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Rates of Vertical Groundwater Movement Estimated from the Earth's Thermal Profile[1]

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Abstract.—An analytical solution is developed describing vertical steady flow of groundwater and heat through an isotropic, homogeneous, and fully saturated semiconfining layer. A type-curve method for estimating groundwater velocities from temperature data is presented.

Introduction. Geophysicists have long recognized that moving groundwater can affect the flux of heat within the Earth. *Van Orstrand* [1934, p. 996] indicated that the transfer of heat by migrating water could cause variations of temperature gradients within the Earth. He discussed these variations at some length, using illustrative data from the Wall Creek Sands of Salt Creek Oil Field, Wyoming.

Because the natural heat-flux density from the Earth is usually small, the upward thermal gradient within the Earth may be affected by groundwater movement. *Stallman* [1960] presented the basic equations for the simultaneous transfer of heat and water within the Earth and suggested that temperature measurements might provide a means of measuring rates of groundwater movement.

We are presenting a solution to Stallman's general equation for the case of steady-state vertical flow of both groundwater and heat. This solution is of interest to groundwater hydrologists, because it may afford a means of calculating vertical rates of groundwater movement and in some instances, where head relationships are known, vertical permeabilities.

General differential equation. The general differential equation for simultaneous nonsteady heat and fluid flow through isotropic, homogeneous, and fully saturated porous mediums is [*Stallman*, 1960]

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - \frac{c_0\rho_0}{\kappa}\left[\frac{\partial(v_x T)}{\partial x} + \frac{\partial(v_y T)}{\partial y} + \frac{\partial(v_z T)}{\partial z}\right] = \frac{c\rho}{\kappa}\frac{\partial T}{\partial t}$$

where

T	=	Temperature at any point at time t.
c_0	=	Specific heat of fluid.
ρ_0	=	Density of fluid.
c	=	Specific heat of solid-fluid complex.
ρ	=	Density of solid-fluid complex.
κ	=	Thermal conductivity of solid-fluid complex.
v_x, v_y, v_z	=	Components of fluid velocity in the x, y, and z direction.
x, y, z	=	Cartesian coordinates.
t	=	Time since flow started.

In the problem treated below the flow of heat and fluid is one-dimensional (vertical) and steady. Under these conditions the differential equation reduces to

$$(\partial^2 T/\partial z^2) - (c_0\rho_0 v_z/\kappa)(\partial T/\partial z) = 0$$

Analysis. Consider the vertical steady anisothermal groundwater flow through the semiconfining layer of an aquifer system (Figure 1) in which the temperature along a vertical section of the semiconfining layer has been measured at three or more points. These temperature measurements should satisfy the solution to the following boundary-value problem describing both heat and groundwater flow in the semiconfining layer:

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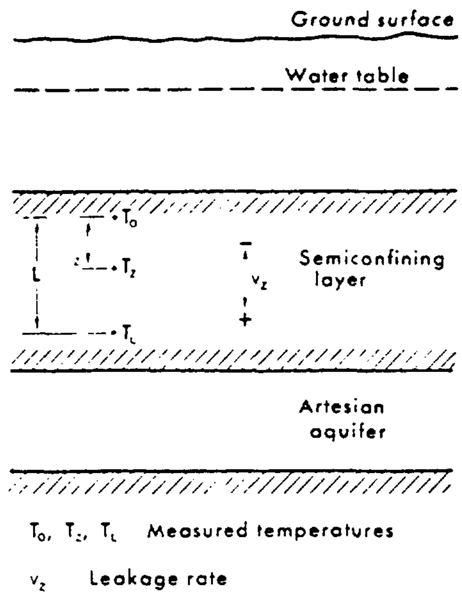

Fig. 1. Diagrammatic sketch of typical leaky aquifer.

$$(\partial^2 T_z/\partial z^2) - (c_0\rho_0 v_z/\kappa)(\partial T_z/\partial z) = 0 \qquad (1)$$

$$T_z = T_0 \qquad \text{at} \qquad z = 0 \qquad (2)$$

$$T_z = T_L \qquad \text{at} \qquad z = L \qquad (3)$$

where

T_z = Temperature measurement at any depth z.
T_0 = Uppermost temperature measurement.
T_L = Lowermost temperature measurement.
L = Length of vertical section over which temperature measurements extend (vertical distance between T_0 and T_L).

and in which the coordinate z is positive downward with origin at T_0, and the groundwater velocity v_z is positive downward.

After solving equation 1 and applying boundary conditions (2) and (3), the solution is

$$(T_z - T_0)/(T_L - T_0) = f(\beta, z/L)$$

where

$$f(\beta, z/L) = [\exp(\beta z/L) - 1]/[\exp(\beta) - 1]$$

and $\beta = c_0\rho_0 v_z L/\kappa$ = a dimensionless parameter that is positive or negative depending on whether v_z is downward or upward.

Table 1 gives values of the function $f(\beta, z/L)$ for a practical range of its parameters. Calculations were made using the U. S. Geological Survey's Burroughs 220 computer.

