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PRELIMINARY

TIME TREND ANALYSIS OF BASALTIC VOLCANISM NEAR THE YUCCA MOUNTAIN SITE

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OBJECTIVE OF THE RESEARCH

The task of quantifying volcanism at Yucca Mountain is as complicated as trying to predict the time of the next catch only based on a few piles of dead fish. [People would debate on the unknown fishing technique(s) used (fishing net, a single hook, etc.) to define a single event. They would also disagree on the freshness of each fish measured.] The issue of the high-level nuclear waste repository at Yucca Mountain has many geological and political considerations. Some proponents of the repository will denounce opposition based on volcanic considerations as farfetched, while others will insist that any risk of site disruption poses an unacceptable threat to population. In the belief that a decision based on available information and educated estimates is preferable to one based on ignorance, the challenge is to address the question: Does the possibility of a volcanic eruption pose a great enough risk to the public to disqualify Yucca Mountain as a nuclear waste repository?

MAJOR RESULTS

I use a preliminary data set based on the Quaternary volcanism (< 1.6 Ma) in the AMRV to demonstrate and check the sensitivity to the models used to predict the future eruptions during the next 10,000 years.

Data

There are seven Quaternary volcanic centers: the sequence of four 1.2-Ma centers in Central Crater Flat, two centers of the 0.28-Ma Sleeping Butte site, and the Lathrop Wells center. The age of the Lathrop Wells center has been refined from

the original 0.27 Ma (Crowe et al. 1982) to 0.01 Ma (Crowe and Perry 1989). The sequence of four 1.2 Ma centers in central Crater Flat includes Red Cone, Northern Cone, Black Cone, and two Little cones. The dates are: 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01.

Sensitivity Analysis

Three models are considered in the following analyses:

1) Based on the assumptions of a simple Poisson model, the estimated recurrence rate is $\hat{\lambda} = E/T = 8/(1.6 \times 10^6) = 5 \times 10^{-6}$/yr (see Appendix I, Ho et al. 1991). The estimated risk ($= \hat{r}$ = probability of at least one major eruption for an isolation time of 10^4 years) is about 4.9%($= 1 - e^{-10^4\hat{\lambda}}$).

2) The second model estimates the instaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and uses a homogeneous Poisson process to predict future eruptions. In this case, $\hat{\lambda} = 5.5 \times 10^{-6}$/yr and $\hat{r} \doteq 5.0\%$ (see Appendix II, Ho 1991).

3) If the prior historical trend continues, the second model can easily be updated to incorporate this requirement. In this case, the estimated risk is about 5.3%, which shows a slight increase as compared to a predictive homogeneous Poisson model.

FUTURE WORK: Modeling of Volcanic Disruption

Crowe et al. (1982) assume that every eruption has the same probability of repository disruption p, and provide a point estimate for p($= a/A$). Their estimated

values of p range from 10^{-4} to 10^{-3}. The calculations are based on a fixed value of a(= area of the repository $\doteq$ 8 km^2), and several choices of A, an area ranging from 1,953 km^2 to 69,466 km^2, corresponding closely to a defined volcanic province and satisfies the requirement of a uniform value of λ. This approach offers computational simplicity. However, the existing data base is inadequate to reasonably constrain A. A more informative approach to model the volcanic disruption of the repository is in progress.

APPENDIX

Copies of Professional Papers and Reports

I. Ho, C.-H. Smith, E.I., Feuerbach, D.L., and Naumann, T.R. 1991. Eruptive Probability Calculation for the Yucca Mountain Site, U.S.A.: Statistical Estimation of Recurrence Rates, Bulletin of Volcanology, 53: (in press)

II. Ho, C.-H. 1991. Time Trend Analysis of Basaltic Volcanism near the Yucca Mountain Site, Journal of Volcanology and Geothermal Research, 46: 61-72

III. Ho, C.-H. 1991. Predictions of Volcanic Eruptions at Mt. Vesuvius, Italy, Journal of Geodynamics, (accepted)

IV. Review of the Los Alamos National Laboratory Study Plan 8.3.1.8.1.1, Probability of Magmatic Disruption of the Repository

V. "Prediction of Volcanic Eruptions: An Application to the Yucca Mountain Site, U.S.A.," abstract presented at the International Conference on Active Volcanoes and Risk Mitigation held in Napoli, Italy, 8/27/91 - 9/1/91.

VI. "Statistical Analysis of Basaltic Volcanism Near the Yucca Mountain Site," presented to the United States Nuclear Waste Technical Review Board in the March 1, 1991 meeting in Tucson on volcanism.

APPENDIX I

I. Ho, C.-H. Smith, E.I., Feuerbach, D.L., and Naumann, T.R. 1991. Eruptive Probability Calculation for the Yucca Mountain Site, U.S.A.: Statistical Estimation of Recurrence Rates, Bulletin of Volcanology, 53: (in press)

ERUPTIVE PROBABILITY CALCULATION FOR THE YUCCA MOUNTAIN SITE, U.S.A.: STATISTICAL ESTIMATION OF RECURRENCE RATES (*)

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ABSTRACT

Investigations are currently underway to evaluate the impact of potentially adverse conditions (e.g., volcanism, faulting, seismicity) on the waste-isolation capability of the proposed nuclear waste repository at Yucca Mountain, Nevada, U.S.A. This paper is the first in a series that will examine the probability of disruption of the Yucca Mountain site by volcanic eruption. In it, we discuss three estimating techniques for determining the recurrence rate of volcanic eruption (λ), an important parameter in the Poisson probability model. The first method is based on the number of events occurring over a certain observation period, the second is based on repose times, and the final is based on magma volume. All three require knowledge of the total number of eruptions in the Yucca Mountain area during the observation period (E). Following this discussion we then propose an estimate of E which takes into account the possibility of polygenetic and polycyclic volcanism at all the volcanic centers near the Yucca Mountain site.

INTRODUCTION

The Yucca Mountain region is located within the Great Basin portion of the Basin and Range physiographic province, a large area of the western United States characterized by alternating linear mountain ranges and alluvial valleys. Crowe and Perry (1989, figure 1) divide the Cenozoic volcanism of the Yucca Mountain region into three episodes that include 1) an older episode of large volume basaltic volcanism (12 to 8.5 Ma) that coincides in time with the termination of silicic

volcanic activity; 2) the formation of five clusters of small volume basalt scoria cones and lava flows (9 to 6.5 Ma), all located north and east of the Yucca Mountain site; and 3) the formation of three clusters of small volume basalt centers (3.7 to .01 Ma), all located south and west of the Yucca Mountain site. The two youngest episodes form northwest-trending zones that parallel the trend of structures in the Spotted Range-Mine Mountain section of the Walker Lane belt. Crowe and Perry (1989) suggest a southwest migration of basaltic volcanism in the Yucca Mountain area based on this structural parallelism, a pattern that may reflect an earlier southwest migration of silicic volcanism in the Great Basin. Smith et al. (1990a) suggest that there is no preferred migration trends for post-6-Ma volcanism in the Yucca Mountain region.

Concern that future volcanism might disrupt the proposed Yucca Mountain repository site motivated the assessment of the volcanic risk to the Yucca Mountain area, located within the Nevada Test Site (NTS). Crowe and Carr (1980) calculate the probability of volcanic disruption of a repository at Yucca Mountain, Nevada using a method developed largely by Crowe (1980). Crowe et al. (1982) refine the volcanic probability calculations for the Yucca Mountain area using the following mathematical model:

$$\Pr[\text{ disruptive event before time } t\ ] = 1 - \exp^{-\lambda t p},$$

where λ is the recurrence rate of volcanic events and p is the probability of a repository disruption, given an event. The parameter p is estimated as a/A, where a is the area of the repository and A is some minimal area that encloses the repository and the area of the

volcanic events. Crowe et al. (1982) develop a computer program to find either the minimum area circle or minimum area ellipse (defined as A) that contains the volcanic centers of interest and the repository site. A is defined to accommodate tectonic controls for the localization of volcanic centers and to constrain λ to be uniform within the area of either the circle or ellipse. The rate of volcanic activity is calculated by determination of the annual rate of magma production for the NTS region and by cone counts using refined age data. Resulting probability values using the refined mathematical model are calculated for periods of 1 year and 100,000 years. Two procedures (explained below) are used for the rate calculations (Crowe et al., 1982). As calculated by Crowe et al. (1982), the probability of volcanic disruption of a waste repository located at Yucca Mountain falls in the range of 3.3×10^{-10} to 4.7×10^{-8} during the first year, which increases approximately linearly with isolation time.

ISSUES THAT ARISE IN CONNECTION WITH THE WORK OF CROWE et al. (1982)

Although the procedure outlined in Crowe et al. (1982) represents a more formal approach to this problem than ever attempted previously, flaws exist. The method must be modified because the existing data base is inadequate to reasonably constrain λ. Despite the fact that there are well-recognized means of gathering data in the NTS region (field mapping; determinations of the eruptive history of basaltic centers; petrology; geochemistry; geochronology, including magnetic polarity determinations; tectonic setting; and geophysical studies) many considerations are still unknown, e.g., age of volcanism and vent counts.

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risk, and probabilities. Reliable historical data make possible the construction of activity patterns for several volcanoes (Wickman, 1966, 1976; Klein, 1982, 1984; Mulargia et al., 1987; Condit et al., 1989; Ho, 1990). Unfortunately, detailed geologic mapping of volcanic centers is in its infancy in the Yucca Mountain area. A formal structure, with conclusions depending on the model assumptions, needs to be developed to evaluate volcanic risk for NTS.

This paper investigates important parameters required to calculate the probability of site disruption and provides estimates for the unknown parameter(s) that are meaningful both statistically and geologically, taking into account the limited availability of precise ages in the NTS region.

The Poisson Model

The application of statistical methods to volcanic eruptions is put onto a sound analytical footing by Wickman (1966, 1976) in a series of papers that discuss the applicability of the methods and the evaluation of recurrence rates for a number of volcanoes. Wickman observes that, for some volcanoes, the recurrence rates are independent of time. Volcanoes of this type are called "Simple Poissonian Volcanoes". Theoretically, the probability formula (Crowe et al., 1982) is derived for this type of volcanic activity from the following assumptions:

1. Volcanic eruptions in successive time periods of length t for each period are independent and should follow a Poisson distribution with a constant mean (average rate) λt, i.e., a simple Poissonian volcano.
2. Every eruption has the same probability of repository disruption p. That is, there is no heterogeneity with respect to disruptiveness.
3. The disruptiveness of the eruptions are independent of one another.

This very brief description is purely mathematical and has no direct interpretation in geologic terms. Since the Poisson model is both the state-of-the-volcanological-art (e.g., Wickman, 1966, 1976) and used in actual risk assessment (e.g., Gardner and Knopoff, 1974; McGuire and Barnhard, 1981), we do not question the above assumptions in this article. Therefore, the following statistical development is based on the assumptions of a simple Poisson model. Of course, exploring alternative models derived from different assumptions based on detailed geologic data and statistical analysis would be valuable, as well (e.g., See Ho, 1991).

Probability Formula

The probability model of Crowe et al. (1982) is based on the following relationship:

$$\Pr[\text{disruptive event before time } t] = 1 - \exp^{-\lambda t p}.$$

The power series expansion for $\exp^{-\lambda t p}$ (Ellis and Gulick, 1986, p. 545)

is:

$$\exp^{-\lambda t p} = \sum_{k=0}^{\infty} \frac{(-\lambda t p)^k}{k!} = 1 - \lambda t p + \frac{(\lambda t p)^2}{2!} - \frac{(\lambda t p)^3}{3!} + \ldots$$

Therefore, the final probability calculation can be simplified as:

$$\Pr[\text{ disruptive event before time } t\ ]$$

$$= \lambda t p - \frac{(\lambda t p)^2}{2!} + \frac{(\lambda t p)^3}{3!} - \ldots$$

$$\doteq \lambda t p, \text{ for small } \lambda \text{ and } p \text{ relative to } t.$$

The approximation is reasonable and is true for virtually all of the calculations in Crowe et al. (1982) since all of their estimated values of both λ ($<10^{-5}$) and p ($<10^{-3}$) are small. Therefore, the accuracy of estimating the unknown parameters λ and p directly influences the significance of values for the probability of repository disruption.

Recurrence Rate

The Poisson process is used to describe a wide variety of stochastic phenomena that share certain characteristics and phenomena in which some "happening" takes place sporadically over a period of time in a manner that is commonly thought of as "at random." We will refer to a "happening" as an event. If events in a Poisson process occur at a mean rate of λ per unit time (1 yr, 10^5 yr, etc.), then the expected number (long-run average) of occurrences in an interval of time in t units is λt. In quantifying volcanism at Yucca Mountain, we define a volcanic eruption as an event. Therefore, the collection of data is directly or indirectly based on the number of eruptions.

Crowe et al. (1982) try three methods to calculate λ in their probability calculations. These are: 1) evaluation of intervals of

volcanic activity for evidence of periodicity, 2) counts of volcanic events in Quaternary time, and 3) evaluation of the ratio of magma production rate and mean magma volume. Based on method 1, they conclude that the data suggest no distinct patterns or periodicity of basaltic volcanism in late Cenozoic time. Therefore, the data are insufficient to analyze interval patterns and thus cannot be used to calculate future rates of volcanic activity (Crowe et al., 1982). We believe, however, that according to the Poisson model assumptions, intervals of volcanic activity should follow an exponential distribution and thus λ can be estimated statistically. We shall demonstrate such statistical sampling and estimation techniques in the following section.

