#### U.S NUCLEAR REGULATORY COMMISSION DIVISION OF WASTE MANAGEMENT

Technical Report #11

#### EVALUATION OF A LARGE-SCALE TRACER TEST CONDUCTED IN GRANDE RONDE BASALT

Basalt Waste Isolation Project Subtask 2.5 Numerical Evaluation of Conceptual Models

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for

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TECHNICAL ASSISTANCE IN HYDROGEOLOGY PROJECT B - ANALYSIS RS-NMS-85-009

MAY, 1987



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#### **1.0 INTRODUCTION**

#### 1.1 GENERAL STATEMENT OF THE PROBLEM

Application of the groundwater travel time (GWTT) criterion is dependent upon several parameters, one of which is effective porosity. At this time, only one effective porosity value has been measured at the BWIP site, and it is the result of a tracer test of limited scale. In order to obtain defensible effective porosity values, particularly along suspected flow paths, additional tracer tests will be required. Ordinarily, tracer tests are considered to De Dest suited for testing relatively short distances. However, effective porosities measured along suspected flow paths at scales comparable to the GWTT distance requirement would greatly reduce the uncertainty in any travel time calculation. Therefore, this technical report attempts to determine to what extent a tracer test can be run at "full scale." The term "full scale" is used to denote a test which measures effective porosity at or near the scale of the 5 kilometer accessible environment boundary which must be incorporated into GWTT calculations. The feasibility of such a test is the subject of this technical report.

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#### 1.2 RELEVANCE TO THE NRC

Calculation of groundwater travel time is a fundamental part of the regulations governing licensing of the High Level Radioactive Waste disposal sites, since pre-emplacement GWTT is a performance objective and average linear velocity is needed for post-emplacement advection/dispersion. Therefore, pre-analysis of possible testing techniques which can provide defensible parameter values for this calculation is necessary to understand how a test might be performed and how to evaluate test designs proposed by DOE.

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#### **1.3 RELATIONSHIP TO OTHER SITE CHARACTERIZATION/REGULATORY TASKS**

NRC will be required to review the BWIP SCP, probably during 1987. It is anticipated that DOE will propose a testing strategy for the determination of effective porosity, which is likely to include tracer testing. In order to prepare for the review, the NRC must pre-determine what data are essential and how they can best be determined, given the hydrogeologic framework at BWIP. This technical report evaluates the feasibility of large scale tracer tests.

#### 2.0 OBJECTIVE

The objective of this technical report is to determine the feasibility of performing tracer tests at a distance approaching the 5 kilometers accessible environment boundary required by the GWTT criterion, based on evaluations using presently available information.

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#### 3.0 EVALUATION

#### 3.1 OPERATIONAL APPROACH

The operational approach of this technical report is made up of four interdependent analyses, as listed below.

- Determine likely horizontal flow-path(s), based on effects of post-emplacement thermally induced vertical gradients. This analysis concentrates on feasibility of a tracer test conducted in the Rocky Coulee Interflow.
- 2. Define a feasible tracer test configuration, using currently available hydraulic parameters related to a flow-path determined in step 1.
- 3. Calculate likely tracer travel time.
- 4. Assess significance of vertical leakage to results of number 3.

The four components of the operational approach were performed in the order listed. Once a possible flow path was identified in step one, hydraulic parameters for that unit were obtained from the TTI database system and used in a well-field simulator to assess various test configurations, including pumping rate, number of wells, distance, test length, and resulting hydraulic gradient. The next step in the analysis was to calculate tracer travel time, assuming a completely conservative tracer, a straightline flow-path, and a range of effective porosities. As a check, a volumetric flow-rate was also calculated. The final step in the analysis was to insure that the gradients calculated in step two were realistic when vertical leakage was considered.

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In order to more clearly present the technical approach, analysis, and conclusion of each component of this technical report, the standard TTI technical report format will be slightly altered. The technical approach, analysis, and conclusion of each component will be presented under the major heading of each component rather than split into three separate headings.

#### 4.0 IDENTIFICATION OF HORIZONTAL FLOW PATH

#### 4.1 TECHNICAL APPROACH

Due to the apparent confining capabilities of basalt flow interiors, it is presumed that radionuclide transport to the accessible environment will take place primarily by horizontal flow within basalt interflows. However, vertical migration of radionuclides will have to occur in vicinity of the repository until one or more interflows having relatively high transmissivity are encountered. In Technical Report #13, it is concluded that the effects of repository heat are significant in determining vertical ground water flux and in fact represent the dominant driving force for vertical flow for at least the first several thousand years after waste emplacement. Since repository heat results in an upward component of ground water flow, likely paths for radionuclide migration initially involve upward vertical movement above the repository until relatively high transmissivity interflows are encountered, followed by horizontal movement along those interflows to the accessible environment (lateral distance of 5 km).

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To determine if the Rocky Coulee Interflow is likely to receive radionuclides within relevant post-emplacement time frames, an approximate analytical method has been developed to estimate travel times through basalt flow interiors in the presence of repository heat. Theoretical development of the approximation is provided in Appendix A.

#### 4.2 ANALYSIS

In evaluating vertical radionuclide migration, it is assumed that radionuclides migrate instantaneously (i.e., have infinite velocity) within the candidate horizon (Cohassett Flow Interior) and in all overlying interflows. This is a conservative assumption which, for the purpose of analysis, maximizes the distance above the repository that radionuclides might reach within relevant post-emplacement time frames. Using this assumption, vertical travel times are based solely on flow velocities within flow interiors above the Cohassett.

To assess the likelihood that radionuclides can reach the Rocky Coulee Interflow and other interflows within reasonable time frames, an arbitrary criterion is used in this evaluation. This criterion considers that in order for a radionuclide to reach the accessible environment via an interflow within 1000 years after emplacement, the radionuclide must reach that interflow before 500 years. The above time specifications are not

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specifically related to post-emplacement performance criteria, but are selected in this analysis to provide an indication of the likelihood that radionuclides can reach interflows within time frames of practical interest. Thus, for a test to be considered in the Rocky Coulee Interflow, it should be determined that it is possible for radionuclides to migrate vertically upward to the Rocky Coulee within 500 years after repository closure.

