HEALTH PHYSICS TECHNOLOGY COURSE

MATH REFRESHER

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OBJECTIVE

This math review handout was recommended by students in previous Health Physics Technology courses. They noted on Course Evaluation Forms that their math skills were “rusty” having been out of school for a while and not having had the opportunity to use them.

The purpose of this Math Review is to familiarize you with the types of mathematical manipulations that you might be expected to use to solve problems during the Health Physics Technology course.

NOTE: This math review handout contains some problems for you to solve. The answers are provided at the end of the handout.

BASIC MATH

Four Basic Mathematical Functions

+	addition	(2 + 3 =)
-	subtraction	(7 - 6 =)
x	multiplication	(5 x 9 =)
÷	division	(8 ÷ 4 =)

NOTE: You will be using your calculator during class and on exams - practice & become familiar will all its functions.

ALGEBRA

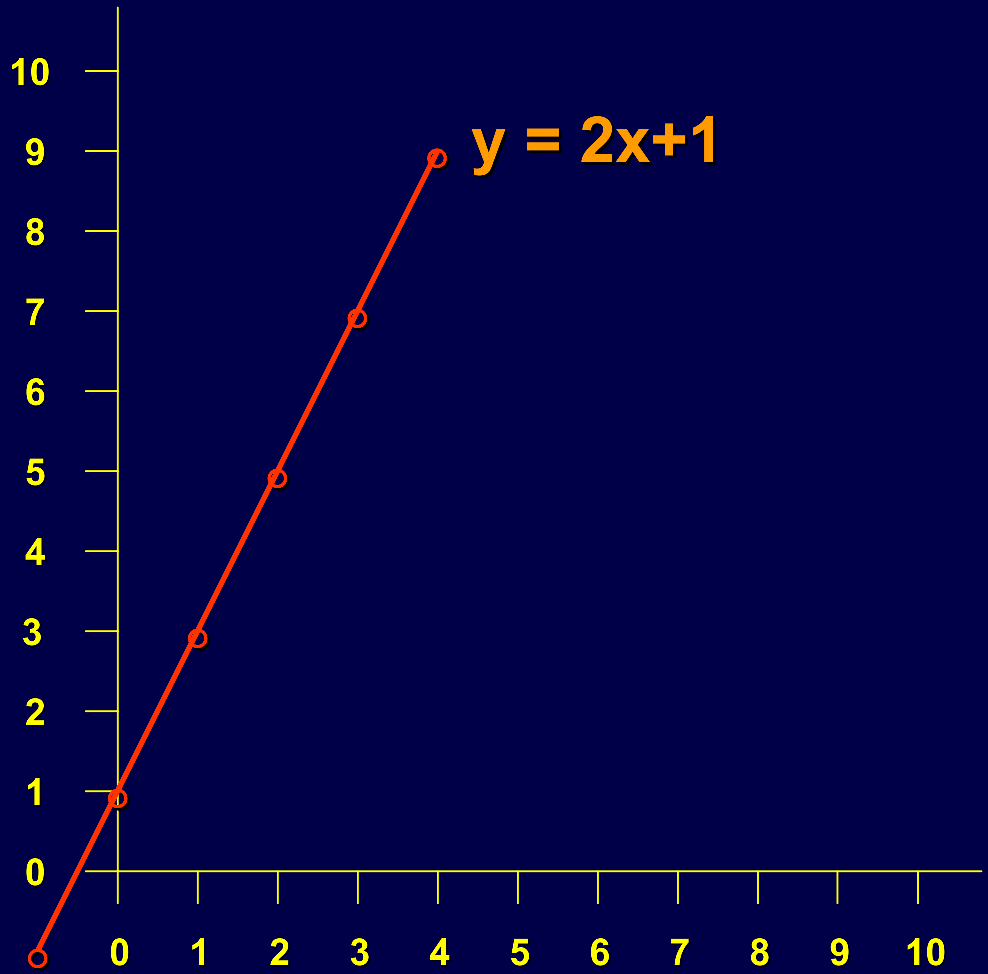
The following notes were taken from the website:

http://library.thinkquest.org/16284/algebra.htm

The material has been modified to fit this handout

Linear Equations

Linear equations are equations that can be graphed as a line.



Linear Equations

Most linear equations are "literal" equations, which means that they use letters as their variables. To solve linear equations, the following methods are useful:

Addition property

For all real numbers a, b, and c, if $a=b$, then $a+c=b+c$.

Multiplication property

For all real numbers a, b, and c, if $a=b$, then $ac=bc$.

Definition of absolute value

For each real number a, $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a<0$

Example

Multiply (a + b) times (c + d)

$= a \times (c + d) + b \times (c + d)$

$= \boxed{ac + ad + bc + bd}$

OR

$$\begin{array}{r} (a + b) \\ \times\ (c + d) \\ \hline ad + bd \\ +\ ac + bc \\ \hline \boxed{ac + ad + bc + bd} \end{array}$$

Example

Solving $ax + b = y$ for "x"

$$6x + 3 = 28.2$$

One equation, one variable equals one solution

Solve for x:

$$10x + 5 = y$$

One equation, two variables equals many solutions

Possible solutions:

x	y

Solving for x:

$$x = (y - 5)/10 = 0.1y - 0.5$$

System of Equations

We have three ways to solve a system of equations:

The first is Graphing

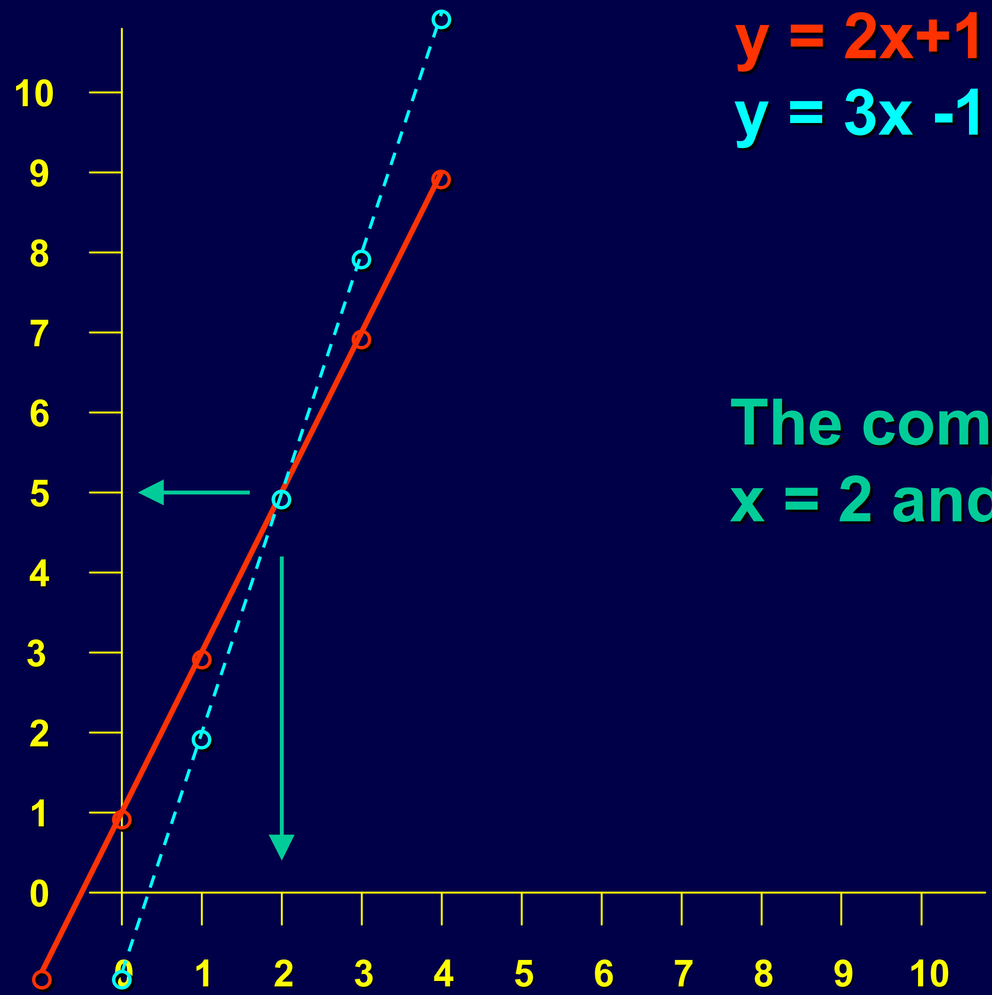
By graphing two or more equations we can find the place(s) where they intersect which is where the answer(s) lie(s). For linear equations, there is always one answer (one intersection) or no answer at all (no intersection).