Based on method 2, Crowe et al. (1982) calculate λ as N/T where N is the number of scoria cones and T is the period of time represented by the age of the cones or some other specified time period. Thus λ is the average number of eruptions per unit time. In their calculation of N/T, they define no statistical sampling technique that is associated with the assumed model. Moreover, they do not provide evidence that counting cones is equivalent to counting eruptive events. Crowe et al. (1989) and Wells et al. (1990) now recognize and classify the Lathrop Wells volcanic center as a polycyclic volcano and hence that some cones may have erupted more than once. Note that the Lathrop Wells volcanic center is only twelve miles away from the proposed Yucca Mountain repository site. We shall introduce a statistical estimation procedure to interpret the estimator.

Based on method 3, Crowe et al. (1982) determine the rate of magma production for the NTS region by fitting a linear regression line to a

data set of four points collected from four volcanic centers. Each value thus represents magmatic volume of a single eruption at a corresponding volcanic center. The mean magma volume for 4 m.y. is calculated by taking the average of these four values. The ratio (rate/mean) is then calculated as an estimate for the annual recurrence rate λ. Similarly, the annual recurrence rate for Quaternary time is obtained using only the two Quaternary data points. We consider this approach questionable, since a simple Poisson model requires a constant rate of occurrence, which is not the same as steady-state magma production in a volume-predictable model (e.g., see Wickman, 1966, 1976; Wadge, 1982). We shall show that such calculations based on magma volume duplicate those of method 2, if the rate of magma volume is constant. Moreover, we shall also point out that, in this case, they apparently assume only four (two) eruptions in the NTS region during the last 4 m.y. (Quaternary time). This apparent assumption explains why their final probabilities based on magma volume are consistently smaller than those based on cone counts (Crowe et al., 1982, tables IV and V).

The rate of volcanic eruption, λ, is a critical parameter for the probability calculation. We shall now examine various statistical methods for calculating λ, how the geologic record of volcanism in the Yucca Mountain can be used to estimate values of λ, and the limitations in calculating λ.

ESTIMATION BASED ON POISSON COUNT DATA

In dealing with distributions, repeating a random experiment several times to obtain information about the unknown parameter(s) is useful. The collection of resulting observations, denoted $x_1, x_2, \ldots, x_n$, is

a sample from the associated distribution. Often these observations are collected so that they are independent of each other. That is, one observation must not influence the others. If this type of independence exists, it follows that $x_1, x_2, \ldots, x_n$ are observations of a random sample of size n. The distribution from which the sample arises is the population. The observed sample values, $x_1, x_2, \ldots, x_n$, are used to determine information about the unknown population (or distribution).

Assuming that $x_1, x_2, \ldots, x_n$ represent a random sample from a Poisson population with parameter λ, the likelihood function is

$$L(\lambda) = \prod_{i=1}^{n} f(x_i;\lambda) = e^{-n\lambda} \lambda^{\sum_{i=1}^{n} x_i} / \prod_{i=1}^{n} x_i!$$

Many good statistical procedures employ values for the population parameters that "best" explain the observed data. One meaning of "best" is to select the parameter values that maximize the likelihood function. This technique is called "maximum likelihood estimation," and the maximizing parameter values are called "maximum likelihood estimates," also denoted MLE, or $\hat{\lambda}$. Note that any value of λ that maximizes $L(\lambda)$ will also maximize the log-likelihood, $\ln L(\lambda)$. Thus, for computational convenience, the alternate form of the maximum likelihood equation,

$$\frac{d}{d\lambda} \ln L(\lambda) = 0$$

will often be used, and the log-likelihood for a random sample from a Poisson distribution is:

$$\ln L(\lambda) = -n\lambda + \sum_{i=1}^{n} x_i \ln\lambda - \ln\left(\prod_{i=1}^{n} x_i!\right).$$

The maximum likelihood equation is:

$$\frac{d}{d\lambda} \ln L(\lambda) = -n + \sum_{i=1}^{n} \frac{x_i}{\lambda} = 0,$$

which has the solution $\hat{\lambda} = \sum_{i=1}^{n} \frac{x_i}{n} = \bar{x}$. This is indeed a maximum because the second derivative

$$\frac{d^2}{d\lambda^2} \ln L(\lambda) = -\sum_{i=1}^{n} \frac{x_i}{\lambda^2},$$

is negative when evaluated at $\bar{x}$.

Let us demonstrate this estimation technique. Let X denote the number of volcanic eruptions for a 10^5-year period for the NTS region from this assumed Poisson process. Then X follows a Poisson distribution with average recurrence rate μ , with $\mu = \lambda t = 10^5\lambda$. If we wish to estimate λ for the Quaternary using the Poisson count data for the NTS region, the successive number of eruptions from the last sixteen consecutive intervals of length 10^5 years ($16 \times 10^5 = 1.6 \times 10^6 \doteq$ Quaternary period) must be estimated. The number of observed eruptions per interval are denoted as $x_1, x_2, \ldots, x_{16}$. Thus, these sixteen values represent a sample of size sixteen from a Poisson random variable with average recurrence rate μ. Estimating the mean of the Poisson variable from these count data gives

$$\hat{\mu} = \bar{x} = \sum_{i=1}^{16} x_i / 16,$$

and $$\hat{\lambda} = \hat{\mu}/10^5 = \sum_{i=1}^{16} x_i/(1.6 \times 10^6)$$

This shows that the estimated annual recurrence rate, $\hat{\lambda}$, is the average number of eruptions during the observation period (in years). Based on this estimation technique, $\hat{\lambda}$ can be defined as

$$\hat{\lambda} = E/T, \tag{1}$$

where E = total number of eruptions during the observation period,

and T = observation period.

Note that for the estimation of λ in this model, an individual observation x_i is not required. However, the distribution of x_i's can provide information for model selection, for model-adequacy checking, and for parameter estimation in general.

ESTIMATION BASED ON REPOSE TIMES

With any Poisson process there is an associated sequence of continuous waiting times for successive occurrences. If events occur according to a Poisson process with parameter λ, then the waiting time until the first occurrence, T_1, follows an exponential distribution, $T_1 \sim \exp(\theta)$ with $\theta = 1/\lambda$. Furthermore, the waiting times between consecutive occurrences are independent exponential variables with the same mean time between occurrences, $1/\lambda$ (Parzen, 1962, p. 135). Several simplifying assumptions must be made in treating eruptions as events in time. Although the onset date of an eruption is generally well-defined as the time when lava first breaks the surface, the duration is harder to determine because of such problems as

slowly-cooling flows or lava lakes, and the gradual decline of activity. We adopt the same definition for repose time as used by Klein (1982). Therefore, we ignore eruption duration, we choose the onset date as the most physically meaningful parameter, and we measure repose times from one onset date to the next. Thus, our definition of "repose time" differs from the classic one as a noneruptive period. This procedure seems justified because most eruption durations are much shorter than typical repose intervals. The mean time between two events (eruptions), θ, is inversely related to the volcanic recurrence rate λ. Assumptions of the Poisson process are rather restrictive, but at least a very tractable and easily analyzed model can be proposed. The maximum likelihood estimator for $\theta = \bar{t}$ (Hogg and Tanis, 1988, p. 336) is:

$$\hat{\theta} = \bar{t} = \sum_{i=1}^{m} t_i/m, \text{ and } \hat{\lambda} = \hat{\theta}^{-1} = \bar{t}^{-1},$$

where $t_1, \ldots, t_m$ represent values of a random sample of size m from an exponential population with parameter θ. For the NTS region, the exact values of t_i's (repose times) are difficult to obtain because the precise date of each eruption is not known. However, based on the definition of repose times we can calculate:

$\sum_{i=1}^{m} t_i$ = time between the first and last eruptions during the observation period,

and

m = total number of repose times = E - 1, which gives

$$\hat{\lambda} = (E-1)/(T_0-T_y), \qquad (2)$$

where

E = Total number of eruptions between T_0 and T_y, inclusive,

with

T_0 = age of the oldest eruption,

T_y = age of the youngest eruption.

Note that the numerical values of E in both Equations 1 and 2 are identical for the same observation period of length T. In practice, however, the observation period for the exponential model (Equation 2) must be trimmed to a period between T_0 and T_y, inclusive, to reflect that exactly m (= E - 1) repose times (t_1 through t_m) are obtained. Theoretically, the two estimates obtained for λ (Equations 1 and 2) should be consistent, but not identical.

ESTIMATION BASED ON MAGMA VOLUME

Let V be the total volume of basaltic magma erupted at the surface in the NTS region during the observation period of length T. From Equation 1, we obtain

$$\hat{\lambda} = E/T = EV/TV = (V/T)/(V/E) = r/\bar{v} \qquad (3)$$

where

$r = V/T$, the annual rate of magma production,

and

$\bar{v} = V/E$, the mean volume of magma during the observation period of length T.

Equation 3 is valid, but it also requires an accurate estimate of E for $\bar{v}$. If E (or r) is underestimated, so is λ. The most efficient way to

calculate r is V/T. Crowe and Perry (1989) present a refined method to calculate r. They evaluate r as the slope of the curve of cumulative magma volume plotted versus time. It is essentially identical to V/T, assuming a constant rate of magma volume (an assumption that Crowe et al. (1982) and Crowe and Perry (1989) have been striving to prove). In this case, the degree of erosional modification of volcanic landforms should be studied to estimate volumes of missing volcanic deposits. The overall error, which is multiplicative, is compounded in the values of Crowe and Perry (1989) for r. Moreover, E must be estimated when calculating $\bar{v}$, the mean volume of magma. Therefore, we see no economy in Equation 3 and consider it to duplicate Equation 1. We derive Equation 3 merely to demonstrate that the estimation procedures used by Crowe et al. (1982) and Crowe and Perry (1989) are flawed and therefore must be modified.

ESTIMATION OF E

All of the statistical estimation methods considered for λ (Equations 1-3) require knowing the value of E (total number of eruptions during the observation period). An accurate count of E is possible for volcanoes with a complete historical record. Identifying E, however, depends strictly on a clear understanding of eruptive processes and reliable dating techniques for the NTS region, since no historical record is available. Scientists differ in their opinions of volcanism at the NTS area. The following is the view of Crowe et al. (1983):

Basalt centers are composed of multiple vents, each marked by a scoria cone. In the NTS region the cones are divided into two categories: large central cones, referred to as the main cones, and satellite cones. The average number of cones at a single center, based on cone counts of seven Quaternary basalt centers in the NTS region, is about 2 to 3 cones. Thus, field data suggest a general eruption pattern where the initial breakthrough of magma to the surface is marked by the development of an eruptive fissure with two or three loci of effusion. Each of these vents becomes the site of small scoria cones. As the eruption proceeds, activity shifts or concentrates at a single vent that becomes the site of the main scoria cone.

The above description indicates that a main scoria cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone. However, the possibility of polygenetic (and polycyclic) volcanism at all the volcanic centers needs to be evaluated. λ would be underestimated if nearby vents have distinguishable ages. We, therefore, estimate E as follows:

Let I denote the number of volcanic centers under investigation, and let J_i be the number of main cones in the ith volcanic center, where i=1, ..., I. The proposed estimate of E is:

$$\hat{E} = \sum_{i=1}^{I} \sum_{j=1}^{J_i} (m_{ij} + e_{ij}), \tag{4}$$

where m_{ij} = number of multiple, time-separate

eruptions of the jth main cone in the ith volcanic center,

and e_{ij} = number of vents that are separate in space and time (with distinguishable age measurements) from the jth main cone in the ith volcanic center.

The rationale for this estimate is that significant information has emerged that some of the volcanic centers are polycyclic volcanoes (e.g., Lathrop Wells center (Wells et al., 1990)). This estimation for parameter E (total number of eruptions) given by Equation 4 takes into account such a possibility for the NTS area. Studies are in progress to attempt to evaluate the values of m_{ij}'s and e_{ij}'s for the Quaternary volcanic centers of the Yucca Mountain.

EMPIRICAL RESULTS

Specifying the observation period (T) is important in modeling the volcanism at NTS. Most of the volcanic risk assessment studies in the Yucca Mountain area are centered around the post-6-Ma (Pliocene and younger) and Quaternary (< 1.6 Ma) volcanism (Crowe et al. 1982, Smith et al. 1990a, and Wells et al., 1990). We shall use a preliminary data set based on the Quaternary volcanism to demonstrate the estimation techniques of the recurrence rate.

According to Crowe and Perry (1989), the younger zone of basaltic activity in the vicinity of Yucca Mountain is characterized by basaltic centers occuring as clusters of scoria cones and lava flows. There are seven Quaternary volcanic centers: the sequence of four 1.2-Ma centers in Central Crater Flat, two centers of the 0.28-Ma Sleeping Butte site,

and the Lathrop Wells center. The age of the Lathrop Wells center has been refined from the original 0.27 Ma (Crowe et al. 1982) to 0.01 Ma (Crowe and Perry 1989). This date (0.01 Ma) is in the range of 0 to 0.02 Ma, the period of the most recent volcanic activity of the Lathrop Wells Cone as reported by Wells et al. (1990). The sequence of four 1.2 Ma centers in central Crater Flat includes Red Cone, Northern Cone, Black Cone, and two Little Cones (Figure 1). Smith et al. (1990a) concentrate on this group of five cinder cone complexes in the central part of Crater Flat in Figure 1. Based on their discussion, the cones form a 12-km-long arcuate chain. Details of vent alignment are best observed on Black Cone and Red Cone in the central part of the chain. In the Black Cone complex, the cinder cone is the most prominent topographic feature (about 100 m high and 500 m in diameter), but it may only account for a small volume of flows. A larger volume of basalt erupted from at least 10 vents located north, south and east of Black Cone. These vents are commonly represented by scoria mounds composed of cinder, ash, and large bombs. Vents are aligned along two sub-parallel zones that strike approximately N35E. One zone includes Black Cone and 4 scoria mounds; the other zone lies 300 m to the southeast of Black Cone and contains at least 7 mounds. Dikes exposed in eroded mounds strike northeast and parallel the trend of the vent zones. The Red Cone complex contains three vent zones; two trend approximately N45E and a third zone strikes N50W (Figure 2). This provides substantive justification of our treatment of the total number of eruptions (E), and demonstrates that data for the Yucca Mountain region are incomplete at this preliminary stage of the site characterization studies.

