NUCLEAR WASTE CONSULTANTS INC.

U.S NUCLEAR REGULATORY COMMISSION
DIVISION OF WASTE MANAGEMENT

Technical Report #10

ANALYSES OF REPOSITORY TEMPERATURE REQUIRED FOR INDUCED THERMAL GROUND-WATER CONVECTION

Salt Repository Project
Subtask 3.5

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for

Nuclear Waste Consultants

TECHNICAL ASSISTANCE IN HYDROGEOLOGY
PROJECT B - ANALYSIS
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Mini-Report Summary

Mini-Report number: 10

Title: Analysis of Repository Temperature Required for Induced Thermal Ground-Water Convection

Objective: The objective of this report is to estimate the minimum repository temperature which would sustain thermal ground-water convection above the repository.

Analysis: A linear stability analysis is performed to estimate the minimum repository temperature required for steady-state ground-water convection in a two-dimensional, isotropic, homogeneous, porous medium. The results indicate that the minimum repository temperature required for thermal convection increases rapidly as permeability decreases.

Conclusion: Because the greatest expected permeability for HSU B is less than 10 md, the minimum repository temperature required for convection is estimated to be greater than 1000°C. The repository temperature is expected to be less than 250°C.

Discussion: The following assumptions were made in order to obtain an analytical solution: homogeneous, isotropic, unfractured, saturated, porous medium with impermeable boundaries. Fractures and unsaturated zones may profoundly affect the ground-water flow system for this problem. Numerical analyses are recommended to investigate the effects of fractures and zones of partial saturation.

ANALYSIS OF REPOSITORY TEMPERATURE REQUIRED FOR INDUCED THERMAL GROUND-WATER CONVECTION

Numerical Evaluation of Conceptual Models
Subtask 3.5
Mini-Performance Assessment #10

February 1987

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1.0 INTRODUCTION

1.1 General Statement of the Problem

Ground water in the sedimentary section above a waste repository will tend to be gravitationally unstable. Buoyancy forces will be generated by thermal expansion of ground water due to increased temperature at depth. Increased temperatures will result from the ambient geothermal gradient and heat generated by radioactive decay within the waste repository. Ground-water convection, which will result when buoyancy forces overcome the viscous resistance of the ground water to flow, would include upward flow paths. Figure 1 illustrates some possible upward flow paths which may result from induced thermal ground-water convection above the repository.

1.2 Statement of Relevance to NRC

The Ogallala Formation, a fresh-water aquifer, and the Dockum Group, which locally yields good quality water, are present within the Deaf Smith County site and lie about 0.5 km above the proposed host horizon, the lower San Andres Unit 4 (Figure 2; DOE, 1986, p. 3-34, 35, 122). Since the Ogallala and Dockum Formations may be considered special sources of ground water, the possibility of upward ground-water flow from the waste repository suggests that ground-water protection requirements and containment requirements specified by the EPA may not be met (see 40CFR191. 12, 13, 16).