To evaluate vertical migration of radionuclides above the repository after closure, it is assumed that the waste canister can effectively contain radioactive waste for a period of 300 years. Thus, any permeable interflow encountered within 200 years after release from the waste canister (300 to 500 years after emplacement) is considered a reasonable candidate for tracer testing.

#### 4.3 CONCLUSION

Calculations presented in Appendix A indicate that for thermal conditions expected to exist between 300 and 500 years after repository closure, radionuclides may possibly encounter the Rocky Coulee, but have less probability of encountering permeable interflows above the Rocky Coulee. Therefore, it is recommended that the Rocky Coulee Interflow is the best hydrostratigraphic unit within which to conduct an initial tracer test to measure horizontal radionuclide transport properties. This conclusion is

based on current hydraulic data for flow interiors which is subject to considerable uncertainty. It is possible that as more hydrologic information is gained on the properties of flow interiors, tracer tests in other basalt interflows may be deemed appropriate.

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#### 5.0 TRACER TEST CONFIGURATION

#### 5.1 TECHNICAL APPROACH

Using a well-field simulator which uses Lotus 123 as its basic framework, an array of 6 wells (three discharge, three injection) were used to simulate a "push-pull" tracer test. The number of wells was selected on the basis of what might be the maximum number practical, given costs and drilling time, yet necessary to produce a sufficient gradient at practical discharge rates. However, the well configuration and number is considered to be for calculation purposes and would thus require refinement as part of the actual design task by DOE. Assumptions used in the analysis are listed below:

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#### Assumptions:

Tested interval	is a fully confined (nonleaky) aquifer.
Transmissivity	= 2.6 ft <sup>2</sup> /day (geometric mean of Rocky Coulee flow top values as determined by DOE (1985))
Storativity	= $1 \times 10^{-5}$ (value commonly used by DOE)
Injection/ Discharge	<ul> <li>75 gpm (25 gpm per well) (various rates were attempted;</li> <li>75 gpm producing sufficient, yet practical drawdowns)</li> </ul>
Time	= variable, as indicated
Distances	= 1000 foot spacing between pumping wells, 1000 foot spacing between injection wells; well fields spaced between 2 and 5 km, as indicated.

Using these assumptions, several runs were made with the well-field simulator, resulting in drawdown data which were both plotted on a horizontal field and as vertical profiles, as indicated in Figures 1-5.

#### 5.2 ANALYSIS

An underlying assumption in the well-field simulator analysis is that a linear gradient must develop along the line between the pumping and injection well-fields. The assumption of a linear gradient is used to maximize groundwater flow between injection and pumping centers and therefore minimize tracer travel time.

At a well-field spacing of 5 km, a linear gradient of .23 develops between 100 and 200 days of pumping/injection (Figures 1 and 2). At 300 days of pumping/injection (Figure 3), the gradient increases only slightly to .25.

With a well-field spacing of 2 km, a linear gradient develops very soon after 50 days of pumping/injection (Figure 4). At 100 days (Figure 5) of pumping/injection, the gradient is .43.

## 5.3 CONCLUSIONS

Based on the assumptions listed in section 5.1, a tracer-test configuration of 3 pumping and 3 injection wells could produce a linear gradient in the Rocky Coulee flow top over distances of 2 to 5 kilometers. Whether or not this gradient is sufficient to transport a tracer between the well-fields in a reasonable time-period, particularly if leakage is considered, will be discussed in later sections.

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#### 6.0 TRACER TRAVEL TIME

#### 6.1 TECHNICAL APPROACH

The hydraulic response to pumping/injection, as determined in Section 5.0, suggests that a tracer test performed at distances of 2 to 5 km might be feasible. As a confirmation of the apparent feasibility of such a tracer test, tracer travel time is calculated in this section. Certain simplifing assumptions are used in this calculation which result in a minimum travel time. Other factors which should eventually be considered are the nature and detectability of the tracer and pumping/injection rate. Potential impacts of vertical leakage are considered in Section 7.0.

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Calculation of a minimum tracer travel time assumes a completely conservative tracer travels a direct flowpath between the pumping and injection well-fields. A form of Darcy's law is used to calculate velocity and travel time, as shown in Appendix B.

As a check on the direct tracer travel time, a more simplistic approach was used. If it is assumed that an injected tracer would fill a cylinder, the axis of which is the injection well(s) and the height is the thickness of the flow top, an approximate filling time can be calculated. Appendix B provides the relationship used in this calculation.

#### 6.2 ANALYSIS

Using equations described in Appendix B and the values for hydraulic conductivity (K) and gradient (i) listed in Appendix B, tracer travel times for 2 and 5 km spacing were calculated and are presented in Table 3.

TABLE 3: CALCULATED TRACER TRAVEL TIME (days)					
SPACING	EFFECTIVE POROSITY				
(Kilometers)	10 <sup>-3</sup>	10 <sup>-4</sup>	10 <sup>-5</sup>		
2	69	6.9	.69		
5	431	43.1	4.31		

As a check on the calculated travel times, the time to fill a given volume (a cylinder centered on the injection wells) with tracer was calculated. Assuming that the injected tracer would fill a cylinder, which conservatively disregards the effects from pumping, the fill time would approximate travel time. This approach resulted in values of 97 and 15 days for 5 and 2 km, respectively, assuming a flow rate of 75 gpm and an effective porosity of  $10^{-4}$ . These values are approximately twice those calculated for the direct travel times (Table 3), as would be expected when pumping (as part of the push-pull system) is disregarded.

