9/30/87

RE-ESTIMATION OF ELEVATIONS WITHIN THE COHASSETT FLOW AT THE REFERENCE REPOSITORY LOCATION

by

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ABSTRACT

The elevations within the Cohassett flow at the reference repository location of the proposed Hanford site in the state of Washington were re-estimated using 10 existing borehole data. The motivation for this work arose during the document review of a report by H.D. Taylor entitled "Geostatistical Estimation of Elevations Within the Cohassett Flow at the Reference Repository Location", which was published in June, 1987.

Re-estimated elevations of the bottom of Cohassett flow-top and the top of Cohassett flow-bottom using universal kriging were quite similar to those given by Taylor. However, re-estimated kriging standard deviations are substantially smaller than Taylor's because of the direct estimation used in this report as compared to the multi-step approach used by Taylor.

As it is well recognized, the most important information that geostatistics provides is the uncertainty of the estimate. Quantifying this uncertainty, however, is much more susceptible to the correct modelling of the variogram parameters than obtaining the estimate itself.

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INTRODUCTION

At the request of the U. S. Nuclear Regulatory Commission and under a subcontract to Engineers International, Inc. of Westmont, IL, a document review of a report by Harold D. Taylor of D.A.S.A. entitled "Geostatistical Estimation of Elevations Within the Cohassett Flow at the Reference Repository Location" was conducted.

According to Taylor, the above report provides the following:

1. A detailed analysis of the current data concerning the Cohassett flow in the vicinity of the proposed repository layout area, especially concerning the location of the proposed repository within that flow.

2. A comparison of different future sampling plans to determine quantitatively their relative abilities to reduce uncertainty in estimation.

3. A detailed description of the methods used so that others may not only check the work presented here, but also extend the work to answer future questions.

Heeding to suggestions given by Taylor, this report provides an extension to his work as well as technical review and comments on his results.

It is assumed that the reader of this report has been exposed to Taylor's report and the reader has a ready access to it for cross-referencing purposes. It is also assumed that he (or she) has a working knowledge of geostatistics.

OBJECTIVES AND SCOPE OF THIS REPORT

There are two main objectives of this report. These are:

1. To provide a technical document review of Taylor's report to the U.S. Nuclear Regulatory Commission.

2. To present different results of estimated elevations and kriging standard deviations within the Cohassett Flow which others may also check and extend in the future.

The scope of this report is limited to re-estimation of elevations and associated kriging standard deviations (i.e., uncertainty) of the bottom of Cohassett flow-top and the top of flow-bottom only. This report does not address the problems of future sampling requirements nor the consequence of unexpected findings, both of which are discussed by Taylor.

SIGNIFICANCE TO NRC WASTE MANAGEMENT PROGRAM

The results as well as most of the comments given by Taylor in the referenced report are essentially correct with one exception. This exception lies in his obtained kriging standard deviations associated with his estimated elevations. As it is explained later in this report, his multi-step approach of estimation produced much conservative estimate of kriging standard deviations. In other words, the confidence intervals given in his report are much wider than necessary at a specified level of confidence (i.e., 80%). A more appropriate confidence interval based on single step (or direct) estimation using Universal Kriging is given in the latter part of this report.

TECHNICAL COMMENTS AND PRACTICAL CONSEQUENCES

Introductory Comments

In reviewing the report, this author was greatly impressed by his professionalism and thoroughness in analysis. This author particularly appreciated his clear and detailed description of the assumptions and the methods employed. This author agrees with the majority of his approaches and obtained results, except those which will be discussed here.

Perhaps, the single most important disagreement lies in his multi-step approach to estimation of elevations of the bottom of flow-top (BFLT) and the top of flow-bottom (TFLB) (See page 5 and page 23 of Taylor's report), as compared to direct estimation by this author. Taylor's justifications for adopting the multi-step approach are as follows:

1. There are much more data available to estimate the top of basalt (TOB) than any other geologic horizons of interest.

2. The top of the vesicular zone should be the best indicator horizon within the Cohassett flow (Taylor, p.5).

3. Combination of deep and shallow boreholes would produce greatest reduction in standard deviations (i.e., uncertainty) for a given drilling budget (Taylor, p.iii).

Although some of his justifications may not be valid, one can assume that they are all valid for the sake of discussion. Even then, there is no particular advantage to his multi-step approach to estimation. This report shows, both theoretically and through actual results, that multi-step estimation gives larger standard deviation (higher uncertainty) than one-step, direct estimation of the above two elevations.

The second important disagreement lies in his method of estimating elevations when there are trends in the data (Taylor, p.5). Taylor used a combination of trend surfaces and kriging of residuals instead of the Universal Kriging (UK) which can account for trends in the data.

In adopting his approach, Taylor appears to have overlooked the uncertainty associated with modelling of the trends in his kriging variance computation. In Universal Kriging, this uncertainty is incorporated into UK kriging variance, as it should be. Thus, in general, Taylor's approach will understate the uncertainty of the estimate. His comments about potential difficulties with UK are essentially correct. Unfortunately, his approach can neither avoid the difficulties that he mentions regarding UK.

The third important disagreement lies in his lack of cross-validation (Knudsen & Kim, 1978, p.174-176) of selected variogram parameters. Although only 12 data points were utilized to model three out of the four variogram parameters, the scarcity of data does not justify the omission of cross-validation. In fact, the converse is true. Unfortunately, no mention of cross-validation results is given anywhere in his report. In contrast, this report relied heavily on the results of cross-validation in selecting the final variogram parameters.

Multi-Step vs. Single Step Estimation

In order to estimate elevations of the bottom of flow-top (BFLT) and the top of flow-bottom (TFLB), Taylor first estimates four variables; namely 1) elevation of the top of basalt (TOB), 2) thickness of basalt from the TOB horizon to the top of the vesicular zone within the Cohassett flow (designated as TBVZ), 3) the thickness from the top of vesicular zone to the bottom of flow-top zone (given as AVZ), and 4) top of vesicular zone to the top of flow-bottom zone (given as BVZ). (Taylor, p.5 and Fig.1 on p.6).

Assuming these four variables, TOB, TBVZ, AVZ and BVZ are uncorrelated, i.e, all cross variograms are zero for all lag distances, the variograms for the bottom of flow-top (BFLT) and the top of flow-bottom are given by Equations 1 and 2 below.

$$\gamma_{BFLT}(h) = \gamma_{TOB}(h) + \gamma_{TBVZ}(h) + \gamma_{AVZ}(h) \qquad (1)$$

$$\gamma_{TFLB}(h) = \gamma_{TOB}(h) + \gamma_{TBVZ}(h) + \gamma_{BVZ}(h) \qquad (2)$$

Although Taylor did not directly compute $\gamma_{BFLT}(h)$ and $\gamma_{TFLB}(h)$ in estimating these two elevations, he implicitly obtained these variograms by his adoption of multi-step approach. Since the reliability of estimated variogram parameters for $\gamma_{BFLT}(h)$ and $\gamma_{TFLB}(h)$ is directly influenced by the individual reliabilities of all three variograms and since only 12 data points were used to estimate the last two of the three variogram parameters in Eqs. (1) and (2), he has unknowingly introduced more than the necessary amount of uncertainty in his implicit estimation of the $\gamma_{BFLT}(h)$ (Note: The situation becomes even worse if these four variables are correlated). This is in spite of the fact that estimation of $\gamma_{TOB}(h)$ is much more reliable due to a larger number of samples available.

By directly estimating $\gamma_{BFLT}(h)$ and $\gamma_{TFLB}(h)$ using the same 12 data points, it can be shown that the resulting variograms have much smaller nugget and sill values which, in turn, produce much smaller kriging variance around each estimate.

Possible Justification of Multi-Step Estimation

The main motive for adopting the mutli-step approach by Taylor appears to be based on his belief that "combination of deep and shallow boreholes would produce the greatest reduction in uncertainty of estimates for a given drilling budget." However, it can be shown that any improvement in the estimation of top of basalt elevation using shallow holes will produce relatively small improvement in the estimation of BFLT and TFLB elevations.

Again assuming that the same four variables are uncorrelated, the resulting improvement in kriging variance depends on the proportion of the sill of $\gamma_{TOB}(h)$ over the total sill of $\gamma_{TOB}(h)$ + $\gamma_{TBVZ}(h)$ + $\gamma_{AVZ}(h)$, in case of Eq. 1. Utilizing the variogram parameters given in his report, this proportion can be computed to be,

$$450/(450+695+175) = 450/1320 = .34 \text{ or } 34\%.$$

In other words, whatever improvement one achieves on TOB elevation estimation will impact only 34% of total kriging variance associated with BFLT elevation estimation. For examples, a 50% reduction in kriging variance of TOB will have only (.5 x .34) = .17 or 17% overall reduction.

Thus, it is questionable whether the potential benefits can justify the costs of multi-step approach of estimation, particularly with regards to the unreliable estimation of two key variogram parameters.

The remainder of this report gives a detailed description of methods used to re-estimate BFLT and TFLB elevations and kriging standard deviations.

RE-ESTIMATION OF ELEVATIONS

Initial Data Editing

The only source of data for this study was the deep borehole data to Cohassett flow depth, as given in Table A-1 of Taylor's report (See p.A-3). There were altogether 25 boreholes in Table A-1 and these data were key punched and verified for keypunch errors.

Initial data analysis using histograms and summary statistics clearly indicated that 2 of 25 holes (DH-4 and RSH-1) can be treated as outliers due to their extreme differences in elevation as compared to the rest of data. Therefore they were removed from the data set. To further validate the data, the remaining 23 hole locations were plotted. This plot showed that 7 out of 23 holes are located far from the proposed repository locations. Since one must determine the drift component in the data and since the primary interest of estimation is near the repository location, these 7 data points were also removed. Figure 1 shows the locations of the remaining 16 boreholes. The seven eliminated holes are; DC-6, DC-7, DC-8, DC-14, DC-15, DDH-13, and DH-4.