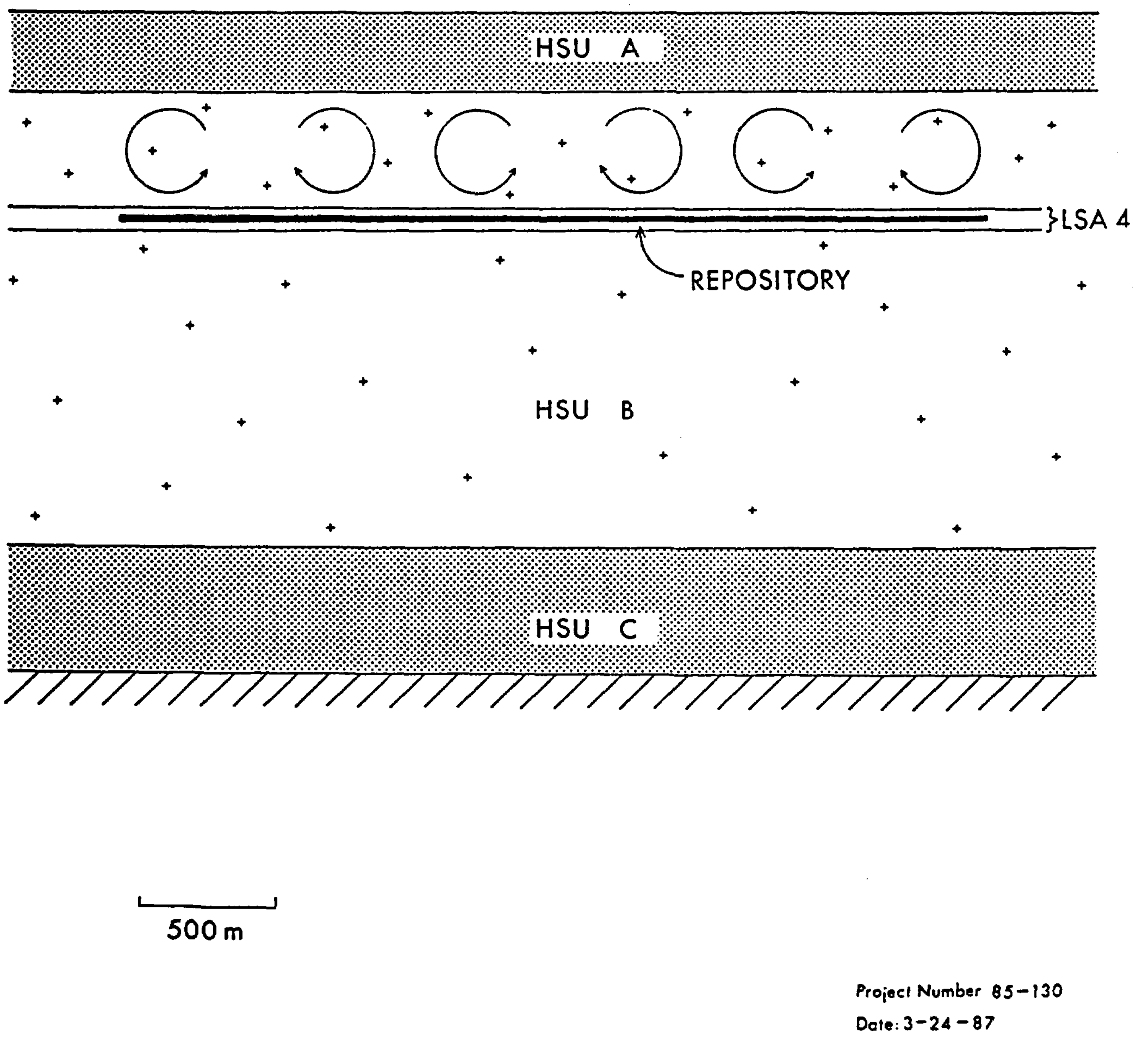

Figure 1. Idealized flow paths of induced thermal ground-water convection above the waste repository. The repository host horizon is the lower San Andres Unit 4 (LSA 4).

ERA	SYSTEM	SERIES	GROUP	FORMATION	HYDROSTRATIGRAPHIC UNIT (HSU)
CENOZOIC	QUATERNARY			RECENT FLUVIAL AEOLIAN AND LACUSTRINE DEPOSITS	FRESHWATER FLOW SYSTEM *HSU A*
	TERTIARY			OGALLALA AND LACUSTRINE DEPOSITS	
MESOZOIC	CRETACEOUS	COMANCHE	WASHITA		
			FREDRICKSBURG		
			TRINITY		
	TRIASSIC		DOCKUM	TRUJILLO (Santa Rosa)	
				TECOVAS	
PALEOZOIC	PERMIAN	OCHOA		DEWEY LAKE (Quartermaster)	SHALE AND EVAPORITE AQUITARD *HSU B*
				ALIBATES	
				SALADO-TANSILL	
		GUADALUPE	ARTESIA (WHITEHORSE)	YATES	
				SEVEN RIVERS	
				QUEEN-GRAYBURG	
			PEASE RIVER	SAN ANDRES (BLAINE)	
		LEONARD		GLORIETA	
			CLEAR FORK	UPPER CLEAR FORK	
				TUBB	
				LOWER CLEAR FORK	
				RED CAVE	
			WICHITA		
		WOLFCAMP			DEEP-BASIN FLOW SYSTEM *HSU C*
	PENNSYLVANIAN	VIRGIL	CISCO		
		MISSOURI	CANYON		
		DES MOINES	STRAWN		
		ATOKA	BEND		
		MORROW			
	MISSISSIPPIAN	CHESTER			
		MERAMEC			
		OSAGE			
	ORDOVICIAN	CANADIAN	ELLENBURGER		
	CAMBRIAN		UNNAMED SANDSTONE		
	PRECAMBRIAN				