Another key issue in the volcanic risk assessment studies is the disagreement over age-dating of the rocks. For example, the K-Ar dates for Red Cone presented by Smith et al. (1990b, table 4) are: 0.98 ± 0.10 Ma for dike, 1.01 ± 0.06 Ma for amphibole bearing unit, and 0.95 ± 0.08 for basalt on top of Red Cone. Until more reliable dating techniques are available, we have no way to distinguish the ages of the cones within each cluster of volcanic centers. Notice that, although an individual observation (x_i or t_i) is not required for the estimation of E developed in this article, the limited availability of precise ages would affect the counts of both m_{ij} and e_{ij} in Equation 4.

Consistent with the notations used in the previous sections, the Quaternary volcanism yields:

$$T = 1.6 \text{ Ma}, \; E = 8, \; T_0 = 1.2 \text{ Ma}, \text{ and } T_y = 0.01 \text{ Ma}.$$

Therefore, based on Equation 1,

$$\hat{\lambda} = E/T = 8/(1.6 \times 10^6) = 5 \times 10^{-6}/\text{yr}$$

Based on Equation 2,

$$\hat{\lambda} = (E-1)/(T_0 - T_y) = (8 - 1)/(1.2 \times 10^6 - 0.01 \times 10^6)$$
$$= 5.9 \times 10^{-6}/\text{yr}$$

Of course, the estimated rate based on Equation 3 is 5×10^{-6}/yr regardless of the value of V, since the magma volume is really never needed in this calculation.

DISCUSSION AND CONCLUSION

The statistical estimation of recurrence rate λ requires a reliable count of distinguishable vents. This approach is based on the geologic record of volcanism at the NTS region. The methods of the approach are supported by sound statistical sampling theory. Crowe and Perry

(1989), however, object that vent counts record only the recognition of a volcanic event, not its magnitude, and so they refine the parameter estimation by concentrating on the cumulative magma volume, which is a continuous variable. Nonetheless, their model assumptions and development are still based on a discrete simple Poisson model, which treats each eruption equally in order to calculate the final probability.

We now conclude this section with a few comments and point to some further work.

a) Their recommended method for estimating λ is to construct a curve of cumulative magma volume versus time, which is also affected by the counts of vents ($\hat{E}$) in the observation period (T). Their ignorance of the critical factor E in Equation 3 leads them to believe that estimation based on magma volume is the most acceptable method (Crowe and Perry, 1989); this questionable belief, in turn, handicaps their estimates for $\bar{v}$ and thus for λ.

b) All of the published results that demonstrate statistical sampling techniques for volcanic activity require a representative sample and a sufficiently large sample size to calculate a reliable long-run average with precision at a desired level (flipping a coin twice does not tell the whole story of the fairness of the coin).

c) Their recognition of the fact that short periods of eruptive activity are bounded by long periods of inactivity at NTS

indicates that their choice of a simple Poisson model should be adequately checked based on more detailed geologic data. So far, the problems of model assumptions and parameter estimations have been treated only separately by Crowe et al. (1982) and Crowe and Perry (1989), despite the fact that the model (simple Poisson, or Volume-predictable model) assumptions and parameter (occurrence rate, or magma effusion rate) estimation methods virtually always depend on each other in volcanic hazard and risk calculations.

Yucca Mountain is remote from human habitation. There is no historical record of volcanism near Yucca Mountain. Therefore, the volcanic record must be developed by detailed field, geomorphic, and geochronologic studies. Precise ages are critical for volcanic rate calculations, but traditional K-Ar dating commonly has a large error in the age range recorded by the volcanoes near Yucca Mountain (1.1 Ma to 20 Ka). Until more precise techniques are developed, there will be uncertainties with regard to the age and duration of volcanism. Since predictions are needed, one possible improvement would be to reconfirm all of the crucial assumptions using data that are the only basis we have for making necessary plans, calculations, and model selections. We have no choice but to form our notion of governing laws on the basis of data and to act accordingly. This is particularly true in volcanic studies, where data are rare and expensive (≐ $300 - $600 per age of a vent at Yucca Mountain). Our efforts for future studies will be devoted to considerably more detailed data collection and statistical modelling. At this preliminary stage of our work, all we can conclude

is that the probabilistic results of Crowe et al. (1982) are based on idealized model assumptions, a premature data base, and inadequate estimates of the required parameters. For the reasons discussed, we think that Crowe et al. underestimate the risk of volcanism at the proposed Yucca Mountain repository site.

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REFERENCES

Condit CD, Crumpler LS, Aubele JC, Elston WE (1989) Patterns of volcanism along the southern margin of the Colorado plateau: the Springerville Field. J Geophys Res 94: 1975-1986

Crowe BM (1980) Disruptive event analysis: Volcanism and igneous intrusion. Battelle Pacific Northwest Laboratory Rep PNL - 2882

Crowe BM, Carr WJ (1980) Preliminary assessment of the risk of volcanism at a proposed nuclear waste repository in the Southern Great Basin. US Geol Survey Open-File Rep 80-375, 15p

Crowe BM, Johnson ME, Beckman RJ (1982) Calculation of the probability of volcanic disruption of a high-level radioactive waste repository within Southern Nevada, USA. Radioactive Waste Management 3: 167-190

Crowe BM, Self S, Vaniman D, Amos R, Perry F (1983) Aspects of potential magmatic disruption of a high-level radioactive waste repository in Southern Nevada. J Geol 91: 259-276

Crowe BM, Harrington C, Turrin B, Champion D, Wells S, Perry F, McFadden L, Renault C (1989) Volcanic hazard studies for the Yucca Mountain Project. Waste Management 89, Volume 1 - High Level Waste and General Interest, 485-491

Crowe BM, Perry FV (1989) Volcanic probability calculations for the Yucca Mountain site: Estimation of volcanic rates. Focus '89, Nuclear waste isolation in the unsaturated zone, September, 1989, hosted by the Nevada section of the American Nuclear Society, pp. 326-334

Ellis R, Gulick D, (1986) Calculus (3rd Ed.). Harcourt Brace Jovanovich, New York.

Gardner JK, Knopoff L (1974) Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian? Bulletin of the Seismological Society of America 64: 1363-1367

Ho CH (1990) Bayesian analysis of volcanic eruptions. J Volcanol Geotherm Res 43: 91-98

Ho CH (1991) Time trend analysis of basaltic volcanism for the Yucca Mountain site. J Volcanol Geotherm Res 45: (in press)

Hogg RV, Tanis EA (1988) Probability and statistical inference (3rd Ed.). Macmillan, New York.

Klein FW (1982) Patterns of historical eruptions at Hawaii volcanoes. J volcanol Geotherm Res 12: 1-35

Klein FW, (1984) Eruption forecasting at Kilauea volcano, Hawaii. J Geophys Res 89: 3059-3073

McGuire RK, Barnhard P (1981) Effects of temporal variations in seismicity on seismic hazard. Bulletin of the Seismological Society of America 71: 321-334

Mulargia F, Gasperini PI, Tinti S, (1987) Identifying different regimes in eruptive activity: an application to Etna volcano. J Volcanol Geotherm Res 34: 89-106

Parzen E (1962) Stochastic processes. Holden-Day, San Francisco

Smith EI, Feuerbach DL, Naumann TR, Faulds JE (1990a) The area of most recent volcansim near Yucca Mountain, Nevada: Implications for volcanic risk assessment. Proceedings of the International Topical Meeting, High-Level Radioactive Waste Management: American Nuclear Society/American Society of Civil Engineers, Vol. 1: pp.81-90

Smith EI, Feuerbach DL, Naumann TR, Faulds JE (1990b) Annual Report of the Center for Volcanic and Tectonic Studies for the period 10-1-89 to 9-30-90. Submitted to the Nuclear Waste Project Office, Report No. 41

Wadge G, (1982) Steady state volcanism: evidence from eruption histories of polygenetic volcanoes. J Geophys Res 87: 4035-4049

Wells SG, McFadden LD, Renault CE, Crowe BM (1990) Geomorphic assessment of late Quaternary volcanism in the Yucca Mountain area, southern Nevada: Implications for the proposed high-level radioactive waste repository. Geology, 18: 549-553

Wickman FE (1966) Repose-period patterns of volcanoes. Ark. Mineral. Geol 4: 291-367

Wickman FE (1976) Markov models of repose-period patterns of volcanoes. In: Merriam DF (Editor) Random processes in geology. Springer, New York, N.Y., pp. 135-161

Figure Titles

Fig. 1. Generalized geologic map of Crater Flat volcanic field area and boundary of proposed radioactive waste repository; inset map shows location of the Crater Flat volcanic field. (Source: Wells et al. 1990, figure 1)

Fig. 2. Geologic map of Red Cone, Crater Flat, Nevada

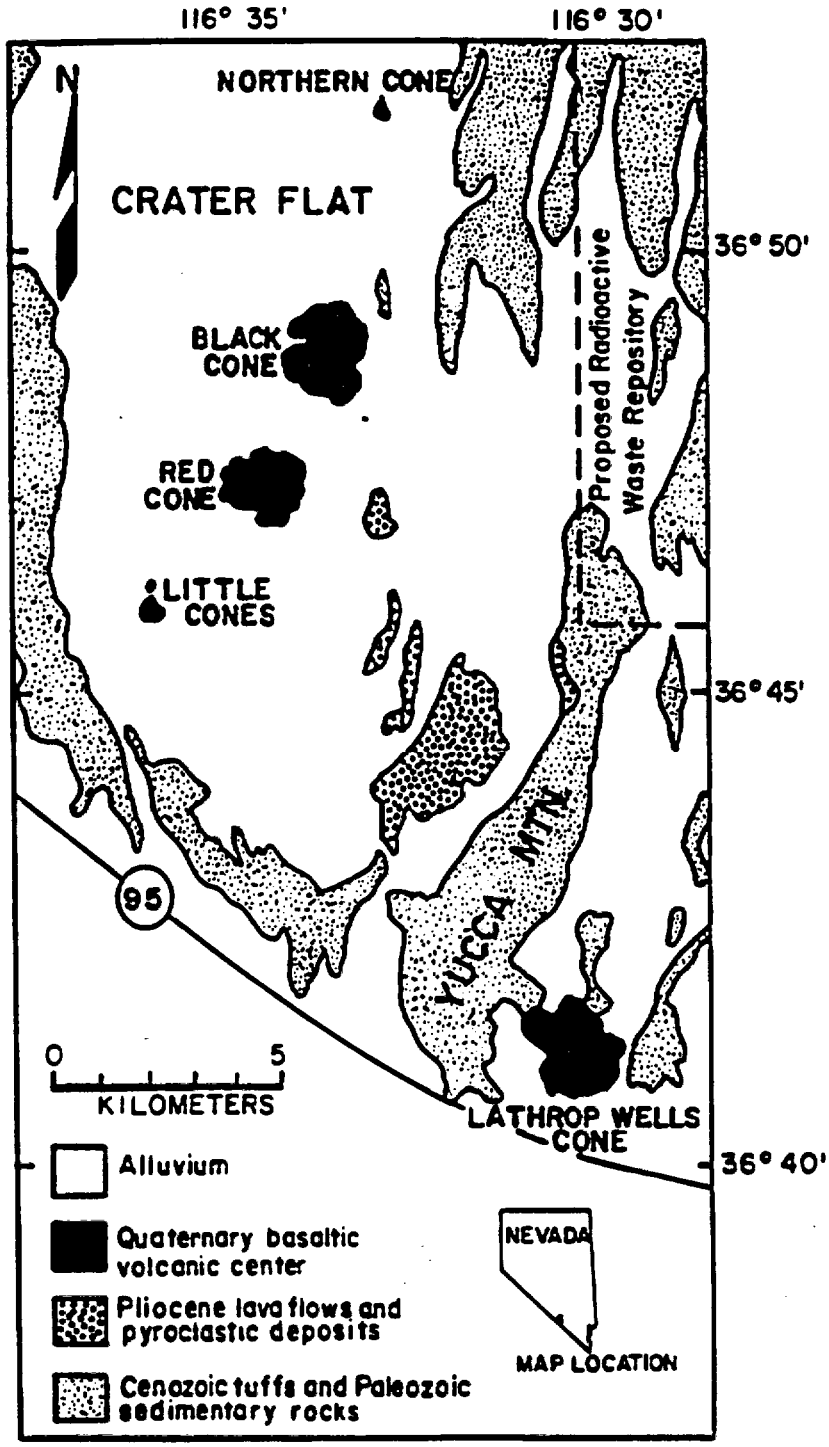

Fig. 1. Generalized geologic map of Crater Flat volcanic field area and boundary of proposed radioactive waste repository; inset map shows location of the Crater Flat volcanic field. (Source: Wells et al. 1990, figure 1)

