Sandia National Laboratories
Albuquerque, New Mexico 87185

WM DOCKET CONTROL
CENTER

'84 APR -2 P :33

WM Record File
A1158

WM Project 10,11,16
Docket No.
PDR ✓
LPDR ✓(C,N,S)

Distribution:
POrnstein

(Return to WM, 623-SS)

March 28, 1984

Mr. Peter Ornstein
Geotechnical Branch
Division of Waste Management
U.S. Nuclear Regulatory Commission
7915 Eastern Avenue
Silver Spring, MD 20910

Dear Mr. Ornstein:

In response to an item, concerning convective heat transfer, in your letter to Dr. Ortiz (March 7, 1984), I have made some modifications to our review of ONWI-495. A copy of the revised review is attached. The nature of these revisions was discussed with you in our telephone conversation last week. Also, you agreed to take a look at Dr. Nataraja's copy of my handwritten notes from the BWIP data review. Presumably, his copy is more legible and would make it unnecessary for us to type them. If, however, you feel that it is still desirable that we submit those notes in a typed format, please let me know.

Sincerely,

Krishan K. Wahi

Krishan K. Wahi
Waste Management Systems
Division 6431

KKW:6431:jm

Copy to:
6430 N. R. Ortiz

WMEG AND WMGT DOCUMENT REVIEW SHEET

FILE NUMBER:

DOCUMENT: "Application of Integral Methods to Prediction of Heat Transfer from a Nuclear Waste Repository," C. J. Blesch, F. A. Kulacki, and R. N. Christensen, ONWI-495, October, 1983.

REVIEWER: Krishan K. Wahi DATE REVIEW COMPLETED: 2/08/84

REVISED: 3/23/84

DATE APPROVED: (WMEG only)

SIGNIFICANCE TO NRC WASTE MANAGEMENT PROGRAM:

NRC is responsible for licensing underground waste repositories for permanent disposal of high-level radioactive waste. A primary concern in the design and performance of a respository is the heat generated by the reprocessed or unreprocessed waste. Whereas many numerical models (computer codes) exist to do sophisticated thermal analyses, they are generally very inefficient compared to analytical solutions. It is useful to have analytical or semi-analytical techniques that can be used to perform sensitivity analyses or scoping calculations. The document reviewed contains a description of a semi-analytical technique to perform far-field thermal analyses of layered media with an imbedded heat source.

In SNL's opinion, it is important for NRC to understand the mathematical assumptions, strengths, weaknesses, and limitations of the proposed technique in the document reviewed here. The license applicant might use this or a similar technique to select design parameters and/or to predict long-term thermal response. Likewise, NRC might want to use this technique as a tool to cross-check the calculations and analyses presented by the applicant.

BRIEF SUMMARY OF DOCUMENT:

An integral method is developed to predict the transient, farfield thermal response in a layered, semi-infinite medium due to a planar heat source at depth. Analytical expressions are formulated for a finite slab, a two-layer semi-infinite body, a two-layer finite slab, and a heat source represented by a sum of exponential decay terms. The solution of the differential equations is obtained numerically using Hamming's predictor-corrector method. The solutions are for a one dimensional formulation with constant conductivities. A constant temperature or a convective heat loss may be prescribed at the boundary. The analysis is applied to a number of stratigraphies representing repositories in different media. In some cases, the results are compared to numerical solutions obtained with HEATING5. Controlling parameters are identified along with estimated errors At the end, an approach is suggested (but not carried through) that can handle temperature-dependence of thermal conductivity as well as improve solution accuracy.