Graphing

Let's graph these two equations

$y = 2x+1$

$y = 3x - 1$



The common solution is $x = 2$ and $y = 5$

System of Equations

The next method is Substitution

We take one of the equations and solve for one variable in terms of the other variable (e.g., $x = 5y - 3$). Then we substitute $(5y-3)$ for the "x" in the other equation. Now we have an equation with only one variable "y". After solving for "y" we can substitute it into either equation to get "x"

Solve one of the equations for one of the variables (x or y) and then substitute the expression into the other equation.

$$ax + by = c$$
$$dx + ey = g$$

Solve for "x":

$ax + by = c$

$ax = c - by$

$x = (c - by)/a$

Substitute:

$dx + ey = g$

$d(c - by)/a + ey = g$

$d(c - by) + aey = ag$

$dc - dby + aey = ag$

$aey - dby = ag - dc$

$y(ae - db) = ag - dc$

$y = (ag - dc)/(ae - db)$

Sample

$$\begin{aligned} 3x + 5y &= 37 \\ 6x - 3y &= 9 \end{aligned}$$

Solve for x and y:

System of Equations

The final method is Linear Combination

In linear combination, we manipulate one or both equations so that each equation has one variable (say “x”) with the same coefficient. Then we subtract the two equations and by doing so, we eliminate that one variable (“x”) in both equations. Now we only have one variable (“y”) to solve for.

Linear Combination

$$ax + by = c$$
$$dx + ey = g$$

We have 2 variables (x and y) and six constants (a, b, c, d, e, g). We also have 2 equations using the same 2 variables. To solve for x and y we need to get rid of one of them.

To get rid of the "x", multiply the first equation by the coefficient of the "x" from the second equation.

Then multiply the second equation by the coefficient of the "x" from the first equation.

NOTE: You could also do this using the coefficients of the "y".

$$ax + by = c$$
$$dx + ey = g$$

$$d \times (ax + by) = c \times d$$
$$-a \times (dx + ey) = g \times -a$$

$$dax + dby = cd$$
$$-adx - aey = -ga$$

$$dax - adx + dby - aey = cd - ga$$

Now just group terms and solve for y

$$(dax - adx) + (dby - aey) = (cd - ga)$$

$$x(da - ad) + y(db - ae) = (cd - ga)$$

$$\boxed{y = \frac{(cd - ga)}{(db - ae)}}$$

SAMPLE:

$$3x + 4y = 18$$
$$5x - 3y = 1$$

Solve for x and y:

Quadratic Equations

The equation $ax^2 + bx^1 + cx^0 = 0$, is called the general quadratic equation in "x". The common form of this equation is $ax^2 + bx + c = 0$. A quadratic equation with no "x^1" term is called a pure quadratic equation.

The square root of a second power number always yields two answers; one positive and one negative. This is called the Square Root Property of Equations. It states that for all real numbers m and n, $n \geq 0$, if $m^2 = n$, then $m = \pm\sqrt{n}$. To solve a quadratic equation, you need to use the quadratic formula which states that the roots of $ax^2+bx+c=0$ are

$$\frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$$

SAMPLE: $x^2 + 8x = -7$

Solve for x:

LAWS OF EXPONENTS

Laws of Exponents

$$10^0 = 1$$

$$x^0 = 1$$

$$e^0 = 1$$

$$x^{-1} = 1/x$$

$$x^{-a} = 1/x^a$$

$$\exp(x) = e^x$$

$$e^{-1} = 1/e$$

$$x^a x^b = x^{a+b}$$

$$(x^a)^b = x^{ab}$$

$$x^a/x^b = x^{a-b}$$

Laws of Exponents

The following expressions cannot be simplified any further:

$x^a\ y^a$ & x^a/y^a

Laws of Exponents

$$e^{-\mu x} = 1/e^{\mu x}$$

$$e^{-\mu x}\, e^{a} = e^{(a-\mu x)}$$

$$e^{-\mu x}\, e^{-\mu x} = e^{-2\mu x}$$

$$x^{1/2} = \sqrt{x}$$

$$x^{1/3} = \sqrt[3]{x}$$

$$(x^{1/2})^{1/3} = x^{1/6} = \sqrt[6]{x}$$

Laws of Exponents

For $\mu x \to \infty$, $e^{\mu x} \to \infty$

For $\mu x \to \infty$, $e^{-\mu x} = 1/e^{\mu x} \to 0$

For $\mu x \to 0$, $e^{-\mu x} \to 1$

For $\mu x \to 0$, $e^{\mu x} \to 1$

Rules of Thumb

For $\mu x << 1$, $e^{-\mu x} = 1 - \mu x$

Good to within:

$\pm$ 5% for $\mu x = 0.3$ and

$\pm$ 1% for $\mu x = 0.2$

DIMENSIONAL ANALYSIS

Dimensional Analysis

The ability to keep track of units, cancel them out properly and ensure that your final answer has the correct units is essential for solving problems.

Problem 1

Attenuation equations are typically expressed in terms of the linear attenuation coefficient (μ) or the mass attenuation coefficient (μ/ρ).

$e^{-\mu x}$ may also be written as $e^{-(\mu/\rho)(\rho)(x)}$

Let x = 3 cm, μ/ρ = 0.021 cm^2/g and ρ = 0.01 g/cm^3

Solve $e^{-\mu x}$

Problem 1 Solution

$$(\mu/\rho)(\rho)(x) = \left(-\ 0.021\ \frac{cm^2}{g}\right)\left(0.01\ \frac{g}{cm^3}\right)(3\ cm)$$

the grams cancel and the cm^2 in the numerator cancels out 2 of the cm^3 in the denominator leaving only one cm in the denominator. But that remaining cm cancels out the cm from the “x” value. The net result is no units left which is what we want since an exponent should not have any dimensions.

$\exp(-\ 0.00063) = 0.99937 \cong 1$

Problem 2

Protective clothing is often sold with units of areal density, e.g., ounces per square yard (oz/yd^2).

Protective clothing is advertised to have an areal density of 40 oz/yd^2.

Calculate what this corresponds to in units of mg/cm^2.

Problem 2 (cont)

1 oz = 28.35 g

1 yd = 3 feet

1 foot = 12 inches

1 inch = 2.54 cm

1 g = 10^3 mg

Problem 2 (solution)

We start out with units of oz/yd^2 and we want to get to units of mg/cm^2:

$$\left(\frac{40\ \text{oz}}{\text{yd}^2}\right)\left(\frac{28.35\ \text{g}}{\text{oz}}\right)\left(\frac{10^3\ \text{mg}}{\text{g}}\right)\left(\frac{1\ \text{yd}^2}{3^2\ \text{ft}^2}\right)\left(\frac{1\ \text{ft}^2}{12^2\ \text{in}^2}\right)\left(\frac{1\ \text{in}^2}{2.54^2\ \text{cm}^2}\right)$$

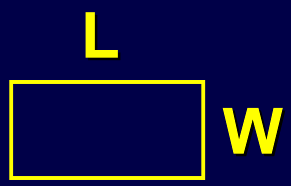
The oz's cancel, the g's cancel, the yd^2 cancel, the ft^2 cancel, the in^2 cancel and we're left with mg in the numerator and cm^2 in the denominator

40 oz/yd^2 = 136 mg/cm^2

GEOMETRY

Length

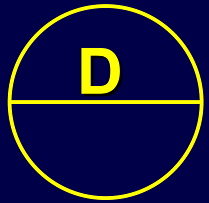
perimeter of a rectangle = (2 x Length) + (2 x Width)



circumference of a circle = π x Diameter

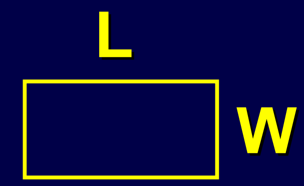
or π x 2 x Radius

where Diameter = 2 x Radius

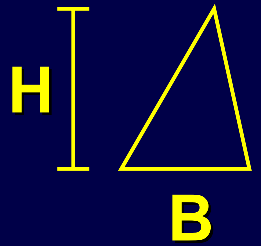


Area

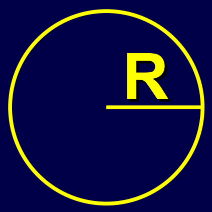
Rectangle = Length x Width



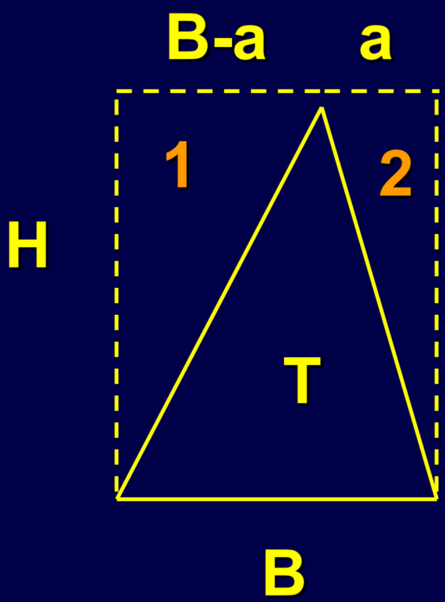
Triangle = ½ Base x Height



Circle = π x Radius^2



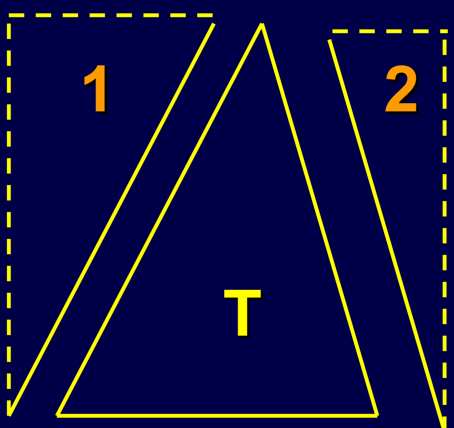
Area of Generic Triangle



Area of rectangle = $B \times H$

Area of triangle (T) = [Area of rectangle] minus

[(Area of right triangle 1) plus (Area of right triangle 2)]



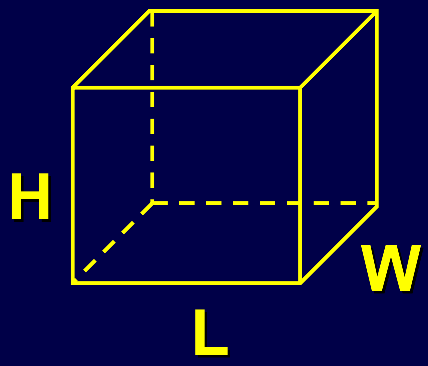
$$= (B \times H) - \{ [\tfrac{1}{2} (B-a) \times H] + [\tfrac{1}{2} a \times H] \}$$

$$= (B \times H) - \tfrac{1}{2} \{ (B \times H) - (a \times H) + (a \times H) \}$$

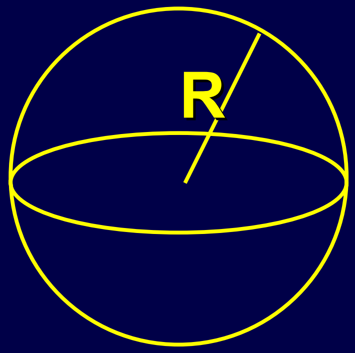
$$= (B \times H) - \tfrac{1}{2} (B \times H) = \tfrac{1}{2} (B \times H)$$

SURFACE AREA

Box = 2 [(L x W) + (L x H) + (W x H)]

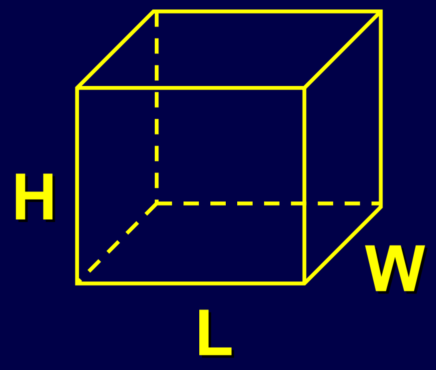


Sphere = 4 π R^2

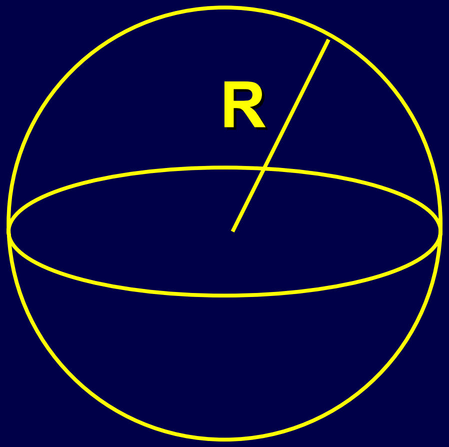


VOLUME

Box = L x W x H



Sphere = $\frac{4}{3} \pi R^3$

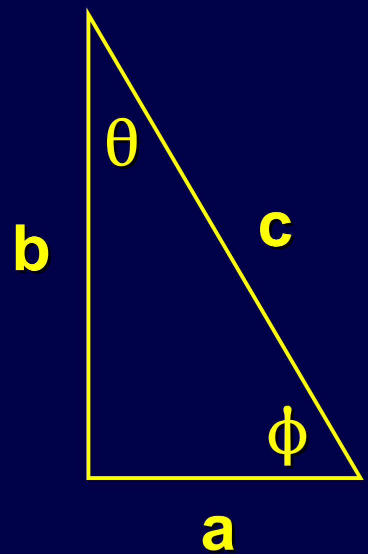


PROBLEMS (calculate the following)

1. area of a circle with a 4 cm diameter? __________
2. surface area of a hemisphere with a radius of 2.4 meters? __________
3. volume of a sphere with a 78 cm diameter? __________
4. volume of a box whose length is 7 cm, width is 3 cm and depth is 12 cm? __________
5. surface area of a box whose length, width and depth are all 12 cm? __________

TRIGONOMETRY

Triangles



$$\text{Sine}\ \theta = \frac{a}{c}$$

$$\text{Cosine}\ \theta = \frac{b}{c}$$

$$\text{Tangent}\ \theta = \frac{a}{b} = \frac{\text{Sine}\ \theta}{\text{Cosine}\ \theta}$$

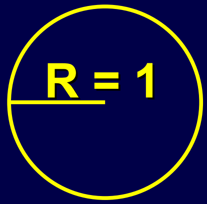
$$c = \sqrt{a^2 + b^2}$$

Degrees or Radians

A complete circle can be thought of as a 360° object

As we saw earlier, the circumference of a circle is $\pi \times D$

D is 2 times the radius ($2 \times R$) so the circumference is $\pi D = 2 \pi R$. But R is arbitrary, so let R = 1



If we go around a circle of radius 1,
we have traveled a distance equal to 2π,
and we have also traveled 360°

Thus $360° = 2\pi = 2 \times 3.1416 = 6.2832$ radians

Degrees	Radians		
0	0π	=	0.0000
30	$(1/6)\pi$	=	0.5236
45	$(1/4)\pi$	=	0.7854
60	$(1/3)\pi$	=	1.0472
90	$(1/2)\pi$	=	1.5708
120	$(2/3)\pi$	=	2.0944
135	$(3/4)\pi$	=	2.3562
150	$(5/6)\pi$	=	2.6180
180	π	=	3.1416
270	$(3/2)\pi$	=	4.7124
360	2π	=	6.2832

Therefore, sine (180°) = sine (π radians)

Angle (degrees)	Angle (radians)	Sine	Cosine	Tangent
0	0	0	1	0
30	0.5236	0.5	0.866	0.577
45	0.7854	0.707	0.707	1
60	1.0472	0.866	0.5	1.732
90	1.5708	1	0	∞

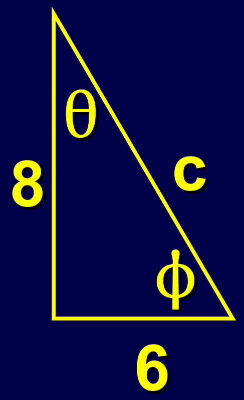
PROBLEMS (calculate the following)

1. Sine (28°) ____________

2. Tangent (0.89 radians) ____________

3. Cosine (88°) ____________

Given this triangle calculate the: →



4. Hypotenuse (c) ____________

5. Cosine of θ ____________

INVERSE TRIGONOMETRIC FUNCTIONS

The sine of an angle is just a number with no dimensions.

If you know the number and want to find the angle corresponding to that number, you can use the inverse trigonometric function. This would be the inverse sine or inverse cosine or inverse tangent. They are sometimes called the arcsine, arccosine and arctangent.

Example:

sine 30° = sine (π/6 radians) = 0.5

inverse sine (0.5) = 30° or 0.5236 radians

The inverse sine is probably displayed as the secondary function sin $^{-1}$ on your calculator so you may have to hit a 2nd key before hitting the sin key

Or, you may have to hit an inv key before hitting the sin key to get the inverse sine

IMPORTANT: Remember that if you input a number and ask for the inverse sine, cosine or tangent, you will get an answer which may be either degrees or radians. You must check what your calculator is set for. Most calculators have a DRG key which toggles between DEG, RAD and GRAD. If you want your answer to be in radians (as we will for the line source equation) be sure your calculator is set for RAD.

EXAMPLE: What is the arcsine of 0.06?

On my calculator, if I punch in 2ndF then sine^{-1} and 0.06, I get 3.44

Is this answer degrees or radians?

In this case it's degrees because my calculator is showing DEG at the top of the display.

If I wanted radians, I could just toggle the DRG key to RAD and I would get 0.06 radians

To avoid problems always check your DRG key setting before tackling inverse trigonometric functions. When you get your answer, look at the units on your display.

PROBLEMS (calculate the following)

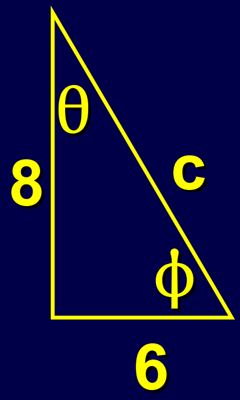
1. arctan (0.666) in degrees ________

2. inverse cosine (0.17) in radians ________

3. inverse sine [cosine (24°)] in degrees ________

4. arcsine (0.44) in degrees ________

Given this triangle what is:



5. ϕ (in degrees) ________

LOGARITHMS

Common Logarithm

Base 10 ($\log_{10}$)

or

Natural Logarithm

Base e ($\ln_e$)

($e = 2.7183$)

The answer "Y" to "$\log_{10} X = Y$" can be thought of as:

"How many times must 10 be multiplied by itself to yield X ?"

or in a slightly different way

"What power must 10 be raised to (i.e., 10^Y) to get X ?"

Y is the answer !

Example: $\log_{10} (100) = 2$ because $10 \times 10 = 10^2 = 100$

($X = 100$ $Y = 2$)

Similarly

"$\ln_e X = Y$" can be thought of as follows

"How many times must "e" be multiplied by itself to yield X ?"

or "What power must "e" be raised to (i.e., e^Y) to get X ?"

Y is the answer !

Example: $\ln_e (10) = 2.3$ (X = 10 Y = 2.3)

"e" must be multiplied by itself 2.3 times ($e \times e \times e^{0.3}$) to get 10

$$(e)^{2.3} = (2.7183)^{2.3} \approx 10$$

NOTE: since "e" is rounded and the answer (2.3) is also rounded, everything is an approximation

Number	$\log_{10}$	$\ln_e$
0	--------	--------
e	0.4343	1
10	1	2.3026
100	2	4.6052
1000	3	6.9078
10000	4	9.2103

Since “10” and “e” are both just numbers, there is a fixed relationship between $\log_{10}$ and $\ln_e$.

$\ln_e$ of any number is always 2.3 times the $\log_{10}$ of that same number.

Example: $\ln_e 45 = 3.807$

$\log_{10} 45 = 1.653$

$(\ln_e 45)/(\log_{10} 45) = (3.807/1.653) = 2.3$

On your calculator you should have two keys:

one labeled log which is base 10 and
one labeled ln which is base e

Each key may be associated with an inverse which might require you to press a 2ndF key

The inverse of log is a power of 10: 10^x
The inverse of ln is an exponential: e^x

$\log_{10} A = B$ $\quad 10^B = A$

$\ln_e C = D$ $\quad e^D = C$

PROBLEMS (Calculate the following)

1. the common logarithm of 88 _________
2. the natural logarithm of 123 _________
3. the ratio of the natural logarithm of 452 to the common logarithm of 452 _________
4. the natural logarithm of the common logarithm of 333 _________
5. the common logarithm of X is 3.2, what is X _________
6. the natural logarithm of Y is 6.4, what is Y _________

Manipulating Logarithms

RULE	EXAMPLE
$\ln (x^a) = a \ln x$	$\ln (10^2) = 2 \ln 10$ $\ln (100) = 4.605$ $2 \ln (10) = 2 (2.303) = 4.606$
$\ln (a \times b) = (\ln a) + (\ln b)$	$\ln (55 \times 32) = \ln (55) + \ln (32)$ $\ln (1760) = 7.473$ $\ln (55) = 4.007$ $\ln (32) = 3.466$ $4.007 + 3.466 = 7.473$

RULE	EXAMPLE
$\ln(x/y) = (\ln x) - (\ln y)$	$\ln(77/22) = \ln(77) - \ln(22)$ $\ln(3.5) = 1.253$ $\ln(77) = 4.344$ $\ln(22) = 3.091$ $4.344 - 3.091 = 1.253$
$\ln(e^a) = a \ln e = a$	ln (e4.9) = 4.9 ln (e) = 4.9 $\ln(134.29) = 4.9$
$\ln(x/y) = -\ln(y/x)$	$\ln(86/38) = -\ln(38/86)$ $\ln(2.263) = 0.82$ $\ln(0.442) = -0.82$

NOTE: the ln or log of any number greater than zero but less than 1 is negative. You cannot take a ln or log of zero or a negative number