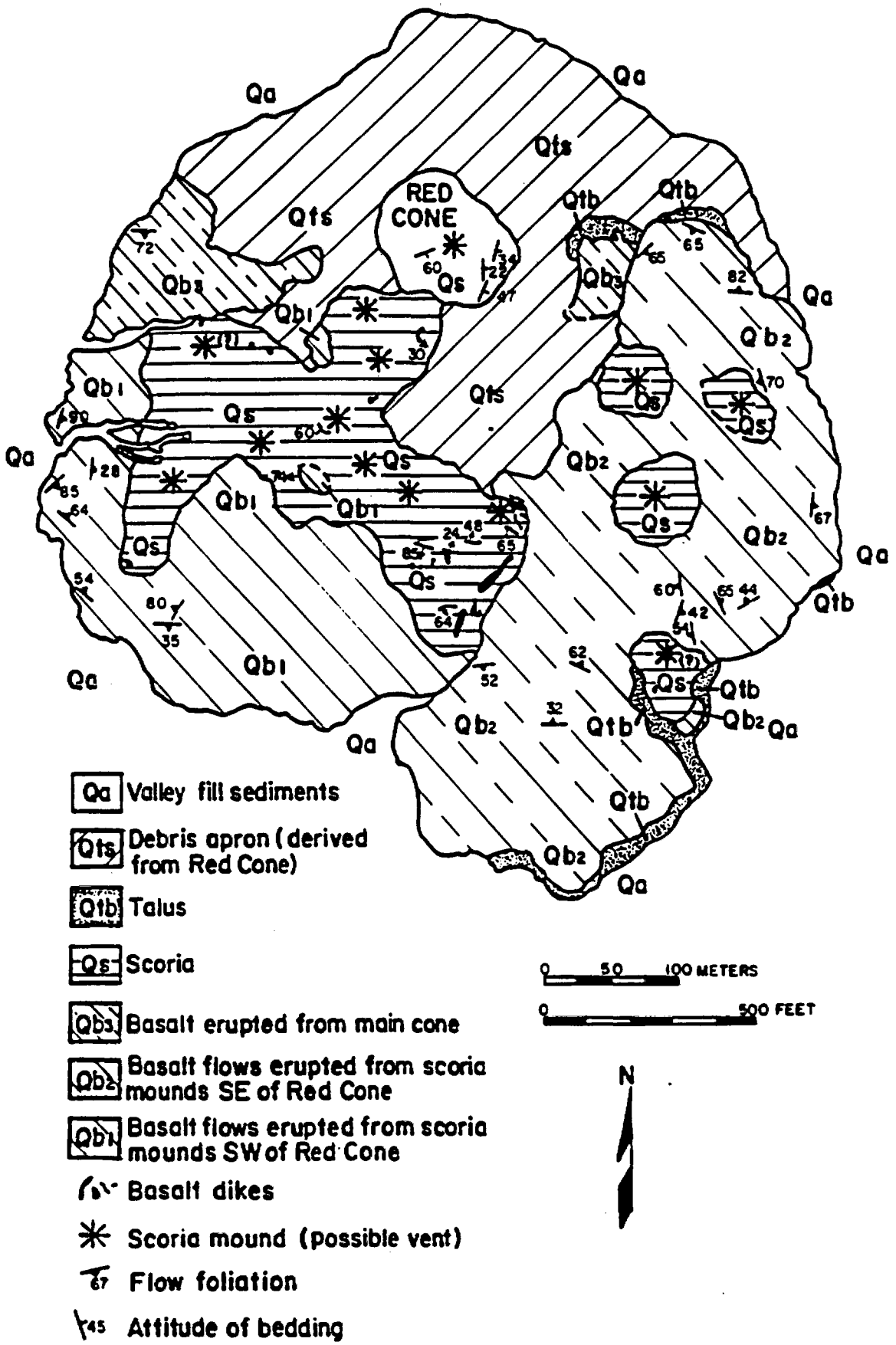

Fig. 2. Geologic map of Red Cone, Crater Flat, Nevada

APPENDIX II

II. Ho, C.-H. 1991. Time Trend Analysis of Basaltic Volcanism near the Yucca Mountain Site, Journal of Volcanology and Geothermal Research, 46: 61-72

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Time trend analysis of basaltic volcanism for the Yucca Mountain site

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ABSTRACT

Ho, C.-H., 1991. Time trend analysis of basaltic volcanism for the Yucca Mountain site. *J. Volcanol. Geotherm. Res.*, 46: 61–72.

The possible recurrence of volcanic activity near the proposed nuclear waste repository at Yucca Mountain, Nevada, U.S.A. is evaluated by estimating the instantaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and by using a homogeneous Poisson process to predict future eruptions. Analysis on the post-6-Ma volcanism near the Yucca Mountain region indicates a moderate developing time trend (p-value = 0.011) of volcanic activity. A similar time trend is obtained by trimming the observation period to 3.7 Ma and younger (period of the youngest episode). Data from the Quaternary basaltic volcanism also show a slight developing time trend, although the developing volcanic activity is not significant at the 0.05 level. Thus, it would oversimplify the assessment of the volcanic risk to the proposed Yucca Mountain repository site if a simple Poisson model were used to model the volcanism. Based on the Quaternary data, the estimated instantaneous recurrence rate is about 5.5×10^{-6}/yr. An estimate of the mean time to the next eruption is about 1.8×10^5 years from now, if it is assumed that the intensity remains constant thereafter. Also, the risk (probability of at least one major eruption during the projected time frame) increases approximately linearly with the time frame chosen as the required interval for radioactive waste to decay to an acceptable level. Our study concludes that the estimated risk for an isolation time of 10^4 years is about 5%, which increases to 42% if 10^5 years is used as the required isolation time.

Introduction

In the United States, spent fuel and high-level radioactive waste will be permanently disposed of in a geologic repository. Disposal of the spent fuel and high-level waste is scheduled to begin in the year 2010. The candidate site for the first U.S. geologic repository is located at Yucca Mountain, Nevada, approximately 100 miles, or about 160 kilometers, northwest of Las Vegas, Nevada. Comprehensive studies are underway on the potential host rock formation. These studies are called site characterization.

An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site (Department of Energy, 1986) because of the presence of multiple basalt centers of Quaternary age.

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risks, and probabilities. Reliable historical data make possible the construction of activity patterns for several

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volcanoes (Wickman, 1966, 1976; Klein, 1982, 1984; Mulargia et al., 1985, 1987; Condit et al., 1989; Ho, 1991). Unfortunately, there is no historical record of volcanism near Yucca Mountain, located within the Nevada Test Site (NTS). The volcanic record must therefore be developed by detailed field, geomorphic, and geochronologic studies.

Concern that future volcanism might disrupt the proposed Yucca Mountain repository site motivated the assessment of the volcanic risk to the Yucca Mountain area. Crowe and Carr (1980) calculate the probability of volcanic disruption of a repository at Yucca Mountain using a method developed largely by Crowe (1980). Crowe et al. (1982) refine the volcanic probability calculations for the Yucca Mountain area using the following mathematical model:

$$Pr\,[\text{no disruptive events before time } t]$$
$$= \sum_{n=0}^{\infty} Pr\,[\text{there are } n \text{ eruptions}] \times Pr\,[\text{none are disruptive}]$$
$$= \exp(-\lambda t p) \qquad (1)$$

where λ is the recurrence rate of volcanic events and p is the probability of a repository disruption, given an event (a volcanic eruption). Theoretically, the probability formula (Crowe et al., 1982) is derived from the following assumptions:

> Volcanic eruptions in successive time periods of length t for each period are independent and should follow a Poisson distribution with a constant mean (average rate) λt, i.e., a simple Poissonian volcano (see Wickman, 1966).

Although the simple Poisson model has proved successful in a wide range of situations, it might be inadequate to model the volcanism at NTS for the following reasons:

(a) A simple Poisson model does not allow for the possibility of a waning (or developing) volcanic time trend, which is one of the major concerns in quantifying the volcanism at the Yucca Mountain region. It should be obvious that the chronological order in which the volcanic eruptions occur is an extremely important aspect of a historical eruptive data set. In order to demonstrate this point, we extend the idea of Ascher (1983) to the volcanic studies. In Figure 1, we use the pseudo-data provided by Ascher (1983). For example, even an eyeball analysis of Figure 1 is adequate to strongly suggest that volcanic activities are "waning", "random", and "developing", since as time increases, the eruptions occur less frequently, about as frequently, and more frequently, respectively. The simple Poisson model, however, assumes that the average recurrence rate (λ) is constant throughout the entire life of the volcanic activity. Once this assumption is made, the model would treat these data sets as equivalent and, therefore, would take the average of the five numbers (14, 34, 42, 72, and 244) as the estimated repose time and its reciprocal as the estimated recurrence rate (λ). It is emphasized that this would occur even for an arbitrarily large number of repose times, rather than just the five repose times in the data sets used here for illustrative purposes.

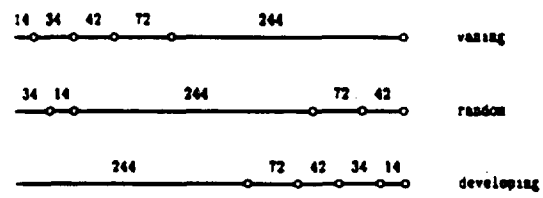


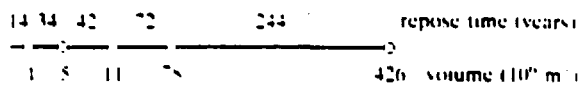
Fig. 1. Times between successive eruptions of three volcanoes in their chronological orders.

A new development in volcanic studies is the possibility that the scoria cone of the Lathrop Wells volcanic center, one of the youngest volcanic centers in the Yucca Mountain region, is significantly younger than associated lava flows (Wells et al., 1988). Further studies have shown that at least three, and possibly more, of the seven Quaternary vol-

canic centers in the region exhibit polycyclic activity (Crowe et al., 1989; Wells et al., 1990). It is therefore of interest to explore alternative model(s) derived from less restrictive model assumptions to see how the time trend could be evaluated.

(b) Given the extremely limited nature of the geologic data at the Yucca Mountain area, the use of the most elementary statistical model (i.e., a simple Poisson model) is hard to justify. As has been mentioned earlier, there is a large and growing body of literature on probabilistic modeling for volcanism. Much of the debate in the literature is centered on the choice of distribution models (principally homogeneous Poisson versus nonhomogeneous Poisson models). There are several variations possible in goodness-of-fit testing. For the homogeneous Poisson model, the chi-square goodness-of-fit test (e.g., see Steel and Torrie, 1980, p. 529) based on count data is often not reliable because of low degrees of freedom or low expected cell counts for some volcanic eruptive data. The Kolmogorov-Smirnov test (e.g., see Steel and Torrie, 1980, p. 535) is considered more reliable and is based on the repose times between eruptions, but does not take into account the relative positions of repose times (Ho, 1991). In other words, any random permutations of the same data set of repose times (e.g., data in Fig. 1) yield the same result as applying the Kolmogorov-Smirnov test. Thus, in general, relatively large samples are usually required to verify the validity of a specified model (at some probability level).

There is a developing literature on volcanic recurrence models that include plots of cumulative volume of volcanic events versus time (Shaw, 1980, 1987; Bacon, 1982; Wadge, 1982). Crowe and Perry (1989) use the curve of cumulative magma volume plotted versus time to evaluate whether the volcanism at NTS indicates steady-state eruptive behaviour in magma production. They then evaluate the slope of the curve as an estimate of the annual rate of magma production. We consider this approach questionable. First, a simple Poisson model requires a constant rate of occurrence, which is not the same as magma production. Also, the trend of magma volume has not proved relevant to that of the frequency of the volcanic events (eruptions) at the NTS area. For example, the data set which has a waning time trend can also have an increasing trend in magma volume. The following data demonstrate this possibility:



Second, the degree of erosional modification of volcanic landforms for the Yucca Mountain region should be studied to estimate volumes of missing volcanic deposits. Third, Crowe et al. (1982) determine the rate of magma production for the NTS region by fitting a linear regression line to a data set of four points collected from four volcanic centers. Each value thus represents magma volume of a single eruption at a corresponding volcanic center. The mean magma volume of 4 m.y. is calculated by taking the average of these four values. The ratio (rate/mean) is then calculated as an estimate ($\hat{\lambda}$) for the annual recurrence rate λ. Ho et al. (1989) show that the magma volume is really never needed in this calculation. Let V be the total volume of basaltic magma erupted at the surface in the NTS region during the observation period of length T. The annual rate of magma production, calculated as the slope of the curve of cumulative magma volume plotted versus time, is essentially identical to V/T, assuming a constant rate of magma volume [an assumption that Crowe et al. (1982) and Crowe and Perry (1989) have been striving to prove]. Thus:

$$\hat{\lambda} = \frac{\text{the annual rate of magma production}}{\text{the mean volume of magma during the observation period}}$$

$$= \frac{(V/T)}{(V/E)} \qquad \text{(Ho et al., 1989, eqn. 3)}$$

$$= E/T \qquad \text{(Ho et al., 1989, eqn. 1)}$$

where E is the total number of eruptions during the observation period. Therefore, the method using the cumulative magma volume to calculate the recurrence rate in the Yucca Mountain site reduces to the simple Poisson mean rate of occurrence which is the most efficient method to estimate the Poisson parameter (λ) based on the maximum likelihood principle (Ho et al., 1989).