Utilizing the 16 data points given in Figure 1, the process of determining the drift (or trend) in the data was initiated. Following the similar procedure used by Taylor (Taylor, p.8), a linear drift (or plane surface) was fitted and the resulting residuals and the variograms of residuals were computed. Based on both the magnitude of a residual and its impact on the obtained variogram, three additional holes were eliminated because they could be considered as possible outliers in the set of residuals.

The three eliminated holes are; DC-12, McGee, and DC-19c. Please note that none of the three holes is located close to the repository area. In eliminating the above three holes, the same justification as given by Taylor was also used. Two holes, DC-1 and DC-2, which are located approximately 35,000' from the center of the repository were not eliminated, mainly because their presence did not adversely affect the residuals nor the variograms. The remaining 13 boreholes were used to estimate the final variogram model parameters of both TFLB and BFLT elevations using Universal Kriging (Olea, 1975). Table 1 gives a listing of this final data set used for subsequent analysis.

Variogram Estimation and Cross Validation

Using the data set given in Table 1, the task of estimating the underlying variogram was next initiated using the University of Arizona's drift determination program called DRIFT.IBM (Kim, 1982). This program has the option of using either the unbiased method of Olea or the biased method used by Taylor, in determining the drift component in the data. In Olea's method, both the drift and underlying variograms are jointly estimated (hence the

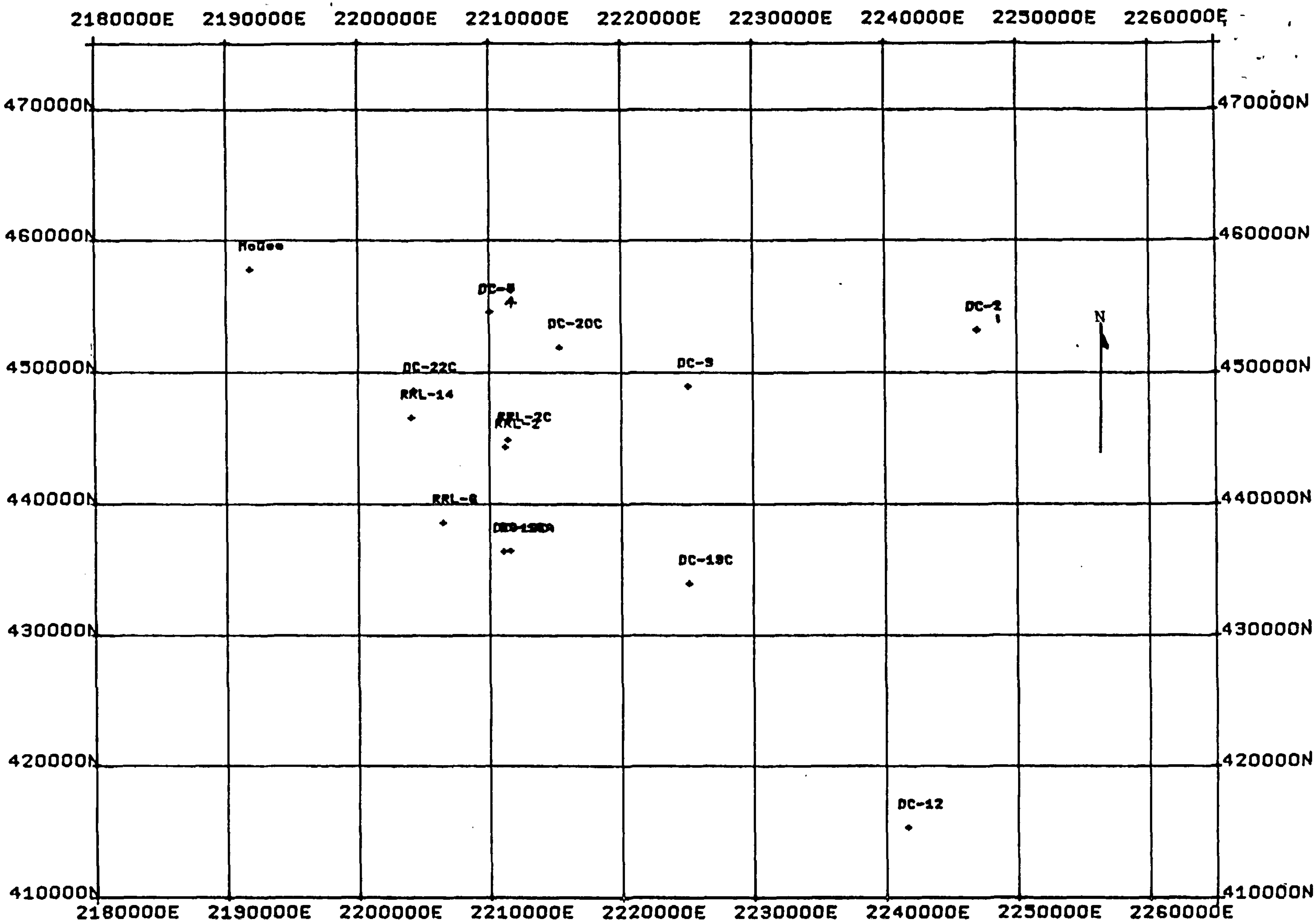


Figure 1. Locations of 16 initially selected boreholes Scale: 1"=3000'

TABLE 1

LISTING OF 13 BOREHOLE DATA USED IN THE ANALYSIS

HOLE ID	NORTH	EAST	BFLT	TFLB
DC-2	453146.	2246947.	-1791.	-2054.
DC-4	454469.	2209991.	-2236.	-2489.
RRL-2	444298.	2211184.	-2365.	-2621.
RRL-6	438580.	2206423.	-2459.	-2700.
RRL-14	446541.	2203991.	-2459.	-2665.
DC-16A	436403.	2211516.	-2422.	-2656.
DC-20C	451882.	2215285.	-2222.	-2483.
DC-22C	448600.	2204190.	-2394.	-2642.
DC-3	448924.	2225057.	-2127.	-2384.
DC-1	453174.	2246998.	-1786.	-2050.
DC-5	454537.	2210068.	-2233.	-2488.
DC-16C	436376.	2211006.	-2401.	-2655.
RRL-2C	444824.	2211376.	-2348.	-2597.

unbiasedness), whereas the residual variogram obtained from the trend surface method used by Taylor is biased.

Both approaches were repeatedly tried and the resulting residual variograms were cross-validated using VALUK.IBM program. This VALUK program estimates each known data point using "leave-one-out" approach and taking into account both the drift and the underlying variogram being verified through cross-validation. As such, it is similar to the UK program. It was learned that the residual variograms using either approach were quite similar, although there existed a significant difference in the constant term of the obtained drift components. It was also learned that the finally selected variogram (e.g., Fig.2) based on the best cross validation results, in turn, produced the best fit to the residual experimental variograms obtained using both approaches.

Variogram For The Bottom of Flow-Top Elevation - Figure 2 shows the residual variogram of the bottom of the flow-top elevation which resulted in the best cross validation using the data set given in Table 1. Figure 3 is the variogram cloud (Chauvet, 1982) of the the same variogram. Figure 4 shows the fitted theoretical model that was used to obtain the residual variogram of Figure 2. The drift model which resulted in this variogram as well as the assumed underlying variogram parameters are given in Table 2 which is simply the output from DRIFT.IBM program.

As can be seen from Table 2, the resulting drift using the assumed variogram parameter (i.e., C_o=125, C=425, a=12000) was

$$-35800 + (0.0131)X + (0.0102)Y \qquad (3)$$

where
x,y = coordinates.

The mean of residuals of 13 points is 1.152 and the variance of residuals is 18.74. By using the biased approach, only the constant term of the drift model changed to -2522 and the mean and the variance of residuals became -0.0230 and 20.52, respectively.

During cross validation using VALUK program, the maximum search radius (RMAX) for including the nearby samples was set to 15,000 feet and the maximum number of samples to include (NK) in kriging was set to 8. However, not all of the points were kriged using 8 nearby samples. If there were less than 4 nearby samples where 4 corresponds to the number of drift terms plus 1, the program did not validate this point. It is for this very reason that only 10 out of 13 points were validated as shown in the second half of Table 3. From Table 3, one can also observe that only 1 out of 10 predictions was outside 2 times the kriging standard deviation (RRL-14). The mean of prediction errors is -2.776 which is nearly zero and weighted square errors and average kriging variances are nearly identical, thus satisfying the desired criteria of cross validation.

```
                              VARIOGRAM

BOTTOM OF FLOW-TOP ELEVATIONS. NO OUTLI

RESIDUALS - BOTTOM OF FLOW TOP ELEVATION. NO OUTLIER.
                                                DATA USED IN CALCULATIONS
AZIMUTH =   0.  DIP =   0.  WINDOW =  90.   MEAN            =  .115E+01
CLASS SIZE   =8000.                         VARIANCE        =  .351E+03
MAX DISTANCE =   160000.                    STD DEVIATION =  .187E+02
LOGARITHMS -NO   RELATIVE VARIOGRAM -NO     NO.OF SAMPLES =        13
COORDINATE SELECTION-NORTH( 410000. 470000.)   EAST(2190000.2250000.)
                  ELEVATION(      0.      0.)
ASSAY DATA SELECTION -AMIN(       -9999.000)   AMAX(        999999.0)

    DISTANCE   # PAIRS     DRIFT       GAMMA (H) MOMENT CENT AVER DIST
     0 -  8000     15.    -.177E+01    .333E+03    .394E+03    4801.6
  8000 - 16000     27.    -.882E+01    .529E+03    .514E+03   10868.8
 16000 - 24000     16.    -.626E+00    .247E+03    .237E+03   18670.8
 24000 - 32000      2.    -.202E+01    .200E+02    .200E+02   31713.3
 32000 - 40000     12.     .106E+02    .199E+03    .200E+03   37711.8
 40000 - 48000      6.     .219E+00    .291E+03    .293E+03   43201.3

     .529E+03 +    #
     .503E+03 +
     .476E+03 +
     .450E+03 +
     .423E+03 +
  G  .397E+03 +
  A  .370E+03 +
  M  .344E+03 + #
  M  .317E+03 +            #
  A  .291E+03 +
     .265E+03 +     #
     .238E+03 +
  #  .212E+03 +          #
  H  .185E+03 +
  #  .159E+03 +
     .132E+03 +
     .106E+03 +
     .794E+02 +
     .529E+02 +
     .265E+02 +        #
              +----+----+----+----+----+----+----+----+----+----+
             0.   32000.    64000.    96000.    ######    ######
```

