UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

October 22, 1987

MEMORANDUM FOR: Daniel Galson
Operations Branch
Division of High Level Waste Management, NMSS

FROM: Lee Abramson
Probabilistic Risk Analysis Branch
Division of Reactor Accident Analysis, RES

SUBJECT: REVIEW OF NUREG/CR-3964, VOLUME 1

In response to your request, I have reviewed Volume 1 of the draft report NUREG/CR-3964, "Techniques for Determining Probabilities of Events and Processes Affecting the Performance of Geologic Repositories: Literature Review." My concerns are with two related areas - the concept of probability and the use of subjective opinion.

The concept of probability used by most mathematicians and statisticians today is based on the Kolmogorov axiomatic approach (this is not the same as the axiomatic approach discussed in the report). In the Kolmogorov approach, probability is defined as a number P(E) associated with an event E (a possible outcome of an experiment) which satisfies certain axiomatic properties ($0 \leq P \leq 1$ and P(E or F) = P(E) + P(F) for disjoint events E and F). As such, the probability of an event is not the result of an experiment but rather a preassigned number which is supposed to characterize the result of the experiment. It must be emphasized, however, that the Kolmogorov approach says nothing about the value of P(E); it simply asserts the existence of a number P(E). The distinction between the probability of an event and an estimate of it is analogous to the distinction between a physical constant (e.g., the speed of light) and a measurement of it. To determine or estimate P(E), it is necessary to make additional assumptions (e.g., to assume that a coin is fair), to perform an experiment, or to use subjective opinion.

With this definition of probability, the theory of probability bears the same relation to the real world as, say, does the theory of relativity. Both theories are logical constructs which purport to predict the results of experiments with physical objects. As with any other scientific theories, these theories are accepted as valid to the extent that their predictions actually describe the results of real experiments. For example, the theory of relativity asserts that the velocity of light is an absolute constant, c. Many experiments have established the validity of this assertion (within experimental error) and have estimated the value of c by measurement. Similarly, the theory of probability asserts that the probability that a die will come up even is a constant, p. Countless experiments have demonstrated the validity of this assertion. If the die is assumed to be fair, then the theoretical value of p = 0.5. However, only experimentation can demonstrate that real dies behave in accordance with the theory. If the die is not fair, then either experimentation, modeling, or subjective opinion can be used to estimate the value of p.

In the section on "Methods of Determining Probabilities" (pp 43-45), the report discusses various approaches to "determining" probabilities. Although not explicitly stated (except for the axiomatic approach), it is clear that "determining" means "estimating." However, the report does not consider precisely what is being estimated, and this omission leads to several misleading or incorrect statements. For example, in the second paragraph on page 43, it is stated that "Mathematically, the probability of a random event is the limiting value of relative frequency of occurrence reached in an infinitely long sequence of observations of that event." First, this statement is not a definition of probability -- it is rather a theorem in the theory of probability, i.e., it follows from the Kolmogorov axioms. Second, there are special very serious problems in trying to use this statement as a definition of probability, including the impossibility of actually observing an infinite sequence of observations and the possibility that such a sequence might not approach a limit. Third, the implication that probability can only be defined for an indefinitely repeatable event is far too restrictive.

This unnecessary requirement for repeatability leads to a second example of a misleading statement in the report. In the third paragraph on page 44, it is stated that geologic events and processes are not random phenomena because, in addition to being essentially deterministic, they "rarely occur repeatedly under identical conditions" and consequently "rarely, if ever, can constitute an infinitely long sequence from which theoretically correct probabilities could be determined." This assertion is immediately followed by the statement that "For this reason, Bayesian methods may be appropriate in many cases of interest to performance assessment." While it is true that Bayesian methods may be appropriate, it is not because the probabilities (in the Kolmogorov sense) of geologic events cannot be defined. Rather, Bayesian methods have the potential for utilizing information about the events which is not available from the very limited observation data. In other words, Bayesian methods have the potential for improving the estimates of the probabilities in question.

This brings me to my second concern -- the use of subjective opinion. While the report does recognize (page 45) that "subjective probability determinations... are subject to manipulation or bias," it does not adequately reflect the demonstrated difficulties in eliciting and aggregating expert opinion. In addition, it does not seem to realize that classical Bayesian methods rely on expert opinion for the choice of prior and are therefore subject to the drawbacks of using subjective judgments. It is for this reason that I characterized Bayesian methods as having the potential for improving estimates in the previous paragraph. It should be recognized that while "Bayesian probabilistic assessments may provide an evaluation of uncertainty associated with a probability estimate that is superior to that provided by objective probability determinations" (page 45), the measure of uncertainty provided by a Bayesian posterior may be dominated by bias introduced by the prior. The reader should be warned about this possibility and should be strongly urged to use a sensitivity analysis to evaluate the possible effects of errors in the prior when using a Bayesian approach.

I would be pleased to discuss these issues further with you. Please call me at X37987 if you have any questions.

Lee Abramson

Lee Abramson
Senior Research Statistician
Probabilistic Risk Analysis Branch
Division of Reactor Accident Analysis, RES

cc: J. Murphy
H. VanderMolen