Crowe and Perry (1989), however, object that cone counts record only the recognition of a volcanic event, not its magnitude, and so they refine the parameter estimation by concentrating on the cumulative magma volume which is a continuous variable. Nonetheless, their model assumptions and development are still based on a discrete simple Poisson model, which treats each eruption equally in order to calculate the final probability (see eqn. 1). So far, the problems of model assumptions and parameter estimations have been treated only separately by Crowe et al. (1982) and Crowe and Perry (1989), despite the fact that the model assumptions and parameter estimation methods virtually always depend on each other in volcanic hazard and risk calculations (Ho et al., 1989).

For the reasons discussed, a formal structure, with conclusions depending on the model assumptions, needs to be developed to ensure that volcanic risk assessment is based on an adequate model and a complete volcanic record of the Yucca Mountain region. In this paper, we evaluate the time trend of volcanic activity near the proposed nuclear waste repository at Yucca Mountain by estimating the instantaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and by using a homogeneous Poisson process to predict future eruptions.

The Weibull process

A simple Poisson process is more specifically known as a *homogeneous Poisson process* (HPP) since the rate λ is assumed independent of time t. The homogeneous Poisson model generally gives a good fit to many volcanoes for forecasting volcanic eruptions. If eruptions occur according to a homogeneous Poisson process, the repose times between consecutive eruptions are independent exponential variables with mean $\theta = 1/\lambda$. The exponential distribution is applicable when the eruptions occur "at random" and are not due to aging, etc. If we replace the constant λ with a function of t, denoted by $\lambda(t)$, then another type of Poisson process can be derived, known as a *nonhomogeneous Poisson process* (NHPP). If $X(t)$ denotes the number of occurrences in a specified interval $[0, t]$ for a NHPP, then it can be shown that $X(t)$ is distributed as a nonhomogeneous Poisson random variable (Parzen 1962, p. 138) with parameter $\mu(t)$, where:

$$\mu(t) = \int_0^t \lambda(s)\, ds$$

The choice for the nonhomogeneous intensity function, $\lambda(t)$, is important in modeling the volcanism at the Yucca Mountain area. In this paper, our choice of $\lambda(t)$ is:

$$\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$$

which gives:

$$\mu(t) = (t/\theta)^{\beta}$$

In this case, the time to first occurrence follows a Weibull distribution, WEI(θ, β). This intensity parameter is an increasing function of t if $\beta > 1$ and a decreasing function of t if $\beta < 1$. Of course the Weibull process is a generalization of the exponential case ($\beta = 1$, which assumes a no-memory property), so it is useful for situations which entail waning, growth, etc. (Ho, 1991). For example, the birth process (new volcanoes) and the death process (extinction) of volcanoes are

included. Clearly, the Weibull model does include the simple Poisson model, since when $\beta = 1$ the Weibull reduces to the exponential (a simple Poisson model). Therefore, our approach in modeling the volcanism at NTS is to consider a general family of distribution, such as WEI(θ, β), and then to decide whether some subset of this family such as WEI(θ, 1) = EXP(θ) is valid. Thus, in this case, the test of $H_0 : \beta = 1$ may be considered a goodness-of-fit test. We shall demonstrate that this test is possible even when the geologic data at the Yucca Mountain area are extremely limited.

In a Weibull process the time to first occurrence, say T_1, follows a Weibull distribution, WEI(θ, β). The time to second occurrence, or the time between occurrences, does not follow a Weibull distribution. This is in contrast to the exponential case in which the times between occurrences are also exponentially distributed. Since the successive times of occurrences from a single Weibull process are of main interest, some statistical results in this framework are discussed in the next section.

Statistical analysis for the Weibull model

The Weibull emerged in the 1960s and 1970s as perhaps the most widely used life distribution. Research was particularly heavy during this period. Many of the statistical methods developed for this model are now routinely used in life testing and reliability work. Books by Bain (1978), Lawless (1982), Mann et al. (1974), and Nelson (1982) review much of the work in this area. The Weibull process will be referred to as failure-truncated, in reliability terminology, if it is observed until the first n failure times, $t_1, \ldots, t_n$, have occurred, and it will be referred to as time-truncated if it is observed for a fixed time t. Ho (1991) has recently used the failure-truncated Weibull model to analyze the following five volcanoes: Aso, Etna, Kilauea, St. Helens, and Yake-Dake. He concludes that the Weibull model can be considered a goodness-of-fit test for a simple exponential model (a homogeneous Poisson model) and that it is preferable for practical use in volcanic studies. For volcanic eruptive forecasting near the Yucca Mountain region, the time-truncated case makes more sense, since t can be extended to the present date to include the repose time following the last eruption.

Suppose we assume that the successive volcanic eruptions (or eruptive cycles) at the Yucca Mountain region follow a simple Weibull process. For a time-truncated Weibull process, let t be predetermined and suppose $n > 1$ eruptions are observed during $[0, t]$ at times $0 < t_1 < t_2 < \ldots < t_n$. The maximum likelihood estimates (MLE) are given (Crow, 1974) by:

$$\hat{\beta} = n / \sum_{i=1}^{n} \ln(t/t_i)$$

$$\hat{\theta} = t/n^{1/\hat{\beta}}$$

These are similar to the failure-truncated case if t is replaced by t_n. Simple calculations yield the following estimates for the data sets in Figure 1:

Volcano	$\hat{\beta}$
waning	0.63
random	0.99
developing	5.4

The $\hat{\beta}$ estimated for the simple Poissonian volcano (random) clearly is consistent with $\beta = 1$, that is with the homogeneous Poisson process. Since the recurrence rate is proportional to $t^{\beta-1}$, the $\hat{\beta}$'s estimated for the waning and developing volcanoes imply decreasing and increasing recurrence rates at which eruptions are occurring, respectively. These results are in complete agreement with an eyeball analysis of Figure 1.

The gain of a Weibull model therefore is obtained with virtually no loss in predictive ability. In sharp contrast, if we fitted the

simple Poisson model to these data sets, we would obtain exactly the same parameter estimates for all volcanoes. This demonstrates the rationale of our choice of a Weibull intensity to model the volcanism at the Yucca Mountain region. Basic properties of the time-truncated Weibull model that are useful for volcanic eruptive studies will be considered further in the remainder of this section.

Goodness-of-fit test

In practice, we may not be willing to assume that the volcanic recurrence rate is strictly monotonic during the observation period (i.e., the volcanism is waning or developing), but we may want to test this hypothesis by statistical means. Observe, in particular, that testing the hypothesis that the volcanic eruptions follow a HPP is equivalent to testing $H_0 : \beta = 1$ vs. $H_A : \beta \neq 1$. To do this we use the result (Crow, 1974) that:

$$2n\hat{\beta}/\beta \sim \chi^2(2n)$$

a chi-square random variable with $2n$ degrees of freedom.

Prediction of future eruptions

Suppose a Weibull model is assumed during the observation time period $[0, t]$. At time t, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$. Furthermore, assuming that the intensity, $\lambda(t)$, remains constant thereafter, then the subsequent repose times between eruptions are independent exponential variables with recurrence rate $\lambda(t)$ and the mean time to the next eruption at cumulative observation time t is expressed as $M(t) = 1/\lambda(t)$. In the application of the Weibull process model to volcanic eruptive forecasting, estimates and confidence intervals for $M(t)$ are of considerable practical interest since $M(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. (In reliability terminology, $M(t)$ is defined as the achieved mean time between failures at cumulative test time t.) Crow (1982) derives the MLE for $M(t)$ as:

$$\hat{M}(t) = [\hat{\lambda}(t)]^{-1} = [(\hat{\beta}/\hat{\theta})(t/\hat{\theta})^{\hat{\beta}-1}]^{-1}$$

and he provides a table (Crow, 1982, table 2) for constructing confidence intervals for this quantity. Since the number of eruptions during some specified length of time t_0 would be distributed as a homogeneous Poisson random variable with constant rate $\lambda(t)t_0$, estimates of probability of future eruptions are readily available from $\hat{\lambda}(t)$ and the Poisson probability distribution function. Note that while we use historical eruptive data during $[0, t]$ to estimate the instantaneous recurrence rate $\lambda(t)$ at time t based on a NHPP with Weibull intensity, we then use a HPP to predict future eruptions based on a recurrence rate $\lambda(t)t_0$ for future time, $[t, t + t_0]$. In other words, we incorporate the time trend (developing or waning) into our estimate of the recurrence rate and description of the general trend, but we take a neutral position (i.e. ignore time trend) when predicting future eruptions. The rationale for this procedure is that, although eruptions are caused by specific physical events or processes, there might be many causal factors with random influences on the sequence of eruptions. As a result, the future time trend could be assumed to be described by a HPP for forecasting purposes.

Geological setting

Yucca Mountain is located in the south-central part of the Southwestern Nevada Volcanic Field (SNVF), a major volcanic province of the southern Great Basin first defined by Christiansen et al. (1977) and extended by Byers et al. (1989). Interested readers are referred to the paper of Byers et al. (1989) for the location of geographic features of the SNVF.

Crowe and Perry (1989, fig. 1) divide the Cenozoic volcanism of the Yucca Mountain region into three episodes that include: (1) an older episode of large-volume basaltic volcanism (12 to 8.5 Ma) that coincides in time with the termination of silicic volcanic activity; (2) the formation of five clusters of small-volume basalt scoria cones and lava flows (9 to 6.5 Ma), all located north and east of the Yucca Mountain site; and (3) the formation of three clusters of small-volume basalt centers (3.7 to 0.01 Ma), all located south and west of the Yucca Mountain site. The two youngest episodes form northwest-trending zones that parallel the trend of structures in the Spotted Range–Mine Mountain section of the Walker Lane belt. Crowe and Perry (1989) suggest a southwest migration of basaltic volcanism in the Yucca Mountain area based on this structural parallelism, a pattern that may reflect an earlier southwest migration of silicic volcanism in the Great Basin. Smith et al. (1990) provide a different point of view of the migration trends of volcanism in the Yucca Mountain region.

The important questions that we attempt to answer for assessment of volcanic risk for the Yucca Mountain site include:

(1) What is the overall time trend of the volcanic activity?

(2) Can the next episode be predicted?

(3) What does the youngest episode (3.7 to 0.01 Ma) tell us? Is the time trend increasing?

(4) Can the probabilities of future eruptions be predicted?

Applications of the time-truncated Weibull model are shown in the next two sections.

Time trend analysis based on episodes and age cycles

Episodes as major events

Regarding the beginning of the oldest episode, 12 Ma, as time zero (i.e., t = 12 Ma), the midpoints (10.25, 7.75 and 1.855) of the intervals of three episodes are recalculated from this date. The data, therefore, are time-truncated. The MLE of β is computed, giving $\hat{\beta}$ = 0.96. Based on $\hat{\beta}$, the data suggest a waning trend in the overall volcanic activity through time from 12 Ma until present for the Yucca Mountain region. However, the p-value ($\doteq$ 0.79) indicates that the waning time trend is not significant. Furthermore, the data yield $\hat{\theta} = 3.81 \times 10^{6}$, $\hat{\lambda}(t) = 2.40 \times 10^{-7}$, and $\hat{M}(t) = 4.18 \times 10^{6}$. Therefore, an estimate of the midpoint of the time interval for the next episode is about 4.18 Ma from now.

Age cycles of the youngest episode as major events

According to Crowe and Perry (1989), the youngest zone of basaltic activity is characterized by basaltic centers occurring as clusters of scoria cones and lava flows. These clusters include the 3.7-Ma basalts in southeastern Crater Flat, 2.8-Ma basalt of Buckboard Mesa, the sequence of four 1.2-Ma centers in Central Crater Flat, two centers of the 0.28-Ma Sleeping Butte site, and the Lathrop Wells center. The age of the Lathrop Wells center has been refined from the original 0.27 Ma (Crowe et al., 1982) to 0.01 Ma (Crowe and Perry, 1989). This date (0.01 Ma) is in the range of 0 to 0.02 Ma, period of the most recent volcanic activity of the Lathrop Wells Cone, reported by Wells et al. (1990). If we divide the basalts of the youngest episode into five age cycles of 3.7, 2.8, 1.2, 0.28, and 0.01 Ma, then the time trend can be analyzed, and the next age cycle can be predicted. We are ready to fit a time-truncated Weibull model to the above five age cycles. Again, we define the beginning of the youngest episode (3.7 Ma) as time zero, so t = 3.7 Ma. The data yield $\hat{\beta}$ = 2.12, $\hat{\theta} = 1.92 \times 10^{6}$, $\hat{\lambda}(t) = 2.29 \times 10^{-6}$, and $\hat{M}(t) = 0.44 \times 10^{6}$. Based on $\hat{\beta}$, the data suggest a nonsignificant developing time trend (p-value $\doteq$ 0.25). An estimate of the mean time to the next eruptive cycle is 0.44

TABLE 1

Summary of time trend analysis

Observation period	Event	$\hat{\lambda}$ (p-value)	$\hat{M}$ (Ma) (90% C.I.)	Risk Isolation period (yr)		
				1	10^4	10^5
12.0 Ma - present	Episode	0.96 (0.79)	4.18			
3.7 Ma -	Cycle	2.12 (0.25)	0.44 (0.14, 2.62)	2.3×10^{-6}	0.02	0.21
6.0 Ma -	Cone	2.29 (0.01)	0.20 (0.11, 0.45)	5.0×10^{-6}	0.05	0.39
3.7 Ma -	Cone	2.55 (0.02)	0.16 (0.07, 0.44)	6.2×10^{-6}	0.06	0.46
1.6 Ma -	Cone	1.09 (0.90)	0.18 (0.08, 0.55)	5.5×10^{-6}	0.05	0.42

Ma, if it is assumed that the intensity, $\lambda(t)$, remains constant thereafter. A 90% confidence interval for $M(t)$ has the form $\Pi_1\hat{M}(t) < M(t) < \Pi_2\hat{M}(t)$, where $\hat{M}(t)$ (= 0.44 Ma) is the MLE of $M(t)$ and Π_1 and Π_2 are the values given by Crow (1982, table 2). The corresponding 90% confidence interval for $M(t)$ is: $0.312 \times 0.44 < M(t) < 5.947 \times 0.44$, or (0.14 Ma, 2.62 Ma). In other words, a 90% confidence interval for the next eruptive cycle near the Yucca Mountain site is between 0.14 and 2.62 Ma from now. Furthermore, if it is assumed that each of the five age cycles represents a major event, then the number of occurrences of such an event $X(t_0)$ in $[0, t_0]$ follows a Poisson random variable:

$$f(x) = \frac{\exp[-\lambda(t)t_0][\lambda(t)t_0]^x}{x!}; \quad x = 0, 1, \ldots$$

The probability of at least one event during the next t_0 years is of considerable practical interest and is quoted as "risk" in Table 1, which summarizes the hierarchical time trend analysis.