Figure 2. Residual variogram of the bottom of Cohassett flow-top elevation

```
                         VARIOGRAM CLOUD

 SYMBOLS USED TO REPRESENT THE NUMBER OF PAIRS

 1= 1   2= 2   3= 3   4= 4   5= 5   6= 6   7= 7   8= 8   9= 9   #=10
 a=11   b=12   c=13   d=14   e=15   f=16   g=17   h=18   i=19   j=20
 k=21   l=22   m=23   n=24   o=25   p=26   q=27   r=28   s=29   t=30
 *: MORE THAN 30

    .487E+04+  1
    .470E+04+
    .454E+04+
    .438E+04+  1
    .422E+04+
    .406E+04+  1
    .389E+04+
    .373E+04+
    .357E+04+
    .341E+04+
C   .324E+04+
L   .308E+04+
O   .292E+04+
U   .276E+04+
D   .260E+04+
    .243E+04+  1
    .227E+04+1
*   .211E+04+
H   .195E+04+  1         1
*   .178E+04+1 2
    .162E+04+            1
    .146E+04+  1  1
    .130E+04+  1  1
    .114E+04+1 2  3    1
    .973E+03+2
    .811E+03+2 1       2
    .649E+03+1 2  1    2
    .487E+03+2 1  1    2
    .324E+03+1 1  3    1
    .162E+03+4 a  6 2  4 4
            +----+----+----+----+----+----+----+----+----+----+
          0.    32000.    64000.    96000.    ######    ######
```

Figure 3. Variogram cloud of the residual variogram shown in Figure 2 - bottom of flow-top

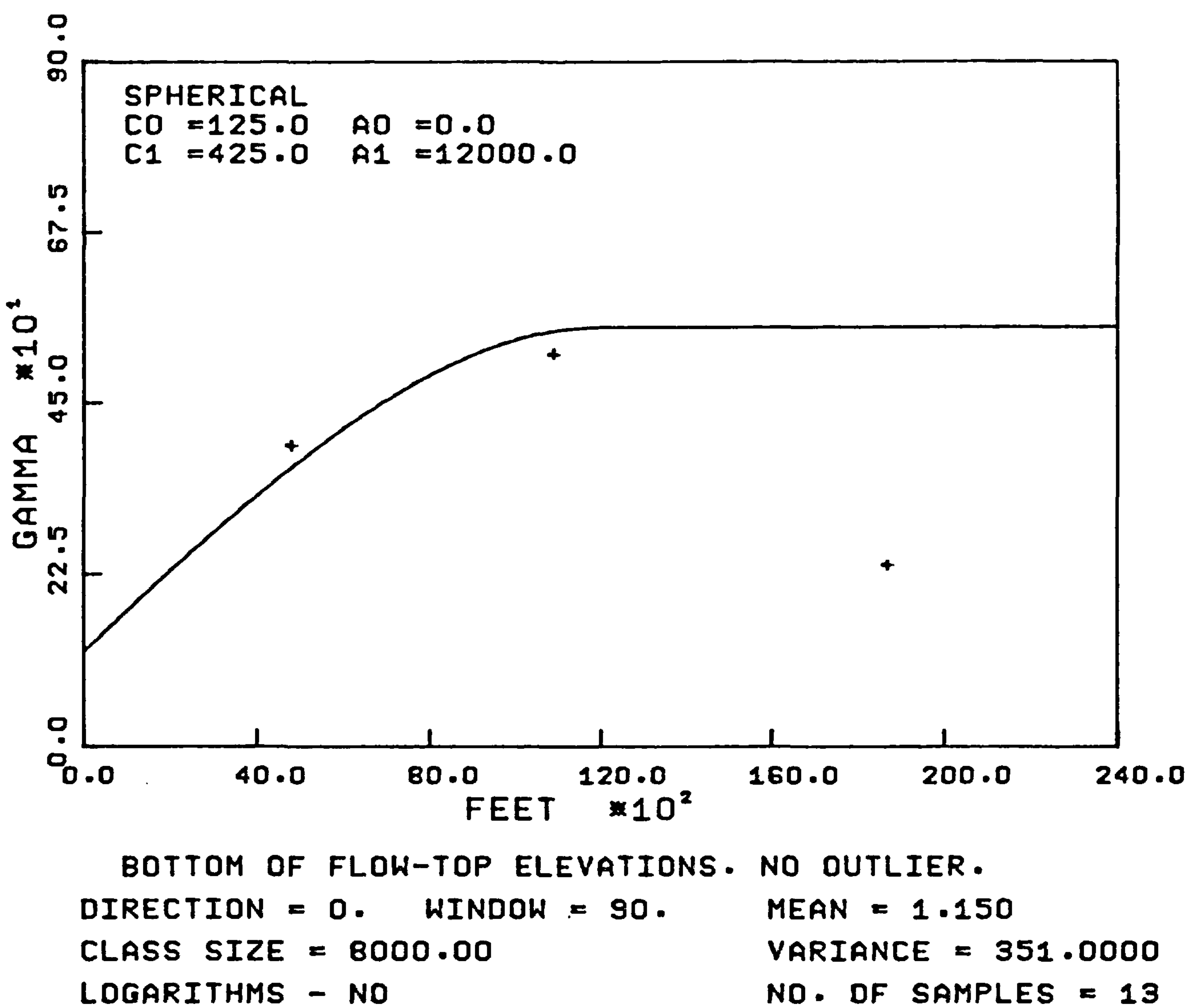


Figure 4. Fitted spherical model - bottom of flow-top

TABLE 2

DRIFT PROGRAM OUTPUT FOR THE BEST CROSS-VALIDATED RESIDUAL VARIOGRAM MODEL

```
EXECUTION OF PROGRAM DRIFT

    BOTTOM OF FLOW-TOP ELEVATIONS. NO OUTLI

    DRIFT COEFFICIENT ESTIMATION OPTION SWITCH =  1


RUN CONTROL PARAMETERS
         IDBUG =    1      ILIST =    0
         ILAM  =    0      IMU   =    0
         IRES  =    1      ND    =    3


MINIMUM NORTHING BOUNDARY =    410000.
MAXIMUM NORTHING BOUNDARY =    470000.
MINIMUM  EASTING BOUNDARY =   2190000.
MAXIMUM  EASTING BOUNDARY =   2250000.

VARIOGRAM PARAMETERS

VARIOGRAM TYPE:   SPHERICAL


    VARIOGRAM  PARAMETERS
         SILL                          .5500E+03
         ANGLE OF ROTATION                   .00
         HOR. ANISOTROPY                    1.00
         VERT. ANISOTROPY                   1.00
         MODEL   C VALUE     RANGE
           1     125.000        0.
           2     425.000    12000.

          DRIFT TERM EXPONENTS ARE

          TERM   1 OF X- AND Y-EXPONENETS ARE     0.   0.
          TERM   2 OF X- AND Y-EXPONENETS ARE     1.   0.
          TERM   3 OF X- AND Y-EXPONENETS ARE     0.   1.

DRIFT EQUATION
```

TERM	X-EXPONENT	Y-EXPONENT	COEFFICIENT	T-STAT
1	.0	.0	-.358E+05	------
2	1.0	.0	.131E-01	.1899E+02
3	.0	1.0	.102E-01	.6512E+01

ID	Y-CORD	X-CORD	Z-CORD	GRADE	DRIFT	RESIDUALS
DC-2	453146.	2246947.	0.	-.1791E+04	-.1790E+04	-.1282E+01
DC-4	454469.	2209991.	0.	-.2236E+04	-.2260E+04	.2357E+02
RRL-2	444298.	2211184.	0.	-.2365E+04	-.2348E+04	-.1713E+02
RRL-6	438580.	2206423.	0.	-.2459E+04	-.2469E+04	.9556E+01
RRL-14	446541.	2203991.	0.	-.2459E+04	-.2419E+04	-.3996E+02
DC-16A	436403.	2211516.	0.	-.2422E+04	-.2424E+04	.2183E+01
DC-20C	451882.	2215285.	0.	-.2222E+04	-.2217E+04	-.5248E+01
DC-22C	448600.	2204190.	0.	-.2394E+04	-.2395E+04	.1398E+01
DC-3	448924.	2225057.	0.	-.2127E+04	-.2119E+04	-.7841E+01
DC-1	453174.	2246998.	0.	-.1786E+04	-.1789E+04	.2765E+01
DC-5	454537.	2210068.	0.	-.2233E+04	-.2258E+04	.2486E+02
DC-16C	436376.	2211006.	0.	-.2401E+04	-.2431E+04	.3013E+02
RRL-2C	444824.	2211376.	0.	-.2348E+04	-.2340E+04	-.8015E+01

```
MEAN OF THE RESIDUALS   =   .1152E+01
STD. DEV. OF RESIDUALS  =   .1874E+02
```

TABLE 3

VALUK PROGRAM OUTPUT FOR THE VARIOGRAM OF

FIGURE 2

RESID. VARIOG. VALIDATION. BOTTOM OF FLOW-TOP, NO OUTLIERS.

