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IDENTIFICATION OF EFFECTIVE CONDUCTIVITY TENSOR IN RANDOMLY HETEROGENEOUS AND STRATIFIED AQUIFERS

by

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EXTENDED ABSTRACT:

The effective macroscale hydraulic conductivity of heterogeneous and stratified geologic formations can be identified by ad hoc inverse methods, or by more generic, direct methods such as homogenization. We propose here a direct approach based on a simple homogenization relation expressing the effective conductivity tensor of randomly heterogeneous flow systems under certain conditions of statistical homogeneity and statistical anisotropy, given the microscale conductivity field $K(x_1,x_2,x_3)$. Imperfectly stratified and anisotropic geologic structures are described by means of directional fluctuation scales, while other features such as degree of variability, bimodality, etc, are conveyed by a probability distribution. The dimensionality of the flow system is also an important factor. We discuss below the general case of a D–dimensional flow system (D = 1, 2, or 3).

The proposed homogenization relation expresses the principal components of the D–dimensional effective conductivity tensor by means of a tensorial power–average operator:

$$\hat{K}_{ii} = \langle K^{p_i} \rangle^{1/p_i} \qquad (i=1,...,D) \tag{1}$$

where the angular brackets < > designate the operation of averaging. In this equation, the p_i's are directional averaging exponents. They are expressed in terms of the directional fluctuation scales ℓ_i, as follows:

$$p_i = 1 - \frac{2}{D}\frac{\ell_H}{\ell_i} \qquad (i=1,...,D) \tag{2}$$

here ℓ_H is the D–dimensional harmonic mean fluctuation scale:

$$\ell_H = \left[\frac{1}{D} \sum_{i=1}^{i=D} \ell_i^{-1} \right]^{-1} \tag{3}$$

Note that the averaging exponents are constrained to lie within the interval [–1,+1], and that they sum up to unity. To summarize, equations (1)–(3) give an analytical

relationship for the D–dimensional effective conductivity tensor in terms of the single–point probability distribution, the principal directions, and the directional fluctuation scales of the microscale log–conductivity field. Note that the microscale data required for implementation of equations (1)–(3) are all of a statistical nature. For technical reasons, we prefer to use here the statistics of log–conductivity rather than conductivity.

The power–average effective conductivity tensor (1)–(3) can be expressed in closed form for several usual types of log–conductivity distributions, such as gaussian, binary, etc. In the case of a "gaussian medium" with normally distributed ℓnK, applying equations (1)–(3) leads to:

$$\hat{K}_{ii} = K_g \exp\left\{\frac{\sigma_Y^2}{2}\left[1 - \frac{2}{D}\frac{\ell_h}{\ell_i}\right]\right\} \qquad (i=1,...,D) \tag{4}$$

when σ_Y^2 is the log–conductivity variance, and K_g is the geometric mean conductivity. This relation was initially developed by Ababou (1988, Vol. 1, Eq. 4.48) in the equivalent form:

$$\hat{K}_{ii} = (K_a)^{\alpha_i}(K_h)^{1-\alpha_i} \qquad (i=1,...,D) \tag{5}$$

where $\alpha_i = (D - \ell_h/\ell_i)/D$, and K_a and K_h represent the arithmetic and harmonic mean conductivities, respectively. Another case of interest is that of a binary medium, made up of a mixture of two distinct conductive phases α and β, present in the proportions (ρ) and ($1-\rho$) respectively. For instance, phase α could be a sandstone matrix, and phase β a set of shale lenses or shale clast inclusions (Desbarats 1987, Bachu and Cuthiell 1990). The conductivity distribution of such a composite medium is of the form:

$$\begin{aligned} &\mathrm{Prob}\{K(x_1,x_2,x_3)=K_\alpha\} = \rho \\ &\mathrm{Prob}\{K(x_1,x_2,x_3)=K_\beta\} = 1-\rho \end{aligned} \tag{6}$$

As before, we assume in first approximation that the spatial anisotropy of the random structure can be defined by three fluctuation scales ℓ_1,ℓ_2,ℓ_3. Specializing equations (1)–(3) for the binary distribution (6) gives:

$$\hat{K}_{ii} = \{\rho K_\alpha^{\,p_i} + (1-\rho)K_\beta^{\,p_i}\}^{1/p_i} \qquad (i=1,...,D) \tag{7}$$

with averaging powers (p_i) are as given previously in equation (2). In the case of a three–dimensional isotropic binary medium, let $D = 3$ and $\ell_1 = \ell_2 = \ell_3$. This yields $p_i = 1/3$ (i=1,2,3) in equation (7). In the case of a two–dimensional isotropic binary medium, let $\ell_1 = \ell_2$ for horizontal isotropy, and $D = 2$ for restriction to two–dimensional space, or equivalently $D = 3$ with $\ell_3 \longrightarrow +\infty$ for two–dimensional horizontal flow through a vertically homogeneous medium. Either case yields $p_i \longrightarrow 0$ for $i = 1$ and 2. Inserting this in (7) and using Taylor developments leads to:

$$\hat{K}_{ii} = (K_\alpha)^\rho (K_\beta)^{1-\rho} \qquad (i=1 \text{ and } 2) \tag{8}$$

where ρ represents the concentration of phase α, and $1-\rho$ the concentration of phase β.

Although the general form of the effective conductivity model (1)–(3) remains conjectural at this stage, many specialized forms of this relation appear to be confirmed by other results. We are exploring the range of validity of the model in several ways: firstly, by comparison with exact bounds and with available homogenization solutions in cases of lower dimensionality, statistical isotropy, symmetric distribution, binary distribution, etc; secondly, by comparison with analytical solutions based on linearized and/or perturbation approximations; and thirdly, by comparison with direct numerical simulations of flow in randomly heterogeneous porous media. The preliminary results of such analyses are encouraging. Some of the relevant effective conductivity results used for comparisons can be found in Ababou (1988), Ababou et al. (1989), Desbarats (1987), Gelhar and Axness (1983), Kohler and Papanicolaou (1982), and Matheron (1967), among others.

Finally, let us briefly indicate some of the possible applications of the tensorial power–average conductivity model in the area of parameter estimation. First, the model can be used as a convenient tool for direct identification of the effective conductivity tensor, given reasonable estimates of the microscale conductivity distribution, principal axes, and fluctuation scales. Unlike other approaches to inverse problems, the present approach does not require numerical procedures. Furthermore, the model can also be used to estimate geostatistical parameters from large–scale flow tests. To illustrate this second type of application, we implemented a parameter identification procedure previously developed for the Oracle fractured granite site by Neuman and Depner (1988). Their goal was to estimate the conductivity correlation scales (which play an important role in contaminant macrodispersion) based on a combination of "microscale" single–hole packer tests and "macroscale" cross–hole tests. The principal axes of macroscale anisotropy, as inferred from the cross–hole data, appeared to be directly related to the orientations of major fracture sets. Now, given the measured macroscale conductivity tensor and an independent estimate of vertical correlation scale, the remaining correlation scales of the Oracle site can be identified by inverting the effective conductivity model (1)–(3). Again, this is feasible without recourse to numerical optimization procedures.

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KEYWORDS:

Groundwater Flow, Darcy Equation, Anisotropy, Effective Conductivity, Conductivity Tensor, Stochastic Flow, Random Fields, Random Media, Homogenization, Parameter Estimation.