Time trend analysis based on main cones

In order to answer the last question (can the probabilities of future eruptions be predicted?), some other relevant issues have to be addressed. An accurate count of the number of eruptions is possible for volcanoes with a complete historical record. To identify the number of eruptions, however, depends on clear understanding of eruptive processes and reliable dating techniques for the Yucca Mountain region, since no historical record is available. Crowe et al. (1983) indicate that a main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone. Therefore, for the next stage of the hierarchical trend analysis, we count each widely recognized main cone as a major event, but do not require that the main cones in each center (or cluster of centers) be of separate ages, since traditional K-Ar dating commonly produces large errors in the age ranges recorded by the volcanoes near Yucca Mountain which would mask the differences of dates and would lead to an underestimation of the recurrence rate. For instance, the 3.7-Ma basalts include at least four volcanic centers (as indicated by a referee). To be consistent with our discussion, we count four main cones in the 3.7-Ma units (an estimate suggested by geologist Daniel Feuerbach). The sequence of four 1.2-Ma centers in central Crater Flat includes Red Cone, Northern

Cone, Black Cone, and two Little Cones (see Vaniman et al., 1982, table 1). Now jointly with two Sleeping Butte Cones, one Lathrop Wells Cone and the basalt of Buckboard Mesa, we form a slightly more detailed set of data for the time trend analysis. Until more reliable dating techniques are available, we have no way to distinguish the ages of the cones within each cluster but to assign the respective cycle age to each cone. The dates then are: 3.7, 3.7, 3.7, 3.7, 2.8, 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01. This may slightly affect the estimation of $\hat{\beta}$ since, in contrast to the exponential model, the Weibull model is sensitive to the locations, numbers, and relative sizes (to t) of the ordered t_i's. If early sparse t_i's are accompanied later by dense t_i's toward t, then $\hat{\beta}$ would be large, showing a developing trend through time, and vice versa (e.g., see Ho, 1991). We shall use these dates to investigate the time trend of volcanism during the following three observation periods: Pliocene and younger (<6.0 Ma), youngest episode (<3.7 Ma), and Quaternary (<1.6 Ma).

Analysis on Pliocene and younger volcanism

Let the beginning of the Pliocene period ($\doteq$ 6.0 Ma) be time zero, so t = 6.0 Ma. The data yield $\hat{\beta}$ = 2.29, which suggests a significant developing time trend (p-value $\doteq$ 0.01) at the 0.05 level. Thus, it would oversimplify the assessment of the volcanic risk to the proposed Yucca Mountain repository site if a simple Poisson model were used to model the post-6-Ma volcanism near the Yucca Mountain region. The estimated instantaneous recurrence rate is about 5×10^{-6}/yr. An estimate of the mean time to the next eruption is 2×10^5 years from now, if it is assumed that the intensity remains constant thereafter. Also, the estimated risk for an isolation time of 10^4 yr is about 5%, increasing to 39% if 10^5 yr is the required isolation time. Table 1 shows the results.

Analysis of volcanism during the youngest episode (<3.7 Ma)

It is necessary to study the time trend during the shortened observation period which ignores the inactivity period during 6.0 Ma to 3.7 Ma. Thus, let the beginning of the youngest episode (3.7 Ma) be time zero, then t = 3.7 Ma. In this case, all four dates evaluated at 3.7 Ma must be discarded, since the recalculated cumulative times are zero. The data (2.8, 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01) yield $\hat{\beta}$ = 2.55, which also suggests a significant developing time trend (p-value $\doteq$ 0.02). The estimated instantaneous recurrence rate is about 6×10^{-6}/yr. Also, the estimated risk for an isolation time of 10^4 yr is about 6%, which increases to 46% if 10^5 yr is the required isolation time. Results are summarized in Table 1.

Analysis on Quaternary volcanism

In this case, t = 1.6 Ma, and the dates are: 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01. Data from Quaternary basaltic volcanism show a slight developing trend ($\hat{\beta}$ = 1.09, p-value $\doteq$ 0.90) of volcanic activity, although the developing volcanic activity is not significant at the 0.05 level. The estimated instantaneous recurrence rate is about 5.5×10^{-6}/yr. An estimate of the mean time to the next eruption is 1.8×10^5 years from now, if it is assumed that the intensity remains constant thereafter. Also, the estimated risk for an isolation time of 10^4 yr is about 5%, which increases to 42% if 10^5 yr is the required isolation time. The risks are slightly lower than those obtained from the data set of the youngest episode treating main cones as major events. Again, Table 1 shows the results.

Projection of the time trend incorporating polycyclic volcanism

So far, we have not taken into account the possibility of polycyclic volcanism, since

at this preliminary stage of our work data for the Yucca Mountain region are incomplete. However, Crowe et al. (1989) and Wells et al. (1990) now recognize and classify the Lathrop Wells volcanic center as a polycyclic volcano, i.e. the Lathrop Wells Cone may have erupted more than once. If we extend our analysis one step further, assuming that there are three additional eruptions associated with this cone, the data become: 1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01, 0.01, 0.01, 0.01. This hypothetical data set yields $\hat{\delta} = 1.50$, suggesting a stronger developing time trend although it is still not significant (p-value $\doteq 0.25$) at the 0.05 level. The estimated instantaneous recurrence rate is 10^{-5} yr, about twice as large as the previous calculation. An estimate of the mean time to the next eruption is 9.7×10^4 years from now, about 50% sooner than the previous estimate. Also, the estimated risk for an isolation time of 10^4 yr is about 10%, increasing to 64% if 10^5 yr is the required isolation time. The numerical results demonstrate that the Weibull model reflects the time trend and the probability calculations properly.

Summary and conclusions

Either 10^4 or 10^5 years is recommended as the required isolation period during which radioactive waste may decay to an acceptable level (see Crowe, 1986). Thus, this period is the minimum length of time for which future volcanic hazards must be forecasted. The evaluation of probabilities are based on this time frame. We now conclude this section with a few comments and point to some further work.

(1) Analysis on the post-6-Ma volcanism near the Yucca Mountain region indicates a moderate developing time trend (p-value $\doteq 0.01$) of volcanic activity. A similar trend is obtained by trimming the observation period to 3.7 Ma and younger (period of the youngest episode). Thus, it would oversimplify the assessment of the volcanic risk to the proposed Yucca Mountain repository site if a simple Poisson model were used to model the volcanism.

(2) Data from the Quaternary basaltic volcanism also show a slight developing time trend, although the developing volcanic activity is not significant at the 0.05 level. A point estimate of the next major eruption is 1.8×10^5 yr, which is approximately twice as long as the required time frame (10^5 yr), and the lower bound of a 90% confidence interval for the next eruption is 8×10^4 yr.

(3) The risk (probability of at least one major eruption) during the isolation time, t_0 yr, is linear in t_0 for $t_0 \lesssim 10^4$:

$$Pr\text{ (at least one major eruption during } t_0 \text{ yr)}$$
$$= 1 - \exp[-\hat{\lambda}(t)t_0]$$
$$= \hat{\lambda}(t)t_0 - \frac{[\hat{\lambda}(t)t_0]^2}{2!} + \frac{[\hat{\lambda}(t)t_0]^3}{3!} + \ldots$$
$$\doteq \hat{\lambda}(t)t_0\ [= t_0/\hat{M}(t)], \quad \text{for } t_0 \lesssim 10^4$$

(4) Based on the Quaternary data, the numerical value of the estimated instantaneous recurrence rate, 5.5×10^{-6}, is the same as that of the risk during the first year. However, this probability should be interpreted with caution. It should not be stated and evaluated on the yearly basis in volcanic risk assessment. For example, the following statements are misleading:

> The **annual** disease-free survival rate after operation of lung cancer patients is *xx* percent.
> *The estimated **annual** probability of future volcanic eruptions is 5.5×10^{-6}.*

The following statements, which reflect the projected time frame, are more informative.

> The overall cumulative five-year disease-free survival rate is *xx* percent.
> *The estimated risk for an isolation time of 10^4 years is about 5%, which increases to 42% if 10^5 years is the required isolation time.*

(5) Despite the fact that there are well-recognized means of gathering data in the NTS region, many considerations are still unknown (e.g., the precision and accuracy of dates, number of buried volcanic centers, etc.). The following points are extremely important for statistical modeling: Crowe et al. (1989) and Wells et al. (1990) now recognize and classify the Lathrop Wells volcanic center as a polycyclic volcano so some cones may have erupted more than once; a referee points out that the 3.7-Ma units include at least 4 centers; and, according to geologist Daniel Feuerbach (pers. commun., 1989), there are 8–12 vents at Red Cone volcanic center. The last point indicates that the recurrence rate would be underestimated if these nearby vents have distinguishable ages (see Ho et al., 1989). We have also shown the estimated recurrence rate would be doubled, provided the Lathrop Wells volcano has erupted 4 times. These considerations are valuable. Further developments are necessary to complete and document those points previously mentioned to ensure that volcanic risk assessment is based on an adequate characterization of the volcanic record of the Yucca Mountain region. In this article, the time trend analysis does not completely take into account such possibilities as polygenetic and polycyclic volcanism, since at this preliminary stage of our work data for the Yucca Mountain region are incomplete. Our efforts for future studies will be devoted to considerably more detailed data collection and statistical modeling of site disruption.

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References

Ascher, H., 1983. Discussion on Statistical Methods in Reliability, by J.F. Lawless. Technometrics, 25: 305–335.

Bacon, C.R., 1982. Time-predictable bimodal volcanism in the Coso Range, California. Geology, 10: 65–69.

Bain, L.J., 1978. Statistical Analysis of Reliability and Life-Testing Models. Marcel Dekker, New York, N.Y., 450 pp.

Byers Jr., F.M., Carr, W.J. and Orkild, P.P., 1989. Volcanic centers of southwestern Nevada: evolution of understanding: 1960–1988. J. Geophys. Res., 94: 5908–5924.

Christiansen, P.W., Lipman, P.W., Carr, W.J., Byers Jr., F.M., Orkild, P.P. and Sargent, K.A., 1977. The Timber Mountains–Oasis Caldera complex of southern Nevada. Geol. Soc. Am. Bull., 88: 943–959.

Condit, C.D., Crumpler, L.S., Aubele, J.C. and Elston, W.E., 1989. Patterns of volcanism along the southern margin of the Colorado plateau: the Springerville field. J. Geophys. Res., 94: 7975–7986.

Crow, L.H., 1974. Reliability analysis for complex, repairable systems. In: F. Proschan and R.J. Serfling (Editors), Reliability and Biometry. SIAM, Philadelphia, Penn., pp. 392–410.

Crow, L.H., 1982. Confidence interval procedures for the Weibull process with applications to reliability growth. Technometrics, 24: 67–72.

Crowe, B.M., 1980. Disruptive Event Analysis: Volcanism and Igneous Intrusion. Battelle Pacific Northwest Laboratory Rep. PNL - 2882.

Crowe, B.M., 1986. Volcanic hazard assessment for disposal of high-level radioactive waste. In: Active Tectonics: Impact on Society. National Academy Press, Washington, D.C., pp. 247–260.

Crowe, B.M. and Carr, W.J., 1980. Preliminary Assessment of the Risk of Volcanism at a Proposed Nuclear Waste Repository in the Southern Great Basin. U.S. Geol. Surv., Open-File Rep. 80-357, 15 pp.

Crowe, B.M. and Perry, F.V., 1989. Volcanic probability calculations for the Yucca Mountain site: estimation of volcanic rates. Focus '89, Nuclear Waste Isolation in the Unsaturated Zone, September, 1989, hosted by the Nevada section of the American Nuclear Society, pp. 326–334.

Crowe, B.M., Johnson, M.E. and Beckman, R.J., 1982. Calculation of the probability of volcanic disruption of a high-level radioactive waste repository within southern Nevada, U.S.A. Radioactive Waste Management, 3: 167–190.

Crowe, B.M., Self, S., Vaniman, D., Amos, R. and Perry, F., 1983. Aspects of potential magmatic disruption of a high-level radioactive waste repository in southern Nevada. J. Geol., 91: 259–276.