RUN CONTROL PARAMETERS

IDBUG	=	1	ILIST	=	1
IPNCH	=	1	ITYPE	=	1
NTYPE	=	1	ISEL	=	1
NUSED	=	0	NK	=	8
RMAX	=	15000.			

MINIMUM NORTHING BOUNDARY	=	410000.
MAXIMUM NORTHING BOUNDARY	=	470000.
MINIMUM EASTING BOUNDARY	=	2190000.
MAXIMUM EASTING BOUNDARY	=	2250000.

SPHERICAL
VARIOGRAM PARAMETERS

SILL	.5500E+03
ANGLE OF ROTATION	.00
HOR. ANISOTROPY	1.00
VERT. ANISOTROPY	1.00

MODEL	C VALUE	RANGE
1	125.000	.0
2	425.000	12000.0

DRIFT PARAMETERS

TERM	X-EXPONENT	Y-EXPONENT
1	.0	.0
2	1.0	.0
3	.0	1.0

KNOWN GRADE VERSUS ESTIMATED GRADE

HOLE ID	KNOWN	ESTIMATED	DIFF	KRIG STD	STD DIFF
DC-4	-.2236E+04	-.2237E+04	.7949E+00	.1564E+02	.5083E-01
RRL-2	-.2365E+04	-.2353E+04	-.1170E+02	.1635E+02	-.7154E+00
RRL-6	-.2459E+04	-.2467E+04	.7556E+01	.2960E+02	.2553E+00
RRL-14	-.2459E+04	-.2417E+04	-.4207E+02	.2029E+02	-.2074E+01
DC-16A	-.2422E+04	-.2397E+04	-.2500E+02	.1717E+02	-.1456E+01
DC-20C	-.2222E+04	-.2204E+04	-.1842E+02	.2456E+02	-.7501E+00
DC-22C	-.2394E+04	-.2421E+04	.2668E+02	.2061E+02	.1295E+01
DC-5	-.2233E+04	-.2236E+04	.2741E+01	.1570E+02	.1746E+00
DC-16C	-.2401E+04	-.2427E+04	.2648E+02	.1656E+02	.1599E+01
RRL-2C	-.2348E+04	-.2353E+04	.5165E+01	.1643E+02	.3143E+00

TABLE 3 Cont'd

VALUK PROGRAM OUTPUT FOR THE VARIOGRAM OF FIGURE 2

SUMMARY STATISTICS OF KRIGING ERRORS

MEAN	-.27767820E+01
VARIANCE	.47759320E+03
STD.DEVIATION	.21853910E+02
AVE.SQ.ERROR	.43754440E+03
WEIGHTED SQ.ERR.	.39469600E+03
SKEWNESS	-.26640980E+00
KURTOSIS	-.72961090E+00
NO. OF ASSAYS	10
AVE KRIG VARIANCE	.39127720E+03

HISTOGRAM OF KRIGING ERRORS

```
                                   **HISTOGRAM**

OBSV RELA  CUML    UPPER
FREQ FREQ  FREQ  CELL LIM. 0         20        40        60        80        100
                           +    +    +    +    +    +    +    +    +    +    +
   1 .100  .100 -.3748E+02 ++++++                                            +
   0 .000  .100 -.3290E+02 +    C                                            +
   0 .000  .100 -.2832E+02 +    C                                            +
   1 .100  .200 -.2373E+02 ++++++    C                                       +
   0 .000  .200 -.1915E+02 +         C                                       +
   1 .100  .300 -.1457E+02 ++++++         C                                  +
   1 .100  .400 -.9986E+01 ++++++              C                             +
   0 .000  .400 -.5403E+01 +                   C                             +
   0 .000  .400 -.8202E+00 +                   C                             +
   2 .200  .600  .3763E+01 +++++++++++                   C                   +
   2 .200  .800  .8346E+01 +++++++++++                             C         +
   0 .000  .800  .1293E+02 +                                       C         +
   0 .000  .800  .1751E+02 +                                       C         +
   0 .000  .800  .2209E+02 +                                       C         +
   2 .200 1.000       INF  +++++++++++                                       C
----                       +    +    +    +    +    +    +    +    +    +    +
  10                       0         20        40        60        80        100
```

HISTOGRAM OF ERRORS WITHIN 2 STANDARD DEVIATIONS

```
                                   **HISTOGRAM**

OBSV RELA  CUML    UPPER
FREQ FREQ  FREQ  CELL LIM. 0         20        40        60        80        100
                           +    +    +    +    +    +    +    +    +    +    +
   0 .000  .000 -.3000E+01 +                                                 +
   1 .100  .100 -.2000E+01 ++++++                                            +
   1 .100  .200 -.1000E+01 ++++++    C                                       +
   2 .200  .400  .0000E+00 +++++++++++         C                             +
   4 .400  .800  .1000E+01 +++++++++++++++++++++                   C         +
   2 .200 1.000  .2000E+01 +++++++++++                                       C
   0 .000 1.000  .3000E+01 +                                                 C
   0 .000 1.000       INF  +                                                 C
----                       +    +    +    +    +    +    +    +    +    +    +
  10                       0         20        40        60        80        100
```

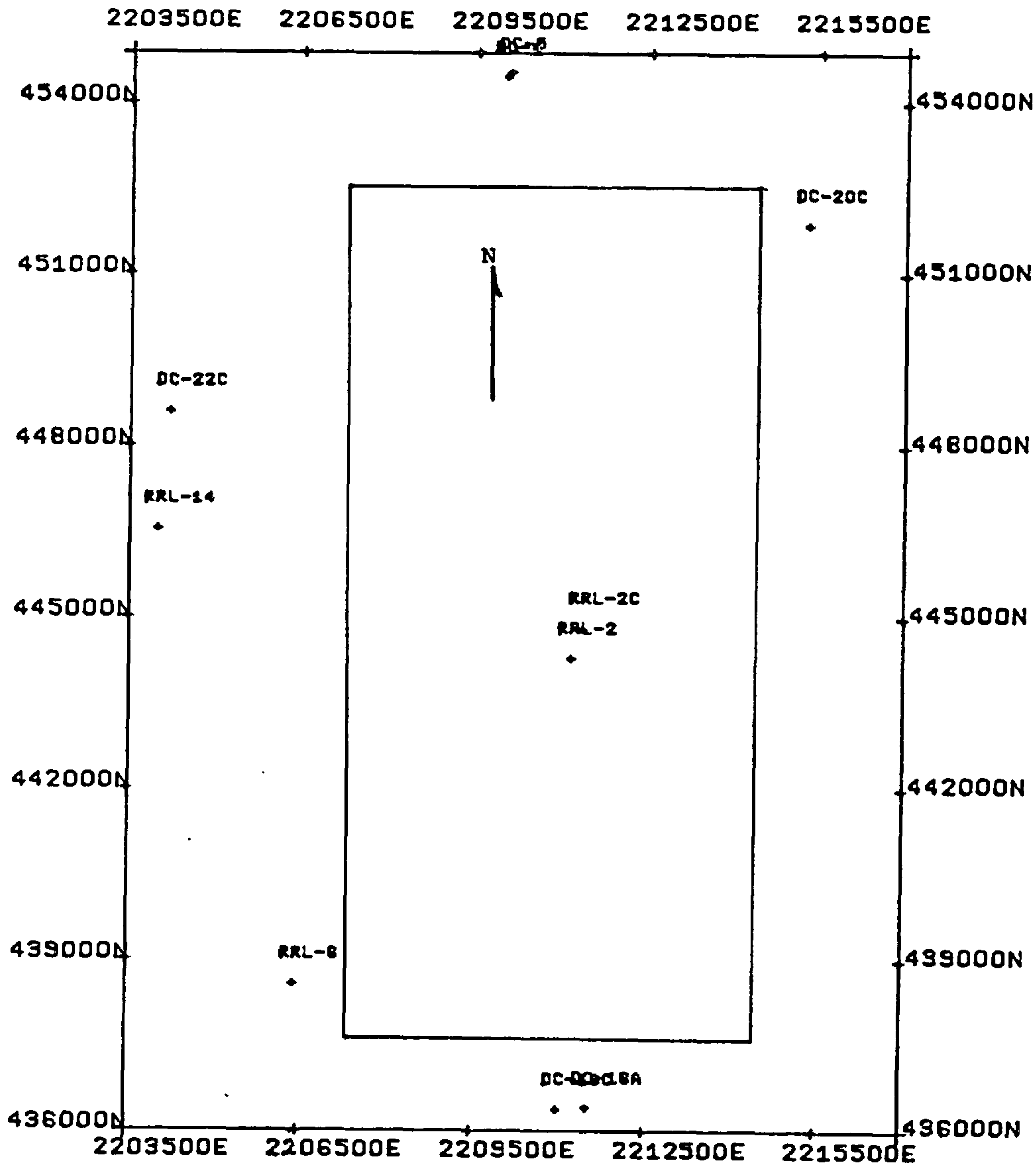


Figure 5. Locations of 10 cross-validated points
Scale: 1"= 3000'

Fig. 5 shows the locations of 10 cross validated points. Please note that 3 holes (DC-1, DC-2, DC-3) shown in Fig. 1 are no longer shown in this plot.

Variogram For the Top of Flow-Bottom Elevation - The same general procedures were repeated to obtain the residual variogram of the top of flow-bottom elevation (See Figure 6). Figure 7 shows the fitted theoretical model that were used to obtain the residual variogram of Figure 6. Table 4 gives the output from the VALUK.IBM program. Again, the cross-validation result given in Table 4 was the best one among many different variogram model parameters that were tried.

Universal Kriging Results

Utilizing the developed variogram models, the elevations of the same area as given by Taylor were estimated using universal kriging. Due to voluminous nature of the output, only a partial listing of the universal kriging program is given in Table 5.

During kriging, the same RMAX and NK values were used and only the 10 samples given in Figure 5 were actually utilized to krig everyone of 500' x 500' grid points. In other words, point kriging instead of block kriging was performed which in turn resulted in larger kriging variance at each grid point. Although Taylor does not mention in his report, it is assumed that he also kriged each grid point.

Figures 8 and 9 are contour maps of the kriged elevation for the bottom of flow-top and the top of flow-bottom. As such, Figures 8 and 9 correspond to Figures 12 and 13 of Taylor (pp.25-26). The obtained contours of Figures 8 and 9 are quite similar to Taylor's, except the southwest portion of the area. This difference is primarily caused by Taylor's adoption of multi-step estimation process and also by using the contour map of top of basalt produced by Rockwell Hanford Operations, rather than his own kriged map. (Please note that, this decision by Taylor further complicates the problem of quantifying the uncertainty of the estimates).

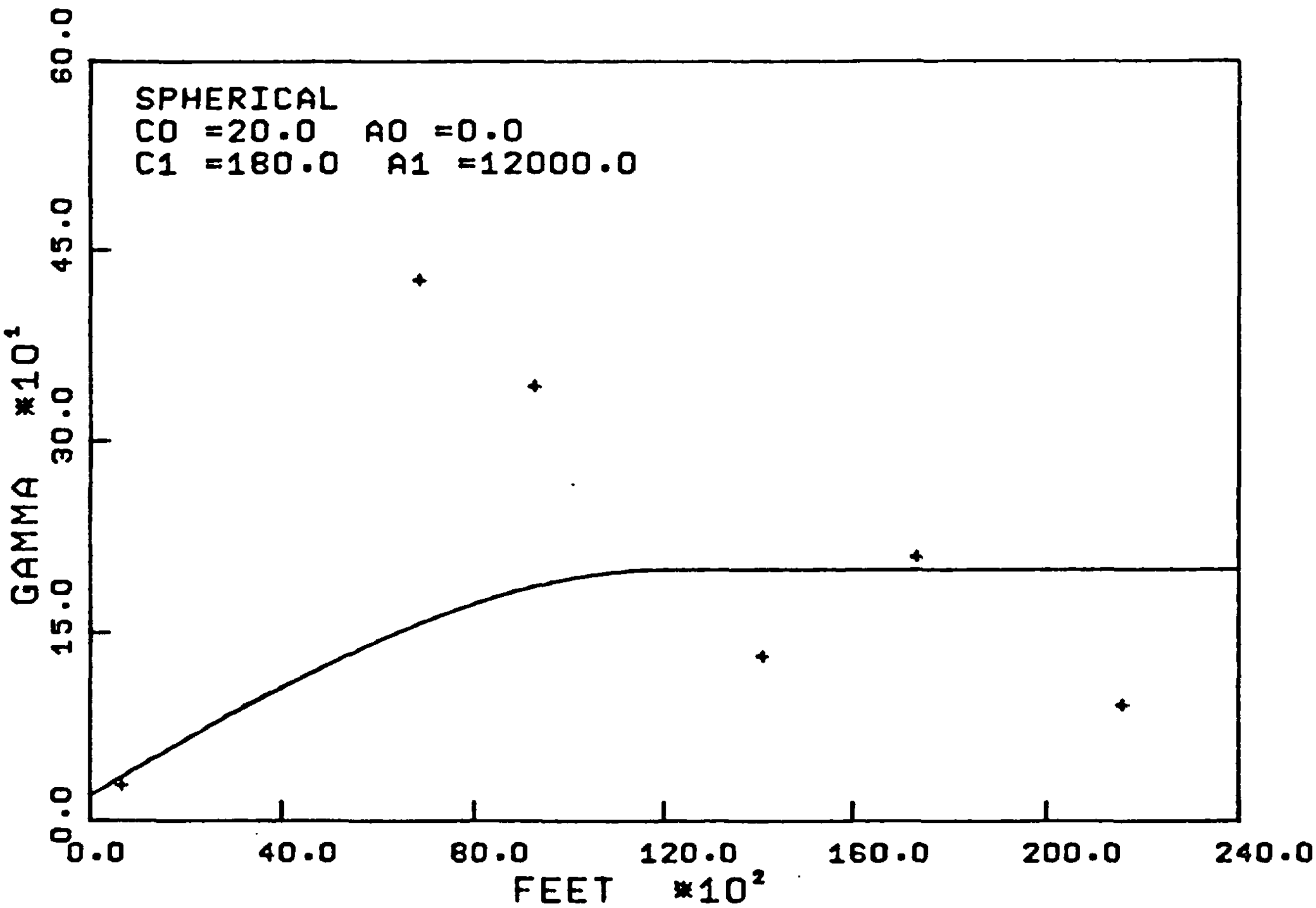
Figures 10 and 11 are contour maps of kriging standard deviation of the bottom of Cohassett flow-top and the top of flow-bottom. As such, these correspond to Figures 14 and 15 of Taylor's report (pp.27-28). Although the general shapes are similar between the two, there exists a large difference in values. This is particularly true between Fig. 11 of this report and Fig. 15 of Taylor's. The values given in this report are less than half of Taylor's. Again, this discrepancy is caused by his multi-step approach of estimation which carried along the uncertainties associated with estimating 3 different variables.

```
                              VARIOGRAM

 TOP OF FLOW-BOTTOM ELEVATIONS. NO OUTLI

RESIDUAL VARIOGRAM -                      NO OUTLIERS.
                                             DATA USED IN CALCULATIONS
 AZIMUTH =   0.  DIP =   0.  WINDOW =  90.  MEAN           =  .112E+01
 CLASS SIZE   =4000.                        VARIANCE       =  .215E+03
 MAX DISTANCE =    80000.                   STD DEVIATION  =  .147E+02
 LOGARITHMS -NO   RELATIVE VARIOGRAM -NO    NO.OF SAMPLES  =        13
 COORDINATE SELECTION-NORTH( 410000. 470000.)   EAST(2190000.2250000.)
                  ELEVATION(      0.       0.)
 ASSAY DATA SELECTION -AMIN(       -9999.000)   AMAX(         999999.0)
```

DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 4000	5.	-.269E+01	.344E+02	.282E+02	660.0
4000 - 8000	10.	.315E+01	.410E+03	.427E+03	6872.3
8000 - 12000	18.	-.223E+01	.331E+03	.343E+03	9274.7
12000 - 16000	9.	-.121E+02	.127E+03	.131E+03	14057.1
16000 - 20000	11.	.228E+01	.216E+03	.210E+03	17337.0
20000 - 24000	5.	-.731E+01	.902E+02	.910E+02	21604.9
28000 - 32000	2.	-.154E+01	.184E+03	.184E+03	31713.3
36000 - 40000	12.	.753E+01	.172E+03	.169E+03	37711.8
40000 - 44000	6.	-.515E+00	.200E+02	.201E+02	43201.3

```
       .410E+03 +    #
       .390E+03 +
       .369E+03 +
       .349E+03 +     #
       .328E+03 +
   G   .308E+03 +
   A   .287E+03 +
   M   .267E+03 +
   M   .246E+03 +
   A   .226E+03 +          #
       .205E+03 +
       .185E+03 +                    #   #
   #   .164E+03 +
   H   .144E+03 +        #
   #   .123E+03 +
       .103E+03 +             #
       .820E+02 +
       .615E+02 +
       .410E+02 +#
       .205E+02 +                            #
                +----+----+----+----+----+----+----+----+----+----+
                0.     16000.     32000.     48000.     64000.     80000.
```

Figure 6. Residual variogram of the top of Cohassett flow-bottom elevation



TOP OF FLOW-BOTTOM ELEVATIONS. NO OUTLIER.

DIRECTION = 0. WINDOW = 90. MEAN = 1.120

CLASS SIZE = 4000.00 VARIANCE = 215.0000

LOGARITHMS - NO NO. OF SAMPLES = 13

Figure 7. Fitted spherical model - top of flow-bottom

TABLE 4

VALUK PROGRAM OUTPUT FOR THE VARIOGRAM OF FIGURE 6

RESID. VARIOG. VALIDATION. TOP OF FLOW-BOTTOM. NO OUTLIERS.

RUN CONTROL PARAMETERS

IDBUG =	1	ILIST =	1
IPNCH =	1	ITYPE =	1
NTYPE =	1	ISEL =	1
NUSED =	0	NK =	8
RMAX =	15000.		

SPHERICAL
VARIOGRAM PARAMETERS

SILL	.2000E+03
ANGLE OF ROTATION	.00
HOR. ANISOTROPY	1.00
VERT. ANISOTROPY	1.00

MODEL	C VALUE	RANGE
1	20.000	.0
2	180.000	12000.0

DRIFT PARAMETERS

TERM	X-EXPONENT	Y-EXPONENT
1	.0	.0
2	1.0	.0
3	.0	1.0

KNOWN GRADE VERSUS ESTIMATED GRADE

HOLE ID	KNOWN	ESTIMATED	DIFF	KRIG STD	STD DIFF
DC-4	-.2489E+04	-.2491E+04	.1915E+01	.6567E+01	.2915E+00
RRL-2	-.2621E+04	-.2602E+04	-.1884E+02	.7711E+01	-.2443E+01
RRL-6	-.2700E+04	-.2695E+04	-.4808E+01	.1780E+02	-.2701E+00
RRL-14	-.2665E+04	-.2664E+04	-.1162E+01	.1084E+02	-.1072E+00
DC-16A	-.2656E+04	-.2651E+04	-.4996E+01	.7952E+01	-.6283E+00
DC-20C	-.2483E+04	-.2460E+04	-.2333E+02	.1457E+02	-.1601E+01
DC-22C	-.2642E+04	-.2636E+04	-.5802E+01	.1106E+02	-.5245E+00
DC-5	-.2488E+04	-.2489E+04	.6204E+00	.6594E+01	.9409E-01
DC-16C	-.2655E+04	-.2663E+04	.7802E+01	.7621E+01	.1024E+01
RRL-2C	-.2597E+04	-.2610E+04	.1297E+02	.7750E+01	.1673E+01

SUMMARY STATISTICS OF KRIGING ERRORS

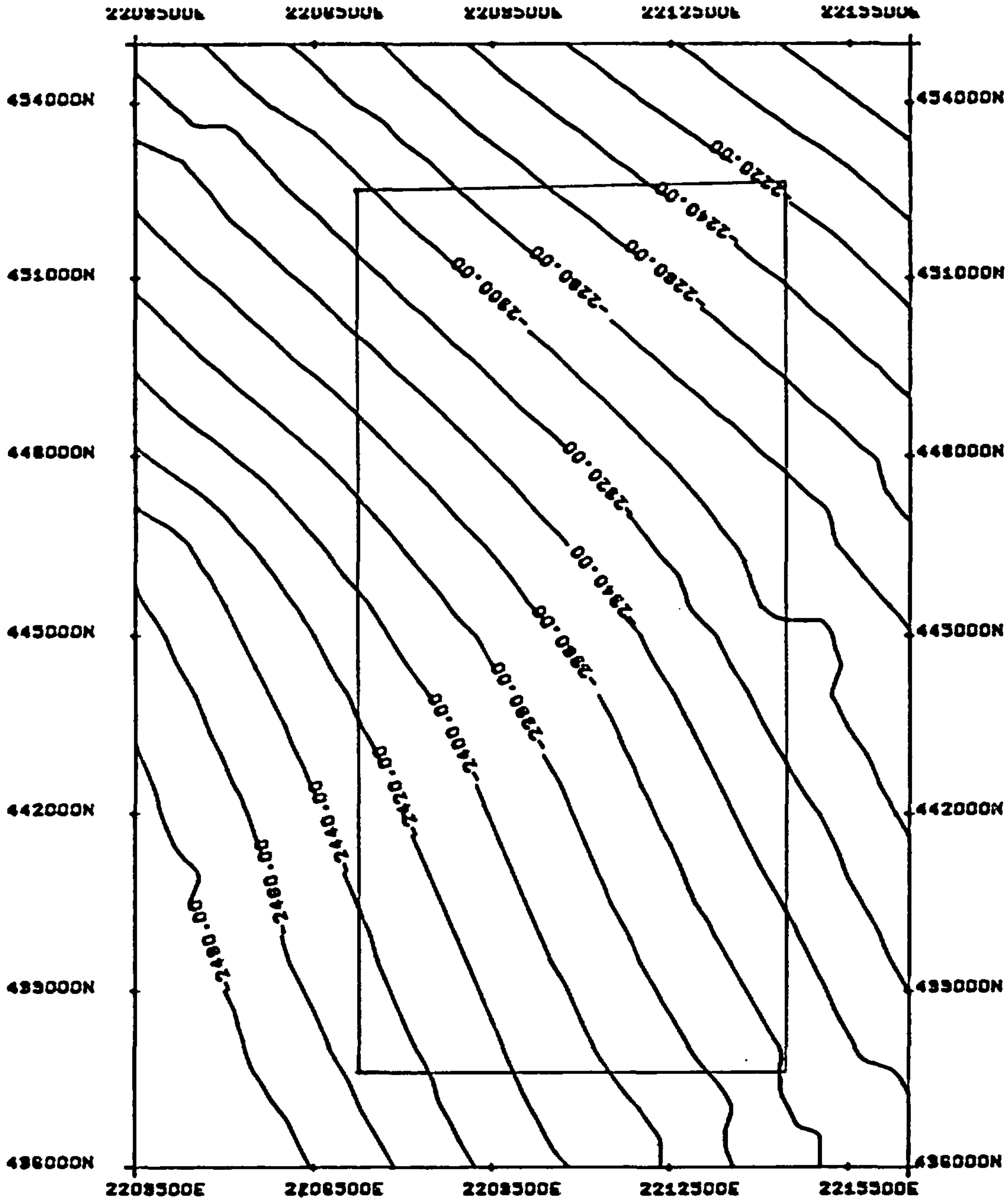
MEAN	-.35626950E+01
VARIANCE	.12091210E+03
STD.DEVIATION	.10996000E+02
AVE.SQ.ERROR	.12151370E+03
WEIGHTED SQ.ERR.	.96376400E+02
SKEWNESS	-.44601580E+00
KURTOSIS	-.46980140E+00
NO. OF ASSAYS	10
AVE KRIG VARIANCE	.10963450E+03

TABLE 5

PARTIAL PROGRAM OUTPUT OF UNIVERSAL KRIGING
BOTTOM OF FLOW-TOP

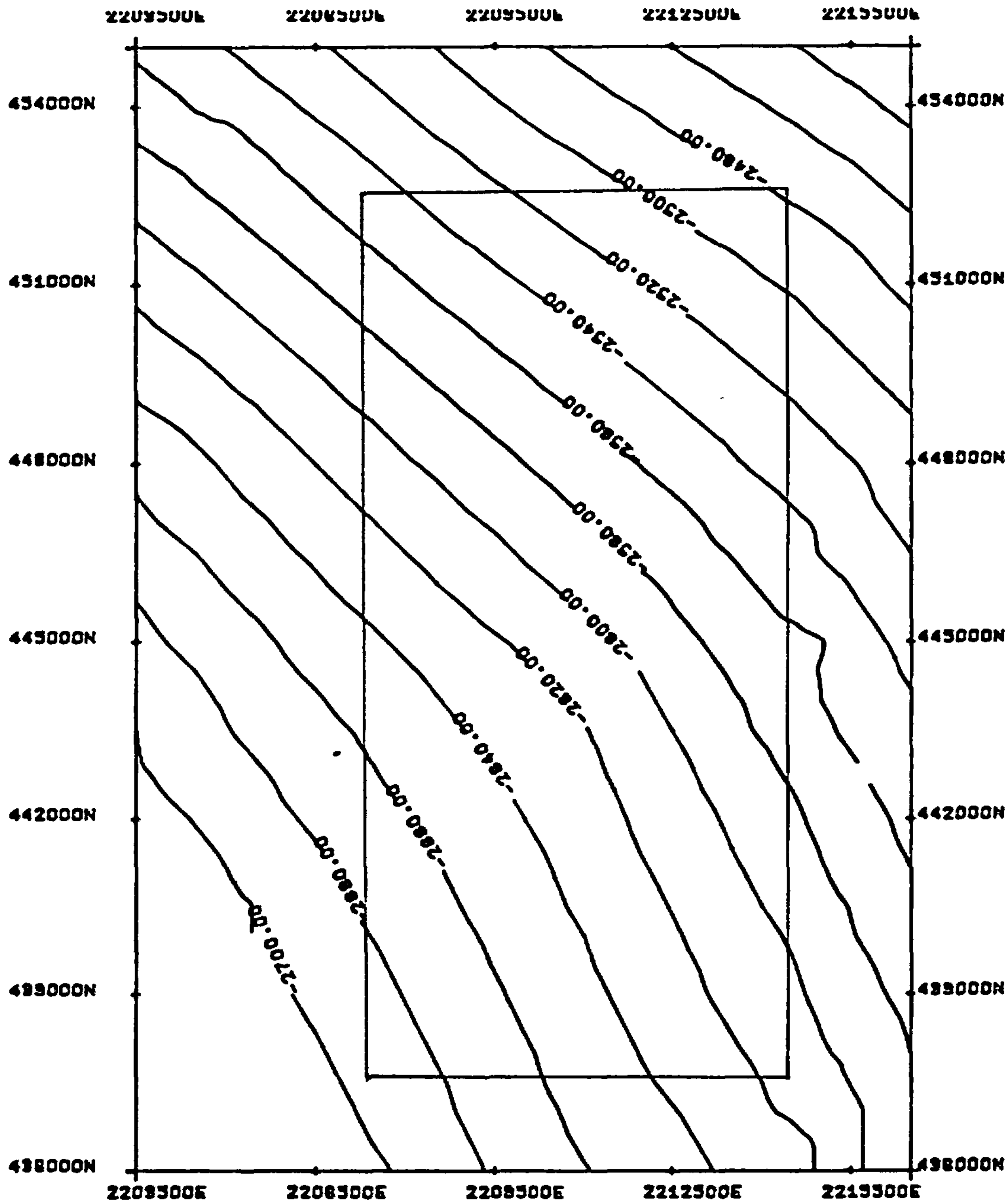
```
EXECUTION OF PROGRAM UNIVKRG

    COHASSETT FLOW - BOTTOM OF FLOW-TOP ELEV

RUN CONTROL PARAMETERS
          IDBUG =    1      ILIST =    1
          IPNCH =    1      IPBK  =    1
          NK    =    8      ND    =    3
          NBC   =    1      NEC   =   26
          NBR   =    1      NER   =   38

 VARIOGRAM PARAMETERS

 VARIOGRAM TYPE:    SPHERICAL


     VARIOGRAM  PARAMETERS
          SILL                        .5500E+03
          ANGLE OF ROTATION                 .00
          HOR. ANISOTROPY                  1.00
          VERT. ANISOTROPY                 1.00
          MODEL    C VALUE     RANGE
              1    125.000        0.
              2    425.000    12000.

     MAX HORIZONTAL DISTANCE TO INCLUDE A HOLE        15000.
      MAX VERTICAL DISTANCE TO INCLUDE A HOLE            10.

     MAX NO. OF HOLES USED TO KRIG A GRID OR A BLOCK     8

     DESCRIPTION OF AREA TO BE KRIGED
          NORTHING OF ROW 1              455000.
          EASTING  OF COL 1             2203500.
          ELEVATION                           0.
          KRIGED AREA IS BOUNDED BY
          COLUMNS                        1 AND   26
          ROWS                           1 AND   38
          GRID OR BLOCK DIMEN. ARE (   500.X    500.)

           DRIFT TERM EXPONENTS ARE

           TERM   1 OF X- AND Y-EXPONENETS ARE     0.    0.
           TERM   2 OF X- AND Y-EXPONENETS ARE     1.    0.
           TERM   3 OF X- AND Y-EXPONENETS ARE     0.    1.


          ***OUTPUT OF KRIGED RESULTS***
```

COLUMN	ROW	Y-COORD.	X-COORD.	ELEV.	VARIABLE 1	VARIANCE 1
1	1	455000.	2203500.	0.	-.2313E+04	.1000E+04
1	2	454500.	2203500.	0.	-.2320E+04	.9249E+03
1	3	454000.	2203500.	0.	-.2328E+04	.8540E+03
1	4	453500.	2203500.	0.	-.2337E+04	.7883E+03
1	5	453000.	2203500.	0.	-.2349E+04	.7124E+03
1	6	452500.	2203500.	0.	-.2356E+04	.6564E+03
1	7	452000.	2203500.	0.	-.2362E+04	.6021E+03



U.K. ELEVATION - BOTTOM OF FLOW-TOP
SCALE: 1 INCH = 3000.00 FEET

Figure 8. Contour map of kriged elevation of the bottom of Cohassett flow-top (Contour interval = 20')



U.K. ELEV. TOP OF FLOW-BOTTOM.COHASETT
SCALE: 1 INCH = 3000.00 FEET

Figure 9. Contour map of kriged elevation of the top of Cohassett flow-bottom (Contour interval = 20')

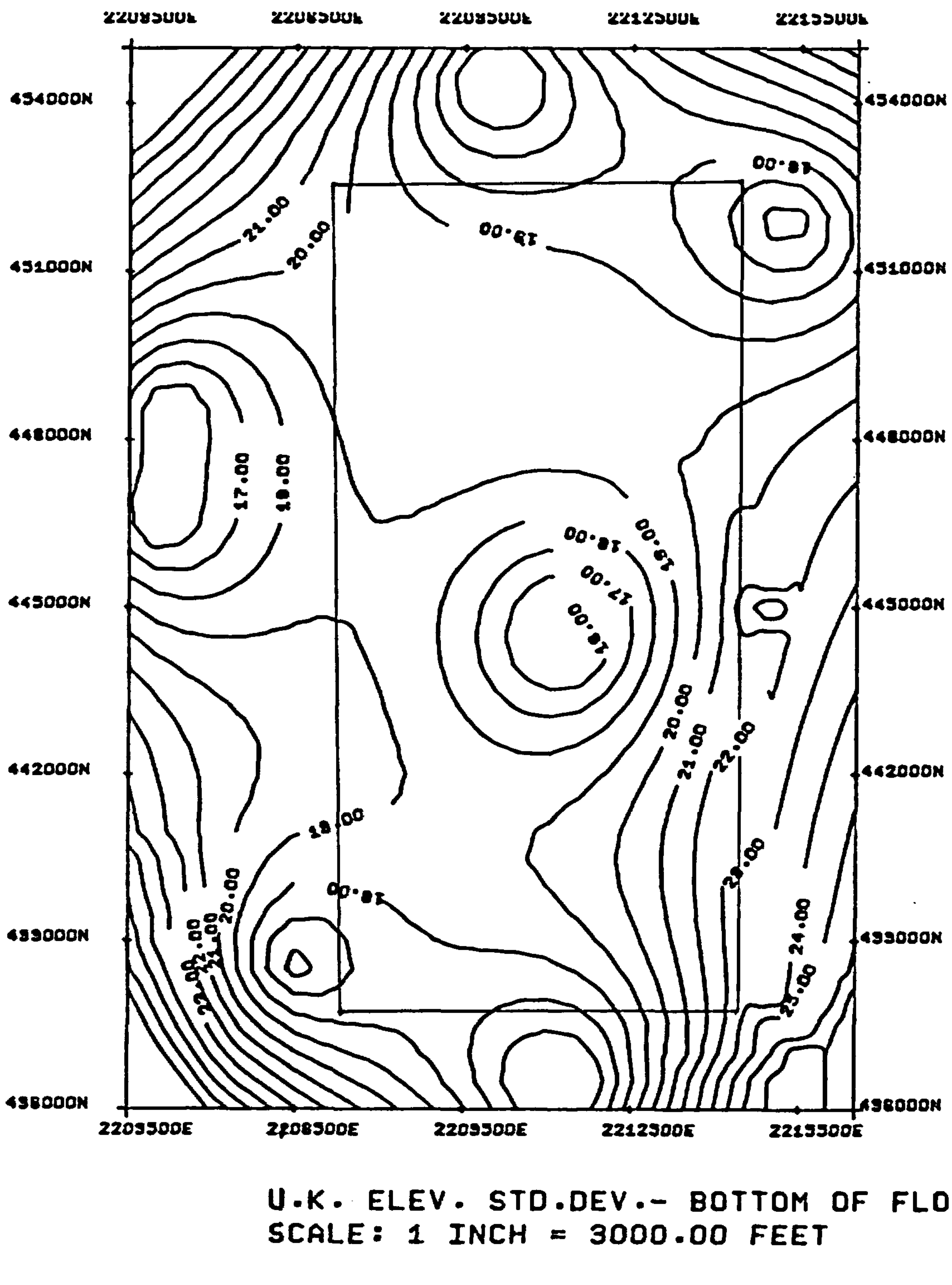
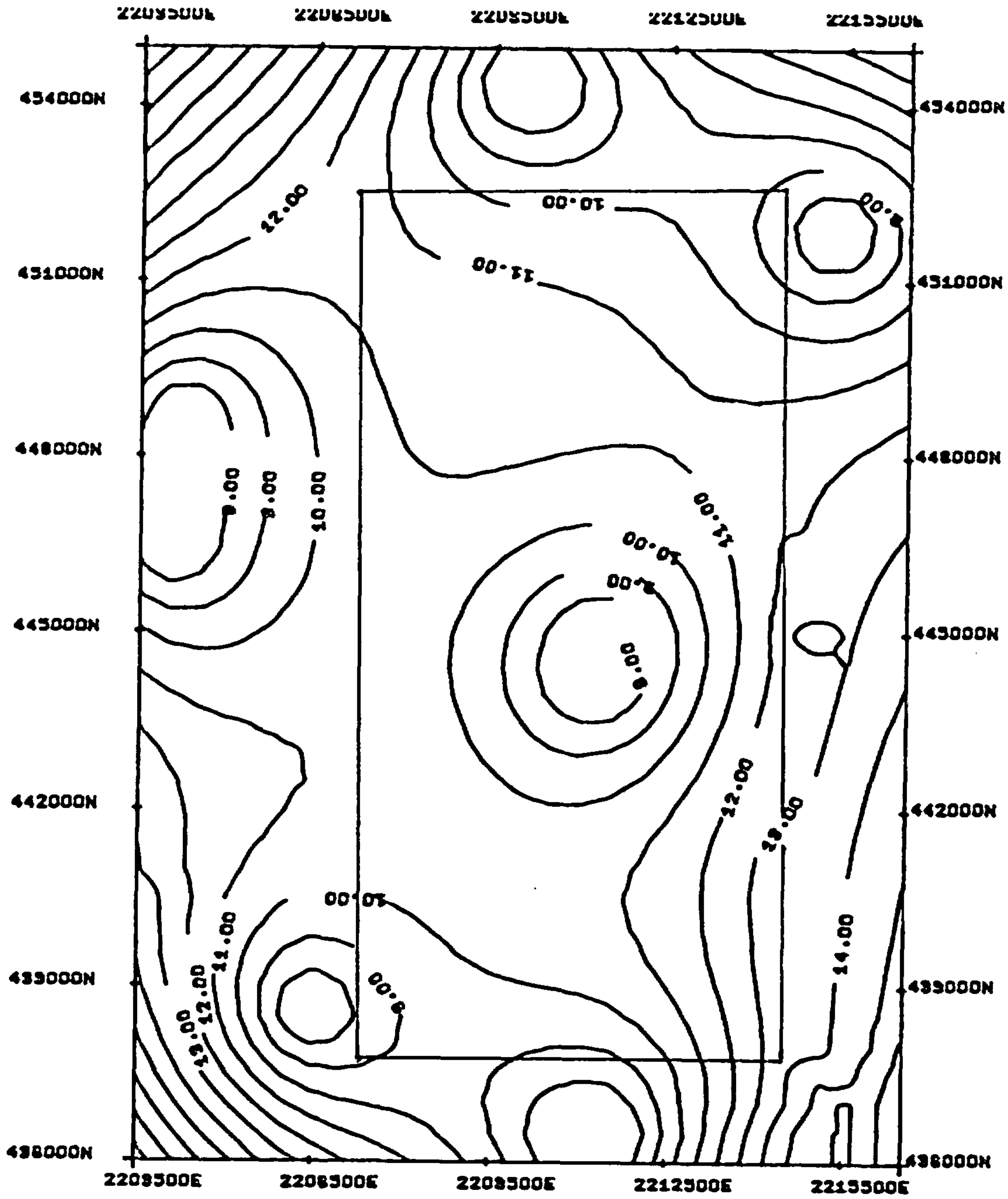


Figure 10. Contour map of kriging standard deviation
Bottom of flow-top (Contour interval = 1')



U.K. ELEV.STD.DEV.- TOP OF FLOW BOTTOM
SCALE: 1 INCH = 3000.00 FEET

Figure 11. Contour map of kriging standard deviation Top of flow-bottom (Contour interval = 1')

Using the kriged elevations and associated kriging standard deviations, the minimum differences in elevations at a pre-specified level of confidence were computed next at each grid point and these values were contoured. This difference is computed using the following equation:

$$\text{DIFFERENCE} = (\text{BFLT} - Z*\text{KRG1.STD}) - (\text{TFLB} + Z*\text{KRG2.STD}) \quad (4)$$

where

KRIG1.STD = kriging standard deviation of BFLT elevation
KRIG2.STD = kriging standard deviation of TFLB elevation
Z value is obtained from standard normal tables.

Figure 12 is the contour of the minimum differences at 80% confidence level (i.e.,Z = 0.84) assuming normality of errors and using one-sided rather than two-sided interval. From this figure, a minimum thickness of approximately 212' is found along the western edge of the repository layout area, which is different than Taylor's results (See Figure 19 on p. 33). At 95% confidence, the minimum thickness is now reduced to approximately 188' which is still thick enough to satisfy buffer zone requirements of the repository design (See Figure 13).

CONCLUSIONS

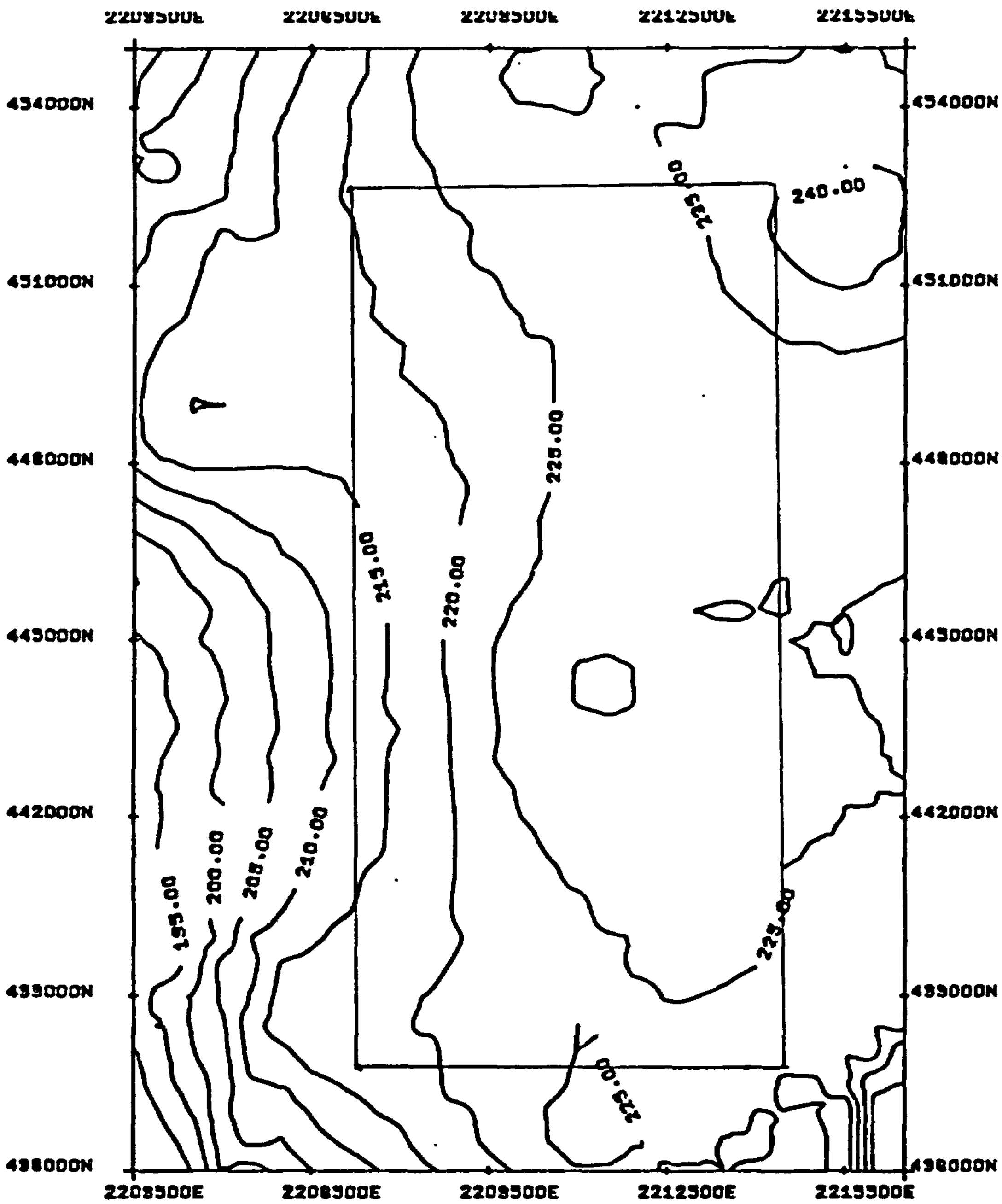
1. Re-estimated elevations of the bottom of Cohassett flow-top (BFLT) and the top of Cohassett flow-bottom (TFLB) using universal kriging are quite similar to those given by Taylor.

2. Re-estimated kriging standard deviations given in this report are substantially smaller than Taylor's because of the direct estimation used here instead of the multi-step approach used by Taylor.

3. As Taylor states in his conclusions, the most important information that geostatistics provides is the uncertainty (or the risk) of the estimate. Quantifying this uncertainty, however, is much more susceptible to the correct modelling of the variogram parameters than obtaining the estimate itself.

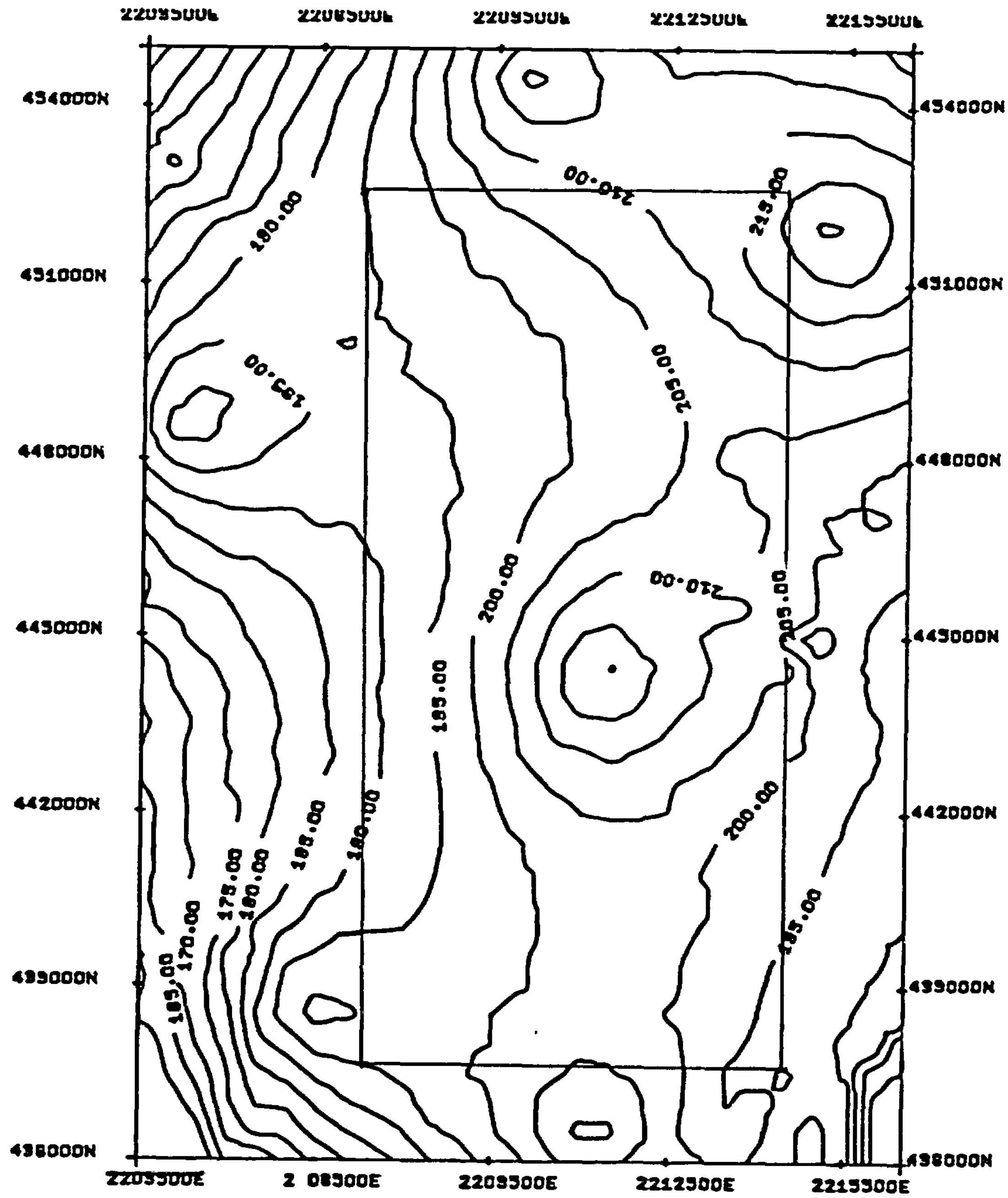
4. The results given in this report should be accepted with some degree of caution, simply because only 13 data points were used for variogram modelling and only 10 data points were used for cross-validation and universal kriging.

5. Assuming the variogram models represent true underlying spatial correlation of samples and also assuming that there will not be any unexpected findings in the future, the obtained minimum difference in elevations at one-sided, 95% confidence interval is



MIN. DIFFERENCE AT 80% CONFIDENCE - 1 SI
SCALE: 1 INCH = 3000.00 FEET

Figure 12. Minimum difference in elevations at one-sided 80% confidence interval (Contour interval = 5')



MIN. DIFFER. AT 95% CONFIDENCE- 1 SIDED
SCALE: 1 INCH = 3000.00 FEET

Figure 13. Minimum difference in elevations at one-sided 95% confidence interval (Contour interval = 5')

sufficient to allow the buffer zone requirements of the repository design.

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Taylor, H.D., 1987, Geostatistical Estimation of Elevations Within the Cohassett Flow at the Reference Repository Location, Report Prepared for Rockwell Hanford Operations, D.A.S.A. of Denver, Colorado.

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