Explanation

———— Unconformity

----- Boundary in Dispute

Figure 2. Generalized hydrostratigraphic column for the Palo Duro Basin (DOE, 1986).

Upward flow paths resulting from induced thermal convection constitute a potentially adverse condition since this potential change in hydrologic conditions would affect the migration of radionuclides to the accessible environment (10CFR60). According to NRC regulations (10CFR60.122) potentially adverse conditions must be investigated.

1.3 Relationship to Other Analyses, Documents, Tasks and Subtasks

Thermally induced ground-water convection may provide a mechanism for the transport of radionuclides to the accessible environment and the contamination of special sources of ground water through upward ground-water flow (Figure 1). However, according to the DOE (p. 3-143, 1986) flow in the Permian evaporite aquitard (HSU B), if it does exist, is vertically downward. Most analyses of hydrodynamic relationships within the Palo Duro basin also indicate downward flow through HSU B (Stephens & Assoc., 1986a).

Stephens & Assoc. (1986b, c) have investigated upward ground-water movement resulting from natural geothermal convection and convection induced by repository heating. One report (Stephens & Assoc., 1986b) concluded that buoyancy forces generated by the relatively low natural geothermal gradient will probably not overcome the viscous resistance to flow in the low permeability evaporite section. Thus natural geothermal convection is not expected to occur in the HSU B of Palo Duro Basin. Another report (Stephens & Assoc., 1986c) estimated the velocity profile of the thermal ground-water plume generated by heat produced by radioactive decay within the repository. However, simplifying assumptions (required to obtain an analytical solution) limit the usefulness of the analysis. Therefore, this additional analysis is being performed to investigate the possibility of ground-water convection induced by radiogenic heating of the waste repository.

2.0 FORMAL STATEMENT OF OBJECTIVE

The objective of this mini-report is to estimate the minimum repository temperature which would sustain thermal ground-water convection above the repository. The estimated temperature required for convection can then be compared with any available values of expected repository temperature.

3.0 OPERATIONAL APPROACH - CONCEPTS AND GENERAL ASSUMPTIONS

3.1 Geology

Because no exploratory wells have been drilled within the Deaf Smith County site, the actual site stratigraphy is not known (DOE, 1986). However, the DOE (1986, Table 3-3) presents a generalized stratigraphic column for the site based on data from nearby wells and regional trends. The upper fresh-water flow system (HSU A) consists of the fluvial and lacustrine deposits of the Ogallala and Dockum Formations (Figure 2). The Permian aquitard (HSU B) which underlies HSU A is a sedimentary sequence of shales, evaporites, carbonates and siltstones (DOE, 1986).

The stratified nature of HSU A and HSU B suggests that there may be a large degree of variability and anisotropy in hydrogeologic parameters. In this analysis the Permian evaporite aquitard is modeled as a 2-dimensional, isotropic, porous medium. This highly simplified model will allow the use of a relatively simple, analytical solution.

3.2 Ground-Water Flow and Thermal Systems

In this analysis ground-water flow is assumed to occur in response to buoyancy forces generated by the thermal expansion of ground water. The analytical solution also assumes flow occurs in a two-dimensional, isotropic, homogeneous unfractured porous medium. The upper and lower flow system boundaries for this analysis are the upper contact of HSU B and the lower San Andres Unit 4 (the host horizon). Although these boundaries are not impermeable, this assumption is made for the present analysis.

Many of the assumptions listed above may lead to an inaccurate estimate of the repository temperature required to sustain induced thermal convection. At present, analytical solutions exist for the assumptions given in this analysis (Turcotte and Schubert, 1982). More complex models, for example those which incorporate an anisotropic flow system with permeable boundaries, would probably require numerical solutions.