### 6.3 CONCLUSIONS

Tracer travel times calculated in this section represent minimum travel time between the pumping and injection well-fields. However, the relatively short travel times on the order of a few days, particularly at a spacing of 2 km, suggests that a test at such a scale may be feasible. This conclusion will be discussed in more detail in section 8.0.

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#### 7.0 SIGNIFICANCE OF VERTICAL LEAKAGE

#### 7.1 TECHNICAL APPROACH

Well field simulations performed in this study assume that the test interval (Rocky Coulee Interflow) is totally confined, which implies that negligible vertical leakage occurs from adjacent aquitards (basalt flow interiors). Although flow interiors have relatively low hydraulic conductivity at the BWIP site, significant vertical groundwater flow into the test interval may be possible due to the large planimetric area over which leakage can operate. This section provides an evaluation of the effect of vertical leakage on hydraulic gradients existing between the two centers of pumping for a proposed large-scale tracer test. Analyses are performed in a sensitivity manner to determine at what point flow interior hydraulic conductivity becomes sufficiently large to affect hydraulic gradients between the centers of injection/withdrawal, and hence have an effect on tracer travel time.

To evaluate the vertical leakage associated with pumping or injection wells, use is made of the steady-state analysis described in Appendix C. This model considers a multiple aquifer/aquitard system in which the middle aquifer is pumped. Aquitards above and below the pumped aquifer have finite permeability and can transmit groundwater by vertical leakage. For the solution used in this study, the unpumped aquifers are assumed to be maintained at constant head. The steady-state nature of this formulation implies that sufficiently large times have passed so that transient effects of pumping have dissipated.

The proposed tracer test scheme calls for a total of six wells; three withdrawal and three injection wells. For the purpose of this analysis, it is assumed that the three-well injection cluster can be simulated as a single injection well and three-well withdrawal cluster is represented as one withdrawal well. To determine hydraulic response in the pumped aquifer, the combined effects of both pumping centers must be considered. This is accomplished through the principal of superposition, a common analytical technique used in well hydraulics.

#### 7.2 ANALYSIS

Hydraulic buildup is determined along a line connecting the two pumping centers. In this evaluation, two spacings for the injection/withdrawal centers are considered; 5 km and 2 km. Relevant input parameters used in the simulations are summarized in Appendix C.

A general relationship is observed between vertical hydraulic conductivity and resulting hydraulic gradients between the injection and withdrawal wells. For relatively small values of aquitard conductivity, hydraulic gradients

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between the pumping centers are relatively unaffected. In this case, the apparently small magnitude of vertical leakage results in a flow system which for all practical purposes can be considered totally confined. For such low values of aquitard conductivity, leakage need not be considered in evaluating the feasibility of a large scale tracer test. As aquitard conductivity increases, the hydraulic gradient near the central portion of the test area becomes less. Smaller gradients would result in longer travel times required for a tracer to travel between the injection/withdrawal pumping centers. For intermediate values of aquitard conductivity, the proposed tracer test may still be feasible, but the effects of leakage will have to be considered in design of the test. Finally, for some cases of relatively high aquitard conductivity, hydraulic gradients attain very small or near-zero values in the area midway between the pumping centers. In this case the tracer would probably not be able to travel between the injection/withdrawal centers within any reasonable time frame for conducting a large scale tracer test.

#### 7.3 CONCLUSIONS

For a five kilometer spacing between injection and withdrawal pumping centers, analyses in Appendix C indicate the following:

o Vertical leakage does not need to be considered for aquitard hydraulic conductivities less than  $10^{-12}$  m/s.

- o For aquitard conductivities of 10-12 to 10-10 m/s, the tracer test may be feasible, but effects of leakage will have to be considered in design and pre-analysis of the test.
- o The proposed test is probably not feasible for aquitard conductivities greater than 10-10 m/s due to excessive tracer travel times.

For a two kilometer spacing between injection and withdrawal pumping centers, the following conclusions are made:

- o Vertical leakage does not need to be considered for aquitard hydraulic conductivities less than 10-11 m/s.
- o For aquitard conductivities of  $10^{-11}$  to  $10^{-9}$  m/s, the tracer test may be feasible, but effects of leakage will have to be considered in design and pre-analysis of the test.
- o The proposed test is probably not feasible for aquitard conductivities greater than  $10^{-9}$  m/s due to excessive tracer travel times.

The proposed LHS testing program will provide an indication of the bulk vertical hydraulic conductivity of selected flow interiors. Once

characteristic conductivity values are obtained, the feasibility (with respect to leakage) of a large scale tracer test in the Rocky Coulee can be assessed.

#### 8-0 CONCLUSIONS

Based on the assumptions provided in this document, a "push-pull" tracer test at a scale of 2 to 5 km may be feasible. The particularly short travel times calculated for the 2 km test for a reasonable range of effective porosities indicates that if various logistical considerations prove to be feasible, a 2 km test could be successfully performed at the BWIP site. The calculated travel times for the same range of effective porosities for the 5 km test suggest that this test could be performed, however, the total time required for the test might be an important consideration in developing test designs and schedules. The specific conclusions of this document are listed:

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- For thermal conditions which are expected to exist for 300 to 500 years after repository closure, vertical gradients could introduce radionuclides to the Rocky Coulee flow top. The probability of radionuclides reaching interflows above the Rocky Coulee is considerably less. Therefore, the Rocky Coulee flow top is considered to be a likely horizontal flow path for radionuclide transport.
- 2. A "push-pull" tracer test configuration of 3 pumping and 3 injection wells could produce sufficient gradients in the Rocky Coulee over distances of 2 to 5 km. This well configuration, however, is considered to be for calculation purposes only and would require refinement during the actual design of such a tracer test.