THEORETICAL CONSIDERATIONS:

The integral methods can be used to obtain solutions to heat transfer problems. These methods are collectively referred to as the "Method of Weighted Residuals" (MWR). A special case of the MWR is the Heat Balance Integral (HBI) method which has a weight function of unity. The one-dimensional equation for transient heat conduction is:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \qquad (1)$$

where,

T = temperature,
ρ = density
c = specific heat capacity,
k = thermal conductivity.
t = time

A residual R may be defined such that

$$R = \rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) . \qquad (2)$$

The fundamental concept of the MWR is to force R to be zero in an average sense over the interval of interest. The basic feature of the HBI method is that a one-parameter solution is sought over the "penetration distance" in a semi-infinite domain. The penetration distance. $\delta(t)$ represents the extent to which diffusion has created a change in internal energy (i.e., the distance heat has travelled in time t). The formal integration of Equation (2) and the use of an approximate temperature distribution results in an ordinary differential equation for $\delta(t)$. A trial function of the form:

$$T = \sum_{n=0}^{N} a_n(t) x^n \qquad (3)$$

is selected. The physical boundary conditions and certain "smoothing conditions" are used to evaluate the time-dependent coefficients $a_n(t)$.

PROBLEMS, DEFICIENCIES OR LIMITATIONS OF REPORT:

The report starts out with the impression that two-dimensional formulations with temperature-dependent thermal conductivities will be provided to be solved numerically. This is not the case. The differential equations derived are strictly one-dimensional with constant, isotropic thermal conductivities. The novelty of this analytical method lies in the fact that it allows more than one layer, thus overcoming the common assumption of a homogeneous medium. Due to the one-dimensional nature of the solution, the temperature gradient in the horizontal direction cannot be described, and the solution of temperature in the vertical direction is adequate for only 100 years or so. The model will, however, provide an upper bound on the far-field temperatures. Although a constant temperature or a convective boundary condition may be specified, the practical limitation on the validity of the solution beyond a 100 years makes the choice of boundary condition meaningless for repository type of applications. Since cavities (tunnels, etc.) cannot be included in the geometric description, the convective losses in the un-backfilled regions during the operational phase cannot be modeled.The heat source must be approximated as a sum of exponentials. The number of layers allowed is very limited. The report does not describe the selection of trial functions (i.e., approximating polynomials) in sufficient detail. No information (not even the program name) or reference is provided about the computer program that is used to solve the differential equations numerically. The only mention of a program is on page 44 where the following statement is made: "The subroutine that implements this method is a modification of the subroutine DHPCG from the IBM FORTRAN Scientific Package [19], which includes error checking and automatically adjusts the time step." Reference [19] mentioned in the above quote is an ASME Paper (No. 80-WA/HT-55).

Some specific, non-trivial errors contained in the document, and some important omissions are:

p. 7 Eqn. (1-7) is missing a negative sign on the right hand side.

p. 9 In paragraph 3, "...the rule is "mostly" incorrect..." , should be: "...the rule is "mostly" correct...".

p. 28 Last line, "When Equation (2-2) is substituted ...", should read: "When Equation (2-5) is substituted..."

p. 32 In Equation (2-15), " $-3a_3r^2$ " on the left hand side should be " $+3a_3r^2$ ".

In Equation (2-16), " $6a_3r^2$ " should be " $6a_3r$".

p. 33 The origin of Equations (2-22) through (2-27) is not mentioned. It is not clear how these definitions were established.

p. 34 Equation (2-30) has some terms missing, the correct equation should be

$$M = - \quad KA(1-r)^3 + 3Ar(1-r)^2 + 3Kr^2(1-r) +$$

$r^3 \quad -1$

p. 36 In Equation (2-35), the term " $3a_3s^3$ " on the left hand side should be " $3a_3s^2$ ".

p. 47 In paragraph 2, "...depth of 100 m beneath..." should be "...depth of 1000 m beneath..."

p. 53 In paragraph 1, the statement, "The similarity in the trends of the numerical and approximate solutions is apparent, especially at $x/L = 1$, i.e., at the source planes.", is unsubstantiated since the figures do not show the numerical solution at that location.

p. 63 First line, "Beyond 200 y..." should be "Beyond 2000 y...".

ACTION TAKEN:

ACTION RECOMMENDED: