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September 14,1985
4441 E. Holmes St.
Tucson, AZ 85711

Mr. John Greeves
U. S. Nuclear Regulatory Commission
623-SS
Washington, D.C. 20555

Dear Mr. Greeves:

Enclosed herewith is a copy of my report entitled " An Executive Summary on the Feasibility Study of Applying Geostatistics to Repository Site Characterization", and dated September 14, 1985. I hope that you will find the enclosed report interesting.

Sincerely,

Young C Kim

Young C. Kim

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An Executive Summary on the Feasibility Study of Applying Geostatistics to Repository Site Characterization

by

Young C. Kim

September 14, 1985

Department of Mining & Geological Engineering
The University of Arizona
Tucson, Arizona 85721

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PURPOSE OF THIS REPORT

The purpose of this report is to document the results of a feasibility investigation of applying geostatistics to repository site characterization. The variable chosen for this investigation is the thickness of a geologic formation. The respository site selected is the Hanford site in the state of Washington. The data used in this study were provided by Mr. John Buckley of Waste Management Engineering Branch, U. S. Nuclear Regulatory Commission.

This investigation was conducted by the Department of Mining and Geological Engineering, University of Arizona, primarily to satisfy an academic curiosity. No external funding was involved for this study.

Prior to presenting the actual results of this investigation, a capsule view of the geostatistical ore reserve estimation method is first provided in this executive summary. This is done so that the remaining discussion on the results, however brief, will be meaningful to NRC management. Consequently, this portion on the geostatistical ore reserve estimation method can be skipped if one wishes.

A CAPSULE VIEW OF GEOSTATISTICAL ORE RESERVE ESTIMATION METHOD

A. Introduction

The geostatistical ore reserve estimation method (or geostatistics in short) is a method developed by G. Matheron in France during the early 1960's. The underlying mathematical basis of the method is called "The Theory of Regionalized Variables," and the method somewhat resembles the classical statistical approach.

However, there exists one fundamental difference between geostatistics and the classical statistics. In the latter, the samples taken to infer the unknown population are always assumed to be random samples. Furthermore, they are assumed to be independent of each other; i.e., obtaining one sample does not affect the outcome of the next sample.

Since the above conditions are rarely satisfied when one takes drill hole assay samples, geostatistics does not demand that samples be independent. On the contrary, this method assumes that adjoining samples are correlated to each other spatially (or in space). It also assumes that the particular relationship expressing the extent of the above correlation can be analytically and statistically captured in a function known as "variogram function" or "variogram" in short.

Geostatistics, therefore, has a special appeal to a geologist or to a mining engineer, not only because of the above explicit recognition of the spatial relationship existing between adjoining samples, but also due to its ability to provide this relationship in quantitative terms.

B. Variogram

The variogram is used to describe the spatial correlation between grades (or any other characteristics such as the thickness of mineralization) within an ore deposit. Thus, it is the basic tool of geostatistics. To a mining engineer or geologist, the variogram is of interest since it incorporates several geological features that are important in ore deposit evaluation, such as the continuity of mineralization, zones of influence, and whether the zones of influence are different along different directions.

These interesting features are captured by analyzing the similarity or dissimilarity between grades for some distance (h) apart. Specifically, the squared difference between two grades (h) feet apart is calculated. To get a generalized relationship applicable to the area of interest, the calculation is repeated for all the samples that are (h) distance apart and the average squared difference obtained. This is the definition of the variogram.

Figure 11 shows an example of a variogram calculated for the Cohassett Flow dense interior thickness of Basalt at the Hanford Site. On this graph, distance (h) is on the X axis and the one-half average squared difference of the thickness in $(m)^2$, which is called $\gamma(h)$, is plotted on the Y axis. As can be seen from this variogram, the values of $\gamma(h)$ steadily increase with increasing distance, approximately 75000' in this instance. This tells us what we intuitively knew; i.e., the samples nearby have similar values and that samples farther away are likely to have quite different values. The variogram thus numerically describes the continuity of mineralization.

The distance at which the variogram levels off is called the range and is simply the traditional geological notion of range of influence. It means that beyond the range the samples are no longer correlated; i.e., they are independent of one another.

The value of the variogram where it levels off is called the sill value. In Figure 11, this sill value of 70.0 $(m)^2$ is also the variance of all the samples used in variogram development. This is no accident because the computation of the average squared difference between samples (h) distance apart is similar to calculating a variance in classical statistics.

Just as the statistical variance of an estimate is frequently viewed as the uncertainty associated with the given estimate, the variogram can be viewed as the measurement of uncertainty that exists when a sample assay is extended some distance. This uncertainty exists mainly due to the variability between grades, as has been captured in the variogram.

The value of the variogram at distance (h) = 0 is called the nugget value. Ideally, this nugget value should be zero simply because two samples from the same point (i.e., h = 0) should have the same grade. A positive nugget value, on the other hand, can have real physical meaning or be due to errors in sampling or assaying. In general, the nugget

should be relatively small if correct sampling and assaying procedures have been used. In Figure 11 the nugget value is relatively small, in the neighborhood of about 7.0 $(m)^2$.

A third item of geological interest that can be obtained from the variogram is whether there is anisotropy in the mineralization, i.e., is the mineralization more continuous in one direction than another? Anisotropy is determined by comparing the range of influence of the variograms, computed along different directions.

It is hoped that the above brief description of the variogram gives at least some feeling of how the variogram captures and displays several of the geologic features.

C. Kriging

There are at least two basic steps that must be performed for a geostatistical site characterization. These two basic steps are: 1) developing the representative variogram model(s) of the geologic horizon and 2) assigning local block grades using an inference (or weighting) scheme known as "Kriging".

The first step is mandatory because it is a basic tool of geostatistics. Performing the first step is sufficient for obtaining a global estimate and associated confidence limits of the estimate, i.e., the average thickness of the entire horizon. The second step of kriging blocks within the horizon is needed if one is interested in knowing the estimated thickness at those block locations.

The attractiveness of the second step, kriging, compared to other methods of assigning block grades is that kriging provides an unbiased estimate having minimum estimation error. It does this by utilizing the variogram to assign weights to each sample grade in such a manner that the sample information is used to best advantage.

In Figure 16, the average thickness of the Cohassett Flow dense interior thickness around the proposed reference repository location (RRL) was estimated using the variogram shown in Figure 11 and using the entire 20 data points that are currently available. As can be seen directly under the heading of "output" in Figure 16, the estimated average thickness of the block whose dimensions are 25,000' X 30,000' with the S-W coordinate of 43,000N and 220,000E is 46.84 meters. Furthermore, this estimate of 46.84 meter is accurate within 2 times square root of 1,447 $(m)^2$, or 2.41 m, with 95% confidence (and assuming that estimation errors are normally distributed with mean "zero" and variance equal to "kriging variance").

The estimated average thickness of 46.84 meter is the weighted average of the 20 data points. The particular weight given to each assay is also shown in Figure 16. Please note that more weights are given to nearer samples whereas practically zero weights are given to samples farther away. Also note the influence of one abnormally high sample thickness of 71.7 meter (Hole DC-16A) on the kriged estimate.

Since one is just as much interested in knowing the thickness variability within the RRL, this information in terms of standard deviation of point samples within RRL can be computed and is given as 4.908 meter in Figure 16. In other words, the thickness can vary 2 times 4.908 meter, or 9.816 meter, with 95% confidence.

The kriging method used above is called "Ordinary Kriging". In addition, there are many other variations of this ordinary kriging such as: 1) simple kriging, 2) universal kriging, 3) co-kriging, 4) disjunctive kriging, 5) lognormal kriging, 6) indicator kriging, 7) probability kriging, and 8) "Bayesian" or soft kriging.

DATA ANALYSIS AND VARIOGRAM MODELING

Prior to actual computations of variograms, the keypunched data for the 20 DDH's were plotted and verified for accuracy. Figure 1 shows the listing of the data, their respective locations on Washington State Coordinate system, and the approximate location of RRL block that was kriged. Figure 2 shows; 1) the listing of thickness sorted by increasing order, 2) a summary statistics, and 3) the histogram plot of the data. From Figure 2, one can immdiately see an abnormally high thickness value of 71.7 meter for hole #DC-16A. Comparisonof thicknesses surrounding this hole all indicate that this value may be an outlier. (This assay does not appear on pages 6-156 of Draft EA document for the Hanford site).

A. Initial Attempt on Variogram Model Development

Obtaining good and representative variograms involves a considerable amount of computation even using a computer, due to the empirical as well as subjective nature of the work. The magnitude of the task increases as the complexity of the geologic structure increases. As the first step, the average variograms in all directions using a window of ±90 degrees were computed for different class sizes.

Figures 3 through 5 show the computed variograms using the entire 20 data points. Figure 3 is using a class size of 10,000', i.e., samples that fall within this interval were used to compute the variogram value for that class size. Figure 4 is based on 15,000' class size whereas 20,000' is used in Figure 5.

Unfortunately, none of these 3 variogram models are satisfactory either due to their randomly fluctuating patterns as seen in Figures 3 and 4 or due to entirely horizontal pattern for the first four points of Figure 5.

B. Renewed Attempt on Variogram Model Development

Since there appeared to be a possible outlier in the data set (i.e., Hole DC-16A), another variogram development attempt was made excluding this assay. Figure 6 shows the resulting statistics of 19 data points. The mean thickness is now 38.07 meter instead of 39.75 meter in Figure 2. Similarly, the variance is reduced to 70.0 $(m)^2$ from 120 $(m)^2$ in Figure 2.

The computed experimental variogram using a class size of 10,000' and 19 data points is given in Figure 7, which is quite satisfactory for our purposes. In other words, the variogram shows a nice linear increasing trend up to the sample pair distance of about 40,000'. Similar results were also obtained using a class size of 20,000' (See Figure 10). Consequently, it was decided to use the results of Figures 7 and 10 for the subsequent geostatistical application.

C. Fitting a Theoretical Model to the Obtained Results

After obtaining experimental variograms, theoretical variogram models that duplicate the experimental variograms (or results) most closely had to be decided. There are two main reasons for replacing the actual model (i.e., the experimental model) by the theoretical one. These are: 1) to guarantee the so-called "positive definite" conditions of the variance-covariance matrix used in kriging, and 2) to recognize the fact that the experimental results are simply one "unique realization" of the underlying random process.

A theoretical variogram model known as the "Spherical Model" was fitted to the experimental results given in Figures 7 and 10. In fitting a theoretical model, both visual fit and cross validation technique were used to verify the selected theoretical model parameters.

The cross-validation technique is basically to use hindsight to choose the best parameter. It is an iterative technique and it is as follows:

> First, take each known data point and predict (or estimate) a value for this very point by kriging, pretending you do not know this data point and using a given model parameter. In other words, only the surrounding data points are used to krig this data point, while excluding the data point being kriged. Next, calculate the error between the predicted value and the true known value for each data point. Repeat the above two steps while varying the model parameters in a manner to reduce the prediction error. Obviously, the "correct" parameter to be chosen is the one giving the least amount of the prediction error.

Figures 8 and 11 are the two best fitted models to the results of Figures 7 and 10, respectively. For those readers who are more familiar with the cross-validation technique, Figures 9 and 12 give the cross-validation results for the respective fit given in Figures 8 and 11.

KRIGING OF REFERENCE REPOSITORY LOCATION

Simply stated, kriging refers to a statistical weighting scheme to weight the surrounding DDH (or any other) assays to obtain an estimated block (or panel) grade. The weighting of the surrounding information is done in such a manner as to provide an unbiased estimate having the minimum estimation error. Furthermore, the scheme provides the confidence limits about each estimated block grade in the form of "kriging variance" for that estimate.

The kriged block for this study covered an area of 25,000' X 30,000' with the S-W coordinate of 43,000N and 220,000E (See Figure 1). There are altogether 9 assays within the block including the one possible outlier of Hole DC-16A. Figure 13 summarizes the statistics of these 9 assays, whereas Figure 14 summarizes the statistics of 8 assays excluding that of DC-16A.

Figure 15 is the kriged results using the variogram of Figure 8, whereas Figure 16 is the kriged results using the variogram of Figure 11. The estimated average thickness for the entire RRL is 46.72 meter in Figure 15 and 46.84 meter in Figure 16. These two numbers can be compared with the numerical average thickness of 45.82 meter as given in Figure 13. The associated kriging variances are 1.719 $(m)^2$ and 1.447 $(m)^2$, respectively.

Since one is just as much interested in knowing the thickness variability within the RRL, this information in terms of standard deviation of (point) samples within RRL is also given in Figure 15 as 5.317 meter and in Figure 16 as 4.908 meter. These numbers can, in turn, be compared with the standard deviation of 3.89 meter given in Figure 14.

Since a standard deviation of 3.89 meter is much smaller than the other two, the use of the classical statistical technique will tend to be overly optimistic by ignoring the redundancy aspect of correlated samples.

CONCLUDING COMMENTS

In the final analysis, the results obtained during this feasibility investigation must be taken with a grain of salt. This is because, the true underlying variogram of the random process is rather difficult to capture with only 19 data points. However, this fact should in no way hinder the application of geostatistics, simply because one can nevertheless perform sensitivity analysis on the variogram itself. Fortunately, kriging is a rather robust technique which can tolerate a considerable amount of variation on the variogram parameter.

Bayesian or soft kriging which utilizes both the actual data and subjective data of the experts is another potential tool that can be used for repository site characterization.

DDH-3	371500.	2309000.	39.6
DC-2	453000.	2247000.	48.2
DC-3	449000.	2225000.	46.0
DC-4	454500.	2210000.	45.1
DC-6	459500.	2277500.	38.6
DC-8	420000.	2280500.	46.0
DC-12	415500.	2241500.	38.1
DC-14	489500.	2261000.	33.2
DC-15	390000.	2310000.	41.2
DC-16A	433000.	2210500.	71.7
DC-19C	434000.	2225000.	42.7
DC-20D	452000.	2215000.	45.4
DC-22C	448500.	2204500.	37.6
DH-4	521500.	2284500.	19.8
DH-5	519500.	2175000.	25.3
RRL-2	444500.	2211000.	45.1
RRL-6	438500.	2206500.	43.2
RRL-14	446500.	2204000.	35.6
RSH-1	402000.	2180000.	23.5
MCGEE	458000.	2192000.	29.1