Application. To estimate the groundwate velocity through the semiconfining layer fro temperature data it is necessary to solve equa tion 4 for the velocity v_z, which appears in th dimensionless parameter β. This is done graphic ally by a 'type-curve' procedure. A set of typ curves consisting of arithmetic plots of $f(\beta, z/L)$ against z/L for different values of β, shown o Figure 2, was prepared from Table 1.

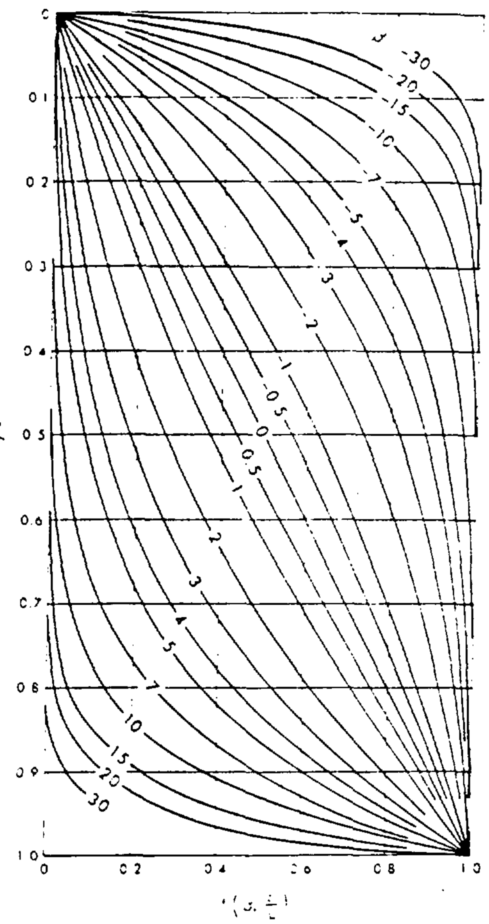

Fig. 2. Type curves of the function $f(\beta, z/L)$.

Ratios $(T_z - T_0)/(T_L - T_0)$, calculated from measured temperature data, are plotted against the depth factor z/L at the same scale as the set of type curves. The $(T_z - T_0)/(T_L - T_0)$ against z/L plot is superimposed on the type-curve set holding the coordinate axes in coincidence. The value of β is interpreted from the type curve that best matches the field data curve.

TABLE 1. Values of the Function $f(\beta, z/L)$

β, z/L	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.001	0	0.1000	0.1999	0.3000	0.3999	0.4999	0.5999	0.6999	0.7999	0.9000	1.0000
0.01	0	0.0996	0.1992	0.2990	0.3988	0.4988	0.5988	0.6989	0.7992	0.8995	1.0000
0.10	0	0.0956	0.1921	0.2896	0.3880	0.4875	0.5680	0.6894	0.7919	0.8954	1.0000
0.25	0	0.0891	0.1805	0.2742	0.3703	0.4688	0.5898	0.6733	0.7795	0.8884	1.0000
0.50	0	0.0790	0.1621	0.2495	0.3413	0.4378	0.5393	0.6460	0.7581	0.8760	1.0000
0.75	0	0.0697	0.1449	0.2259	0.3132	0.4073	0.5088	0.6181	0.7360	0.8630	1.0000
1.0	0	0.0612	0.1289	0.2036	0.2862	0.3775	0.4785	0.5900	0.7132	0.8495	1.0000
2.0	0	0.0347	0.0770	0.1287	0.1918	0.2689	0.3631	0.4782	0.6187	0.7904	1.0000
3.0	0	0.0183	0.0431	0.0765	0.1216	0.1824	0.2646	0.3755	0.5252	0.7272	1.0000
4.0	0	0.0092	0.0229	0.0433	0.0738	0.1192	0.1870	0.2882	0.4391	0.6642	1.0000
5.0	0	0.0014	0.0117	0.0236	0.0433	0.7586	0.1295	0.2179	0.3636	0.6039	1.0000
6.0	0	0.0020	0.0058	0.0125	0.0249	0.4743	0.0885	0.1632	0.2995	0.5477	1.0000
7.0	0	0.0009	0.0028	0.0065	0.0141	0.2931	0.0600	0.1217	0.2459	0.4961	1.0000
8.0	0	0.0004	0.0013	0.0034	0.0079	0.1799	0.0404	0.0904	0.2016	0.4491	1.0000
9.0	0	0.0002	0.0006	0.0017	0.0044	0.1099	0.0272	0.0671	0.1652	0.4065	1.0000
10	0	0.0001	0.0003	0.0009	0.0024	0.0067	0.0183	0.0497	0.1353	0.3679	1.0000
15	0	0.0000	0.0000	0.0000	0.0001	0.0006	0.0025	0.0111	0.0498	0.2231	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0025	0.0183	0.1353	1.0000
30	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0025	0.0498	1.0000
100	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
−0.001	0	0.1000	0.2001	0.3001	0.4001	0.5001	0.6001	0.7001	0.8001	0.9000	1.0000
−0.01	0	0.1005	0.2008	0.3011	0.4012	0.5012	0.6012	0.7010	0.8008	0.9004	1.0000
−0.10	0	0.1046	0.2081	0.3106	0.4120	0.5125	0.6120	0.7104	0.8079	0.9044	1.0000
−0.25	0	0.1116	0.2205	0.3267	0.4302	0.5312	0.6297	0.7258	0.8195	0.9109	1.0000
−0.50	0	0.1240	0.2419	0.3540	0.4607	0.5622	0.6587	0.7505	0.8379	0.9210	1.0000
−0.75	0	0.1369	0.2640	0.3819	0.4912	0.5927	0.6868	0.7741	0.8551	0.9303	1.0000
−1.0	0	0.1505	0.2868	0.4100	0.5215	0.6225	0.7138	0.7964	0.8711	0.9388	1.0000
−2.0	0	0.2096	0.3813	0.5218	0.6369	0.7311	0.8082	0.8713	0.9230	0.9653	1.0000
−3.0	0	0.2728	0.4748	0.6245	0.7354	0.8176	0.8784	0.9235	0.9569	0.9817	1.0000
−4.0	0	0.3358	0.5609	0.7118	0.8130	0.8808	0.9262	0.9567	0.9771	0.9908	1.0000
−5.0	0	0.3961	0.6364	0.7821	0.8705	0.9241	0.9567	0.9764	0.9883	0.9956	1.0000
−6.0	0	0.4523	0.7005	0.8368	0.9115	0.9526	0.9751	0.9875	0.9942	0.9980	1.0000
−7.0	0	0.5039	0.7541	0.8783	0.9400	0.9707	0.9859	0.9935	0.9972	0.9991	1.0000
−8.0	0	0.5509	0.7984	0.9096	0.9596	0.9820	0.9921	0.9966	0.9987	0.9996	1.0000
−9.0	0	0.5935	0.8348	0.9329	0.9728	0.9890	0.9956	0.9983	0.9994	0.9998	1.0000
−10	0	0.6321	0.8647	0.9503	0.9817	0.9933	0.9976	0.9991	0.9997	0.9999	1.0000
−15	0	0.7769	0.9502	0.9889	0.9975	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000
−20	0	0.8647	0.9817	0.9975	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
−30	0	0.9502	0.9975	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
−100	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The groundwater velocity is calculated from the relation