Crowe, B.M., Harrington, C., Turrin, B., Champion, D., Wells, S., Perry, F., McFadden, L. and Renault, C., 1989. Volcanic hazard studies for the Yucca Mountain Project. Waste Management 89, Volume 1—High Level Waste and General Interest, pp. 485–491.

Department of Energy, 1986. Final Environmental Assessment: Yucca Mountain site, Nevada Research and Development, Nevada, Washington, D.C., DOE/RW-0073.

Ho, C.-H., 1990. Bayesian analysis of volcanic eruptions. J. Volcanol. Geotherm. Res., 43: 91–98.

Ho, C.-H., 1991. Nonhomogeneous poisson model for volcanic eruptions. Math. Geol., 23: 167–173.

Ho, C.-H., Smith, E.I., Feuerbach, D.L. and Nauman, T.R., 1989. Eruptive probability calculation for the Yucca Mountain site, U.S.A.: statistical estimation of recurrence rates. Unpubl. paper submitted to Bull. Volcanol.

Klein, F.W., 1982. Patterns of historical eruptions at Hawaii volcanoes. J. Volcanol. Geotherm. Res., 12: 1–35.

Klein, F.W., 1984. Eruption forecasting at Kilauea Volcano, Hawaii. J. Geophys. Res., 89: 3059–3073.

Lawless, J.F., 1982. Statistical Models and Methods for Lifetime Data. John Wiley, New York, N.Y., 580 pp.

Mann, N.R., Schafer, R.E. and Singparwalla, N.D., 1974. Methods for Statistical Analysis of Reliability and Lifetime Data. John Wiley, New York, N.Y., 564 pp.

Mulargia, F., Tinti, S. and Boschi, E., 1985. A statistical analysis of flank eruptions on Etna volcano. J. Volcanol. Geotherm. Res., 23: 263–272.

Mulargia, F., Gasperini, P. and Tinti, S., 1987. Identifying different regimes in eruptive activity: an application to Etna volcano. J. Volcanol. Geotherm. Res., 34: 89–106.

Nelson, W.B., 1982. Applied Life Data Analysis. John Wiley, New York, N.Y., 634 pp.

Parzen, E., 1962. Stochastic Processes. Holden-Day, San Francisco, Calif., 324 pp.

Shaw, H.R., 1980. The fracture mechanisms of magma transport from the mantle to the surface. In: R.B. Hargraves (Editor), Physics of Magmatic Transport. Princeton University Press, Princeton, N.J., pp. 201–264.

Shaw, H.R., 1987. Uniqueness of volcanic systems. In: R.W. Decker, T.L. Wright and P.H. Staufer (Editors), Volcanism in Hawaii. U.S. Geol. Surv., Prof. Pap., 1350: 1357–1394.

Smith, E.I., Feuerbach, D.L., Naumann, T.R. and Faulds, J.E., 1990. The area of most recent volcanism near Yucca Mountain, Nevada: implications for volcanic risk assessment. Proc. Int. Topical Meeting, High-Level Radioactive Waste Management. Am. Nucl. Soc./Am. Soc. Civ. Eng., 1: 81–90.

Steel, R.G.D. and Torrie, J.H., 1980. Principles and Procedures of Statistics (2nd ed.). McGraw-Hill, New York, N.Y., 633 pp.

Vaniman, D.T., Crowe, B.M. and Gladney, E.S., 1982. Petrology and geochemistry of hawaiite lavas from Crater Flat, Nevada. Contrib. Mineral. Petrol., 80: 341–357.

Wadge, G., 1982. Steady state volcanism: evidence from eruption histories of polygenetic volcanoes. J. Geophys. Res., 87: 4035–4049.

Wells, S.G., McFadden, L.D., Renault, C.E., Turrin, B.D. and Crowe, B.M., 1988. A geomorphic assessment of Quaternary volcanism in the Yucca Mountain area, Nevada Test Site, southern Nevada. Proc. Geol. Soc. Am., Cordilleran Section, Las Vegas, Nevada, March, Vol. 20, p. 242.

Wells, S.G., McFadden, L.D., Renault, C.E. and Crowe, B.M., 1990. Geomorphic assessment of Late Quaternary volcanism in the Yucca Mountain area, southern Nevada: implications for the proposed high-level radioactive waste repository. Geology, 18: 549–553.

Wickman, F.E., 1966. Repose-period patterns of volcanoes. Ark. Mineral. Geol., 4: 291–367.

Wickman, F.E., 1976. Markov models of repose-period patterns of volcanoes. In: D.F. Merriam (Editor), Random Processes in Geology. Springer, New York, N.Y., pp. 135–161.

APPENDIX III

III. Ho, C.-H. 1991. Predictions of Volcanic Eruptions at Mt. Vesuvius, Italy, Journal of Geodynamics, (accepted)

PREDICTIONS OF VOLCANIC ERUPTIONS AT Mt. VESUVIUS, ITALY

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ABSTRACT

We present a mathematically-based model for predicting the likelihood of future eruptions of Vesuvius and, by implication, other volcanoes. The volcanic activity of Vesuvius in the period 1630-1989 is described by a compound Poisson distribution with a gamma compounding density. The frequency distribution of eruptions in any given interval of equal length follows a negative binomial distribution (NBD). The assumptions of the NBD model are less restrictive, so the observation time can be extended to include the long repose following the 1944 eruption of Vesuvius. Moreover, this exceedingly flexible model has only two parameters which can be determined easily from the eruptive count data. The future probability of x number of eruptions is predicted on the aggregate behavior of past volcanic activity. This capability would be useful for long-term planning, such as for land-use development, although not for short-term forecasts of volcanic hazards.

INTRODUCTION

Vesuvius is a volcano on the shore of the Bay of Naples in central Italy. Located in an area of Europe that has been populated for almost 3000 years, more is known of the eruptive history of Vesuvius than of almost any other volcano (Bullard, 1984).

Since AD 79, Vesuvius has been rather active, with major eruptions occurring in 472, 512, 685, 993, 1036, 1049, 1138, 1139, and 1631. The eruption of 1631 is particularly significant, as previously, Vesuvius had a pattern of long periods of repose between eruptions. Since 1631, however, Vesuvius has been in a constant state of activity, with

noteworthy large eruptions in 1779, 1794, 1822, 1838, 1850, 1872, 1906, and 1944. There has been no major activity since 1944, although that eruption did not seem to signal the end of a cycle. In view of the last years of the history of Vesuvius, the repose following the 1944 eruption is so long that a complete probability model of the Vesuvius activity in the last 360 years (1630-1989) needs to be developed.

Wickman (1966) uses a series of repose states characterized by time-independent rate parameters to describe the repose-period patterns of Vesuvius. In other words, Vesuvius is modeled as a sequence of activity states (Markov chains), with the duration of the states being random variables distributed according to an exponential probability density function. Wickman's assumptions are verified by Carta *et al.* (1981), who divide the periods into a series of eruptive cycles. These cycles are characterized by four states: repose (R), persistent activity (A), intermediate eruption (IE), and final eruption (FE). To model the duration of the states, Carta *et al.* (1981) favor Wickman's hypothesis that they are random variables from exponential distributions of form $f_s(x) = \lambda_s e^{-\lambda_s x}$ where s labels the states. To determine λ_s, Carta *et al.* (1981) suggest two methods: $\bar{x} = s^2 = 1/\lambda_s$, where $\bar{x}$ is the sample mean, and s^2 is the sample variance; and minimizing $F(t) - \int_0^t f(x)dx$, where f(x) is the exponential probability density function, and F(t) is the cumulative distribution function of the data. However, the models presented by Wickman (1966) and Carta *et al.* (1981) are based on the assumption that the conditions of Vesuvius

mainly changed stepwise. Moreover, the period from 1944 to the present is excluded, because their aim is obviously not the prediction of the outbreaks of Vesuvius, but rather a description of its general pattern of activity. In spite of these limitations, their results have positive relevance and serve as the foundation of the present work, which provides a more general model for Vesuvius.

COMPOUND POISSON MODEL

As observed by Wickman (1966) and Carta *et al.* (1981), Vesuvius has complex distribution functions which can be reproduced, at least qualitatively, by simple statistical models. These models are characterized by chains of different states of the volcanic activity (Markov chains). The times of permanence in each state are assumed to be distributed according to an exponential distribution and the transition probabilities from one state to the subsequent one are assumed to be time independent. In other words, the number of volcanic eruptions in [0,t] follows a Poisson process with recurrence rate $\mu=\lambda t$; however, the average recurrence rate in unit time λ may be different from one state to the subsequent one, or, more generally, from one period of time to the subsequent one. Notice that we have directly transformed their results from a sequence of repose times between eruptions to Poisson count data, because if eruptions are occurring according to a Poisson process, the repose times between consecutive eruptions are independent exponential variables. The following mathematical development is based on the eruptive count data, which are more physically meaningful, reliable, and consistent than measurements

based on repose times, magmatic volume, or other geologic variables. In searching for a more flexible model for Vesuvius, we use the idea of Ho (1990) and assume that λ is a continuous random variable that follows a probability density function $g(\lambda)$. There is one more reason that suggests this generalization, which is also suitable for other volcanoes. Although eruptions are caused by specific physical events or processes, there might be many causal factors with random influences on the sequence of eruptions. As a result, parameter λ is a random variable. We now turn to the modification of the Poisson scheme which is derived by supposing that the average recurrence rates in successive time periods of equal length are not the same. Without loss of generality, each observation period will be treated as unit time, i.e. t=1. One naturally commences with the assumption that $g(\lambda)$ has a normal distribution. But the assumption of a normal distribution cannot be justified because values of λ may range from zero far in the positive direction so that $g(\lambda)$ would be skewed. The distribution of λ we adopt is the gamma distribution. This is plausible, because such a distribution is fairly flexible (having two adjustable parameters) and the right shape (*i.e.* a continuous distribution for non-negative values that is reversed-j-shaped or hump-backed and always positively skewed). If λ follows a gamma distribution, GAM(r,α), then

$$g(\lambda) = \frac{\alpha^r}{\Gamma(r)} \lambda^{r-1} e^{-\alpha\lambda}, \ \lambda > 0; \ r, \ \alpha > 0.$$

Treating the average eruptive rate as a random variable λ means that the probability distribution function $f(x;\lambda)$ is actually a conditional

probability, the condition being that λ is in state λ. Thus, when using a probability distribution for λ, it is more appropriate to use the notation $f(x|\lambda)$ for the data X, so

$$f(x|\lambda) = e^{-\lambda}\lambda^{x}/x!, \; x = 0, 1, \ldots .$$

From the conditional distribution of X and the given distribution for λ, we can calculate the joint distribution of (x,λ):

$$f(x,\lambda) = f(x|\lambda)g(\lambda),$$

and the corresponding marginal or absolute distribution of X, with probability

$$\begin{aligned} P(x) &= E_g[f(x,\lambda)] = \int f(x|\lambda)g(\lambda)d\lambda \\ &= \int_0^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!} \frac{a^{r}}{\Gamma(r)} \lambda^{r-1}e^{-a\lambda}d\lambda \\ &= \frac{\Gamma(r+x)}{\Gamma(r)x!} \left[\frac{a}{a+1} \right]^{r} \left[\frac{1}{a+1} \right]^{x}, \; x = 0, 1, \ldots, \qquad (1) \end{aligned}$$

which is a form of negative binomial distribution (abbreviated NBD) that is referred to as a compound Poisson distribution with a gamma compounding density. Interest in the marginal distribution centers around the fact that, if X has the conditional density $f(x|\lambda)$ and λ actually is random with density $g(\lambda)$, P(x) is the probability according to which x will actually occur. For this reason, P(x) is sometimes called the predictive distribution for X because it describes what one would "predict" that X would be. If the future observation time interval increases by a factor of t, a in the right-hand side of (1) is replaced by at.

EMPIRICAL RESULTS OF VESUVIUS

In spite of the fact that maximum likelihood estimates for the NBD

were given (implicitly) by Anscombe (1950), the two common techniques for estimation of (r,a) [required to evaluate (1)] have been the method of moments and fitting the mean and zeros (Anscombe, 1950). The second method is more attractive in terms of efficiency. The technique is to equate the observed proportion of zero counts P_0 to its theoretical value, *i.e.* to write

$$\left[\frac{\hat{a}}{\hat{a}+1} \right]^{\hat{a}\bar{x}} = P_0, \tag{2}$$

and

$$\hat{r} = \hat{a}\bar{x}, \tag{3}$$

where $\bar{x}$ is the observed sample mean. Equation 2 is solved for $\hat{a}$ by iteration. Once $\hat{a}$ and $\hat{r}$ are estimated, the probability of future eruptions can be calculated based on (1). The eruptive data of Vesuvius (Simkin *et al.*, 1981) for the period 1630-1989 (Table 1) will be analyzed in detail. Based on Table 1, the year 1630 was taken as a starting point and the period 1630-1989 was divided into 72 five-year intervals. The empirical distribution provides sample variance s^2 = 0.873, sample mean $\bar{x}$ = 0.833, and P_0 = 0.444. We then use the generalized model (NBD) to fit this data set. Equations 2 and 3 yield $\hat{a}$ = 19.104 and $\hat{r}$ = 15.914, and the observed and theoretical values of P(x) based on Equation 1 are summarized (Table 2). Note that the observed and theoretical frequencies at x = 0 are always identical (except for some rounding errors) because the parameter estimates are obtained by fitting the sample mean and observed proportion of zero eruptions. Therefore, this model reflects 100 percent accuracy for

fitting the theoretical value of P(0) to the observed data, and thus fully incorporates the information provided by the present repose time from 1944 till present. Though the fit looks extremely good, a chi-square goodness-of-fit test (*e.g.*, Steel and Torrie, 1980, p. 529) is performed to check the adequacy of the fit of the NBD. The test statistic is 0.02, which leads to p-value $\doteq 1$, and indicates that the fit is near perfect (a rule of thumb suggests that the test is not reliable if some expected counts are less than 5 or if the degrees of freedom are too small, which is often the case for volcanic eruptive data).

