APPENDIX B

ASSESSMENT OF ROD-TO-ROD AND ROD-TO-THIMBLE THERMAL RADIATION IN THE FLECHT REFLOOD HEAT TRANSFER DATA

Fundamental to any thermal radiation problem is the characteristics of the surfaces which comprise the enclosure for heat transfer. This Attachment describes a method for assessing the thermal rod-to-rod radiation for a given rod array or enclosure.

A thermal radiation enclosure features a detailed, multi-surfaced representation of the hot rod and its surroundings. Many surfaces comprise the radiation enclosure, including items such as control rod guide tubes, core shims, as well as the fuel rods surrounding the hot rod. The model used to provide an assessment of the thermal rod-to-rod radiation heat transfer during the reflooding of a hot FLECHT fuel bundle is based on the net radiation method[1]. The method is described below.

The outgoing energy flux, B_i, from surface i is composed of direct emitted energy plus the reflected portion of the incident energy flux, H_i. Thus

$$B_i = \varepsilon_i \sigma T_i^4 + \gamma_i H_i = \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i) H_i \qquad (1)$$

where

T_i = the surface absolute temperature (°R)
σ = Stefan-Boltzman constant (0.1712×10^{-8} BTU/hr - ft^2 - °R)
ε_i = emissivity

The flux incident upon surface i is composed of contributions from all the surfaces within the enclosure that can see surface i, including surface i itself, if it is concave. The energy incident on surface i for an enclosure containing n surfaces is then

$$A_i H_i = \sum_{j=1}^{n} A_j F_{ji} B_j = \sum_{j=1}^{n} A_i F_{ij} B_j = A_i \sum_{j=1}^{n} F_{ij} B_j \qquad (2)$$

and

$$H_i = \sum_{j=i}^{n} F_{ij} B_j \qquad (3)$$

where the reciprocity relation, $A_i F_{ij} = A_j F_{ji}$, has been employed in the simplification of Eq. 2. Use of Eq. 3 eliminates the term H_i in Eq. 1 to yield

$$B_i \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i) \sum_{j=1}^{n} F_{ij} B_j \qquad (4)$$

The net heat transferred per unit time from a surface, Q_i, is the difference between the emitted radiation and the absorbed portion of the incident radiation:

$$Q_i = A_i \varepsilon_i \sigma T_i^4 - A_i \alpha_i H_i = A_i(\varepsilon_i \sigma T_i^4 - \varepsilon_i H_i) \tag{5}$$

Solving for H_i produces

$$H_i = \sigma T_i^4 - \frac{Q_i}{\varepsilon_i A_i} \tag{6}$$

and substitution of Eq. 6 into Eq. 1 results in

$$B_i = \sigma T_i^4 - (\frac{1-\varepsilon_i}{\varepsilon_i})\frac{Q_i}{A_i} \tag{7}$$

Elimination of B_i from Eq. 4 using Eq. 7 results in

$$\sigma T_i^4 - \frac{Q_i}{A_i \varepsilon_i} = \sum_{j=1}^{n} F_{ij} \sigma T_i^4 - \sum_{j=1}^{n} F_{ij}(\frac{1-\varepsilon_i}{\varepsilon_i})\frac{Q_i}{A_i} \tag{8}$$

After collecting terms and allowing the index i to take on values from i to n, the following set of equations is obtained where:

$$\sum_{j=1}^{n}(F_{ij} - \delta_{ij})\sigma T_i^4 = \sum_{j=1}^{n}(F_{ij}\frac{1-\varepsilon_j}{\varepsilon_j} - \delta_{ij})\frac{Q_j}{A_j}, i = 1,\ldots,n \tag{9}$$

where

δ_{ij} is the Kronecker function defined as

$$\delta_{ij} = \begin{Bmatrix} 1 \; if \; i=j \\ 0 \; if \; i \neq j \end{Bmatrix}$$

Eq. 9 is the general steady-state equation for determining radiation exchange in a gray, diffuse enclosure of n surfaces by the net radiation method. Eq. 9 represents a set of n linear algebraic equations (i.e., linear in T^4) containing n surface temperatures and n heat fluxes. With the n known surface temperatures, Eq. 9 can be solved for the net heat transferred, Q_i, for the n surfaces. With Q_i computed from Eq. 9, the equivalent heat transfer coefficient for thermal radiation contribution can be computed from

$$h_{rad} = \frac{Q_i}{[A_i(T_i - T_{sat})]} \tag{10}$$

where h_{rad} is in BTU/hr-ft^2-°F and is based on the FLECHT fluid saturation temperature, T_{sat}.
Appendix A contains the FORTRAN program that evaluates h_{rad} for radiation enclosures extracted from the FLECHT top skewed power distribution reflood heat transfer tests. The enclosure consists of a square cylindrical rod array which can be evaluated for n+1 surfaces, when the boundary enclosure surface is also included. An emissivity of 0.8[2] was used for oxidized stainless steel. The FLECHT rod radius is 0.374 in.[3] with a pitch for the bundle of 0.496 in. [3] For simplicity, the thimbles were assumed to be the same diameter as the heater rods.