$$v_z = \kappa\beta / c_0\rho_0 L \qquad (5)$$

Discussion and conclusions. The type curves (Figure 2) indicate that under isotropic and steady-state conditions with no groundwater flow ($\beta = 0$) the thermal gradient is linear with depth. As groundwater movement occurs, the thermal profile curves and the plot of $(T_z - T_0)/(T_L - T_0)$ against z/L is convex upward or downward, depending upon the direction of water movement. The curvature of the thermal profile increases with increasing groundwater velocity.

The lowest value at which we might expect to detect a curvature in the $(T_z - T_0)/(T_L - T_0)$ against z/L plot is $|\beta| = 0.5$. That is, the lower limit of detectable groundwater velocities is

$$v_z = 0.5\kappa / c_0\rho_0 L$$

Typical values for the thermal conductivity of water-saturated clay are approximately $\kappa = 2 \times$

10^{-3} calorie/cm sec °C [*Birch*, 1942, p. 259]. The quantity $\kappa/c_0\rho_0$ is therefore approximately

$$\frac{\kappa}{c_0\rho_0} = \frac{2 \times 10^{-3}\ \text{cal/cm} - \text{sec} - °C}{1\ \text{cal}/g - °C \times 1\ g/\text{cm}^3}$$

$$= 2 \times 10^{-3}\ \text{cm}^2/\text{sec}$$

With this value of $\kappa/c_0\rho_0$ the least magnitude of v_z that can be determined is

$$|v_z| = \frac{0.5 \times 2 \times 10^{-3}\text{cm}^2/\text{sec}}{L}$$

$$= \frac{1 \times 10^{-3}\text{cm}^2/\text{sec}}{L}$$

The magnitude of v_z depends on L; if, for example, $L = 10$ meters, the least magnitude is

$$|v_z| = \frac{1 \times 10^{-3}\ \text{cm}^2/\text{sec}}{1 \times 10^3\ \text{cm}}$$

$$= 1 \times 10^{-6}\ \text{cm/sec}$$

or approximately 0.1 cm/day.

From Darcy's law

$$|v_z = K\ dh/dz|$$

where K = permeability (hydraulic conductivity) of the semiconfining layer, and h = head at any depth z; we can also suggest a lower limit of detectable permeability by assuming a head gradient. For a gradient of 1°C (1 cm/m), which is easily observed in the field

$$dh/dz = 1 \times 10^{-2}\ \text{cm/cm}$$

and $v_z = 1 \times 10^{-6}\ \text{cm/sec}$

$$K = \frac{1 \times 10^{-6}}{1 \times 10^{-2}} = 1 \times 10^{-4}\ \text{cm/sec}$$

Permeabilities of this order of magnitude are in the range commonly associated with semiconfining layers. As Stallman suggests, it appears that temperature measurements afford a means of detecting vertical groundwater movement, a problem that continues to trouble groundwater hydrologists.

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