Manipulating Power Functions

RULE	EXAMPLE
$x^a \times x^b = x^{(a + b)}$	$8^2 \times 8^3 = 8^{(2 + 3)}$ $64 \times 512 = 32768$ $8^5 = 32768$
$\dfrac{x^a}{x^b} = x^{(a - b)}$	$\dfrac{7^5}{7^3} = 7^{(5 - 3)}$ $7^5 = 16807$ $7^3 = 343$ $16807/343 = 49$ $7^2 = 49$

RULE

$(x^a)^b = x^{(a \times b)}$

EXAMPLE

$(8^2)^3 = 8^{(2 \times 3)}$

$(64)^3 = 262144$

$8^6 = 262144$

PROBLEMS (Calculate the following)

1. $\log(2.9^3)$ __________

2. $\ln(12.7 \times 6.8)$ __________

3. $\log(74.3/31.7)$ __________

4. $\ln(e^{7.2})$ __________

5. $-\log(0.5/15)$ __________

6. $4.5^{3.7} \times 4.5^{1.6}$ __________

7. $\dfrac{6.2^{7.3}}{6.2^{2.9}}$ __________

INTERPOLATION

NOTE: Some of the following slides list pages in the Health Physics Technology Reference Manual for looking up Table values. The values are given to you. However, if you wish to look them up yourself and you do not have access to this Reference Manual you can use any available Health Physics source such as the Radiological Health Handbook.

The simplest form of interpolation is the “guesstimate”.

Let’s say you have a set of numbers like these:

speed (mph)	stopping distance (feet)
10	10
20	40
30	70
40	95
50	120
60	150

If you are asked what is the stopping distance for a car travelling 45 mph, you can calculate the exact value or you can “eyeball” the answer (about 105 feet).

A set of data from a Table can be considered points on a line where each point consists of an "x" value and a "y" value.

For example, let's look at the Table on page MISC-26. The "x" values are listed in the first column (Photon Energy (MeV)) and the "y" values are listed in any one of the other columns (Mass Energy Absorption Coefficients (μ_{en}/ρ)). For simplicity let's look at the column labeled "Compact Bone"

Energy	μ_{en}/ρ
0.01	19.0
0.1	0.0386
1.0	0.0297
10.0	0.0159

The “line” represented by the “x” and “y” values doesn’t have to be “straight”. In fact, the values listed in the table on the previous page definitely do not represent a “straight” line.

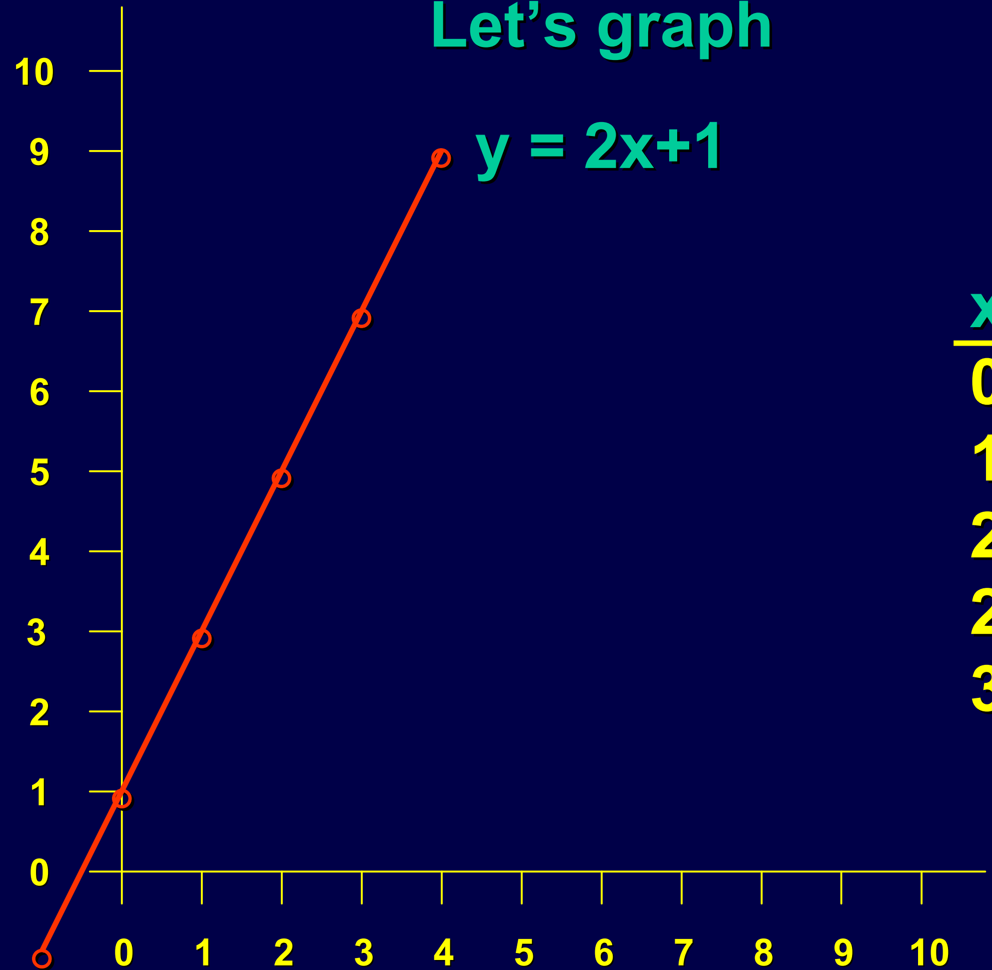
However, if we select two consecutive values from a data table, we can hope that we can approximate the curved “line” by a “straight” line (i.e., if the points are close enough, even a curved line looks straight).

If this approximation is valid we can use “linear interpolation”.

Sample

Let's graph

$y = 2x+1$

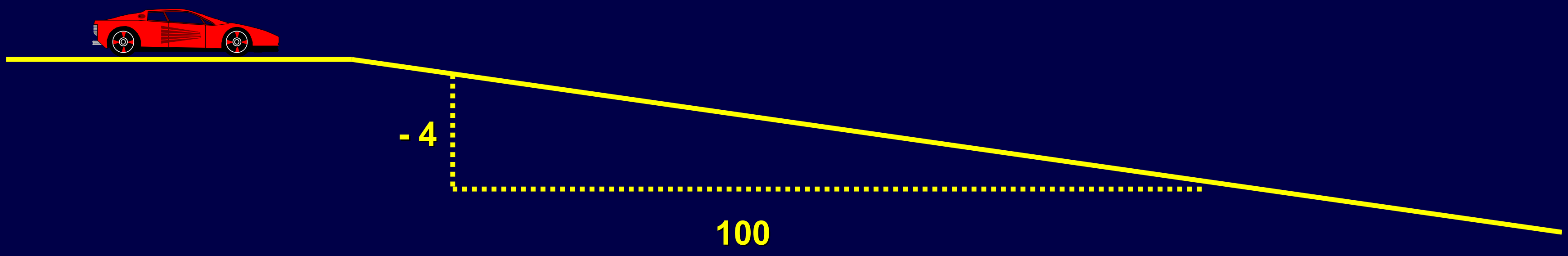


x	y
0	1
1	3
2	5
2.5	?
3	7

The slope of a straight line is just the change in “y” divided by the change in “x”.

You can compare that to the slope of a steep hill where a road sign might state “Caution: 4% grade”.

What the sign is telling you is that the road drops 4 feet for each 100 feet forward that you drive. Or, to put it in our terms, for every 100 units of “x”, “y” changes by - 4 units.



Back to our original problem, how do we determine the "y" value associated with x = 2.5?

Of course we could look at the graph but typically we don't have a graph, we have a table of numbers.

Let's use linear interpolation. $$\frac{\Delta y}{\Delta x} = \frac{7-5}{3-2} = \frac{2}{1} = 2$$

The slope of the line is 2. No matter which two "x" and "y" values we select, the slope will always be 2.

So let's select the following two "x-y" combinations and repeat the same calculation we used for the slope:

x	y
3	7
2.5	?

$$\frac{7-y}{3-2.5} = 2$$

$$\frac{7-y}{0.5} = 2$$

$$7-y = 1$$

$$7-1 = 6 = y$$

So when $x = 2.5$, then $y = 6$

Sample Problem

Determine μ_{en}/ρ for 7 MeV photons traversing compact bone

Looking up some adjacent values from the Table on page MISC-26 yields

MeV	μ_{en}/ρ
6	0.0178
7	?
8	0.0165

Using linear interpolation we get:

$$\frac{y - 0.0178}{7-6} = \frac{0.0165 - 0.0178}{8-6}$$

$$y - 0.0178 = \frac{-0.0013}{2}$$

$$y = 0.0178 - 0.00065 = 0.0172$$

Quick check since 7 MeV is half way between 6 MeV and 8 MeV:

$$\frac{0.0178 + 0.0165}{2} = 0.0172$$

Another Problem

Determine μ_{en}/ρ for ^{60}Co photons traversing muscle

First we need to determine the energy of the photons for ^{60}Co. Look on page Isotopes-6

^{60}Co emits 1.17 MeV and 1.33 MeV photons 100% of the time.

Looking up some adjacent values from the Table on page MISC-26 yields:

MeV	μ_{en}/ρ
1.0	0.0308
1.17	?
1.5	0.0281

MeV	μ_{en}/ρ
1.0	0.0308
1.33	?
1.5	0.0281

$$\frac{0.0281 - 0.0308}{1.5 - 1.0} = \frac{0.0281 - y}{1.5 - 1.17}$$

Same equation except that 1.17 is replaced by 1.33

$$\frac{-0.0027}{0.5} = \frac{0.0281 - y}{0.33}$$

$$-0.00178 = 0.0281 - y$$

$$y = 0.0299$$

$$\frac{-0.0027}{0.5} = \frac{0.0281 - y}{0.17}$$

$$-0.00092 = 0.0281 - y$$

$$y = 0.0290$$

$$y = 0.0295$$

One Last Problem

Determine the buildup factor for ^{60}Co photons if $\mu x = 3$ in lead.

The values for the buildup factors are located on page MISC-35.

Let's assume that ^{60}Co emits a single photon with an energy of 1.25 MeV.

Unfortunately, we have a double interpolation.

Material	MeV	μx*						
		1	2	4	7	10	15	20
Lead	0.5	1.24	1.42	1.69	2.00	2.27	2.65	(2.73)
	1.0	1.37	1.69	2.26	3.02	3.74	4.81	5.86
	2.0	1.39	1.76	2.51	3.66	4.84	6.87	9.00
	3.0	1.34	1.68	2.43	2.75	5.30	8.44	12.3
	4.0	1.27	1.56	2.25	3.61	5.44	9.80	16.3
	5.1097	1.21	1.46	2.08	3.44	5.55	11.7	23.6
	6.0	1.18	1.40	1.97	3.34	5.69	13.8	32.7
	8.0	1.14	1.30	1.74	2.89	5.07	14.1	44.6
	10.0	1.11	1.23	1.58	2.52	4.34	12.5	39.2

We have to interpolate between μx = 2 and μx = 4 and also between energies of 1 MeV and 2 MeV.

Let's start with interpolating between μx = 2 & 4

For 1 MeV

μx	BU
2	1.69
3	?
4	2.26

$$\frac{2.26 - 1.69}{4 - 2} = \frac{2.26 - y}{4 - 3}$$

$$\frac{0.57}{2} = \frac{2.26 - y}{1}$$

$$y = 2.26 - 0.285 = 1.975$$

For 2 MeV

μx	BU
2	1.76
3	?
4	2.51

$$\frac{2.51 - 1.76}{4 - 2} = \frac{2.51 - y}{4 - 3}$$

$$\frac{0.75}{2} = \frac{2.51 - y}{1}$$

$$y = 2.51 - 0.375 = 2.135$$

Now we have buildup values for μx = 3

3

1.975

Material	MeV	μx*						
		1	2	4	7	10	15	20
Lead	0.5	1.24	1.42	1.69	2.00	2.27	2.65	(2.73)
	1.0	1.37	1.69	2.26	3.02	3.74	4.81	5.86
	2.0	1.39	1.76	2.51	3.66	4.84	6.87	9.00
	3.0	1.34	1.68	2.43	2.75	5.30	8.44	12.3
	4.0	1.27	1.56	2.25	3.61	5.44	9.80	16.3
	5.1097	1.21	1.46	2.08	3.44	5.55	11.7	23.6
	6.0	1.18	1.40	1.97	3.34	5.69	13.8	32.7
	8.0	1.14	1.30	1.74	2.89	5.07	14.1	44.6
	10.0	1.11	1.23	1.58	2.52	4.34	12.5	39.2

2.135

So let's interpolate between 1 MeV and 2 MeV

MeV	BU
1	1.975
1.25	?
2	2.135

$$\frac{2.135 - 1.975}{2 - 1} = \frac{2.135 - y}{2 - 1.25}$$

$$\frac{0.16}{1} = \frac{2.135 - y}{0.75}$$

$$y = 2.135 - (0.16 * 0.75) = 2.135 - 0.12 = 2.015$$

So the final answer is:

3

1.25

2.015

Material	MeV	μx^*						
		1	2	4	7	10	15	20
Lead	0.5	1.24	1.42	1.69	2.00	2.27	2.65	(2.73)
	1.0	1.37	1.69	2.26	3.02	3.74	4.81	5.86
	2.0	1.39	1.76	2.51	3.66	4.84	6.87	9.00
	3.0	1.34	1.68	2.43	2.75	5.30	8.44	12.3
	4.0	1.27	1.56	2.25	3.61	5.44	9.80	16.3
	5.1097	1.21	1.46	2.08	3.44	5.55	11.7	23.6
	6.0	1.18	1.40	1.97	3.34	5.69	13.8	32.7
	8.0	1.14	1.30	1.74	2.89	5.07	14.1	44.6
	10.0	1.11	1.23	1.58	2.52	4.34	12.5	39.2

SOLVING FOR EMBEDDED VARIABLES

RULES:

1. You can multiply both sides of an equation by the same value
2. You can divide both sides of an equation by the same value
3. You can add the same value to both sides of an equation
4. You can subtract the same value from both sides of an equation
5. You can multiply any side of an equation by 1 or any ratio which equals 1 (e.g. a / a)
6. You can "flip" both sides of an equation (i.e. reverse numerator and denominator) provided there is only one denominator on each side, e.g. ($1/a = 1/b \rightarrow a = b$) but ($1/a + 1/b = 1/c \not\rightarrow a + b = c$)
7. $\ln(b^a) = a \ln(b)$ {if $b = e$, $\ln e = 1$ so $\ln(e^a) = a$} also $e^{(\ln b)} = b$
8. If sine $a = b$ then inverse sine $b = a$

Examples (in each case solve for "C"):

$$A = \frac{B \times D}{C \times E}$$

$$A = \frac{B}{C} + \frac{D}{E}$$

$$A = \frac{B + C}{D + E}$$

$$A = B \times e^{\frac{C \times D}{E}}$$

$$A = B \times \ln \left[\frac{C}{D} \right]$$

$$\frac{F}{C} = \frac{D}{A} + \frac{E}{B}$$

$$A = B^C$$

$$A = B \times \text{sine } C$$

$$A = (B + C) \times (D + E)$$

$$A = B \times C^{n}$$

Extracting Information from Tables

To solve exam problems you will be required to look up numbers from reference tables. If you look up the wrong value, you will surely get the wrong answer. To avoid this, always be sure of the following:

- you are on the correct page
- you are looking down the correct column
- you are looking across the correct row

The following page shows a sample conversion table from the reference manual.

To convert from “A” to “B” find the “A” column along the top row. Drop down to the box containing the number “1". Move across to the “B” column. Multiply your “A” value by this number to get “B”.

LENGTH

µm	mm	cm	m	in	ft	yd	mi
1	1.000e-03	1.000e-04	1.000e-06	3.937e-05	3.281e-06	1.094e-06	6.214e-10
1.000e+03	1	1.000e-01	1.000e-03	3.937e-02	3.281e-03	1.094e-03	6.214e-07
1.000e+04	1.000e+01	1	1.000e-02	3.937e-01	3.281e-02	1.094e-02	6.214e-06
1.000e+06	1.000e+03	1.000e+02	1	3.937e+01	3.281e+00	1.094e+00	6.214e-04
2.540e+04	2.540e+01	2.540e+00	2.540e-02	1	8.333e-02	2.778e-02	1.578e-05
3.048e+05	3.048e+02	3.048e+01	3.048e-01	1.200e+01	1	3.333e-01	1.894e-04
9.144e+05	9.144e+02	9.144e+01	9.144e-01	3.600e+01	3.000e+00	1	5.682e-04
1.609e+09	1.609e+06	1.609e+05	1.609e+03	6.336e+04	5.280e+03	1.760e+03	1

Sample: Convert 64 feet to mm

ANSWERS

See page 10

Example

Solve $ax + b = y$ for "x"

$$6x + 3 = 28.2$$

One equation, one variable equals one solution

Solve for x:

$$6x = 28.2 - 3 = 25.2$$

$$\boxed{x = 25.2/6 = 4.2}$$

$$10x + 5 = y$$

One equation, two variables equals many solutions

Possible solutions:

x	y
0	5
1	15
2	25
3	35

Solve for x:

$$x = (y - 5)/10 = 0.1y - 0.5$$

See page 15

Sample (test your knowledge)

$$3x + 5y = 37$$
$$6x - 3y = 9$$

Solve for "x":

$3x + 5y = 37$

$3x = 37 - 5y$

$x = (37 - 5y)/3$

$3x + 5x5 = 37$

$3x = 12$

$\boxed{x = 4}$

Substitute into:

$6x - 3y = 9$

$6[(37 - 5y)/3] - 3y = 9$

$6(37 - 5y) - 9y = 27$

$222 - 30y - 9y = 27$

$-30y - 9y = 27 - 222$

$y(-30 - 9) = -195$

$y = -195/-39 = 5$

$\boxed{y = 5}$

See page 19

SAMPLE
(test your knowledge)

$$3x + 4y = 18$$
$$5x - 3y = 1$$

$$5 \times (3x + 4y) = (18) \times 5$$
$$3 \times (5x - 3y) = (1) \times 3$$

$$
\begin{array}{rr}
 & 15x + 20y = 90 \\
- & 15x - 9y = 3 \\
\hline
 & 29y = 87
\end{array}
$$

$$\boxed{y = 87/29 = 3}$$

Now if we plug y = 3 into either of the two original equation, we can solve for $\boxed{\text{x which equals 2}}$

See page 21

SAMPLE

(test your knowledge)

$$x^2 + 8x = -7$$

$a = 1$

$b = 8$

$c = 7$

$$\frac{-8 \pm \sqrt{8^2 - 4*1*7}}{2*1} = \frac{-8 \pm \sqrt{64 - 28}}{2} = \frac{-8 \pm \sqrt{36}}{2} = \frac{-8 \pm 6}{2}$$

$$\frac{-14}{2} = \boxed{-7} \qquad \frac{-2}{2} = \boxed{-1}$$

See page 41

PROBLEMS (calculate the following)

1. area of a circle with a 4 cm diameter? $12.6\ cm^2$

2. surface area of a hemisphere with a radius of 2.4 meters? $36.2\ m^2$

3. volume of a sphere with a 78 cm diameter? $248475\ cm^3$

4. volume of a box whose length is 7 cm, width is 3 cm and depth is 12 cm? $252\ cm^3$

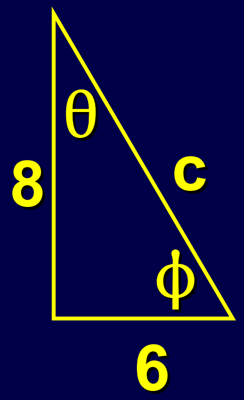
5. surface area of a box whose length, width and depth are all 12 cm? $864\ cm^2$

See page 47

PROBLEMS (calculate the following)

1. Sine (28°) — 0.47
2. Tangent (0.89 radians) — 1.235
3. Cosine (88°) — 0.035

Given this triangle calculate the: →



4. Hypotenuse (c) — 10
5. Cosine of θ — 0.8

See page 52

PROBLEMS (calculate the following)

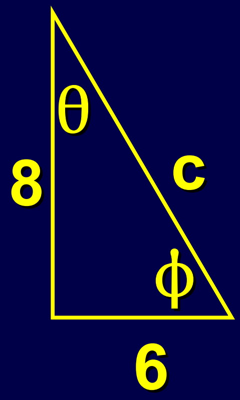
1. arctan (0.666) in degrees **33.7**

2. inverse cosine (0.17) in radians **1.4**

3. inverse sine [cosine (24°)] in degrees **66**

4. arcsine (0.44) in degrees **26.1**

Given this triangle what is:



5. ϕ (in degrees) **53.1**

See page 60

PROBLEMS (Calculate the following)

1. the common logarithm of 88 — 1.94
2. the natural logarithm of 123 — 4.81
3. the ratio of the natural logarithm of 452 to the common logarithm of 452 — 2.3
4. the natural logarithm of the common logarithm of 333 — 0.93
5. the common logarithm of X is 3.2, what is X — 1585
6. the natural logarithm of Y is 6.4, what is Y — 602

See page 65

PROBLEMS (Calculate the following)

1.	$\log(2.9^3)$	1.387
2.	$\ln(12.7 \times 6.8)$	4.459
3.	$\log(74.3/31.7)$	0.37
4.	$\ln(e^{7.2})$	7.2
5.	$-\log(0.5/15)$	1.477
6.	$4.5^{3.7} \times 4.5^{1.6}$	2897.52
7.	$\dfrac{6.2^{7.3}}{6.2^{2.9}}$	3065.66

Examples (in each case solve for “C”):

See page 86

$$\boxed{A = \frac{B \times D}{C \times E}}$$

We want to isolate “C” on one side all by itself

Multiply both sides by C

$$C \times A = \frac{B \times D}{E}$$

Divide both sides by A

$$\boxed{C = \frac{B \times D}{A \times E}}$$

Sample problem of this type:

$$\frac{\text{dose}}{\text{time}} = \text{dose rate}$$

Solve for time

Answer:

$$\text{time} = \frac{\text{dose}}{\text{dose rate}}$$

$$\text{hour} = \frac{\text{mrem}}{\frac{\text{mrem}}{\text{hour}}}$$

See page 87

$$\boxed{A = \frac{B}{C} + \frac{D}{E}}$$

Subtract D/E from both sides.

$$A - \frac{D}{E} = \frac{B}{C}$$

Multiply the "A" by 1 (E/E)

$$\frac{AE}{E} - \frac{D}{E} = \frac{B}{C}$$

Now we have a common denominator on the left so we can add the numerators

$$\frac{AE - D}{E} = \frac{B}{C}$$

To isolate "C" we can multiply both sides by "C" and multiply both sides by "E" and then divide both sides by (AE - D).