View factors were computed for each pair of surfaces in the enclosure and are based on the work of D.A. Mandell[4]. Because all of the rods have essentially the same diameter, a given rod can view only rods that are not more than one row or one column away. For a 5x5 rod array, for example, rod 1 in Fig. B-1 can see rods 2, 6, 7 through 10, and the rods in the column headed by rod 7 plus rods 12 through 22. The view factors from rod 1 to the other rods are zero (i.e., $F_{1\text{-}6}=F_{1\text{-}11}=F_{1\text{-}16}=F_{1\text{-}21}=0.0$). View factors are described in Appendix C.

The top skewed powèr distribution FLECHT Test NO. 13404, where the average rod temperature was found to be 2010 °F, with a temperature scatter of 51 °F[5]. From this information, the hottest rod is computed to be 2061 °F[5]. The thimble temperatures were approximately 1750 °F at the time of the peak clad temperature, as shown in Fig. B-3.

Figs. B-1 and B-2 display two 5x5 enclosures that were evaluated for the top skewed power test No. 13404. Results of the calculations are summarized below for the central rod (i.e., rod no. 13).

h_{rad} (BTU/hr-ft^2 - °F)

Enclosure	5x5 array, rod 13
1	2.95
2	3.57

APPENDIX D

This Appendix contains a description of the view factor formulation for a square array of cylindrical rods.

The geometrical view factors for the rod array given in Fig. B-1 consist of computing three surface-to-surface view factors:

1) adjacent rods,
2) diagonal rods, and
3) next nearest diagonal rods

Adjacent rod view factors are computed from

$$F_{1-2} = \frac{1}{\pi}(\sqrt{x^2} - 1 - x + \frac{\pi}{2} - \cos^{-1}\frac{1}{x})$$

where x = pitch/diameter = P/D

For the nearest diagonal rods

$$F_{1-7} = \frac{X_2 - 2x_1 - A_2 + 2A_1}{\pi D}$$

where

$$x_1 = \sqrt{P^2 - D^2},$$
$$x_2 = \sqrt{2P^2 - D^2},$$
$$A_1 = \frac{D}{2}(\tan^{-1}\frac{x_1}{D} + 0.7854 - \tan^{-1}\frac{x_2}{D}), and$$
$$A_2 = \frac{D}{2}(\frac{\pi}{2} - 2\tan^{-1}\frac{x_1}{D})$$

View factors for all of the next nearest neighbors, i

$$F_{1i} = \frac{x_2 + x_3 - 2(x_1 + A_1 - A_2)}{2\pi D}$$

where

$$x_1 = \sqrt{[(\Delta i - 1)^2 + 1]P^2 - D^2},$$
$$x_2 = \sqrt{[(\Delta i)^2 + 1]P^2 - D^2},$$
$$x_3 = \sqrt{[(\Delta i - 2)^2 + 1]P^2 - D^2},$$
$$A_1 = \frac{D}{2}[\tan^{-1}\frac{x_3}{D} - \tan^{-1}\frac{x_1}{D} - \tan^{-1}(\Delta 1 - 1) + \tan^{-1}(\Delta i - 2)], and$$
$$A_2 = \frac{D}{2}[\tan^{-1}\frac{x_1}{D} - \tan^{-1}(\Delta i - 1) - \tan^{-1}\frac{x_2}{D} + \tan^{-1}(\Delta i)]$$

$\Delta i = i - 1$, where i is the number of rods away from rod 1 in a given row or column. For example, in Fig. 1, for $F_{1=8}$, i=2 and for $F_{1\text{-}10}$, i=4.

The view factors, computed using the above formulation for the adjacent, diagonal, and next closest diagonal rods are given below for the 5x5 array:

Rod Location	View Factor
Adjacent Rod (1-2)	0.127030
Diagonal Rod (1-7)	0.085721
Next Closest Diagonal Rod (1-12)	0.015797
Next Closest Diagonal Rod (1-17)	0.001805
Next Closest Diagonal Rod (1-22)	0.000501

Conservation of energy requires

$$\sum_{j-1}^{n} F_{ij} = 1$$

And, the view factor for the boundary surface n+1 to each rod i is simply

$$F_{(n+1)-i} = F_{i-(n+1)}\frac{A_i}{A_b}$$

where A_i is the rod surface area and A_b is the boundary area.

Also

$$F_{(n+1)-(n+1)} = 1 - \sum_{j=1}^{n} F_{(n+1)-j}$$

Also note that the reciprocity rule is applied to determining the view factors from the boundary to the rod array surfaces.