3.3 Repository and Source Term

In this analysis the repository is modeled as an impermeable, isothermal boundary. The standard solution is for steady-state ground-water flow. Because the decay rate of the radionuclides is proportional to the amount of parent material present, the actual temperature of the repository will not be constant. Rather, following waste emplacement the average repository temperature will increase to some maximum and then decrease as the radioactive decay rate decreases (for example see Figure 6-8, DOE, 1986; Figure 4.4, Wagner and others, 1986).

4.0 TECHNICAL APPROACH

4.1 Formal Statement of Problem

A linear stability analysis is performed to estimate the minimum repository temperature required for thermal ground-water convection to occur above the repository. The conceptual model for this analysis is a layer of fluid-saturated, permeable material which is situated between impermeable, isothermal boundaries heated from below (Figure 3).

4.2 Identification of Solution Techniques

The solution utilizes the "Boussinesq approximation" (Turcotte and Schubert, 1982) where fluid is considered to be

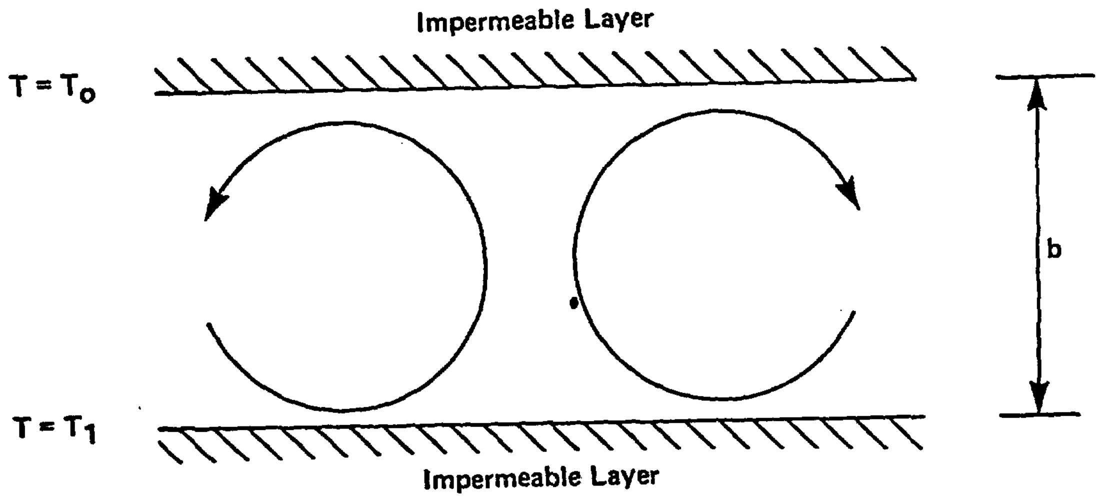

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Figure 3. Two-dimensional cellular convection in a layer of fluid-saturated permeable medium between impermeable isothermal boundaries heated from below.

incompressible except for a buoyancy term (density variation due to thermal expansion) in the vertical Darcy velocity component. Darcy's law (for vertical and horizontal flow), the equation of conservation of mass for the flow of a viscous incompressible fluid (rigid matrix), and the equation of conservation of energy for 2-dimensional flow of an incompressible fluid in a porous medium are solved subject to boundary conditions appropriate for isothermal, impermeable boundaries. The solution allows one to obtain the appropriate Rayleigh number for the onset of thermal convection in a layer of permeable material heated from below (see section 4.3, Appendix). Substitution of the appropriate hydraulic parameters yielding a Rayleigh number greater than a critical value suggests that this specific combination of parameters represents a condition under which thermal convection will occur (in a uniform permeable medium heated from below). The mathematical statement and analysis of the problem are presented in the Appendix.

4.3 Definitions and Assumptions

The appropriate Rayleigh number of thermal convection in a layer of saturated porous material heated from below is:

$$Ra \equiv \frac{\alpha_f g \rho_f^2 c_{p_f} k b (T_1 - T_0)}{\mu \lambda_m} \tag{1}$$

(Wooding, 1957; Donaldson, 1962; Turcotte & Schubert, 1982) where

α_f = coefficient of volumetric expansion of fluid [t^{-1}]

g = gravitational acceleration [LT^{-2}]

ρ_f = fluid density [ML^{-3}]

c_f = specific heat of convecting fluid [$EM^{-1}t^{-1}$]

k = permeability [L^2]

b = thickness for porous layer [L]

$(T_1 - T_0)$ = temperature difference between isothermal boundaries [t]

μ = dynamic viscosity [$ML^{-1}T^{-1}$]

λ_m = thermal conductivity of saturated medium [$ET^{-1}L^{-1}t^{-1}$]

(dimensions; M = mass; L = length;, T = time; t = temperature; E = energy).

List of Assumptions:

1. Two-dimensional, homogeneous, isotropic, unfractured, porous medium bounded by impermeable, isothermal surface.

2. Fluid incompressible, except for buoyancy term (due to density variation caused by thermal expansion), also known as Boussinesq approximation; fresh water. The effects of using a density appropriate for saline water is apparent in equations 1 and 2. Spatial variability of fluid density cannot be incorporated in this simple analysis and may be pursued in a subsequent mini-report, perhaps through numerical simulation.

3. Temperature difference between conduction solution and

convection solution, as well as the horizontal and vertical Darcy velocities, are infinitesimal at the onset of convection. (Linear stability analysis; this assumption allows the non-linear terms to be ignored).

4.4 Identification of Expected Output and Judgement Criteria

Permeability is the only parameter in the standard solution which is expected to vary by much more than an order of magnitude and thus is also expected to be a critical parameter in this analysis. This analysis will determine the repository temperature T (for which thermal convection in a uniform permeable layer will occur) as a function of permeability. This calculated temperature can then be compared with available estimates of repository temperatures.

4.5 Standard Solution

Rearranging the Rayleigh number equation (Wooding, 1957; Donaldson, 1962; Turcotte and Schubert, 1982),

$$T_1 = T_o + \frac{\mu \lambda_m R_a}{\alpha_f \; g \; \rho_f^2 c_f b} k^{-1} \qquad (2)$$

indicates the minimum repository temperature for which thermal convection will occur.

5.0 ANALYSIS

Given the following parameter values considered representative of the aquitard and geothermal system, the minimum repository temperature T_1 for which thermal convection is likely to occur is calculated as a function of permeability k:

α_f	$= 2.4 \times 10^{-4}\,°C^{-1}$	(T = 23°C, CRC, 1978; p.F-5)
g	$= 9.78$ m s^{-2}	(Turcotte and Schubert, 1982;p.430)
ρ_f	$= 10^6$ g m^{-3}	(Hillel, 1980; p.294)
c_f	$= 4.2$ W s $g^{-1}\,°C^{-1}$	(Hillel, 1980; p.294)
k	=	(variable, units: m^2)
b	= 468 m	(DOE, 1986; Table 3-3)
T_0	= 19 °C	(estimated, see DOE, 1986; p.3-101)
μ	$= 0.93$ g m^{-1} s^{-1}	(T = 23°C, CRC, 1978; p.F-51)
λ_m	$= 3$ W $m^{-1}\,°C^{-1}$	(Reiter and Tovar, 1982; DOE, 1986, p. 3-101-104)
Ra	$= 4\,\pi^2$	(Turcotte and Schubert, 1982)

6.0 RESULTS

The results of the analysis are presented in Figure 4. The minimum repository temperature required for thermal convection increases rapidly as permeability decreases.

7. CONCLUSION

Horner plot analyses of DST data suggest that permeability within the Lower San Andres Unit 4 ranges from 10^{-17} to

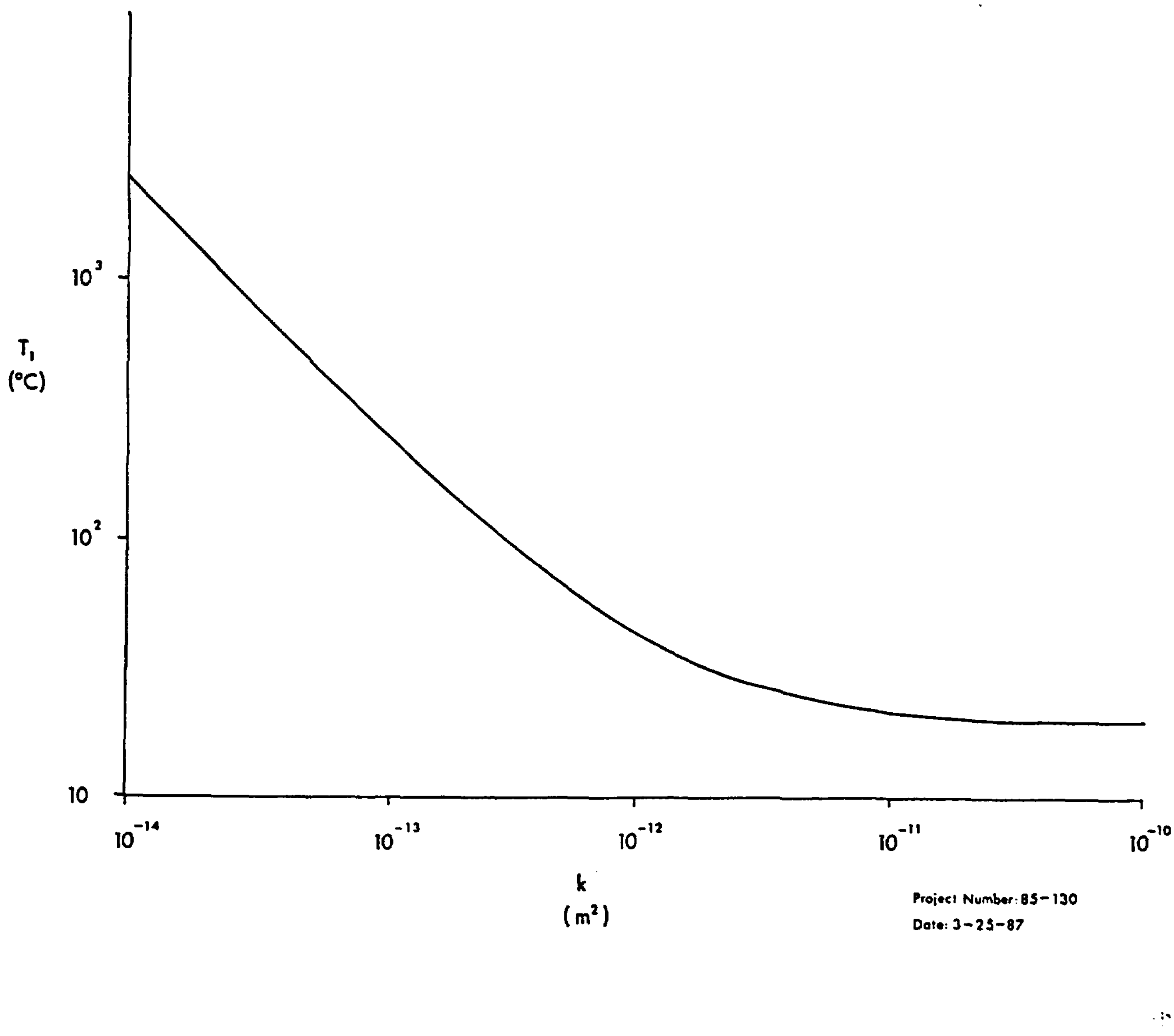

Figure 4. Minimum Repository Temperature Required for Convection as a Function of Permeability

3×10^{-15} m^2 (0.01 to 3.05 md); one analysis indicates that permeability in the Queen Grayburg Formation may be 2×10^{-15} m^2 (1.56 md, DOE, 1986; p. 3.142). According to the DOE (1986, p. 3-143) no permeability testing of the predominant shales and evaporites of HSU B has been done. Freeze and Cherry (1979) report permeabilities for shales range from about 10^{-20} to 10^{-16} m^2 (10^{-5} to 10^{-1} md). The DOE (1986, p. 3-143) cites a range of 3×10^{-17} to 7×10^{-19} m^2 (0.03 to 0.0007 md) for the permeability of the Salado salt in New Mexico. Therefore, because the greatest expected permeability for HSU B is less than 10^{-14} m^2 (10 md) the minimum repository temperature required for induced thermal convection is estimated to be greater than 1000 °C (see Figure 4).

8.0 DISCUSSION

According to the DOE (1986; p. 6-211, Figure 6-8) the expected maximum temperature on the surface of a waste package is 228°C and occurs 5 years after emplacement; the temperature decreases to 150 and 75°C at 50 and 500 years, respectively. Another study (Wagner and others, 1986) suggests that the maximum temperature in and near the repository range from 96 to 137°C. Thus the expected repository temperatures are much less than the estimated minimum temperature required for convection (Figure 4). Therefore, within the limits of the assumptions made in this analysis, thermal ground-water convection will not result for radiogenic heating of the waste repository.

In order to obtain the analytical solution for the problem presented in this report the simplifying assumptions for a homogeneous, isotropic, unfractured, saturated, permeable medium bounded by isothermal, impermeable boundaries were made. Some of the assumptions clearly contradict observations of the actual flow system of the Palo Duro basin. For example, the stratified sediments in the basin should exhibit considerable heterogeneity and anisotropy. Other studies (Gustavson and Budnik, 1985; Stephens & Assoc., 1986b) suggest that HSU B may contain fracture zones and zones of partial saturation.

It is recommended that numerical simulations be applied to the problem of induced thermal ground-water convection. In particular, studies which investigate the effects of fracture zones and partial saturation on the potential for thermal convection should be pursued. Zones of partial saturation may provide discontinuities in the flow system which will act as barriers to radionuclide transport. Alternatively, fracture zones may increase the potential for ground-water convection and radionuclide transport along zones of enhanced permeability.

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APPENDIX

APPENDIX
Mathematical Analysis

The following mathematical analysis (taken from Turcotte and Schubert, 1982) is not meant to be a rigorous derivation, but rather is intended to be a guide illustrating the method of solution. The presentation of the governing equations is followed by a list of the major assumptions and the boundary conditions.

1.0 GOVERNING EQUATIONS

Darcy's law, horizontal

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial x} \qquad (1)$$

Darcy's law, vertical

$$v = -\frac{k}{\mu}\left(\frac{\partial p}{\partial y} - \rho_f\, g\right) \qquad (2)$$

Conservation of Mass (see assumptions)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

Conservation of Energy (see assumptions)

$$\rho_m\, c_m \frac{\partial T}{\partial t} + \rho_f\, c_f \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \lambda_m\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

where:

u, v = horizontal, vertical Darcy velocity (LT^{-1})

k = permeability (L^2)

μ = viscosity ($ML^{-1}T^{-1}$)

p = pressure (FL^{-2})

ρ_f, ρ_m = fluid, matrix density (ML^{-3})

g = fravitational acceleration (LT^{-2})

c_f, c_m = fluid, matrix specific heat ($EM^{-1}t^{-1}$)

T = temperature (t^{-1})

λ_m = thermal conductivity of saturated medium ($ET^{-1}L^{-1}t^{-1}$)

(dimensions; M = mass; L = length; T = time; t = temperature; E = energy; F = force)

2.0 ASSUMPTIONS AND BOUNDARY CONDITIONS

1. homogeneous, isotropic, 2-dimensional, permeable, rigid, unfractured, saturated medium.

2. fluid incompressible, except for buoyancy term in vertical Darcy velocity.

3. physical properties of fluid, except density, are constant, i.e. unaffected by temperature and pressure variations.

4. upper boundary, y = 0, at temperature T_0 ; impermeable.

5. lower boundary, y = b, at temperature, $T_1 > T_0$; impermeable.

3.0 APPROACH

In the absence of convection the temperature distribution is given by the conduction solution

$$T_c = T_0 + \frac{T_1 - T_0}{b} y \qquad (5)$$

where b is the layer thickness (Figure 3). At the onset of convection, the temperature distribution may be expressed as the conduction solution plus some infinitesimal temperature disturbance T'.

$$T = T_0 + \frac{T_1 - T_0}{b} y + T' \qquad (6)$$

(prime symbol indicates value is infinitesimal). Likewise, at the onset of convection the Darcy velocity components are also infinitesimal. In terms of the infinitesimal quantities, the appropriate forms of equations 1 through 4 are:

$$u' = -\frac{k}{\mu}\frac{\partial p'}{\partial x} \qquad (7)$$

$$v' = -\frac{k}{\mu}\left(\frac{\partial p'}{\partial y} + \alpha_f \rho_f g T'\right) \qquad (8)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \qquad (9)$$

$$\rho_m c_m \frac{\partial T'}{\partial t} + \rho_f c_f v' \frac{(T_1 - T_0)}{b} = \lambda_m \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2}\right) \qquad (10)$$

To obtain eq.(10) substitute eq. (6) into eq. (4) and neglect the nonlinear infinitesimal terms (nonlinear terms involve products of the infinitesimal quantities u', v', and T', e.g. $u' \frac{\partial T'}{\partial x}$ Since T', u' and v' are small quantities, the nonlinear terms will be much smaller than the linear terms and thus can be neglected; such "linearized analysis" is a standard mathematical approach).

The critical condition for the onset of convection can be determined from steady-state considerations. Differentiate eq. (7) with respect to y and eq. (8) with respect to x; subtract to get

$$\frac{\partial u'}{\partial y} - \frac{\partial v'}{\partial x} = \frac{k\, \alpha_f\, \rho_f\, g}{\mu} \frac{\partial T'}{\partial x} \qquad (11)$$

Similarly, cross-differentiate and then subtract eq. (9) and (11) to get

$$\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2} = \frac{-k\, \alpha_f\, \rho_f\, g}{\mu} \frac{\partial^2 T'}{\partial x^2} \qquad (12)$$

Solving the steady-state eq.(10)

$$\rho_f\, c_f\, v' \left(\frac{T_1 - T_0}{b}\right) = \lambda_m \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2}\right) \qquad (13)$$

for v' and substituting into eq. (12) results with

$$\frac{\partial^4 T'}{\partial x^4} + 2 \frac{\partial^4 T'}{\partial x^2 \partial y^2} + \frac{\partial^4 T'}{\partial y^4} = - \frac{k\, \alpha_f\, \rho_f^2\, g\, c_f\, (T_1 - T_0)}{\mu\, \lambda_m\, b} \frac{\partial^2 T'}{\partial x^2} \qquad (14)$$

Using the boundary conditions, since $T' = 0$ at $y = (0, b)$ then

$$\frac{\partial^2 T'}{\partial x^2} = 0 \text{ at } y = 0, b \qquad (15)$$

Also, $v' = 0$ at $y = (0, b)$ (16)

Substituting eq.(15) and (16) into eq.(13) one determines

$$\frac{\partial^2 T'}{\partial y^2} = 0 \text{ at } y = 0, b \qquad (17)$$

$$T' = T_0' \sin\frac{\pi y}{b} \quad \sin\frac{2\pi x}{\lambda} \qquad (18)$$

satisfies eq. (14) and the boundary conditions where T_0' is the amplitude and λ is the wavelength of T'. Substituting eq.(18) into eq.(14), then

$$\frac{\{ (\frac{2\pi b}{\lambda})^2 + \pi^2 \}^2}{(\frac{2\pi b}{\lambda})^2} = \frac{\alpha_f \; g \; \rho_f^2 \; c_f \; k \; b \; (T_1 - T_0)}{\mu \; \lambda_m} \equiv Ra \qquad (19)$$

defines the appropriate Rayleigh number for thermal convection in a layer of permeable material heated from below.

The minimum value of Ra for which convection can occur is obtained differentiating the left side of eq.(19) with respect to λ and setting the result equal to zero to get

$$\lambda = 2b. \qquad (20)$$

Substituting eq.(20) into eq.(19) one determine the minimum critical Rayleigh number for the onset of convection.

$$\min (Ra_{cr}) = 4\pi^2 \qquad (21)$$

To determine whether convection can occur under given conditions, substitute appropriate parameters into eq.(19). A resulting Rayleigh number greater than the minimum critical value of $4\pi^2$ indicates that convection can occur.