- 3. Tracer travel times calculated for the 2 and 5 km tests indicate tracer tests performed at this scale are feasible, excluding the effects of vertical leakage.
- 4. The extent to which vertical leakage impacts tracer travel time is dependent upon the aquitard hydraulic conductivity. The results of calculations preformed in this document suggest that for a 5 km test, vertical leakage need be considered for aquitard conductivities greater than  $10^{-12}$  m/s. For a 2 km test, leakage is significant for aquitard conductivities greater than  $10^{-11}$  m/s.

#### 9.0 DISCUSSION

Although the results of this analysis are encouraging for the performance of "large scale" tracer tests, several questions regarding their feasibility must still be answered. These open questions are listed below:

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- The feasibility of producing the gradients indicated would have to be addressed. This may be particularly important in the injection wells to insure hydrofracturing would not occur at the required pressures. However, at the short travel times shown for the 2 km test, lower drawdown and injection pressure could still produce a feasible test.
- The detectability of the tracer is a critical issue which would have to be addressed. The detectability is dependent upon dilution and chemical reactions along the travel route.
- 3. The methods used to inject and "collect" the tracer are important issues for an accurate determination of effective porosity. The only existing tracer test on the BWIP site produced results which have been somewhat in question due to the quick travel time in the formation compared to travel time within the boreholes.
- 4. There are additional practical considerations that would have to be addressed in such a test. The actual design of the well fields would

have to consider costs and schedule, since few, if any, existing wells would serve as either pumping or injection sites.

5. Recent information (summarized in TTI, 1987) concerning possible areas of high vertical leakage could become the critical issue regarding the feasibility of a large scale tracer test. Questions regarding the nature and magnitude of vertical leakage at the BWIP site must be resolved prior to any serious effort to design a tracer test or review the feasibility of such a test.

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FIGURE 1. ROCKY COULEE TRACER TEST EVALUATION - 100 DAYS AT 5 KM

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FIGURE 4. ROCKY COULEE TRACER TEST EVALUATION - 50 DAYS AT 2 KM



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# APPENDIX A

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#### **APPENDIX A:**

#### IDENTIFICATION OF HORIZONTAL FLOW PATH

#### A. IDENTIFICATION OF HORIZONTAL FLOW PATH

#### A.1 TECHNICAL APPROACH

Due to the apparent confining capabilities of basalt flow interiors, it is presumed that radionuclide transport to the accessible environment will take place primarily by horizontal flow within basalt interflows. However, vertical migration of radionuclides will have to occur in vicinity of the repository until one or more interflows having relatively high transmissivity are encountered. In Technical Report #13, it is concluded that the effects of repository heat are significant in determining vertical ground water flux and in fact represent the dominant driving force for vertical flow for at least the first several thousand years after waste emplacement. Since repository heat results in an upward component of ground water flow, likely paths for radionuclide migration initially involve upward vertical movement above the repository until relatively high transmissivity interflows are encountered, followed by horizontal movement along those interflows to the accessible environment (lateral distance of 5 km).

# A.1.1 Vertical Flow Velocity

To determine if the Rocky Coulee Interflow is likely to receive radionuclides within relevant post-emplacement time frames, an approximate analytical method has been developed to estimate travel times through basalt flow interiors in the presence of repository heat. This analysis makes use of Darcy's law for a variable density fluid (Runchal et al, 1985) to determine vertical ground water velocity:

$$vz = -K (dH + R)$$
(A-1)
  
ne dz

where:

$$H = \underline{p} + E$$
 (A-2)

$$R = \underline{D} - 1$$
 (A-3)  
Do

vz = average vertical fluid velocity [L t-1]
K = hydraulic conductivity [L t-1]
ne = effective porosity [ ]
p = fluid pressure [M L-1 t-2]

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E = elevation above an arbitrary datum [L]

- g = acceleration of gravity [L t<sup>-2</sup>]
- D = fluid density [M L-3]
- Do = arbitrary reference density [M L<sup>-3</sup>]
- R = Duoyancy factor [ ]

The parameter H is sometimes referred to as fresh water head. However, as discussed in Technical Report #10, H is related to fluid pressure and is not necessarily a true hydraulic head. In Equation A-1, hydraulic conductivity is assumed to be a constant even though this parameter depends on fluid density and viscosity, both of which are temperature dependent. Treating K as a constant is considered justified for this analysis because variations associated with anticipated temperature conditions (factors on the order of two) are small compared to the overall uncertainty in appropriate values of this parameter for basalt flow interiors (orders of magnitude).

To estimate vertical fluid velocity through a basalt flow interior, reference is made to Figure A1. It is assumed that a flow interior is situated between two interflows, each having relatively high permeabilities. Vertical flow velocity is estimated by applying a finite difference approximation to Equation A-1:

$$vz = -K (H2 - H1 + R)$$
 (A-4)  
ne D

where:

H1 = fresh water head at bottom of flow interior [L]
H2 = fresh water head at top of flow interior [L]
D = thickness of flow interior [L]

and the buoyancy parameter R is treated as a constant (averaged) value within the flow interior.

For pre-emplacement conditions, it is assumed that the ground water flow system is static (vz = 0) and that the local flow system is characterized by a constant density fluid set equal to the reference density (Do). Based on Equations A-1 and A-3, these assumptions would imply that:

$$H1 = H2$$
 (A-5)

for pre-emplacement conditions. Furthermore, if transmissivities of the interflows are considered sufficiently large that pressure changes resulting from temperature induced flow can be neglected, then Equation A-5 applies to post-emplacement conditions as well. Substituting Equation A-5 into A-4 results in:

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$$vz = -\frac{KR}{ne}$$
 (A-6)

To evaluate the parameter R (Equation A-3), Do is set equal to the average fluid density within the flow interior for pre-emplacement conditions and D is equal to average fluid density at a prescribed time after repository closure. The parameter R represents the buoyancy force driving vertical flow. For selected test problems, velocities determined from the above analytical approximation compared favorably with results of the onedimensional numerical model described in Technical Report #7.

#### A.1.2 Fluid Density

Evaluation of Equation A-3 to determine R requires that the average fluid density within the flow interior be determined for pre- and post-emplacement temperature conditions. Fluid density is computed using the following empirical equation presented in Technical Report #10:

$$D = A(T) + B^* + S$$
 (A-7)

where:

- $A(T) = (+999.83952 + 16.945176 T 7.9870401 X 10^{-3} T^{2} -46.170461 X 10^{-6} T^{3} + 105.56302 X 10^{-9} T^{4} -280.54253 X 10^{-12} T^{5}) / (1 + 16.879850 X 10^{-3} T) / 1000$ (A-8)
- D = density of water (g cm-3) T = temperature (C) B\* = pressure correction (0.00233 g cm-3) S = salinity (1000 mg/1 = .001 g cm-3)

Note that the above empirical equations require consistent use of [ gram - centimeter - degrees Celsius ] units.

#### A.1.3 Temperature

The following equation is used to determine pre-emplacement temperatures, based on a linear geothermal gradient:

$$To = .0333 d + 15$$
 (A-9)

where:

To = pre-emplacement temperature (C)

d = depth below ground surface (m)

The above equation specifies a temperature of 15 degrees C at ground surface (depth equal zero) and a temperature of 47 degrees at the midpoint of the repository horizon (depth of 960 meters). Note that the above equation requires consistent use of [ meters - degrees Celsius ] units.

Using the one-dimensional heat conduction analysis presented in Technical Report #3, the change in temperature resulting from repository heat is represented by:

$$Tr = f(x,t) \tag{A-10}$$

where:

Tr = change in temperature resulting from repository heat [T]
x = vertical distance above or below repository [L]
t = time after repository closure [t]

and f is an analytical function incorporating thermal properties and time varying rates of heat generation within the repository. Equation A-10 is evaluated using the HP-41 computer program and input parameters presented in Technical Report #3. A listing of the algorithm is given in Technical Report #3. Finally, the rock temperature existing after repository closure is given by:

$$Tn = To + Tr$$
(A-11)

where:

Tn = post-emplacement temperature [T]

#### A.2 ANALYSIS

In evaluating vertical radionuclide migration, it is assumed that radionuclides migrate instantaneously (i.e., have infinite velocity) within the candidate horizon (Cohassett Flow Interior) and in all overlying interflows. This is a conservative assumption which, for the purpose of analysis, maximizes the distance above the repository that radionuclides might reach within relevant post-emplacement time frames. Using this assumption, vertical travel times are based solely on flow velocities within the interiors above the Cohassett.

#### A.2.1 Flow Velocity

Figure A2 shows a stratigraphic section indicating the locations and depths of the first three flow interiors above the candidate horizon. These include, in descending order, the Frenchman Springs, Grande Ronde 1-2, and Rocky Coulee Flow Interiors. The interflow between Grande Ronde 1 and 2 is not well developed so that the dense portions of these flows are considered to represent a single continuous flow interior.

To assess the likelihood that radionuclides can reach the Rocky Coulee Interflow and other interflows within reasonable time frames, an arbitrary criterion is used in this evaluation. This criterion considers that in order for a radionuclide to reach the accessible environment via an interflow within 1000 years after emplacement, the radionuclide must reach that interflow before 500 years. The above time specifications are not specifically related to post-emplacement performance criteria, but are selected in this analysis to provide an indication of the likelihood that radionuclides can reach interflows within time frames of practical interest. Thus, for a test to be considered in the Rocky Coulee Interflow, it should be determined that it is possible for radionuclides to migrate vertically upward to the Rocky Coulee within 500 years after repository closure.

To evaluate vertical migration of radionuclides above the repository after closure, it is assumed that the waste canister can effectively contain radioactive waste for a period of 300 years. Thus, any permeable interflow encountered within 200 years after release from the waste canister (300 to 500 years after emplacement) is considered a reasonable candidate to tracer testing.

Calculations used to determine pre- and post-emplacement temperatures at the midpoint of each interflow are summarized in Table A1. Post-emplacement temperatures are calculated at 300 and 500 years after repository closure. Calculations used to determine pre- and post-emplacement fluid densities and associated buoyancy parameters (R) are summarized in Table A2. Densities are computed using temperatures in Table A1 and thus represent fluid properties at the midpoints of the flow interiors. Such density values are considered reasonable estimates of the average fluid properties for the flow interiors. As shown in Table A2, the buoyancy factor (R) for each flow interior does not change appreciably between 300 and 500 years. The maximum change in buoyancy factor for a flow interior is less than a factor of 2. As indicated in Equation A-6, flow velocity is directly proportional to vertical hydraulic conductivity. Since the uncertainty in conductivity for flow interiors ranges over orders of magnitude, the factor of two variation in the buoyancy factor is not considered significant and average values are used in subsequent computations. Average buoyancy factors between 300 and 500 years are -0.0071, -0.0113, and -0.0213 for the Frenchman Springs No.7, Grande Ronde 1-2, and Rocky Coulee Flow Interiors, respectively.

Equation A-6 is used to calculate vertical flow velocities. These calculations utilize the average buoyancy factors given above with values of hydraulic conductivity and effective porosity equal to  $10^{-11}$  m/s and  $10^{-4}$ , respectively. The conductivity value is of the same order as the geometric mean of test results for single borehole tests conducted within Grande Ronde

flow interiors. The assumed effective porosity is the medium of values typically associated with basalt flow interiors. The resulting flow velocities are summarized below:

Frenchman Springs No. 7:vz = 0.0224 m/yGrande Ronde 1-2:vz = 0.0356 m/yRocky Coulee:vz = 0.0672 m/y

#### A.2.2 Identification of Potential Permeable Interflows

The first relatively permeable interflow above the repository horizon is the Rocky Coulee. To reach this hydrostratigraphic unit, radionuclides must migrate vertically through 46 meters of the Rocky Coulee flow interior. Using the vertical flow velocity of 0.0672 m/y given above for the Rocky Coulee, the vertical distance traveled in 200 years (300 to 500 years after emplacement) is calculated to be 13.4 meters, which is only a small proportion of the total thickness of the flow interior. This travel distance could have substantial errors due to uncertainties in hydraulic parameters for dense basalt (particularly vertical hydraulic conductivity and effective porosity). However, the indication is that radionuclides might possibly reach the Rocky Coulee Interflow, but would have less probability of encountering permeable interflows which are higher in the stratigraphic section (using the assumed 300 to 500 year time criterion).

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#### A.3 CONCLUSION

Calculations presented herein indicate that for thermal conditions expected to exist between 300 and 500 years after repository closure, radionuclides may possibly encounter the Rocky Coulee, but have less probability of encountering permeable interflows above the Rocky Coulee. Therefore, it is recommended that the Rocky Coulee Interflow is the best hydrostratigraphic unit within which to conduct an initial tracer test to measure horizontal radionuclide transport properties. This conclusion is based on current hydraulic data for flow interiors which is subject to considerable uncertainty. It is possible that as more hydrologic information is gained on the properties of flow interiors, tracer tests in other basalt interflows may be deemed appropriate.

#### A.4 REFERENCES

Runchal, A.K., B. Sagar, R.G. Baca and N.W. Kline. 1985. PORFLO - A Continuum Model for Fluid Flow, Heat Transfer, and Mass Transport in Porous Media. Rockwell Hanford Operations, RHO-BW-CR-150P.

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### TABLE A1. PRE- AND POST-EMPLACEMENT TEMPERATURE CONDITIONS

#### a. Pre-emplacement

FLOW INTERIOR	d (m)	To (Celsius)
FS 7	809	41.9
GR 1-2	840	43.0
RC	889	44.6

#### D. Post-emplacement

			t = 300 yrs		t = 500 yrs	
FLOW INTERIOR	To (C)	x (m)	Tr (C)	T (C)	Tr (C)	T (C)
FS 7	41.9	145	12.6	54.5	18.8	60.7
GR 1-2	43.0	114	20.7	63.7	25.6	68.6
RC	44.6	64	37.8	82.4	38.2	82.8

#### Definitions

- d = depth below ground surface
- To = pre-emplacement temperature (geothermal gradient)
- x = distance above repository horizon
- t = time since repository closure
- Tr = temperature increase resulting from repository heat
- T = post-emplacement temperature

#### Notes

- FS 7 Frenchman Springs Flow No. 7
- GR 1-2 Grande Ronde Flows 1 and 2
- RC Rocky Coulee Flow

Tr computed using HP-41 computer program and input parameters described in Technical Report #3.

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#### TABLE A2. PRE- AND POST-EMPLACEMENT FLUID DENSITIES AND BUOYANCY PARAMETERS

#### a. Pre-Emplacement

FLOW INT.	To (C)	Do (g cm-3)
FS 7	41.9	.9948
GR 1-2	43.0	.9944
RC	44.6	•9937

#### D. Post-Emplacement

		t = 300 yrs			t = 500 yrs			
FLOW INT.	Do (g cm-3)	T (C)	D (g cm-3)	R ( )	T (C)	D (g cm-3)	R ()	Av. R ( )
FS 7	.9948	54.5	.9893	0055	60.7	.9862	0086	0071
GR 1-2	.9944	63.7	•9846	0099	68.6	.9819	0126	0113
RC	.9937	82.4	.9736	0212	82.8	.9734	0214	0213

#### Definitions

10 = pre-emplacement temperature at midpoint of flow interior

- Do = reference fluid density (pre-emplacement density)
- T = post-emplacement temperature at mid-point of flow interior
- D = post-emplacement fluid density
- R = buoyancy factor

#### Notes

- FS 7 Frenchman Springs Flow No. 7
- GR 1-2 Grande Ronde Flows 1 and 2
- RC Rocky Coulee Flow





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APPENDIX B

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#### **APPENDIX B:**

#### TRACER TRAVEL TIME

#### B. TRACER TRAVEL TIME

#### **B-1** TECHNICAL APPROACH

The hydraulic response to pumping/injection, as determined in Section 5.0, suggests that a tracer test performed at distances of 2 to 5 km might be feasible. As a confirmation of the apparent feasibility of such a tracer test, tracer travel time is calculated in this section. Certain simplifing assumptions are used in this calculation which result in a minimum travel time. Other factors which should eventually be considered are the nature and detectability of the tracer and pumping/injection rate. Potential impacts of vertical leakage are considered in Section 7.0.

Calculation of a minimum tracer travel time assumes a completely conservative tracer travels a direct flowpath between the pumping and injection well-fields. A form of Darcy's law is used to calculate velocity and travel time:

$$v = \frac{Ki}{ne} = \frac{x}{t}$$
(B-1)

and therefore:

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(B-2)

$$t = x ne$$
  
K 1

where:

v = average horizontal fluid velocity [L/t] K = hydraulic conductivity [L/t] ne = effective porosity [] i = hydraulic gradient [] t = time [t] x = distance [L]

Effective porosity values used in the calculation include a minimum, maximum, and "best guess"  $(10^{-5}, 10^{-3}, \text{ and } 10^{-4} \text{ respectively})$ , based on the little available data from the BWIP site.

As a check on the direct tracer travel time, a more simplistic approach was used. If it is assumed that an injected tracer would fill a cylinder, the axis of which is the injection well and the height is the thickness of the flow top, an approximate filling time can be calculated. Using the following relationship:

$$V = pi r^2 p ne = Qt$$
 (B-3)

where:

```
V = volume of the cylinder [L<sup>3</sup>]
r = radius of the cylinder [L]
D = height of the cylinder (thickness of the Rocky Coulee flow top) [L]
ne = effective porosity [ ]
Q = flow rate [L<sup>3</sup> t<sup>-1</sup>]
t = time [t]
pi = 3.14
```

The equation can be rewritten:

$$r^2 = \underbrace{Q t}{p_1 b ne}$$
(8-4)

Equation (B-4) can then be solved for either the radius of the cylinder at some time (t) or for the time required to fill a cylinder of radius (r).