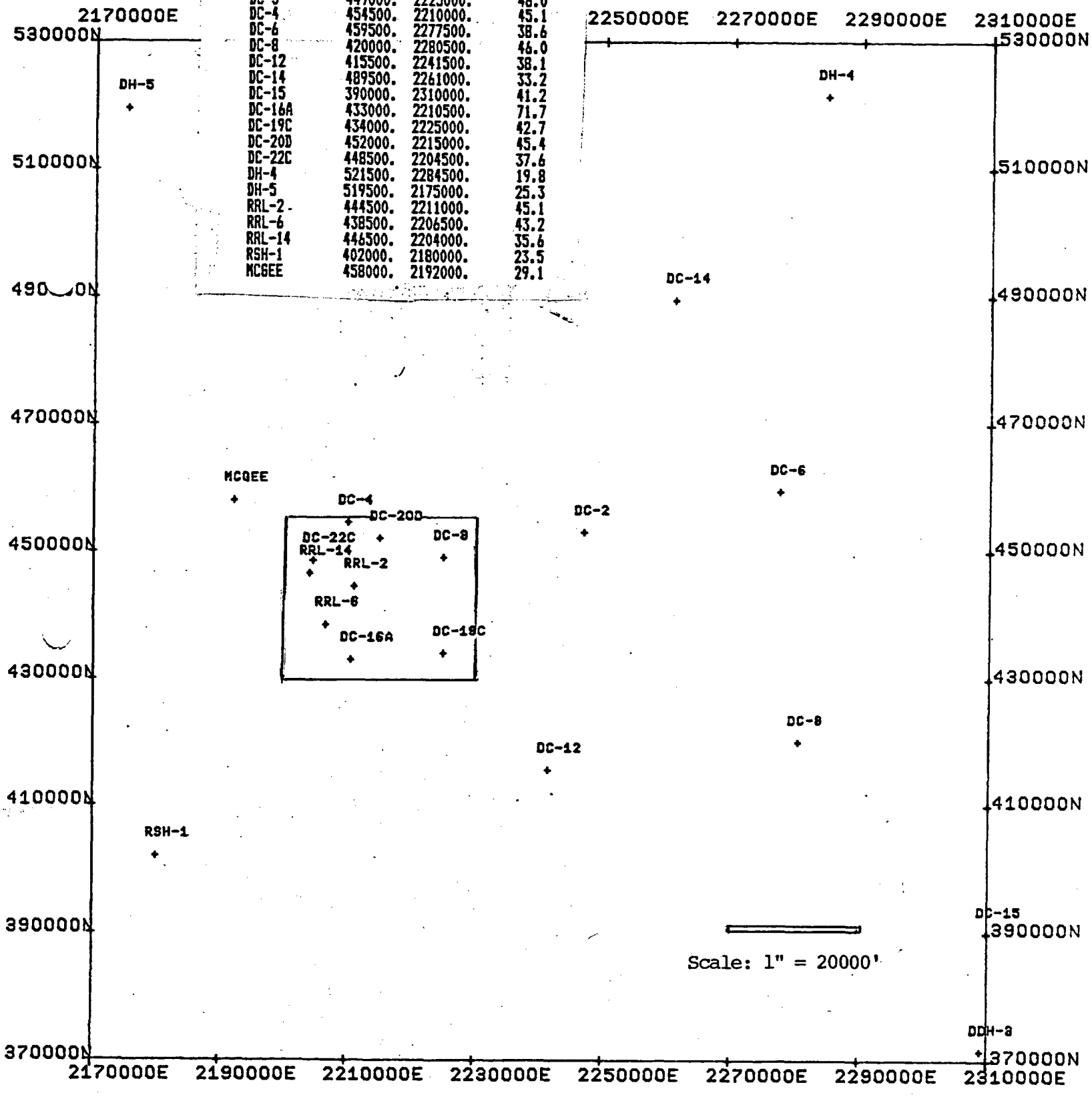

Figure 1

THICKNESS STATISTICS

NUMBER OF ASSAYS = 20

ASSAY DATA - SORTED

19.800 23.500 25.300 29.100 33.200 35.600 37.600 38.100 38.600 39.600
41.200 42.700 43.200 45.100 45.100 45.400 46.000 46.000 48.200 71.700

SUMMARY STATISTICS FOR UNTRANSFORMED DATA

MEAN = .39750010E+02
VARIANCE = .12291050E+03
STD.DEV. = .11086500E+02
SKEWNESS = .69104730E+00
KURTOSIS = .19134950E+01
COEF VAR = .27890560E+00

DATA SELECTION:

MIN. ASSAY VALUE = .000
MAX. ASSAY VALUE = 80.000

HISTOGRAM

THICKNESS STATISTICS

```
OBSV  RELA  CUML   UPPER
FREQ  FREQ  FREQ  CELL LIM. 0         5         10        15        20        25
                            +    +    +    +    +    +    +    +    +    +    +
   1  .050  .050  .2000E+02 +***********                                        +
   0  .000  .050  .2200E+02 +                                                   +
   1  .050  .100  .2400E+02 +***********                                        +
   1  .050  .150  .2600E+02 +***********                                        +
   0  .000  .150  .2800E+02 +                                                   +
   1  .050  .200  .3000E+02 +***********                                        +
   0  .000  .200  .3200E+02 +                                                   +
   1  .050  .250  .3400E+02 +***********                                        +
   1  .050  .300  .3600E+02 +***********                                        +
   1  .050  .350  .3800E+02 +***********                                        +
   3  .150  .500  .4000E+02 +******************************                     +
   1  .050  .550  .4200E+02 +***********                                        +
   2  .100  .650  .4400E+02 +********************                               +
   3  .150  .800  .4600E+02 +******************************                     +
   2  .100  .900  .4800E+02 +********************                               +
   1  .050  .950  .5000E+02 +***********                                        +
   0  .000  .950  .5200E+02 +                                                   +
   0  .000  .950  .5400E+02 +                                                   +
   0  .000  .950  .5600E+02 +                                                   +
   1  .050 1.000    INF     +***********                                        +
  ---                       +    +    +    +    +    +    +    +    +    +    +
  20                        0         5         10        15        20        25
```

Figure 2.

VARIOGRAM

WA

THICKNESS VARIOGRAM

AZIMUTH = 0. WINDOW = 90.
CLASS SIZE = 10000.
MAX DISTANCE = 200000.
LOGARITHMS -NO RELATIVE VARIOGRAM -NO
COORDINATE SELECTION-NORTH(0. 999999.) EAST(0.9999999.)
ASSAY DATA SELECTION -AMIN(.000) AMAX(100.0)

DATA USED IN CALCULATIONS
MEAN = .397E+02
VARIANCE = .123E+03
STD DEVIATION = .111E+02
NO.OF SAMPLES = 20

DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 10000	9.	.550E+01	.600E+02	.613E+02	6876.2
10000 - 20000	22.	.236E+01	.122E+03	.127E+03	14771.3
20000 - 30000	15.	.613E+01	.807E+02	.776E+02	23277.5
30000 - 40000	14.	.516E+01	.145E+03	.137E+03	36091.1
40000 - 50000	14.	.106E+02	.140E+03	.137E+03	44047.5
50000 - 60000	14.	.570E+01	.736E+02	.726E+02	54954.0
60000 - 70000	15.	.585E+01	.879E+02	.860E+02	64498.9
70000 - 80000	22.	.643E+01	.124E+03	.124E+03	74959.9
80000 - 90000	9.	.339E+01	.115E+03	.116E+03	84227.3
90000 -100000	9.	.691E+01	.260E+03	.259E+03	95468.2
100000 -110000	11.	-.136E+00	.199E+03	.199E+03	104811.0
110000 -120000	16.	.321E+01	.175E+03	.176E+03	114734.3
120000 -130000	10.	.548E+01	.188E+02	.187E+02	124802.8
130000 -140000	4.	.168E+02	.147E+03	.147E+03	133312.5
140000 -150000	2.	.156E+02	.135E+03	.135E+03	145261.2
150000 -160000	2.	.118E+02	.101E+03	.994E+02	155367.2
180000 -190000	1.	.159E+02	.126E+03	.126E+03	187070.2
190000 -200000	1.	.143E+02	.102E+03	.102E+03	199649.7

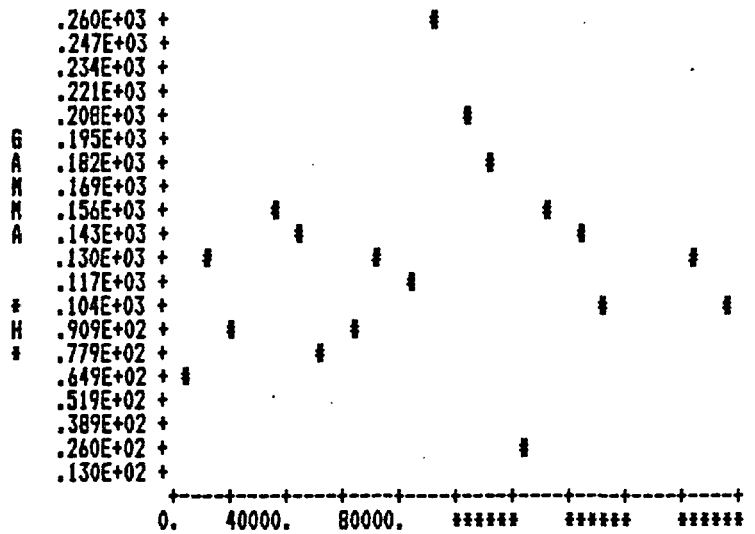

Figure 3

VARIOGRAM

WA

THICKNESS VARIOGRAM

AZIMUTH = 0. WINDOW = 90.
CLASS SIZE = 15000
MAX DISTANCE = 300000.
LOGARITHMS -NO RELATIVE VARIOGRAM -NO
COORDINATE SELECTION-NORTH(0. 999999.) EAST(0.9999999.)
ASSAY DATA SELECTION -AMIN(.000) AMAX(100.0)

DATA USED IN CALCULATIONS
MEAN = .397E+02
VARIANCE = .123E+03
STD DEVIATION = .111E+02
NO.OF SAMPLES = 20

DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 15000	19.	.279E+01	.111E+03	.139E+03	9561.8
15000 - 30000	27.	.520E+01	.866E+02	.848E+02	20531.2
30000 - 45000	23.	.839E+01	.163E+03	.162E+03	38542.3
45000 - 60000	19.	.566E+01	.662E+02	.658E+02	52885.4
60000 - 75000	27.	.735E+01	.944E+02	.932E+02	68332.2
75000 - 90000	19.	.322E+01	.134E+03	.133E+03	80509.5
90000 -105000	15.	.267E+01	.220E+03	.218E+03	98244.1
105000 -120000	21.	.342E+01	.192E+03	.191E+03	113058.3
120000 -135000	13.	.846E+01	.541E+02	.557E+02	126545.1
135000 -150000	3.	.144E+02	.114E+03	.115E+03	142237.8
150000 -165000	2.	.118E+02	.101E+03	.994E+02	155367.2
180000 -195000	1.	.159E+02	.126E+03	.126E+03	187070.2
195000 -210000	1.	.143E+02	.102E+03	.102E+03	199649.7

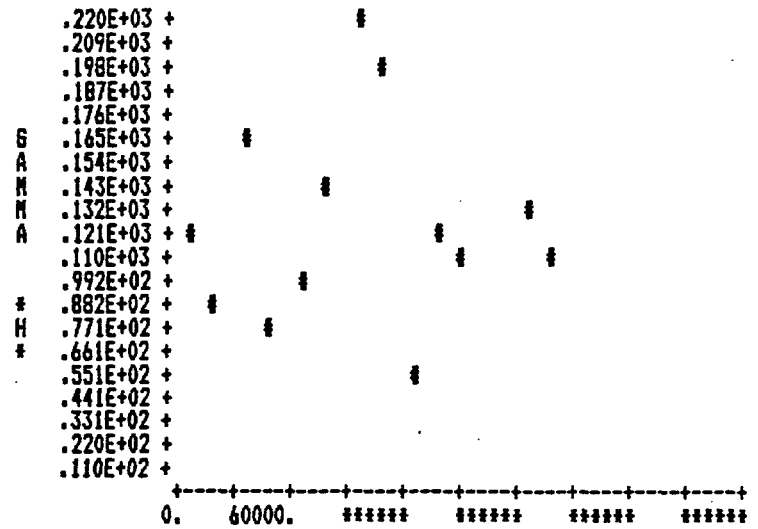

Figure 4

VARIOGRAM

WA

THICKNESS VARIOGRAM

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                                                DATA USED IN CALCULATIONS
AZIMUTH =   0.   WINDOW =  90.                  MEAN          =  .397E+02
CLASS SIZE  =   20000.                          VARIANCE      =  .123E+03
MAX DISTANCE =   400000.                        STD DEVIATION =  .111E+02
LOGARITHMS -NO   RELATIVE VARIOGRAM -NO         NO.OF SAMPLES =        20
COORDINATE SELECTION-NORTH(      0. 999999.)   EAST(      0.9999999.)
ASSAY DATA SELECTION -AMIN(            .000)   AMAX(           100.0)
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DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 20000	31.	.327E+01	.104E+03	.117E+03	12479.1
20000 - 40000	29.	.566E+01	.112E+03	.112E+03	29463.4
40000 - 60000	28.	.816E+01	.107E+03	.101E+03	49500.7
60000 - 80000	37.	.619E+01	.110E+03	.110E+03	70719.0
80000 -100000	18.	.515E+01	.187E+03	.192E+03	89847.8
100000 -120000	27.	.184E+01	.185E+03	.185E+03	110691.5
120000 -140000	14.	.872E+01	.555E+02	.571E+02	127234.1
140000 -160000	4.	.137E+02	.118E+03	.116E+03	150314.2
180000 -200000	2.	.151E+02	.114E+03	.114E+03	193359.9

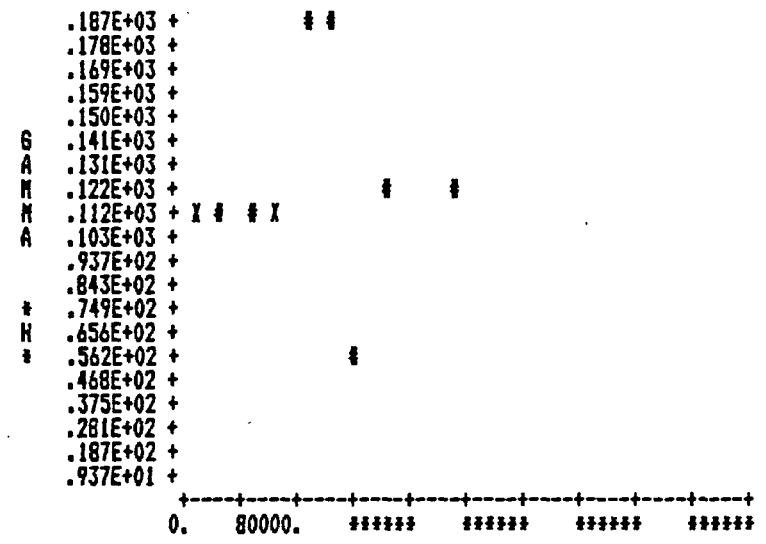


Figure 5

THICKNESS STATISTICS

NUMBER OF ASSAYS = 19

ASSAY DATA - SORTED

19.800 23.500 25.300 29.100 33.200 35.600 37.600 38.100 38.600 39.600
41.200 42.700 43.200 45.100 45.100 45.400 46.000 46.000 48.200

SUMMARY STATISTICS FOR UNTRANSFORMED DATA

MEAN =	.38068430E+02
VARIANCE =	.70042880E+02
STD.DEV. =	.83691630E+01
SKEWNESS =	-.84961370E+00
KURTOSIS =	-.39955710E+00
COEF VAR =	.21984520E+00

DATA SELECTION:

MIN. ASSAY VALUE =	.000
MAX. ASSAY VALUE =	65.000

HISTOGRAM

THICKNESS STATISTICS

OBSV FREQ	RELA FREQ	CUML FREQ	UPPER CELL LIM.	0 5 10 15 20 25
1	.053	.053	.2000E+02	**********
0	.000	.053	.2200E+02	
1	.053	.105	.2400E+02	**********
1	.053	.158	.2600E+02	**********
0	.000	.158	.2800E+02	
1	.053	.211	.3000E+02	**********
0	.000	.211	.3200E+02	
1	.053	.263	.3400E+02	**********
1	.053	.316	.3600E+02	**********
1	.053	.368	.3800E+02	**********
3	.158	.526	.4000E+02	**********
1	.053	.579	.4200E+02	**********
2	.105	.684	.4400E+02	**********
3	.158	.842	.4600E+02	**********
2	.105	.947	.4800E+02	**********
1	.053	1.000	.5000E+02	**********
0	.000	1.000	.5200E+02	
0	.000	1.000	.5400E+02	
0	.000	1.000	.5600E+02	
0	.000	1.000	INF	

19				0 5 10 15 20 25

Figure 6

VARIOGRAM

WA

THICKNESS VARIOGRAM

AZIMUTH = 0. WINDOW = 90.
CLASS SIZE = 10000
MAX DISTANCE = 200000.
LOGARITHMS -NO RELATIVE VARIOGRAM -NO
COORDINATE SELECTION-NORTH(0. 999999.) EAST(0.9999999.)
ASSAY DATA SELECTION -AMIN(.000) AMAX(65.0)

DATA USED IN CALCULATIONS
MEAN = .381E+02
VARIANCE = .700E+02
STD DEVIATION = .837E+01
NO.OF SAMPLES = 19

DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 10000	8.	.263E+01	.168E+02	.187E+02	6885.6
10000 - 20000	17.	.374E+01	.200E+02	.204E+02	14576.0
20000 - 30000	13.	.714E+01	.405E+02	.404E+02	23543.4
30000 - 40000	12.	-.333E+00	.467E+02	.471E+02	36548.0
40000 - 50000	12.	.641E+01	.430E+02	.422E+02	44296.4
50000 - 60000	14.	.570E+01	.736E+02	.726E+02	54954.0
60000 - 70000	15.	.585E+01	.879E+02	.860E+02	64498.9
70000 - 80000	19.	.233E+01	.586E+02	.597E+02	75268.0
80000 - 90000	9.	.339E+01	.115E+03	.116E+03	84227.3
90000 -100000	8.	.197E+01	.158E+03	.159E+03	95714.0
100000 -110000	10.	-.320E+01	.173E+03	.171E+03	104452.8
110000 -120000	14.	.508E+01	.671E+02	.667E+02	114590.4
120000 -130000	10.	.548E+01	.188E+02	.187E+02	124802.8
130000 -140000	4.	.168E+02	.147E+03	.147E+03	133312.5
140000 -150000	2.	.156E+02	.135E+03	.135E+03	145261.2
150000 -160000	2.	.118E+02	.101E+03	.994E+02	155367.2
180000 -190000	1.	.159E+02	.126E+03	.126E+03	187070.2
190000 -200000	1.	.143E+02	.102E+03	.102E+03	199649.7

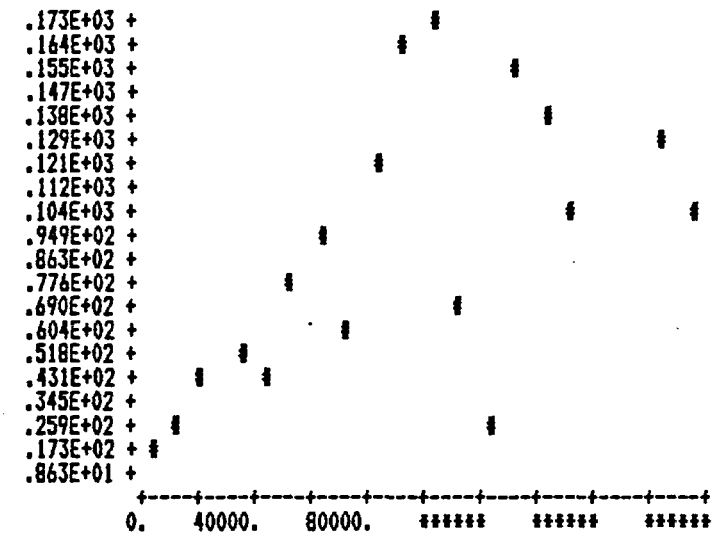
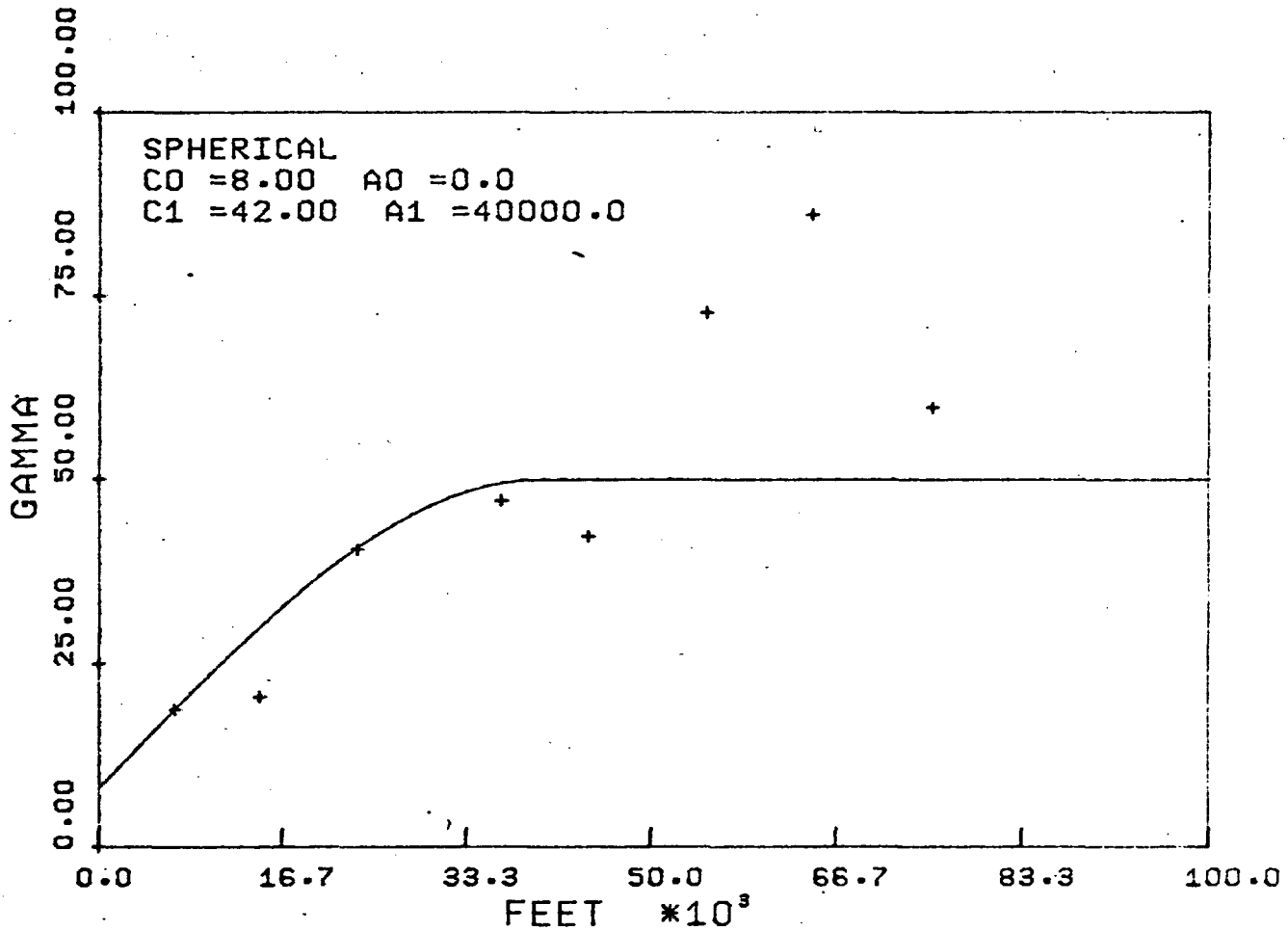


Figure 7



WA

DIRECTION = 0. WINDOW = 90. MEAN = 38.100

CLASS SIZE = 10000.00 VARIANCE = 70.0000

LOGARITHMS - NO NO. OF SAMPLES = 19

Figure 8

```
VALIDATION OF T   NESS VARIOGRAM

RUN CONTROL PARAMETERS
        ILIST =    0     IDBUG =    0
        ITYPE =    0     ISEL  =    1
        IFORM =    0     IVAR  =    0
        NUSED =    0     NK    =   20
        RMAX  =******

INPUT DATA SELECTION:
        MIN. ASSAY VALUE =       .00  MAX. ASSAY VALUE =     65.00

    VARIOGRAM TYPE: SPHERICAL

    VARIOGRAM  PARAMETERS
        SILL                        .5000E+02
        ANGLE OF ROTATION                .00
        HOR. ANISOTROPY                 1.00
        VERT. ANISOTROPY                 .00
        MODEL   C VALUE     RANGE
           1     8.000       0.
           2    42.000    40000.

    SUMMARY STATISTICS OF KRIGING ERRORS

        MEAN                       -.20495970E+01
        VARIANCE                    .67137280E+02
        STD.DEVIATION               .81937340E+01
        AVE.SQ.ERROR                .67804600E+02
        WEIGHTED SQ.ERR.            .42574370E+02
        SKEWNESS                   -.92959520E+00
        KURTOSIS                    .35126680E+01
        NO. OF ASSAYS                          19
        AVE KRIG VARIANCE           .41931430E+02

    HISTOGRAM OF KRIGING ERRORS

                          **HISTOGRAM**

OBSV  RELA  CUML   UPPER
FREQ  FREQ  FREQ  CELL LIM. 0       20       40       60       80       100
                            +    +   +    +   +    +   +    +   +    +   +
   1  .053  .053 -.1935E+02 +***                                          +
   1  .053  .105 -.1712E+02 +*** C                                        +
   0  .000  .105 -.1489E+02 +    C                                        +
   1  .053  .158 -.1266E+02 +***   C                                      +
   0  .000  .158 -.1043E+02 +      C                                      +
   0  .000  .158 -.8202E+01 +      C                                      +
   1  .053  .211 -.5972E+01 +***      C                                   +
   1  .053  .263 -.3742E+01 +***        C                                 +
   1  .053  .316 -.1511E+01 +***          C                               +
   6  .316  .632  .7191E+00 +****************             C               +
   3  .158  .789  .2950E+01 +*********                        C           +
   2  .105  .895  .5180E+01 ******                                   C    +
   1  .053  .947  .7410E+01 ***                                        C  +
   0  .000  .947  .9641E+01 +                                         C  +
   1  .053 1.000    INF     +***                                         C
----                        +    +   +    +   +    +   +    +   +    +   +
  19                        0       20       40       60       80       100
```

HISTOGRAM OF ERRORS WITHIN 2 STANDARD DEVIATIONS

```
                          **HISTOGRAM**

OBSV  RELA  CUML   UPPER
FREQ  FREQ  FREQ  CELL LIM. 0       20       40       60       80       100
                            +    +   +    +   +    +   +    +   +    +   +
   0  .000  .000 -.3000E+01 +                                            +
   2  .105  .105 -.2000E+01 ******                                       +
   2  .105  .211 -.1000E+01 ******     C                                 +
   6  .316  .526  .0000E+00 ****************        C                    +
   8  .421  .947  .1000E+01 ********************                      C  +
   1  .053 1.000  .2000E+01 ***                                          C
   0  .000 1.000  .3000E+01 +                                            C
   0  .000 1.000    INF     +                                            C
----                        +    +   +    +   +    +   +    +   +    +   +
  19                        0       20       40       60       80       100
```

Figure 9

VARIOGRAM

WA

THICKNESS VARIOGRAM

```
                                                DATA USED IN CALCULATIONS
AZIMUTH =   0.   WINDOW =  90.                  MEAN          =  .381E+02
CLASS SIZE   =   20000.                         VARIANCE      =  .700E+02
MAX DISTANCE =  400000.                         STD DEVIATION =  .837E+01
LOGARITHMS -NO   RELATIVE VARIOGRAM -NO         NO.OF SAMPLES =        19
COORDINATE SELECTION-NORTH(      0. 999999.)   EAST(      0.9999999.)
ASSAY DATA SELECTION -AMIN(            .000)   AMAX(            65.0)
```

DISTANCE	# PAIRS	DRIFT	GAMMA (H)	MOMENT CENT	AVER DIST
0 - 20000	25.	.338E+01	.189E+02	.201E+02	12115.1
20000 - 40000	25.	.355E+01	.435E+02	.444E+02	29785.6
40000 - 60000	26.	.603E+01	.595E+02	.602E+02	50035.1
60000 - 80000	34.	.388E+01	.716E+02	.703E+02	70516.9
80000 -100000	17.	.272E+01	.135E+03	.138E+03	89632.8
100000 -120000	24.	.163E+01	.111E+03	.108E+03	110366.4
120000 -140000	14.	.872E+01	.555E+02	.571E+02	127234.1
140000 -160000	4.	.137E+02	.118E+03	.116E+03	150314.2
180000 -200000	2.	.151E+02	.114E+03	.114E+03	193359.9

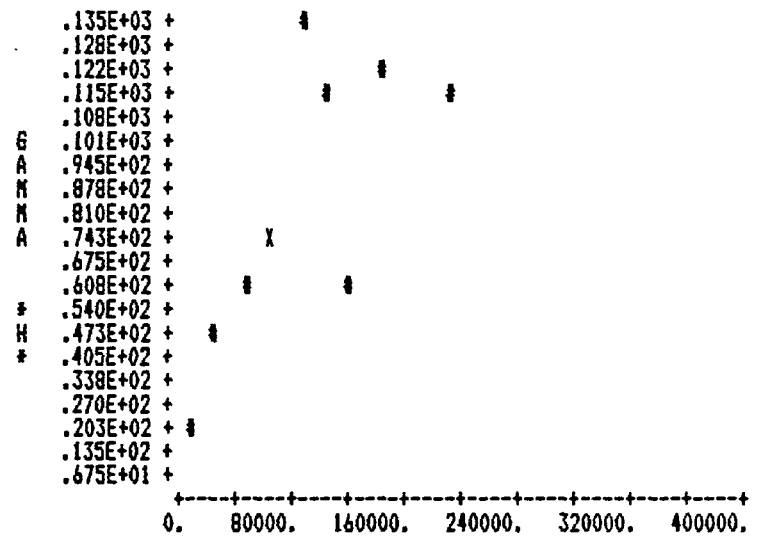
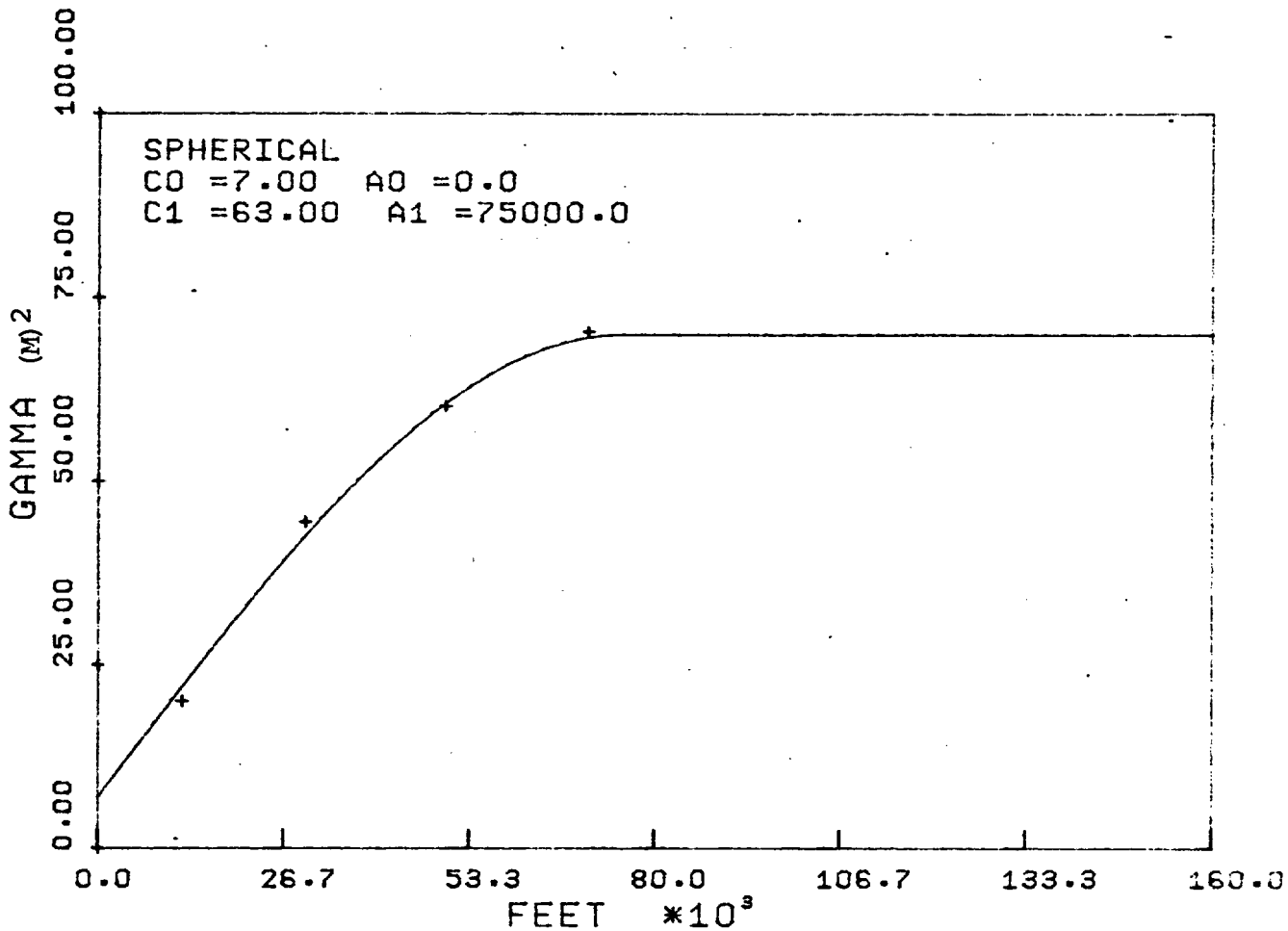


Figure 10



WA

DIRECTION = 0. WINDOW = 90. MEAN = 38.100

CLASS SIZE = 20000.00 VARIANCE = 70.0000

LOGARITHMS - NO NO. OF SAMPLES = 19

Figure 11

VALIDATION OF KNESS VARIOGRAM

RUN CONTROL PARAMETERS

ILIST =	0	IDBUG =	0
ITYPE =	0	ISEL =	1
IFORM =	0	IVAR =	0
NUSED =	0	NK =	20
RMAX	=******		

INPUT DATA SELECTION:
MIN. ASSAY VALUE = .00 MAX. ASSAY VALUE = 65.00

VARIOGRAM TYPE: SPHERICAL

VARIOGRAM PARAMETERS

SILL	.7000E+02
ANGLE OF ROTATION	.00
HOR. ANISOTROPY	1.00
VERT. ANISOTROPY	.00

MODEL	C VALUE	RANGE
1	7.000	0.
2	63.000	75000.

SUMMARY STATISTICS OF KRIGING ERRORS

MEAN	-.21039080E+01
VARIANCE	.46581170E+02
STD.DEVIATION	.68250400E+01
AVE.SQ.ERROR	.48555960E+02
WEIGHTED SQ.ERR.	.24108730E+02
SKEWNESS	-.97053030E+00
KURTOSIS	.32863440E+01
NO. OF ASSAYS	19
AVE KRIG VARIANCE	.43601760E+02

HISTOGRAM OF KRIGING ERRORS

HISTOGRAM

```
OBSV  RELA  CUML   UPPER
FREQ  FREQ  FREQ  CELL LIM. 0       20       40       60       80       100
                            +   +   +   +   +   +   +   +   +   +   +
  1  .053  .053 -.1663E+02 +***                                          +
  1  .053  .105 -.1492E+02 +*** C                                        +
  0  .000  .105 -.1321E+02 +   C                                         +
  1  .053  .158 -.1149E+02 +***   C                                      +
  0  .000  .158 -.9778E+01 +      C                                      +
  0  .000  .158 -.8064E+01 +      C                                      +
  1  .053  .211 -.6350E+01 +***      C                                   +
  0  .000  .211 -.4636E+01 +         C                                   +
  2  .105  .316 -.2922E+01 +*****         C                              +
  2  .105  .421 -.1208E+01 +*****             C                          +
  5  .263  .684  .5057E+00 +************                  C             +
  1  .053  .737  .2220E+01 +***                               C          +
  1  .053  .789  .3933E+01 +***                                 C        +
  3  .158  .947  .5647E+01 +********                                  C  +
  1  .053 1.000    INF     +***                                          C
----                        +   +   +   +   +   +   +   +   +   +   +
 19                         0       20       40       60       80       100
```

HISTOGRAM OF ERRORS WITHIN 2 STANDARD DEVIATIONS

HISTOGRAM

```
OBSV  RELA  CUML   UPPER
FREQ  FREQ  FREQ  CELL LIM. 0       20       40       60       80       100
                            +   +   +   +   +   +   +   +   +   +   +
  0  .000  .000 -.3000E+01 +                                            +
  1  .053  .053 -.2000E+01 +***                                          +
  3  .158  .211 -.1000E+01 +********  C                                 +
  7  .368  .579  .0000E+00 +******************         C                +
  7  .368  .947  .1000E+01 +******************                        C  +
  1  .053 1.000  .2000E+01 +***                                          C
  0  .000 1.000  .3000E+01 +                                            C
  0  .000 1.000    INF     +                                            C
----                        +   +   +   +   +   +   +   +   +   +   +
 19                         0       20       40       60       80       100
```

Figure 12

THICKNESS STATISTICS

NUMBER OF ASSAYS = 9

ASSAY DATA - SORTED

35.600 37.600 42.700 43.200 45.100 45.100 45.400 46.000 71.700

SUMMARY STATISTICS FOR UNTRANSFORMED DATA

MEAN	=	.45822220E+02
VARIANCE	=	.10745460E+03
STD.DEV.	=	.10366030E+02
SKEWNESS	=	.18634980E+01
KURTOSIS	=	.26519580E+01
COEF VAR	=	.22622290E+00

DATA SELECTION:

MIN. ASSAY VALUE	=	.000
MAX. ASSAY VALUE	=	80.000

BOUNDARY COORDINATES OF ASSAY AREA:

MIN. NORTHING	=	430000.
MAX. NORTHING	=	455000.
MIN. EASTING	=	2200000.
MAX. EASTING	=	2230000.
MIN. ELEVATION	=	0.
MAX. ELEVATION	=	0.

HISTOGRAM

THICKNESS STATISTICS

OBSV FREQ	RELA FREQ	CUML FREQ	UPPER CELL LIM.	0 10 20 30 40 50
0	.000	.000	.2000E+02	+
0	.000	.000	.2200E+02	+
0	.000	.000	.2400E+02	+
0	.000	.000	.2600E+02	+
0	.000	.000	.2800E+02	+
0	.000	.000	.3000E+02	+
0	.000	.000	.3200E+02	+
0	.000	.000	.3400E+02	+
1	.111	.111	.3600E+02	**********
1	.111	.222	.3800E+02	**********
0	.000	.222	.4000E+02	+
0	.000	.222	.4200E+02	+
2	.222	.444	.4400E+02	**********
3	.333	.778	.4600E+02	**********
1	.111	.889	.4800E+02	**********
0	.000	.889	.5000E+02	+
0	.000	.889	.5200E+02	+
0	.000	.889	.5400E+02	+
0	.000	.889	.5600E+02	+
1	.111	1.000	INF	**********
---				+ + + + + + + + + +
9				0 10 20 30 40 50

Figure 13

THICKNESS STATISTICS

NUMBER OF ASSAYS = 8

ASSAY DATA - SORTED

35.600 37.600 42.700 43.200 45.100 45.100 45.400 46.000

SUMMARY STATISTICS FOR UNTRANSFORMED DATA

MEAN	=	.42587500E+02
VARIANCE	=	.15181500E+02
STD.DEV.	=	.38963450E+01
SKEWNESS	=	-.95890710E+00
KURTOSIS	=	-.67098860E+00
COEF VAR	=	.91490340E-01

DATA SELECTION:

MIN. ASSAY VALUE	=	.000
MAX. ASSAY VALUE	=	65.000

BOUNDARY COORDINATES OF ASSAY AREA:

MIN. NORTHING	=	430000.
MAX. NORTHING	=	455000.
MIN. EASTING	=	2200000.
MAX. EASTING	=	2230000.
MIN. ELEVATION	=	0.
MAX. ELEVATION	=	0.

HISTOGRAM

THICKNESS STATISTICS

OBSV FREQ	RELA FREQ	CUML FREQ	UPPER CELL LIM.	0 20 40 60 80 100
0	.000	.000	.2000E+02	
0	.000	.000	.2200E+02	
0	.000	.000	.2400E+02	
0	.000	.000	.2600E+02	
0	.000	.000	.2800E+02	
0	.000	.000	.3000E+02	
0	.000	.000	.3200E+02	
0	.000	.000	.3400E+02	
1	.125	.125	.3600E+02	*******
1	.125	.250	.3800E+02	*******
0	.000	.250	.4000E+02	
0	.000	.250	.4200E+02	
2	.250	.500	.4400E+02	**********
3	.375	.875	.4600E+02	**********
1	.125	1.000	.4800E+02	*******
0	.000	1.000	.5000E+02	
0	.000	1.000	.5200E+02	
0	.000	1.000	.5400E+02	
0	.000	1.000	.5600E+02	
0	.000	1.000	INF	

8				

Figure 14

```
      RUN OF SKRIG

RUN CONTROL PARAMETERS
        IDBUG =    1     ILIST =    0
        NK    =   20

    VARIOGRAM  PARAMETERS
        SILL                          .5000E+02
        ANGLE OF ROTATION                   .00
        HOR. ANISOTROPY                    1.00
        VERT. ANISOTROPY                    .00
        MODEL   C VALUE     RANGE
            1     8.000       0.
            2    42.000    40000.

    MAX HORIZONTAL RANGE OF INFLUENCE        100,000

    MAX VERTICAL RANGE OF INFLUENCE            10000.

    MAX NUMBER OF HOLES USED TO KRIG A BLOCK       20

    DESCRIPTION OF AREA TO BE KRIGED
        NORTHING OF ROW 1            430000.
        EASTING  OF COL 1           2200000.
        ELEVATION                         0.
        KRIGED AREA IS BOUNDED BY
        COLUMNS                      1 AND    1
        ROWS                         1 AND    1
        BLOCK DIMENSIONS ARE ( 25000.X 30000.)
        BLOCK VARIANCE              21.71679


        ***OUTPUT OF KRIGED RESULTS***

  COLUMN  ROW     Y-COORD.  X-COORD. ELEVATION       GRADE     GRADE VAR.

     1     1      442500.  2215000.       0.        46.72       1.7190

B:\ >TYPE SKRIG.DBG

 HOLE     NORTH       EAST      ELEV      GRADE      WEIGHT
RRL-2     444500.  2211000.       0.      45.10        .12
RRL-6     438500.  2206500.       0.      43.20        .10
DC-20D    452000.  2215000.       0.      45.40        .09
DC-16A    433000.  2210500.       0.      71.70        .13
RRL-14    446500.  2204000.       0.      35.60        .07
DC-3      449000.  2225000.       0.      46.00        .16
DC-22C    448500.  2204500.       0.      37.60        .05
DC-4      454500.  2210000.       0.      45.10        .05
DC-19C    434000.  2225000.       0.      42.70        .16
MCGEE     458000.  2192000.       0.      29.10        .01
DC-2      453000.  2247000.       0.      48.20        .02
DC-12     415500.  2241500.       0.      38.10        .01
RSH-1     402000.  2180000.       0.      23.50        .01
DC-6      459500.  2277500.       0.      38.60        .01
DC-14     489500.  2261000.       0.      33.20        .01
DC-8      420000.  2280500.       0.      46.00        .01
DH-5      519500.  2175000.       0.      25.30        .01
```

$$\sigma^2_{(\%\,\text{Deposit Area})} = \sigma^2_{(\%)} - \sigma^2_{(\text{Block}\,\%)}$$

$$= 50.0 - 21.72 = 28.28\ (\%)^2$$

$$\sqrt{28.28} = 5.317 \leftarrow \text{Std. Dev. of Point Samples}$$

Figure 15

```
RUN OF  .IG

RUN CONTROL PARAMETERS
        IDBUG =    1     ILIST =    0
        NK    =   20

   VARIOGRAM  PARAMETERS
        SILL                          .7000E+02
        ANGLE OF ROTATION                   .00
        HOR. ANISOTROPY                    1.00
        VERT. ANISOTROPY                    .00
        MODEL   C VALUE     RANGE
            1     7.000       0.
            2    63.000   75000.

   MAX HORIZONTAL RANGE OF INFLUENCE                100000.

   MAX VERTICAL RANGE OF INFLUENCE                   10000.

   MAX NUMBER OF HOLES USED TO KRIG A BLOCK          20

   DESCRIPTION OF AREA TO BE KRIGED
        NORTHING OF ROW 1              430000.
        EASTING  OF COL 1             2200000.
        ELEVATION                           0.
        KRIGED AREA IS BOUNDED BY
        COLUMNS                         1 AND    1
        ROWS                            1 AND    1
        BLOCK DIMENSIONS ARE ( 25000.X 30000.)
        BLOCK VARIANCE                 45.91177


        ***OUTPUT OF KRIGED RESULTS***

 COLUMN  ROW     Y-COORD.  X-COORD. ELEVATION       GRADE     GRADE VAR.

    1    1      442500.  2215000.        0.        46.84       1.4470
```

HOLE	NORTH	EAST	ELEV	GRADE	WEIGHT
RRL-2	444500.	2211000.	0.	45.10	.12
RRL-6	438500.	2206500.	0.	43.20	.11
DC-20D	452000.	2215000.	0.	45.40	.09
DC-16A	433000.	2210500.	0.	71.70	.14
RRL-14	446500.	2204000.	0.	35.60	.07
DC-3	449000.	2225000.	0.	46.00	.15
DC-22C	448500.	2204500.	0.	37.60	.06
DC-4	454500.	2210000.	0.	45.10	.05
DC-19C	434000.	2225000.	0.	42.70	.16
MCGEE	458000.	2192000.	0.	29.10	.01
DC-2	453000.	2247000.	0.	48.20	.02
DC-12	415500.	2241500.	0.	38.10	.02
RSH-1	402000.	2180000.	0.	23.50	.01
DC-6	459500.	2277500.	0.	38.60	.00
DC-14	489500.	2261000.	0.	33.20	.00
DC-8	420000.	2280500.	0.	46.00	.00
DH-5	519500.	2175000.	0.	25.30	.00

$$\sigma^2(\%/\text{Repository Area}) = \sigma^2(\%) - \sigma^2(\text{Block}/\%)$$

$$= 70.0 - 45.92 = 24.09 \quad (m)^2$$

$$\sqrt{24.09} = 4.908$$

← Std. Dev. of Point Samples Within the Repository Area.

Figure 16