CODE VERIFICATION OF FLAC

FOR THERMOMECHANICAL ANALYSIS

Prepared for:

U.S. Nuclear Regulatory Commisssion
Washington, D.C.

Contract No. NRC-02-85-002

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April 1987

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Contract No. NRC-02-85-002
Task Order No. 002, Task 2

Task Description: to perform verification testing of the microcomputer code FLAC for thermomechanical analysis

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INTRODUCTION

This report provides the code verification of the FLAC* microcomputer program for application to thermomechanical analysis. The report presents a set of thermal and mechanical (creep) calculations performed with FLAC. The verification testing is conducted for the thermal and creep components of the code separately; the intention of this study is to demonstrate the accuracy of each component. The numerical solutions for both steady-state and transient problems are compared with analytical solutions and the accuracy of the thermal and creep calculations in the code is reported. Verification testing of the elastic/plastic material models in FLAC are presented in the FLAC User's Manual.

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*"FLAC, Fast Lagrangian Analysis of Continua," developed by Peter A. Cundall and Itasca Consulting Group, Inc. (Suite 210, 1313 5th Street SE, Minneapolis, Minnesota, 55414)

CASE 1: STEADY-STATE TEMPERATURE DISTRIBUTION ALONG A TAPERED FIN

Problem Description:

A very long tapered rectangular steel fin dissipates heat to air, from a wall. The wall temperature is 1100 °C, the surrounding air is at 100 °C, and the convective coefficient is 15 W/m²°C. The fin has a thermal conductivity of 15 W/m°C and tapers from a width of 8.33 cm to 0 cm over a distance of 33.33 cm. Determine the surface temperature distribution along the fin (shown in Fig. 1).

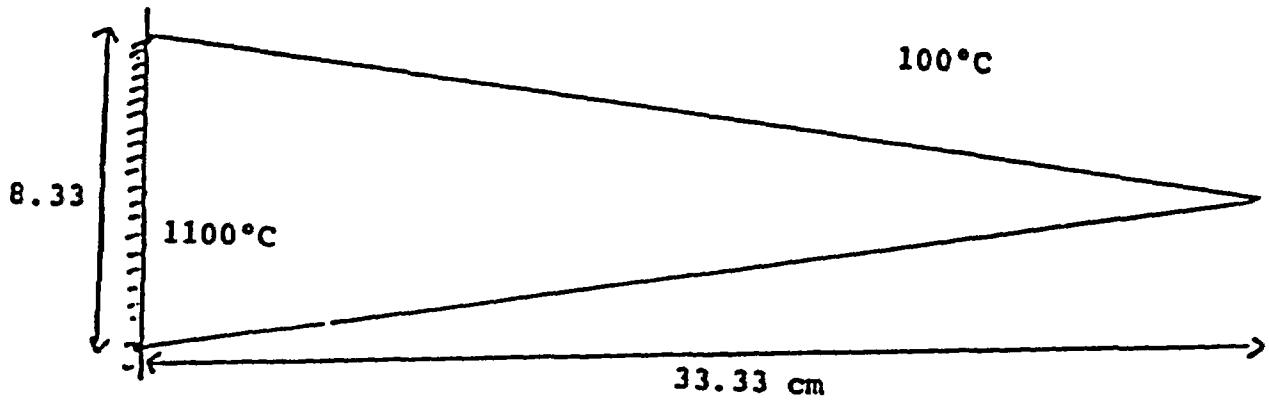

Fig. 1 Temperature Distribution Along a Tapered Fin

The analytical solution for this problem is given by Carslaw and Jaeger (p. 142). The temperatures obtained from FLAC are compared with this solution in Table 1, below. The results from FLAC agree well with the analytical solution.

Reference

Carslaw, H. S., and J. C. Jaeger. Conduction of Heat in Solids (2nd Ed.). London: Oxford University Press, 1959.

FLAC Data File — Case 1

```
*Tapered fin
gr 9,1
mod el
thmod iso
gen 0,0 0,0.0833 0.26664,0.04998 0.26664,0.03332 i=1,9
gen 0.26664,0.03332 0.26664,0.04998 0.3,0.045815 0.3333,0.04165 i=9,10
prop dens 1000 spec 300 cond 15
thapp convec 15 100 j=1
thapp convec 15 100 j=2
ini tem 1100
tfix 1100 i=1
ini tem 500 i=4,10
thsolve ste 1000 tem 1500 clock 1e10
ret
```

Table 1

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION
(l=33.33cm)

T (°C)	x/l								
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Analytical	1100	972	855	750	655	570	493	426	365
FLAC	1100	971	855	750	656	572	498	433	378
Error (%)	none	0.1	none	none	0.1	0.35	1.0	1.6	3.5

CASE 2: ONE-DIMENSIONAL STEADY-STATE HEAT CONDUCTION AND CONVECTION THROUGH A COMPOSITE WALL

Problem Description:

A furnace wall consists of two layers, firebrick and insulating brick. The temperature inside the furnace is 3000°C, and the outside temperature is 25°C. The thermal conductivity of the firebrick is 1.6 W/m°C; that of the insulating brick is 0.2 W/m°C. The convective heat transfer coefficients are 100 W/m²°C in the furnace and 15 W/m²°C on the outside wall. The firebrick is 25cm thick; the insulating brick is 15cm thick. Calculate the temperature at both sides of the insulating brick as well as at the inner surface of the firebrick (see Fig. 2). The results from FLAC agree well with the analytical solution as shown in Table 2.

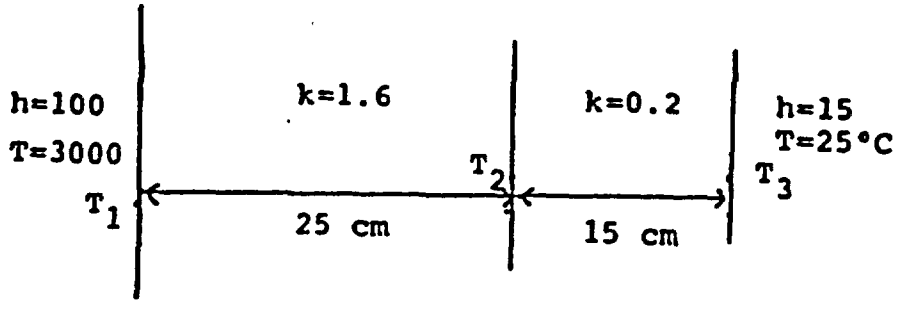

Fig. 2 Composite Wall

FLAC Data File — Case 2

```
* Composite wall
gr 16,1
m e
gen 0,0 0,0.025 0.4,0.025 0.4,0
thmod iso
prop dens=10000 spec=300
prop cond=1.6 i=1,10
prop cond=.2 i=11,16
ini tem=2000
thapp conv=100 3000 i=1
thapp conv=15 25 i=17
thsolv step=2500 clock=1e10 tem=1e5
return
```

Table 2

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION

	Analytical (°C)	FLAC (°C)	Error(%)
T_1	2969.7	2970	0.01
T_2	2496.8	2496	0.03
T_3	226.8	227	0.09

CASE 3: THERMAL RESPONSE OF A HEAT-GENERATING SLAB

Problem Description:

An infinite plate of thickness 1m, initially at a temperature of 60°C, is subjected to a sudden heat generation rate of 40 kW/m³ and a surface temperature of 32 °C. Determine the temperature distribution in the plate after 0.2 sec. The physical properties of the plate are:

Density	500 kg/m³
Specific Heat	0.2 kJ/kg°C
Thermal Conductivity	20 W/m°C

The analytical solution for this problem is given graphically in Schneider (1955, p. 309). The results from FLAC are well within this solution, as shown in Table 3, below.

Reference

Schneider, P. J. Conduction Heat Transfer. Cambridge, Mass.: Addison-Wesley, 1955.

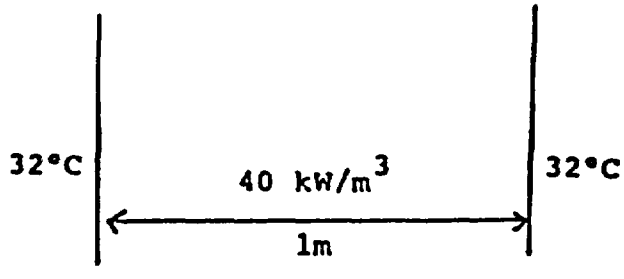

Fig. 3 Heat-Generating Slab

FLAC Data File — Case 3

```
*Heat generating slab
gr 5,1
m e
gen 0,0 0,0.1 0.5,0.1 0.5,0
thmod iso
prop dens=500 cond=20 spec=0.2
ini tem=60
*apply flux of 400 W/element over the entire grid
thapp source=400 0 i=1,5
tfix 32 i=1
thsolve ste 0
set thdt=1e-3
thsolve ste=200 tem=200
return
```

Table 3

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION

x (m)	Analytical T (°C)	FLACT T (°C)	Error (%)
0.0	32	32	none
0.1	75	75.7	0.93
0.2	105	103.9	1.05
0.3	120	120.7	0.58
0.4	129	129.3	0.23
0.5	131	131.9	0.69

CASE 4: INFINITE SLAB WITH APPLIED HEAT FLUX

Problem Description:

A heat flux of $1 W/m^2$ is applied to one edge of a semi-infinite slab, initially at 0 degrees. The slab has a conductivity of 1 W/m°C and a diffusivity of $1 m^2/s$. The material has a Young's modulus, E, of 1 kPa, a Poisson's ratio, ν, of 0.25, and a linear thermal expansion coefficient of 1/°C (i.e., a volumetric thermal expansion coefficient of 3/°C). The bulk modulus, K, and shear modulus, G, are

$$K = E/(3*(1-2\nu)) = 0.667 \text{ kPa}$$
$$G = E/(2(1+\nu)) = 0.4 \text{ kPa}$$

The analytical solutions are given for the temperature and stresses by Carslaw and Jaeger (p. 75) and Timoshenko and Goodier (p. 435), respectively. As shown in Table 4, the results from FLAC are in good agreement with these solutions, especially considering the number of significant figures available in the solutions.

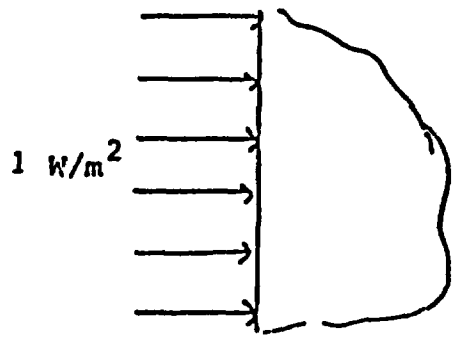

Fig. 4 Infinite Slab with Applied Flux

References

Carslaw, H. S., and J. C. Jaeger. Conduction of Heat in Solids (2nd Ed.). London: Oxford University Press, 1959.

Timoshenko, S. P., and J. N. Goodier. Theory of Elasticity (3rd Ed.). New York: McGraw-Hill, 1970.

FLAC Data File — Case 4

```
*Infinite slab with applied flux
gr 25,1
m e
thmod iso
prop b=.67 s=.4 d=1 cond=1 spec=1 thexp=3
gen 0,0 0,0.2 5,0.2 5,0
thapp flux 1 0 i=1
fix y j=1
fix y j=2
fix x y i=26
set thdt=8e-4
thsolve clock=1e10 tem=1e10 step=250
step 700
save prob4_1.sav
thsolve clock=1e10 tem=1e10 step=1000
step 700
save prob4_2.sav
return
```

Table 4

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION

Temperature:

	x (m)	Analytical T (°C)	FLAC T (°C)	Error (%)
(t=0.2 sec)	0.0	0.505	0.501	0.79
	0.2	0.330	0.326	1.21
	0.6	0.116	0.114	1.72
	1.0	0.031	0.030	3.23
(t=1.0 sec)	0.0	1.128	1.127	0.0887
	0.2	0.940	0.938	0.213
	0.6	0.628	0.627	0.159
	1.0	0.400	0.398	0.50

Stress (σ_{yy}):

	x (m)	Analytical (kPa)	FLAC (kPa)	Error (%)
(t=0.2 sec)	0.1	-0.548	-0.551	0.55
	0.3	-0.347	-0.350	0.865
	0.5	-0.206	-0.208	0.971
	0.7	-0.114	-0.116	1.75
	0.9	-0.059	-0.060	1.69
(t=1.0 sec)	0.1	-1.375	-1.379	0.29
	0.3	-1.138	-1.142	0.35
	0.5	-0.931	-0.934	0.32
	0.7	-0.752	-0.755	0.40
	0.9	-0.599	-0.602	0.50

CASE 5: TRANSIENT TEMPERATURE DISTRIBUTION IN AN ORTHOTROPIC BAR

Problem Description:

A long bar of rectangular cross-section is initially at 500°C. It is suddenly exposed to fluid at 100°C. The thermal conductivity is 20 W/m°C in the x-direction and 3.6036 W/m°C in the y-direction. The bar is 0.3333m in the x-direction and 0.16666m in the y-direction. The specific heat is 9.009e-3 J/kg°C, and the density is 400 kg/m³. The convective heat transfer coefficient is 240 W/m°C. Determine the temperature at points A,B,C,D (see Fig. 5) at $t=8.3333\times10^{-4}$ secs.

Table 5 compares the results from FLAC with the solutions by Schneider (1955, p. 262). The FLAC results compare very well with the analytical solutions.

Reference

Schneider, P. J. Conduction Heat Transfer. Cambridge, Mass.: Addison-Wesley, 1955.

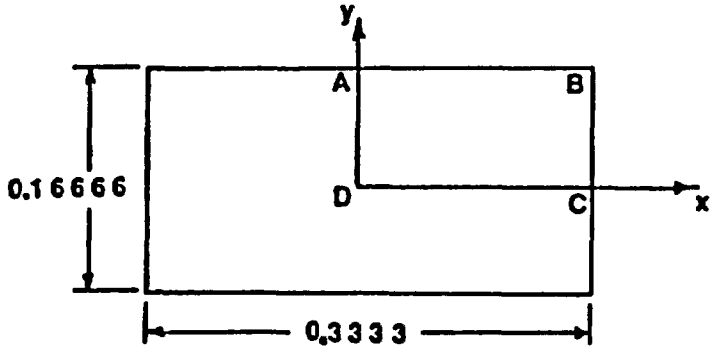

Fig. 5 Orthotropic Bar

FLAC Data File — Case 5

```
* Orthotropic bar
gr 7,5
mod el
thmo aniso
prop de 400 spe 9.009e-3 xcon 20 ycon 3.6036
ini tem 500
thapp convec 240 100 i=7 j=1,2
thapp convec 240 100 i=8 j=3,6
thapp convec 240 100 i=1,8 j=6
gen 0,0 0,2.7e-2 1.66667e-1,2.7e-2 1.666667e-1,0 i=1,7 j=1,2
gen 0,2.7e-2 0,8.3333e-2 1.66667e-1 8.3333e-2 1.66667e-1,2.7e-2
i=1,8 j=3,6
mod null i=7 j=1
mod null i=1,8 j=2
inter 1 aside 8,3 7,3 6,3 5,3 4,3 3,3 2,3 1,3
inter 1 bside 1,2 2,2 3,2 4,2 5,2 6,2 7,2
set tol=0.01
thsolve ste 575 tem 1000
set thdt=6.14333e-7
thsolve ste 1
sav ortho.sav
ret
```

Table 5

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION

x	y	Analytical T (°C)	FLAC T (°C)	Error (%)
0	0	458.7	463.0	0.94
0.1666	0	280.5	280.3	0.07
0	0.083333	198.2	198.7	0.25
0.1666	0.083333	149.4	154.4	3.35

CASE 6: STEADY-STATE HEAT CONDUCTION AND CONVECTION FROM A SLAB

Problem Statement:

Determine the steady-state wall temperature for a composite slab in contact with media at different temperatures at each face. The slab consists of two material types with different thermal properties and different convective coefficients to the surrounding media, as shown in Fig. 6, below.

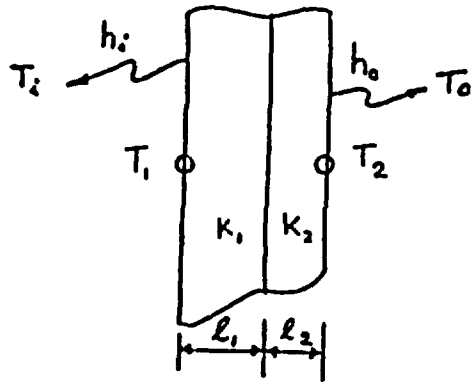

Fig. 6 Geometry of Composite Slab

Slab Properties:

K_1 = 1.3840 W/m°C

K_2 = 0.1730 W/m°C

h_i = 68.139 W/m² °C

h_o - 11.357 W/m² °C

l_1 = 0.233 m

l_2 = 0.127 m

Initial Conditions:

T_i = 1648.90°C

T_o = 26.68 °C

FLAC Data File — Case 6

```
* Heat conduction and convection from a slab
gr 10,15
m e
gen 0,0 0,2.0 .233,2.0 .233,0 i=1,6 j=1,16
gen s s .360,2.0 .360,0 i=6,11 j=1,16
thmod iso
prop bu=5e9 sh=3e9 dens=8950 spec=382 thexp=3e-5
prop cond=1.38424 i=1,5
prop cond=.17303 i=6,10
fix x i=1
fix x i=11
fix y j=1
fix y j=16
ini te=900
thapp conv=68.1396 1648.9 i=1 j=1,16
thapp conv=11.3566 26.67 i=11 j=1,16
tit
Case 2 - composite slab with convective surfaces
thsolv step=5000 clock=1e.0 tem=1e5
save case2.sav
return
```

Table 6

COMPARISON OF FLAC WITH ANALYTICAL SOLUTION

	Analytical (°C)	FLAC (°C)	Error (%)
T_1(°C)	1625	1625	None
T_2(°C)	169	169	No-

CASE 7: INTERNALLY-HEATED SLAB WITH FIXED FACE TEMPERATURES

Problem Statement:

Determine the temperature distribution in a slab with internal heating and fixed wall temperatures, as shown in Fig. 7, below.

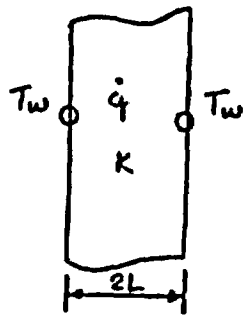

Fig. 7

Analytic Solution:

The steady state analytic solution for the temperature distribution in the slab is:

$$T - T_W = \frac{\dot{q}}{2k}(L^2 - x^2)$$

where k = conductivity;

L = slab half-thickness

$\dot{q}$ = internal heat production;

T = temperature;

T_W = wall temperature; and

x = the distance from the midplane of the slab;

Problem Properties, Initial Conditions:

$k = 1.0$ w/m°C

$L = 0.5$ m

$\dot{q} = 5$ kW/m^3

$T_W = 65$°C

FLAC Data File — Case 7

```
* Internally-heated slab
gr 10,15
m e
gen 0,0 0,3.0 1.0,3.0 1.0,0
thmod iso
prop bu=5e9 sh=3e9 dens=2000 cond=1 spec=1000 thexp=3e-5
tfix 65 i=1
tfix 65 i=11
ini te=65
*apply flux of 100 w/element over the entire grid
thapp source=100.0  0 i=1,10 j=1,15
thsolv step=250 tem=1e5 clock=1e5
tit
Slab with internal heat source , steady state
save case3.sav
return
```

Table 7

COMPARISON OF FLACT WITH ANALYTICAL SOLUITON

x	Analytical (°C)	FLAC (°C)	Error (%)
0	690.00	690.00	None
0.1	665.00	665.00	None
0.2	590.00	590.00	None
0.3	465.00	465.00	None
0.4	290.00	290.00	None
0.5	65.00	65.00	None

CASE 8: TRANSIENT RESPONSE OF A SLAB WHOSE SURFACE TEMPERATURE IS SUDDENLY LOWERED

Problem Statement:

Examine a half-space whose temperature T_o at time t = 0 is 550°C. The surface of that half-space is suddenly lowered to 100°C at t = 0. The problem is to determine how much time is required for a point at a depth of 0.01 meter to reach 200°C (Fig. 1).

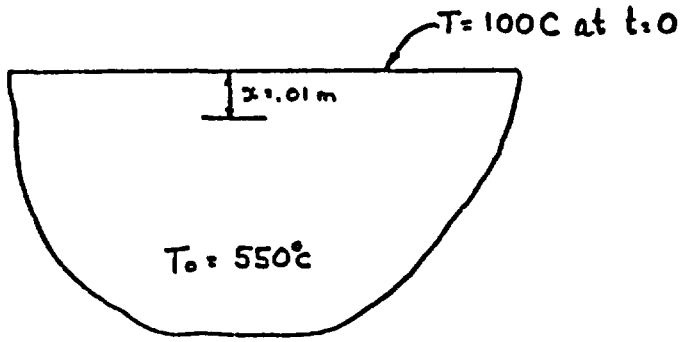

Fig. 8 Illustration of the Half-Space Problem

Properties of the Body: Tungsten Steel, $k = 43$ w/m°C, $\rho = 8826$ kg/m^3, $C_p = 389$ J/kg°C

Solution: First, determine the thermal diffusivity:

$$\alpha = \frac{K}{\rho C_p} = 1.252\text{x}10^{-5}\ \text{m}^2/\text{sec}$$

The problem is one dimensional and can be solved using the method given in Karlekar and Desmond (1982, p. 205). The solution is given by:

$$\text{erf}\ \frac{x}{2\ (\alpha t)^{1/2}} = \frac{T_s(x,t) - T_s}{T_o - T_s} = \frac{200 - 100}{550 - 100} = 0.222$$

where erf = error function; T_o = initial temperature of body; T_s = surface temperature; and $T_s(x,t)$ = temperature at depth x, time t.

From Table 4-2 in Karlekar and Desmond (1982, 207):

$$\frac{x}{2\ (\alpha t)^{1/2}} = 0.2$$

or

$$t = \left[\frac{x}{0.2} \ \frac{1}{2} \right]^2 \cdot \frac{1}{1.252 \times 10^{-5}} = 49.92 \text{ sec}$$

The time calculated by FLAC is 50.32 sec (error = 0.8%).

Reference

Karlekar, B, V., and R. M. Desmond. Heat Transfer (2nd Ed.). St. Paul, Minnesota: West Publishing Company, 1982.

FLAC Data File — Case 8

```
*Slab quenching
gr 10,20
mod e
thmod iso
prop bu=4.7e9 sh=2.8e9 dens=8826 cond=43 spec=389 thexp=3e4
* model of a half space with initial temp=550C and surface
* suddenly cooled to 100C.  Determine the temp change at a depth
* of .01 meters
gen -.2,0 -.2,.1 .2,.1 .2,0
ini y=.5 j=21
ini y=.4 j=20
ini y=.35 j=19
ini y=.3 j=18
ini y=.25 j=17
ini y=.2 j=16
ini y=.15 j=15
ini y=.10 j=14
ini y=.075 j=13
ini y=.065 j=12
ini y=.06 j=11
ini te=550
tfix 100 i=1,11 j=1
thsolve clock=1e10 tem=1e10 step=100
return
```

CASE 9: PARALLEL-PLATE VISCOMETER

Problem Description:

An incompressible fluid of viscosity 10^{10} kg/ms lies between two parallel plates of width 20m, a distance of 10m apart. The two plates move toward each other at a rate of 10^{-4} m/s. Calculate the steady-state stresses and velocities in the fluid (Fig. 9).

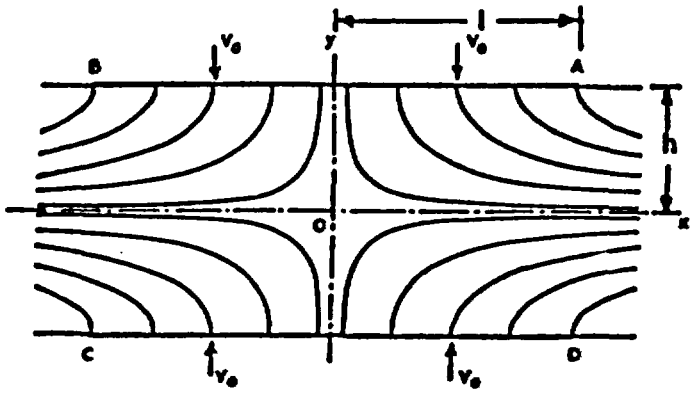

Fig. 9

Jaeger (1969, p. 140) gives the analytical solution to this problem as

$$V_x = 3\, V_o x\, (h^2 - y^2)/2h^3$$

$$V_y = V_o y\, (y^2 - 3h^2)/2h^3$$

$$\sigma_{xx} = 3\eta\, V_o\, [3(h^2 - y^2) + x^2 - l^2]/2h^3$$

$$\sigma_{yy} = 3\eta\, V_o\, (y^2 - h^2 + x^2 - l^2)/2h^3$$

$$\sigma_{xy} = -3\, V_o \eta\, (xy/h^3)$$

where $V_o = 10^{-4}$ m/s, $h = 5$m, $l = 10$m, and $\eta = 10^{10}$ kg/ms.

Reference

Jaeger, J. C. Elasticity, Fracture and Flow With Engineering and Geological Applications (3rd Ed.). London: Methuen & Co. Ltd., 1969.

FLAC Data File — Case 9

```
* Parallel plate test viscous model
g 10 5
m vis
title
parallel-plate viscometer with viscous model
fix x i 1
fix y j 1
fix x y j 6
ini yv -1e-4 j 6
appl xf 4.5e5 i 11 j 1
appl xf 8.64e5 yf -2.4e5 i 11 j 2
appl xf 7.56e5 yf -4.8e5 i 11 j 3
appl xf 5.76e5 yf -7.2e5 i 11 j 4
appl xf 3.24e5 yf -9.6e5 i 11 j 5
prop d 1 sh .5e9 bu 1.5e9 vis 1e10
set dt 1
ste 500
ret
```

Tables 8 and 9 show the velocities and stresses in the FLAC grid and the analytical solutions for the velocities and stresses. Comparison of the two sets of results show a good agreement. The velocity components are all within 1.3% of the analytical solution; the stresses are generally within 5%. When the velocities and stresses are plotted, the analytical solution and FLAC results are virtually indistinguishable (compare Figs. 10 and 11).

Table 8

FLAC RESULTS

x velocity (multiply values below by 10^{-4})

I / J	1	2	3	4	5	6	7	8	9	10	11
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.109	0.218	0.327	0.436	0.545	0.654	0.763	0.872	0.982	1.094
4	0.000	0.194	0.388	0.582	0.776	0.970	1.164	1.357	1.551	1.746	1.940
3	0.000	0.255	0.509	0.764	1.018	1.273	1.527	1.782	2.037	2.291	2.545
2	0.000	0.291	0.582	0.873	1.164	1.455	1.746	2.037	2.328	2.618	2.907
1	0.000	0.303	0.606	0.909	1.212	1.515	1.818	2.122	2.424	2.727	3.028

y velocity (multiply values below by 10^{-4})

I / J	1	2	3	4	5	6	7	8	9	10	11
6	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
5	-0.946	-0.945	-0.946	-0.945	-0.946	-0.945	-0.945	-0.946	-0.945	-0.946	-0.942
4	-0.794	-0.794	-0.794	-0.794	-0.794	-0.794	-0.794	-0.794	-0.794	-0.793	-0.789
3	-0.570	-0.570	-0.570	-0.570	-0.570	-0.570	-0.570	-0.570	-0.569	-0.568	-0.565
2	-0.297	-0.297	-0.297	-0.297	-0.297	-0.297	-0.297	-0.297	-0.297	-0.296	-0.295
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

xx stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-1.043	-1.018	-0.971	-0.896	-0.803	-0.677	-0.537	-0.363	-0.172	0.040
4	-0.752	-0.728	-0.678	-0.608	-0.508	-0.391	-0.241	-0.075	0.120	0.343
3	-0.534	-0.509	-0.462	-0.387	-0.293	-0.168	-0.026	0.146	0.339	0.557
2	-0.388	-0.364	-0.315	-0.243	-0.145	-0.026	0.122	0.290	0.485	0.702
1	-0.316	-0.291	-0.243	-0.170	-0.073	0.049	0.194	0.364	0.557	0.774

yy stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-1.261	-1.236	-1.189	-1.114	-1.021	-0.895	-0.755	-0.581	-0.390	-0.184
4	-1.358	-1.334	-1.284	-1.214	-1.114	-0.996	-0.847	-0.681	-0.488	-0.270
3	-1.431	-1.406	-1.359	-1.284	-1.190	-1.065	-0.923	-0.751	-0.559	-0.338
2	-1.479	-1.455	-1.406	-1.334	-1.236	-1.117	-0.969	-0.801	-0.605	-0.384
1	-1.503	-1.479	-1.431	-1.358	-1.261	-1.139	-0.995	-0.824	-0.629	-0.407

xy stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-0.054	-0.164	-0.273	-0.382	-0.491	-0.600	-0.709	-0.818	-0.927	-1.036
4	-0.042	-0.127	-0.212	-0.297	-0.382	-0.467	-0.552	-0.636	-0.721	-0.802
3	-0.030	-0.091	-0.152	-0.212	-0.273	-0.333	-0.394	-0.455	-0.514	-0.572
2	-0.018	-0.055	-0.091	-0.127	-0.164	-0.200	-0.237	-0.272	-0.308	-0.343
1	-0.006	-0.018	-0.030	-0.042	-0.055	-0.067	-0.079	-0.091	-0.103	-0.114

Table 9

ANALYTICAL SOLUTIONS

x velocity (multiply values below by 10^{-4})

I / J	1	2	3	4	5	6	7	8	9	10	11
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.108	0.216	0.324	0.432	0.540	0.648	0.756	0.864	0.972	1.080
4	0.000	0.192	0.384	0.576	0.768	0.960	1.152	1.344	1.536	1.728	1.920
3	0.000	0.252	0.504	0.756	1.008	1.260	1.512	1.764	2.016	2.268	2.520
2	0.000	0.288	0.576	0.864	1.152	1.440	1.728	2.016	2.304	2.592	2.880
1	0.000	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000

y velocity (multiply values below by 10^{-4})

I / J	1	2	3	4	5	6	7	8	9	10	11
6	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
5	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944	-0.944
4	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792	-0.792
3	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568	-0.568
2	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296	-0.296
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

xx stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-1.026	-1.002	-0.954	-0.882	-0.786	-0.666	-0.522	-0.354	-0.162	0.054
4	-0.738	-0.714	-0.666	-0.594	-0.498	-0.378	-0.234	-0.066	0.126	0.342
3	-0.522	-0.498	-0.450	-0.378	-0.282	-0.162	-0.018	0.150	0.342	0.558
2	-0.378	-0.354	-0.306	-0.234	-0.138	-0.018	0.126	0.294	0.486	0.702
1	-0.306	-0.282	-0.234	-0.162	-0.066	0.054	0.198	0.366	0.558	0.774

yy stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-1.254	-1.230	-1.182	-1.110	-1.014	-0.894	-0.750	-0.582	-0.390	-0.174
4	-1.350	-1.326	-1.278	-1.206	-1.110	-0.990	-0.846	-0.678	-0.486	-0.270
3	-1.422	-1.398	-1.350	-1.278	-1.182	-1.062	-0.918	-0.750	-0.558	-0.342
2	-1.470	-1.446	-1.398	-1.326	-1.230	-1.110	-0.966	-0.798	-0.606	-0.390
1	-1.494	-1.470	-1.422	-1.350	-1.254	-1.134	-0.990	-0.822	-0.630	-0.414

xy stress (multiply values below by 10^{6})

I / J	1	2	3	4	5	6	7	8	9	10
5	-0.054	-0.162	-0.270	-0.378	-0.486	-0.594	-0.702	-0.810	-0.918	-1.026
4	-0.042	-0.126	-0.210	-0.294	-0.378	-0.462	-0.546	-0.630	-0.714	-0.798
3	-0.030	-0.090	-0.150	-0.210	-0.270	-0.330	-0.390	-0.450	-0.510	-0.570
2	-0.018	-0.054	-0.090	-0.126	-0.162	-0.198	-0.234	-0.270	-0.306	-0.342
1	-0.006	-0.018	-0.030	-0.042	-0.054	-0.066	-0.078	-0.090	-0.102	-0.114

FLAC (Version 1.10)

LEGEND

20/ 4/1987 12:57
step 500
Thermal Time 0.0000E+00
Creep Time 5.0000E+02
-5.556E-01 < x < 1.056E+01
-2.222E+00 < y < 8.889E+00

XX-stress contours
Contour interval= 1.000E+05
E: -6.000E+05
R: 7.000E+05

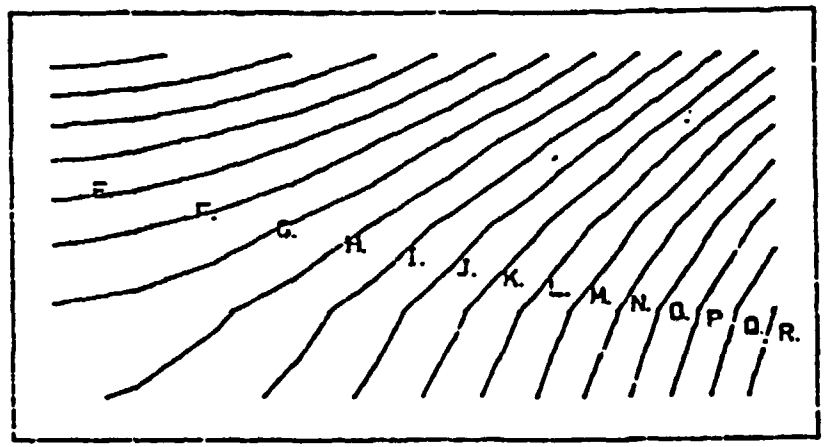

Fig. 10 FLAC Solution

Nuclear Regulatory Commission

LEGEND

20/ 4/1987 12:27

-5.556E-01 < x < 1.056E+01
-2.222E+00 < y < 8.889E+00

XX-stress contours
Contour interval= 1.000E+05
E: -6.000E+05
R: 7.000E+05

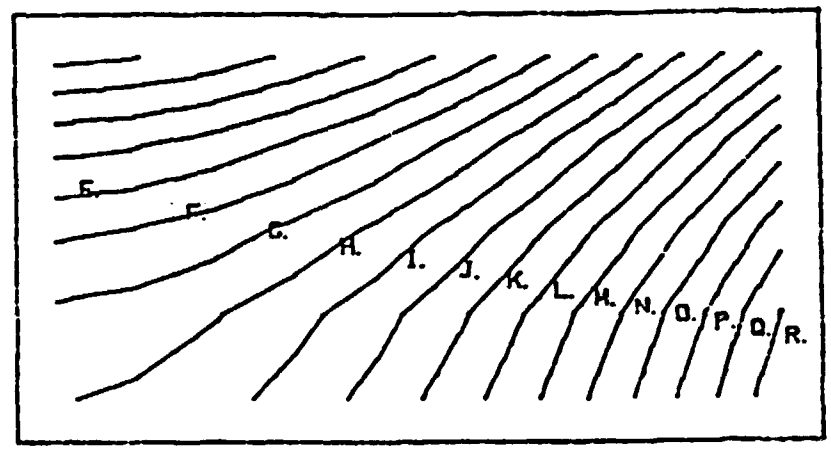

Fig. 11 Analytical Solution

Nuclear Regulatory Commission