the variability between profiles at a site as opposed to relative variability within individual profiles. A comparison of the results of the different methods of analysis presented here is presented in the Discussion section.

Fractal Dimension of velocity profiles by rescaled range analysis

Another method used to examine velocity profiles was rescaled range analysis. This method was developed by Hurst (1951) to calculate the minimum reservoir capacity required to prevent flooding during wet periods and drought during dry periods. From a time series of annual outflow from a lake, Hurst calculated a mean outflow value, and a cumulative sum of departures from the mean. The range was calculated as the difference between the maximum and minimum values of the sum of departures, for a given length of time (also called lag). Hurst observed that the range increased with lag; longer periods of time would include observations and trends that would increase the range of the sum of departures. To compare with other data (e.g. tree rings, varve thicknesses), Hurst divided the range by the standard deviation, yielding a rescaled range (R/S). When these rescaled range values were plotted against the lag (s) (which could be time or distance) on logarithmic graphs, the points fell on a straight line (a power function). Hurst (1951) observed that the rescaled range was proportional to s^H, where H is the slope of the graph, and that H ranged from 0.69 to 0.80. Mandelbrot and Wallis (1969a) introduced a method to simplify and present the data from the rescaled range analysis called pox diagrams. To create pox diagrams, the R/S value is plotted for a variety of starting values in the record, for a series of lag values. This creates a reduced data set that is more manageable, but retains the necessary information.

The average H values for the velocity profiles were calculated from a least-squares fit line through the pox diagram for each profile. The fractal dimension of the self-affine velocity profiles can be estimated from the value of the Hurst exponent as (Feder, 1988)

$$D_P = 2 - H \qquad (5)$$

For the calculated values of H, this yields values of D_P ranging from 1.25 to 1.32 (average 1.28) for the velocity profiles from the high lacunarity model, and from 1.25 to 1.34 (average 1.30) for the velocity profiles from the low lacunarity fractal.

Fractal dimension of velocity profiles by spectral analysis

Another method used to examine the velocity profiles was spectral analysis. The self-affine velocity profiles were characterized as single-valued random functions in space (velocity as a function of node position). These curves were analyzed as functions of stochastic processes using the theories of fractional Brownian motion and fractional Gaussian noise models developed by Mandelbrot and Van Ness (1968).

Ordinary Brownian motion ($B(t)$) is random. Mandelbrot and Van Ness (1968) introduced the concept of fractional Brownian motion ($B_H(t)$) as exhibiting persistent or antipersistent behavior. The parameter H can be divided into three subsections, $0 < H < 0.5$, $H = 0.5$, and $0.5 < H < 1$, which have different effects on the function $B_H(t)$. For $H = 0.5$, fractional Brownian motion becomes ordinary Brownian motion, and $B_H(t) = B(t)$. For the interval $0 < H < 0.5$, the behavior of $B_H(t)$ is termed "antipersistent". An antipersistent $B_H(t)$ function switches directions often; if the curve is increasing for one increment, it is likely to decrease in the next. For the interval $0.5 < H < 1$, the behavior of $B_H(t)$ is termed "persistent". A persistent $B_H(t)$ function does not switch directions as frequently; if the curve is increasing for one increment, it is likely to continue increasing in the next increment.

Although neither ordinary Brownian motion or fractional Brownian motion have derivatives, a semblance of a derivative can be created by taking first differences. In the case of ordinary Brownian motion, taking first differences creates white Gaussian noise; similarly, for fractional Brownian motion, taking first differences creates fractional Gaussian noise. While the trace of fractional Brownian motion is non-stationary (i.e. exhibits trends), taking first differences removes the trends. The relationship between fractional Brownian motion and fractional Gaussian noise is useful in calculating the value of the parameter H from the power spectral density graph. Mandelbrot and Wallis (1969b) present that $B_H(t)$ has a spectral density proportional to $f^{(-2H-1)}$ (always a negative slope between 1 and 3). Differentiating this yields $B_{H'}(t)$ proportional to $f^{(1-2H)}$ (slope between -1 and 1) (Lundahl, 1986). If the slope (ß) of the power spectral density of fractional Brownian motion is plotted versus frequency, then H can be calculated from

$$H = \frac{\beta - 1}{2} \tag{6}$$

Similarly for first differences of fractional Brownian motion (fractional Gaussian noise), the slope of the logarithmic plot of power spectral density versus frequency yields the relationship

$$H = \frac{1 - \beta}{2} \tag{7}$$

The fractal dimension of the fractional Brownian motion and fractional Gaussian noise can be calculated from equation 6. Therefore, for fractional Brownian motion

$$D_F = \frac{5 - \beta}{2} \tag{8}$$

and for fractional Gaussian noise

$$D_F = \frac{3 + \beta}{2}. \tag{9}$$

Taking first differences removes the trends inherent in traces of fractional Brownian motion, therefore, the calculated values for H from the data sets of fractional Gaussian noise should be closer to the actual H than the calculated H from the data sets of the traces of fractional Brownian motion.

The velocity profiles were analyzed as traces of fractional Brownian motion, and first differences of the velocity profiles were analyzed as fractional Gaussian noise. Equations 6 and 7 were used to calculate the value of the Hurst exponent from the slope of the logarithmic plot of power versus frequency. The velocity profiles were not normalized, because normalizing would have no effect on the slope of the plots. The occurrence of calculated values of H outside of the valid range of the Hurst exponent ($0 < H < 1$), and the low values for the coefficient of determination (R^2) for the least-squares fit indicates that there are problems in modeling velocity profiles as fractional Gaussian noise or traces of fractional Brownian motion. Further consideration of this analysis is included in the Discussion section.

Fractal dimension of velocity profiles by threshold analysis

Another method of analysis was to section the normalized velocity profiles at a set threshold value. This yielded a Cantor dust of regions (line segments) of velocity values greater than the threshold value. The velocity profiles were normalized by a large constant value, and sections were taken at the 0.1, 0.2 and 0.3 thresholds. Only the threshold line segments at 0.1 were used, because the higher threshold values yielded too few data for analysis. Velocity profile sections from the higher lacunarity fractal model yielded lower estimated fractal dimensions (average 0.71, range 0.66 to 0.73), because the curves were more "persistent" (i.e. tending to remain above or below a threshold for a larger number of data points). The velocity profile sections from the lower lacunarity models yielded a higher fractal dimensions (average 0.76, range 0.75 to 0.77) because of the greater number of crossings of the threshold value (i.e. less persistent).

Coefficient of variation of velocity profiles

Coefficient of variation measures variation in terms of the size of the expected value; it is the standard deviation of each profile divided by its mean.

$$\tau = \frac{\sigma}{\mu} \tag{10}$$

where:

τ = coefficient of variation
σ = standard deviation
μ = mean

Values of the coefficient of variation do not consistently differentiate velocity profiles from the different lacunarity models: an average of 1.25 for high lacunarity models and 1.24 for low lacunarity models. However, the range of values of the coefficient of variation for the velocity profiles from the lower lacunarity model is smaller and the variability of the coefficient of variation among boreholes at a site may prove to be a useful characterizing parameter (range of coefficient of variation for high lacunarity profiles is 1.18 to 1.37 and the range for low lacunarity models is from 1.22 to 1.27).

Summary of Analysis Results

Characteristics of 1D hydraulic conductivity and velocity profiles were analyzed to evaluate their potential use for characterizing the 2D hydraulic conductivity distribution between boreholes. Table 1 summarizes the values determined in the analyses. Details of the analyses are presented by Zlatev (1991).

Table 1. Summary of Analyses

				High Lacunarity Models		**Low Lacunarity Models**	
Dimension	**Property**	**Characteristic**	**Method**	**Average**	**Range**	**Average**	**Range**
2D	K	D_F	D_s	1.896	-	1.896	-
1D	K	D_F	box-flex	0.846	-	0.863	-
2D	K	Lac	Norm	0.8	0.76-0.83	0.75	0.74-0.76
2D	K	Lac	Non-norm	133.	122-152	16.5	16.7-16.9
1D	K	Lac	Non-norm	142.	114-172	13.5	8-25
1D	V	D_F	box-flex	1.47	1.44-1.49	1.51	1.50-1.52
1D	V	D_F	rescaled	1.28	1.25-1.32	1.30	1.25-1.34
1D	V	D_F	spectral	-	-	-	-
1D	V	D_F	thresh	0.72	0.71-0.73	0.76	0.75-0.77
1D	V	Coef of Var	-	1.18	1.25-1.37	1.24	1.22-1.27

Hydraulic Conductivity Profiles

Given the results of analyses conducted to date, it appears Cantor dusts sets of K from a few boreholes in a field setting will be insufficient to obtain an accurate estimate of the fractal dimension of two- or three-dimensional porous media between the holes. However, we expect that the lacunarity of Cantor dusts sets of K from a few boreholes in a field setting will be sufficient to obtain a reasonable estimate of the lacunarity of two- or three-dimensional porous media between the holes. Because field settings will not have binary hydraulic conductivity fields, this analysis will have to be conducted with regard to hydraulic units above and below selected threshold values. Use of the method of calculating lacunarity developed herein (without normalizing) was found to better differentiate porous media texture. There was more variability of lacunarity between sampled 1D profiles from the model with a high lacunarity of the 2D hydraulic conductivity distribution. This is because the higher lacunarity model has larger homogeneous units and as a result a larger portion of such a model must be examined before the characteristics of the entire model can be identified. Assessment of such variability may be useful in characterizing porous media.

Velocity Profiles

The box-flex counting method for determining fractal dimension of the velocity profiles provided the most consistent relationship between fractal dimension for the velocity profile and lacunarity of the 2D K field. Two normalization schemes were used to fit the self-affine curves to the format for the box-flex method; the 2D models of different lacunarity were better differentiated by D_F from velocity profiles normalized to a constant maximum value of velocity at the site, as contrasted against normalization to the maximum velocity in each profile.

The rescaled range method of determining fractal dimension of the velocity profiles yielded an inconsistent relationship between fractal dimension and 2D lacunarity of the K field. As was anticipated, the average values of fractal dimension of the velocity profiles sampled from the high lacunarity models were smaller than those from the low lacunarity models. However, the results were inconsistent because the maximum and minimum average values of fractal dimension from individual profiles for the high and low lacunarity models overlapped. If the rescaled range method was used to determine fractal dimensions of velocity profiles from a field site, it would not be possible to differentiate the texture of the porous media. The problem is believed to be due to the lack of sensitivity in rescaled range measurements to fractal dimension.

Fractal dimension of the velocity profiles was also analyzed by the spectral density method as traces of fractional Brownian motion and fractional Gaussian noise. The occurrence of calculated values of H outside of the valid range of the Hurst exponent ($0 < H < 1$), and the low values for the coefficient of determination (R^2) for the least-squares fit indicates that there are problems in modeling velocity profiles as fractional Gaussian noise or traces of fractional Brownian motion. The problems with this analysis are probably due to the profiles consisting of areas that are "smooth" (the velocity profile is either high or low velocity for a given section) and "rough" (the velocity profile alternates a great deal within a given section). Malinverno (1989) indicates that for profiles such as this, a mixture of the characteristics of the profile will ensue, unless the profile is divided into "homogeneous" smooth and rough sections. This additional processing was not performed, therefore at this time, the results of this spectral density analysis remain inconclusive.

The threshold analysis yielded a consistent relationship between fractal dimension of velocity profiles and 2D lacunarity; again, the largest average fractal dimension calculated for the velocity profiles from the high lacunarity models was smaller than the smallest average fractal dimension calculated for the velocity profiles from the low lacunarity models.

The coefficient of variation for the velocity profiles was measured. It was hypothesized that the more variable nature of the velocity profiles from the low lacunarity models would have a larger coefficient of variation. However, the results of the analysis were inconsistent; and so it is concluded that the porous medium textures were not distinguishable on the basis of this analysis.

Discussion

Lacunarity of the 1D hydraulic conductivity profiles (without normalization) is a good indicator of the 2D lacunarity of the K distribution. Cantor dust sets of hydraulic conductivity from a few boreholes in a field setting will be insufficient to obtain an accurate estimate of the fractal dimension of two- or three-dimensional porous media between the holes. For every stochastic model, the average fractal dimension of the velocity profiles taken from the low lacunarity model were higher than the average fractal dimension of the velocity profiles taken from the high lacunarity model. The variability of the fractal dimensions calculated by the different methods suggests that further research in the method for determining fractal dimension is required. However, for this application, the relative value (rather than the absolute value) of fractal dimension is of interest, so this is not a substantial problem. Of the four methods used to measure the fractal dimension of the velocity profiles, the box-flex method and the threshold analyses for fractal dimension yielded consistent relationships between the velocity profiles and the 2D lacunarity of the K distribution from which they were taken. These analyses suggest that sparse data sampling provides clues to the nature of the subsurface. Integration of such results will aid subsurface interpretation.

Velocity profiles hold information about the connectedness of the high and low hydraulic conductivity zones. The velocity profile reflects the continuity of heterogeneities between profiles. Knowledge of the continuity of high K units is important because of the control that the connected regions of high hydraulic conductivity have on contaminant transport. In general, hydraulic conductivity profiles may indicate both high and low hydraulic conductivity at a site, but if the velocity profiles show consistently low velocities, it could be concluded that the high hydraulic conductivity zones are not connected, whereas if the velocity varies considerably, one would assume there are regions of connected high hydraulic conductivity.

Currently velocity profiles cannot be measured at the desired level of detail in the field. This study demonstrates their potential usefulness before effort is invested in developing technology to measure velocity profiles in the field.

Porous medium models are used to gain a better understanding of flow and contaminant transport in a physical groundwater system, to perform sensitivity analysis, to examine changes to the recharge/discharge balance (due to pumping), and to predict the performance of ground-water remediation schemes. When limited data are available, a stochastic approach can be used to generate alternative porous medium models that honor the available data. Stochastic simulations require information about the hydraulic conductivity distribution to assign hydraulic conductivity values to unsampled areas. These simulations typically rely on semivariogram analysis of data from boreholes.

The results of this research can be integrated to facilitate creation of more realistic two-dimensional stochastic models of heterogeneous porous mediums by quantifying the degree of heterogeneity from analysis of one-dimensional hydraulic conductivity and velocity profiles. Such an approach will help ground-water hydrologists maximize the interpretations from limited borings of heterogeneous porous media. Combining the information gathered from the hydraulic conductivity profiles with the information from the velocity profiles allows the interpretation of connected high hydraulic conductivity zones between wells. In essence, samples taken from a borehole can provide clues to the distributions of heterogeneities in unsampled interwell regions, thus indicating the degree of connectedness of high hydraulic conductivity zones.

References

Barton, C. C., and LaPointe, P., in press, Applications of fractals in the earth sciences, GSA Special Publication XXX.

Barton, C. C., Schutter, T. A., and Samuel, J. K., 1989, DIMENSION -- A computer program that computes the fractal dimension of lines or points in a plane using the box, box-rotate, box-flex, box-density, and perimeter-area methods., Open-File Report 89-xxxA, USGS, 28 msp.

Falconer, K. J., 1990, Fractal geometry - mathematical foundations and applications., John Wiley and Sons, Inc., New York, 288 p.

Feder, J., 1988, Fractals., Plenum Press, New York, 283 p.

Freeze, R. A., and Cherry, J. A., Groundwater., Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 604 p.

Hurst, H. E., 1951, Long-term storage capacity of reservoirs., Transactions American Society of Civil Engineers, v. 116, pp. 770-808.

Hurst, H. E., Black, R. P., and Simaika, Y. M., 1965, Long-term storage, an experimental study., Constable, London, 145 p.

Lundahl, T., Ohley, W. J., Kay, S. M., and Siffert, R., 1986, Fractional Brownian motion: A maximum likelihood estimator and its application to image texture., IEEE Transactions on Medical Imaging, v. MI-5, n. 3, p. 152-161.

Malinverno, A., 1989, Segmentation of topographic profiles of the seafloor based on a self-affine model., IEEE Journal of Oceanic Engineering, v. 14, n. 4, p. 348-359.

Mandelbrot, B. B., 1967, How long is the coast of Britain? Statistical self similarity and fractional dimension. Science 155, 636-638.

Mandelbrot, B. B., 1983, The fractal geometry of nature., W. H. Freeman and Co., 468 p.

Mandelbrot, B. B., and Van Ness, J. W., 1968, Fractional brownian motions, fractional noises and applications., SIAM Review, v. 10, n. 4, pp. 422-437.
Mandelbrot, B. B., and Wallis, J. R., 1969a, Some long-run properties of geophysical records., Water Resources Research, v. 5, n. 2, pp. 321-340.
Mandelbrot, B. B., and Wallis, J. R., 1969b, Computer experiments with fractional gaussian noises. Part 1, averages and variances., Water Resources Research, v. 5, n. 1, pp. 228-241.
Zlatev, P., 1991, Interpretation of two-dimensional characteristics of stochastic fractal aquifer heterogeneity models from one-dimensional sampling, PhD Dissertation T-4077, Colorado School of Mines.