#### B.2 ANALYSIS

Using equation B-2 and the values for hydraulic conductivity (K) and gradient (i) listed below, tracer travel times for 2 and 5 km spacing were calculated and are presented in Table B-1.

$$K = \frac{T}{D} = \frac{2.6 \text{ f}^2 \text{ d}^{-1}}{16.7 \text{ f}} = 5.5 \text{ x } 10^{-5} \text{ cm s}^{-1}$$

i = 2000 feet of drawdown and 2000 feet of buildup in the respective well fields over a distance of 2 or 5 km (i = .610 and .244, respectively). The drawdown and buildup values are of the same magnitude as the results of the well field simulator.

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SPACING	EFF	· · ·	
(Kilometers)	10 <sup>-3</sup>	10 <sup>-4</sup>	10 <sup>-5</sup>
2	69	6.9	.69
5	431	43.1	4.31

#### TABLE B-1: CALCULATED TRACER TRAVEL TIME (days)

As a check on the calculated travel times, the time to fill a given volume (a cylinder centered on the injection wells) with tracer was calculated. Assuming that the injected tracer would fill a cylinder, which conservatively disregards the effects from pumping, the fill time would approximate travel time. This approach resulted in values of 97 and 15 days for 5 and 2 km, respectively, assuming a flow rate of 75 gpm and an effective porosity of  $10^{-4}$ . These values are approximately twice those calculated for the direct travel times (Table B-1), as would be expected when pumping (as part of the push-pull system) is disregarded.

# APPENDIX C

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#### **APPENDIX C:**

#### SIGNIFICANCE OF VERTICAL LEAKAGE

#### C. SIGNIFICANCE OF VERTICAL LEAKAGE

#### C.1 TECHNICAL APPROACH

Well field simulations performed in this study assume that the test interval (Rocky Coulee Interflow) is totally confined, which implies that negligible vertical leakage occurs from adjacent aquitards (basalt flow interiors). Although flow interiors apparently have very low hydraulic conductivity at the BWIP site, significant vertical groundwater flow into the test interval may be possible due to the large planimetric area over which leakage can operate. This section provides an evaluation of the effect of vertical leakage on hydraulic gradients existing between the two centers of pumping for a proposed large-scale tracer test. Analyses are performed in a sensitivity manner to determine at what point flow interior hydraulic conductivity Decomes sufficiently large to affect hydraulic gradients between the centers of injection/withdrawal, and hence have an effect on tracer travel time.

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#### C.1.1 Well Hydraulics Solution for a Leaky Aquifer

To evaluate the vertical leakage associated with pumping or injection wells, use is made of the steady-state analytical model shown in Figure C1. This model considers a multiple aquifer/aquitard system in which the middle aquifer is pumped. Aquitards above and below the pumped aquifer have finite permeability and can transmit groundwater by vertical leakage. For the solution used in this study, the unpumped aquifers are assumed to be maintained at constant head. The steady-state nature of this formulation implies that sufficiently large times have passed so that transient effects of pumping have dissipated.

For a well with constant rate injection or withdrawal, the steady-state hydraulic buildup or drawdown in the pumped aquifer is given by the following equation (Hantush and Jacob, 1955):

$$s = \underline{Q} \quad Ko(C) \tag{C-1}$$

where:

$$C = r SQR \left[ \frac{1}{1} \left( \frac{K1}{K1} + \frac{K2}{K2} \right) \right]$$
(C-2)  
T D1 D2

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s = hydraulic buildup (injection) or drawdown (withdrawal) in pumped aquifer [L]

$$Q$$
 = injection or withdrawal flow rate in well [L<sup>3</sup> t<sup>-1</sup>]

- C = dimensionless leakage parameter [ ]
- pi = 3.14159
- r = radial distance from pumped well to point of observation [L]
- K1 = vertical hydraulic conductivity of lower aquitard [L t-1]
- K2 = vertical hydraulic conductivity of upper aquitard [L t<sup>-1</sup>]
- p1 = thickness of lower aquitard [L]
- b2 = thickness of upper aquitard [L]

In the above equations, Ko is the modified Bessel function and SQR indicates a square root. For this study, the hydraulic conductivity of the two aquitards (flow interiors) is assumed equal and Equation C-2 simplifies to:

$$C = r SQR \left[ \frac{K (1 + 1)}{T D1 D2} \right]$$
(C-3)

where:

K = aquitard hydraulic conductivity [L t-1]

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For evaluation of leakage effects, use is made of Equations C-1 and C-3 to predict the hydraulic buildup or drawdown in the pumped aquifer resulting from the operation of a single well or pumping center.

#### C.1.2 Superposition for Simulating Multiple Wells

The proposed tracer test scheme calls for a total of six wells; three withdrawal and three injection wells. For the purpose of this analysis, it is assumed that the three-well injection cluster can be simulated as a single injection well and three-well withdrawal cluster is represented as one withdrawal well. To determine hydraulic response in the pumped aquifer, the combined effects of both pumping centers must be considered. This is accomplished through the principal of superposition, a common analytical technique used in well hydraulics:

$$st = si - sw$$
 (C-4)

where:

st = total hydraulic buildup in the pumped aquifer [L]
si = hydraulic buildup associated with injection center [L]
sw = hydraulic drawdown associated with withdrawal center [L]

If the magnitude of the injection rate at one pumping center is equal to the withdrawal rate at the other center, superposition of Equation C-1 results in:

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st =Q [ Ko(Ci) - Ko(Cw)	1	(C-5)
2 pi T		

where:

 $Ci = ri D \tag{C-6}$ 

$$Cw = rw D \tag{C-7}$$

$$D = SQR \left[ \frac{K \left( \frac{1}{L} + \frac{1}{L} \right)}{T \quad D1 \quad D2} \right]$$
(C-8)

Ci = dimensionless leakage parameter associated with injection well [] Cw = dimensionless leakage parameter associated with withdrawal well [] ri = radial distance from injection well to point of observation [L] rw = radial distance from withdrawal well to point of observation [L] D = leakage parameter [L-1]

Equations C-5 through C-8 are used in this evaluation to compute hydraulic buildup within the pumped aquifer during the proposed tracer test.