This produces the same effect as flipping both sides of the equation and then multiplying both sides by "B"

$$\frac{E}{AE - D} = \frac{C}{B}$$

$$\boxed{\frac{BE}{AE - D} = C}$$

See page 88

$$\boxed{A = \frac{B + C}{D + E}}$$

Multiply both sides by (D + E)

$$A \times (D + E) = B + C$$

Subtract "B" from both sides

$$\boxed{A \times (D + E) - B = C}$$

If we wish we may factor the A into the parentheses

$$\boxed{(AD) + (AE) - B = C}$$

See page 89

$$\boxed{A = B \times e^{\frac{C \times D}{E}}}$$

First isolate the exponential by dividing both sides by “B”

$$\frac{A}{B} = e^{\frac{C \times D}{E}}$$

Next we get rid of the exponential by taking the inverse which is the natural logarithm “ln” of both sides

$$\ln \left[\frac{A}{B}\right] = \ln \left[e^{\frac{C \times D}{E}} \right]$$

But we know that the natural logarithm of an exponential is just the exponent

$$\ln\left[\frac{A}{B}\right] = \frac{C \times D}{E}$$

Now we merely have to isolate "C" which we can do by multiplying both sides by E and dividing both sides by D

$$\boxed{\frac{E}{D}\ln\left[\frac{A}{B}\right] = C}$$

If you wished, you could expand the ln of the fraction since we know that ln (A/B) = ln A - ln B.

Sample problem of this type:

The equation for radioactive decay is

$A = A_o \times e^{(-0.693 \times t)/T}$

Solve for t (the time it takes for A_o to decay to A)

Answer:

$$\frac{T}{-0.0693} \ln \left[\frac{A}{A_o}\right] = t$$

See page 90

$$\boxed{A = B \times \ln\left[\frac{C}{D}\right]}$$

To isolate "C" we must get rid of the logarithm. To do this, we employ the inverse which is the exponential. But first we must remove the "B" by dividing both sides by "B".

$$\frac{A}{B} = \ln\left[\frac{C}{D}\right]$$

Now we apply the exponential to both sides.

$$e^{\frac{A}{B}} = e^{\ln\left[\frac{C}{D}\right]}$$

But an exponential and a natural logarithm are inverses so they cancel leaving the variables.

$$e^{\frac{A}{B}} = \frac{C}{D}$$

Finally, we isolate the "C" by multiplying both sides by "D"

$$\boxed{D \times e^{\frac{A}{B}} = C}$$

See page 91

$$\boxed{\frac{F}{C} = \frac{D}{A} + \frac{E}{B}}$$

To isolate “C” we need to get it into the numerator. This will first require us to obtain a common denominator on the right side of the equation. Then we can “flip” both sides. We can get the common denominator by multiplying each fraction on the right side by 1. We multiply the first fraction by “B/B” and the second by “A/A”. This gives us the common denominator “AB”

$$\frac{F}{C} = \frac{DB}{AB} + \frac{EA}{BA} = \frac{DB + EA}{AB}$$

Now with a single denominator on both sides we can “flip”

$$\frac{C}{F} = \frac{AB}{DB + EA}$$

The final step is to remove the “F from the left side by multiplying both sides by “F”

$$\boxed{C = \frac{FAB}{DB + EA}}$$

Sample problem of this type:

The equation for effective half life is:

$$\frac{1}{T_{eff}} = \frac{1}{T_{rad}} + \frac{1}{T_{bio}}$$

$$T_{eff} = \frac{T_{rad} \times T_{bio}}{T_{rad} + T_{bio}}$$

See page 92

$$\boxed{A = B^C}$$

To isolate "C" we need to remove it from the power position and get it back on a normal level. We can do this using logarithms since we know from rule 7 that $\ln(b^a) = a \ln(b)$. Actually we can use either common or natural logarithms. There's no real advantage to either unless "B" is a power of 10 in which case common logarithms will be simpler.

$$\ln A = \ln[B^C] = C \ln B$$

To isolate "C" we simply divide both sides by ln B

$$\boxed{\frac{\ln A}{\ln B} = C}$$

See page 93

$$\boxed{A = B \times \text{sine } C}$$

To isolate "C" we'll need to employ the inverse of the "sine" which is the "sine^{-1}" or "arcsine". But first we'll need to isolate the sine by dividing both sides by "B"

$$\frac{A}{B} = \text{sine } C$$

Applying the inverse sine yields

$$\boxed{C = \text{sine}^{-1}\left[\frac{A}{B}\right] = \text{arcsine}\left[\frac{A}{B}\right]}$$

See page 94

$$\boxed{A = (B + C) \times (D + E)}$$

To isolate "C" we first divide both sides by "(D + E)"

$$\frac{A}{(D + E)} = (B + C)$$

Now we merely subtract "B" from each side

$$\boxed{\frac{A}{(D + E)} - B = C}$$

See page 95

$$\boxed{A = B \times C^{n}}$$

The isolate "C" we need to get rid of the power "n". This can be done using logarithms since rule 7 states that ln (b^a) = a ln (b). But first we need to get rid of the "B" from the right side by dividing both sides by "B"

$$\frac{A}{B} = C^{n}$$

Now we can apply the logarithm (either common or natural) to both sides

$$\ln\left[\frac{A}{B}\right] = \ln\left[C^{n}\right] = n \ln C$$

$$\ln\left[\frac{A}{B}\right] = n \ln C$$

Now we can isolate "C" by first dividing both side by "n" and the applying an exponential to both sides

$$\frac{1}{n} \ln\left[\frac{A}{B}\right] = \ln C$$

But remember that "a ln b" is the same as ln b^a. Here "a" is "A/B" and "b" is "1/n" so we get

$$\ln\left[\left(\frac{A}{B}\right)^{\frac{1}{n}}\right] = \ln C$$

$$\ln\left[\left(\frac{A}{B}\right)^{\frac{1}{n}}\right] = \ln C$$

Taking the exponential of both sides eliminates both logarithms and gives us "C"

$$\boxed{\left(\frac{A}{B}\right)^{\frac{1}{n}} = C}$$

Instead of using logarithms and exponentials, you could just take the "n th" root of both sides of the second equation

$$\frac{A}{B} = C^{n}$$

$$\sqrt[n]{\frac{A}{B}} = \sqrt[n]{C^{n}} = (C^{n})^{\frac{1}{n}} = C$$

$$\sqrt[n]{\frac{A}{B}} = \boxed{\left(\frac{A}{B}\right)^{\frac{1}{n}} = C}$$

Which is the same answer we got using logarithms and exponentials

See page 97

To convert from "A" to "B" find the "A" column along the top row. Drop down to the box containing the number "1". Move across to the "B" column. Multiply your "A" value by this number to get "B".

LENGTH

µm	mm	cm	m	in	ft	yd	mi
1	1.000e-03	1.000e-04	1.000e-06	3.937e-05	3.281e-06	1.094e-06	6.214e-10
1.000e+03	1	1.000e-01	1.000e-03	3.937e-02	3.281e-03	1.094e-03	6.214e-07
1.000e+04	1.000e+01	1	1.000e-02	3.937e-01	3.281e-02	1.094e-02	6.214e-06
1.000e+06	1.000e+03	1.000e+02	1	3.937e+01	3.281e+00	1.094e+00	6.214e-04
2.540e+04	2.540e+01	2.540e+00	2.540e-02	1	8.333e-02	2.778e-02	1.578e-05
3.048e+05	3.048e+02	3.048e+01	3.048e-01	1.200e+01	1	3.333e-01	1.894e-04
9.144e+05	9.144e+02	9.144e+01	9.144e-01	3.600e+01	3.000e+00	1	5.682e-04
1.609e+09	1.609e+06	1.609e+05	1.609e+03	6.336e+04	5.280e+03	1.760e+03	1

Sample: Convert 64 feet to mm

(test your knowledge)

64 * 304.8 = 19507.2 mm