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#### C.2 ANALYSIS

Hydraulic buildup is determined along a line connecting the two pumping centers. For this case, the relationship between ri and rw is as follows:

$$rw = d - ri$$
 (C-9)

where:

d = spacing of injection/withdrawal centers [L]

Due to symmetry, only half the distance between the two centers need be considered because hydraulic buildup near the injection wells will be the mirror image of drawdown near the withdrawal wells. In this evaluation, two spacings for the injection/withdrawal centers are considered; 5 km and 2 km.

Relevant input parameters used in the simulations are summarized in Table C1. The upper aquitard is considered to extend from the test interval (Rocky Coulee Interflow) to the lower-most Frenchman Springs interflow, which is the first interflow above the Rocky Coulee with high transmissivity. The lower aquitard is assumed to extend from the test interval down to the Birkett Interflow, also known to have high transmissivity within the RRL. The same hydraulic

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conductivity assumed for each aquitard and values of 10-13, 10-12, 10-11, 10-10, and 10-9 m/s are considered in the sensitivity analyses.

Results from the leakage analyses are graphically illustrated in Figure C2. The figure shows hydraulic buildup along the line separating the two pumping centers for spacings of five kilometers and two kilometers. For the purpose of presentation, only half the flow system (injection side) is presented. This is because hydraulic buildup within the injection side of the flow system is the mirror image of drawdown on the withdrawal side. Heads are not shown for distances less than 300 meters from the pumping center, because the assumption of a single injection well (adopted for analytical purposes) does not realistically simulate the proposed multiple well test configuration at small radial distances.

As shown in Figure C2 a general relationship is observed between vertical hydraulic conductivity and resulting hydraulic gradients between the injection and withdrawal wells. For relatively small values of aquitard conductivity hydraulic gradients between the pumping centers are relatively unaffected. In this case, the apparently small magnitude of vertical leakage results in a flow system which for all practical purposes can be considered totally confined. For such low values of aquitard conductivity, leakage need not be considered in evaluating the feasibility of a large scale tracer test. As aquitard conductivity increases, the hydraulic gradient near the central portion of the test area (right hand side of graphs) becomes less. Smaller gradients would

result in longer travel times required for a tracer to travel between the injection/withdrawal pumping centers. For intermediate values of aquitard conductivity, the proposed tracer test may still be feasible, but the effects of leakage will have to be considered in design of the test. Finally, for some cases of relatively high aquitard conductivity, hydraulic gradients attain very small or near-zero values in the area midway between the pumping centers. In this case the tracer would probably not be able to travel between the injection/withdrawal centers within any reasonable time frame for conducting a large scale tracer test.

#### C.3 CONCLUSIONS

For a five kilometer spacing between injection and withdrawal pumping centers, Figure C2 indicates the following:

- o Vertical leakage does not need to be considered for aquitard hydraulic conductivities less than 10-12 m/s.
- o For aquitard conductivities of 10-12 to 10-10 m/s, the tracer test may be feasible, but effects of leakage will have to be considered in design and pre-analysis of the test.

o The proposed test is probably not feasible for aquitard conductivities greater than  $10^{-10}$  m/s due to excessive tracer travel times.

For a two kilometer spacing between injection and withdrawal pumping centers, Figure C2 indicates that:

- Vertical leakage does not need to be considered for aquitard hydraulic conductivities less than 10-11 m/s.
- o For aquitard conductivities of 10-11 to 10-9 m/s, the tracer test may be feasible, but effects of leakage will have to be considered in design and pre-analysis of the test.
- o The proposed test is probably not feasible for aquitard conductivities greater than  $10^{-9}$  m/s due to excessive tracer travel times.

The proposed LHS testing program will provide an indication of the bulk vertical hydraulic conductivity of selected flow interiors. Once characteristic conductivity values are obtained, the feasibility (with respect to leakage) of a large scale tracer test in the Rocky Coulee can be assessed.

### C.4 REFERENCES

Hantush, M.S. and C.E. Jacob. 1955. Non-steady Radial Flow in an Infinite Leaky Aquifer. Trans. Amer. Geoph. Union, vol. 36, no. 1, pp. 95-100. BWIP 2.5 - Technical Report #11 -C-11-

TABLE C1. INPUT PARAMETERS USED IN LEAKAGE ANALYSES

Pumped Aquifer (Rocky Coulee Interflow)

Transmissivity (T):  $2.8 \times 10^{-6} \text{ m}^2/\text{s}$  (2.6 ft<sup>2</sup>/d)

Upper Aquitard (extends from Rocky Coulee Interflow to lower-most Frenchman Springs Interflow) Thickness (b1): 61 m (413 ft) Vertical Hydraulic Conductivity (K): 10-13, 10-12, 10-11, 10-10, 10-9 m/s

Lower Aquitard (extends from Rocky Coulee Interflow to Birkett Interflow)

Thickness (b2): 126 m (200 ft) Vertical Hydraulic Conductivity (K): 10-13, 10-12, 10-11, 10-10, 10-9 m/s

Withdrawal/Injection

Flow Rate (Q): .00473 m<sup>3</sup>/s (75 gpm)

Spacing Between Pumping Centers (d): 5000 m, 2000 m

# FIGURE C1. ANALYTICAL MODEL