Present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of future activity to be put forward. Attempts at long-term forecasting can only be made on statistical grounds, using historical records to select an adequate model and to calculate eruptive probabilities. For example, according to Equation 1, if a time span of 5 years is chosen for the future observation,

$P_0 = 0.444$, $P_1 = 0.352$, $P_2 = 0.148$, $P_3 = 0.044$, $P_4 = 0.010$, $P_5 = .002$,

where the subscript indicates the number of eruptions in the interval. Hence, no eruptions most likely will occur for Vesuvius during 1990-1994; however, the probability of at least one eruption is 0.556. Indeed, for a compound Poisson model, the recurrence rate which has been postulated as a random variable need not be observable in any sense. It is only necessary to assume that in any one period of time

the recurrence rates behave as if they were a random sample from a gamma distribution.

CONCLUSION

The evaluation of eruptive probabilities for a given volcano remains an open problem in the definition of volcanic risk. Many volcanoes with repose times of hundreds or thousands of years are commonly regarded as extinct, with activity occurring before written records remembered only in myths and legends. With an unexpected renewal of activity, this lack of awareness often generates a panic which can cause more economic damage than the eruption itself. A knowledge of eruptive mechanisms might provide information about the probability of an eruption. However, the only available defense against an eruption, apart from evaluation, remains the implementation of suitable land management policies such that the potential losses in vulnerable zones are brought to a minimum (Tazieff and Sabroux 1983, Ch. 6). Little has been done successfully to modify the course of an eruption. For Vesuvius, the exact time and consequences of an eruption in the near future are difficult to assess with certainty. The probability model developed in this paper deals with some of the purely statistical studies which can be incorporated into a social framework. Social and economic studies of the problems related to the volcanism of Vesuvius are as important as the study of why Vesuvius behaves as it does.

ACKNOWLEDGEMENTS

The author is grateful to Andy Tsang and David Lerman for excellent computational services and literature review. The work was supported through the Nevada Nuclear Waste Project Office.

REFERENCES

Anscombe, F.J., 1950. Sampling theory of the negative binomial and logarithmic distributions. Biometrika, v. 37, p. 358-382.

Bullard, F.M., 1984. Volcanoes of the earth (2nd ed.). University of Texas Press, Austin, Texas, 629 pp.

Carta, S., Figari, R., Sartoris, G., Sassi, E., and Scandone, R., 1981. A statistical model for vesuvius and its volcanological implications. Bull. Volcanol., v. 44, p. 129-151.

Ho, C.-H., 1990. Bayesian analysis of volcanic eruptions. Journal of Volcanology and Geothermal Research, v. 43, p. 91-98.

Simkin, T., Siebert, L., McClelland, L., Bridge, D., Newhall, C., and Latter, J.H., 1981. Volcanoes of the world. Smithsonian Institution and Hutchinson Ross, Stroudsburg, Pennsylvania, 232 pp.

Steel, R.G.D., and Torrie, J.H., 1980. Principles and procedures of statistics (2nd ed.). McGraw-Hill, New York, 633 pp.

Tazieff, H., and Sabroux, J.C., (Editors), 1983. Forecasting volcanic events. Elsevier, New York, 635 pp.

Wickman, F.E., 1966. Repose-period patterns of volcanoes. Ark. Mineral. Geolo., v. 4, p. 291-367.

TABLE 1

YEARS OF ERUPTIONS OF VESUVIUS*

FOR THE PERIOD 1630 - 1989

1631	1699	1770	1837	1872
1649	1701	1771	1839	1874
1649	1707	1779	1841	1881
1660	1712	1783	1850	1891
1663	1737	1794	1854	1895
1680	1744	1798	1855	1900
1682	1751	1804	1856	1906
1685	1754	1810	1858	1913
1694	1760	1813	1861	1929
1696	1764	1822	1864	1941
1696	1766	1826	1868	1942
1698	1767	1834	1870	1944

* Periods of low-level constant activity are ignored.

TABLE 2

SUMMARY OF THE FIT OF A NEGATIVE BINOMIAL DISTRIBUTION

*Number of Eruptions	Frequencies Observed	Theoretical
0	32	32.0
1	25	25.3
2	11	10.6
3	3	3.2
4	1	0.7
5+	0	0.2

* Based on 72 five-year intervals (1630-1989)

APPENDIX IV

IV. Review of the Los Alamos National Laboratory Study Plan 8.3.1.8.1.1, Probability of Magmatic Disruption of the Repository

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1 Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Can the Issue be Resolved?

Study plan 8.3.1.8.1.1 deals with questions of great importance. There is no doubt that the phenomenon is stochastic, and the answer is necessarily probabilistic.

This study plan has many pitfalls. The plan is compartmentalized into activities without a well-defined procedure for unifying the data obtained in each activity. In some instances, the results of one activity could have a drastic effect on another activity. The plan does not adequately address how it will incorporate the data collected in the various activities into a single model which answers the question at hand. Additionally, some of the mathematical techniques proposed do not apply to this problem. Some of the models are based on faulty assumptions, and others are just plainly misused. There are places where the plan lacks detail as to how it will address certain issues. The plan does not have a complete and convincing method for determining the probability of future disruption by volcanic activity at the nuclear waste repository.

The reviewer suggests a less divided approach to solving this problem. One must be prepared to handle all foreseeable problems such as the detection of the presence of magma. More care needs to be taken in combining the data collected in the different activities. A detailed plan for incorporating all **reasonable** mathematical models must be developed.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1
Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Activity: 8.3.1.8.1.1.1 Location and Timing of Volcanic Events.

Will Proposed Activity Achieve Objective(s)?:

This activity will provide greater knowledge of the volcanic history of the Yucca Mountain region.

Is the Scope Of The Activity Reasonable?:

The activity will provide information from many different sources concerning the location and timing of volcanic activity in the past. The investigators must, however, insure the data collected from other sources are suitable for use in other activities discussed later in the plan.

Will The Proposed Methodologies and/or Techniques Provide Representative Data?:

There really are no methodologies and techniques other than using data from another study (see Los Alamos National Laboratory Study Plan 8.3.1.8.5.1). It is extremely important, however, to insure the data is representative. Depending on the procedures used in the other tests, the data collected may or may not be a fair representation of the location and timing of future volcanic events. There have been criticisms that some of the dating tools mentioned in Los Alamos National Laboratory Study Plan 8.3.1.8.5.1 are untested and controversial. Other techniques are felt to be not as well suited to certain tasks as others available (see Smith, Eugene, "Review of Los Alamos National Laboratory Study Plan 8.3.1.8.5.1 Characterization of Volcanic Features."). Have these criticisms been addressed?

Other Comments:

During this activity, the investigators should never lose sight of their higher goal. That goal is to predict the location and timing of future volcanic events. This is an extremely complicated task and must not be taken lightly. It is not enough to simply quantify the location and timing of past volcanic activity, and even that goal may not be so simple.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1
Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Activity: 8.3.1.8.1.1.2 Evaluation of the Structural Controls of Basaltic Volcanic Activity.

Will Proposed Activity Achieve Objective(s)?:

Various limitations and flaws mentioned below raise doubts as to the applicability of results from this activity as currently planned. The activity assumes sufficient models will be developed to accomplish this task. Some of the models already developed and presented in this plan are based on faulty assumptions.

Is the Scope Of The Activity Reasonable?:

The activity assumes many different models will be tested and used, but it does not mathematically define how it will be determined if a model is good or not (p. 21).

Will The Proposed Methodologies and/or Techniques Provide Reasonable Parameter Values?:

No, some of the models are based on faulty assumptions. Model 1 (p. 22) of the deterministic models mentioned lacks validity. The investigators use a simple linear regression model based on the longitude and latitude of volcanic centers to derive confidence bands for the fitted regression line. The probability of disruption is calculated as $P_{rd} = 1 - C_l$, where C_l is the level of significance of the confidence band that intersects Yucca Mountain. There are several flaws in this approach:

Flaw No. 1 No randomness remains after the investigators construct a particular confidence band. Probability $(1 - C_l)$ in its interpretation as long-term relative frequency makes no sense in this situation.

Flaw No. 2 There is an assumption that all future volcanic activity will occur only along that regression line (with normal variation). There are two problems with this assumption. No geophysical reasons appear to be given to justify this restriction, and it is assumed that the regression lines shown are the best fit. The similarity of the lines (Fig. 3, p. 23), however, is due to the presence of two influential extreme cases. Were these cases treated as outliers and removed, the new regression lines would be far different, probably approaching far closer to Yucca

Mountain. Also, the small number of data points used make it difficult to test normality of the error. In the absence of normality, the 'best fit' need not be the line found by linear regression. In any case, latitude and longitude are both random variables, and while a linear regression can be done of one random variable on another, there are certain restrictions on their underlying conditional probability distributions for such a regression to be statistically valid.

Flaw No. 3 The investigators assume in calculating the probability of repository disruption as $P_{rd} = 1 - C_l$ that only a bullseye volcanic event at the site could cause repository disruption, which is questionable.

Other Comments:

Model 2 (p. 24) may assume correctly that the strike-slip fault of Crater Flat will control future volcanic activity, but the plan does not mention how the data in Table 2 (p. 26) will be used to determine the probability of disruption at the repository. The same argument can be made for the data in Table 3 from Model 3. Legitimate mathematical models must first be developed and shown to be reasonable for predicting future volcanic events. A detailed plan must also be developed to incorporate these models if more than one reasonable model is found.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1 Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Activity: 8.3.1.8.1.1.3 Presence of Magma Bodies in the Vicinity of the Site.

Will Proposed Activity Achieve Objective(s)?:

This activity will provide useful data to detect the presence of magma bodies in the vicinity of the site. However, the plan does not propose a detailed procedure for confirming the presence of magma.

Is the Scope Of The Activity Reasonable?:

The activity uses different types of tests for detecting the presence of magma. The presence of magma would have a significant impact on the results of the rest of the plan. Therefore, a detailed procedure of how the investigators intend to handle the situation of the presence of magma is essential.

Will The Proposed Methodologies and/or Techniques Provide Representative Data?:

Insufficient knowledge to comment.

Other Comments:

How long will it take to develop a plan if the possibility of the presence of magma is determined? What will be the impact on the other activities of positive identification of magma? This seems to be the most critical activity, but the plan does not seem to recognize this fact. The investigators can not fail to keep the "big picture" in mind. The results of this activity could greatly alter the other activities and the final result. This issue needs to be addressed by something more than is currently found in the plan.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1
Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Activity: 8.3.1.8.1.1.4 Probability Calculation and Assessment.

Will Proposed Activity Achieve Objective(s)?:

This activity has many areas that could preclude finding a true answer to the probability of future disruption by volcanic activity at the repository. The probability model developed is incorrect and much of the data to be collected in other activities is suspect (e.g, data from the Cima and Lunar Crater fields, see p. 39). The collection of the data will also be difficult.

Is the Scope Of The Activity Reasonable?:

No, the activity attempts to incorporate many different models to estimate the probability parameters. This will be a difficult task for which there is no plan (e.g., model adequacy checking, parameter estimation techniques related to each model, etc.) developed. The weighting system, by expert opinion (p. 40), is not well-defined. The proposed spreadsheet matrix (p. 40) is not really a plan to assemble the data.

Will The Proposed Methodologies and/or Techniques Provide Reasonable Parameter Values?:

This activity has some major problems and some minor problems. The major problems seriously endanger the cogency of the investigators' work. These are:

1. Equation 2 (p. 30) is not suitable for the stated purpose. The plan calls Equation 2 a conditional probability. It is not a conditional probability because there are no defined events.

2. Equation 3 (p. 31) requires restrictive and hierarchical model assumptions that are treated rather casually by the investigators. The investigators seem to be unnecessarily limiting themselves in the choice of available statistical models to a simple Poisson model even before the completion of Activity 8.3.1.8.1.1.1 (location and timing of volcanic events).

3. The investigators claim that a probabilistic approach requires a random or exponential distribution of the data (p. 42). This is a misleading or erroneous claim. Data, even when random, can show an overall structure, temporally or spatially. Also, the investigators state a uniform rate of activity is required for a

stochastic approach (bottom of p. 42). Wrong. Consider the nonhomogeneous Poisson process as just one counterexample.

4. There is no such thing as "annual probability" (p. 32, 33, etc.) in volcanic risk assessment studies unless the projected life of the geologic repository is only one year.

Other Comments:

The proposed methods of estimating the recurrence rate parameter and the disruption parameter will almost certainly produce a wide range of results. If this is so, the proposed method of combining all the models into a spreadsheet which produces a combined probability distribution is questionable. A probability model suitable for this activity must be developed after the completion of Activity 8.3.1.8.1.1.1. Additionally, a method of combining all the inputs from the different proposed models must be defined.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1
Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

Sequence of Activities:

There is a possibility that the sequence of events could be seriously disrupted. The decision document to amend the study plan (p. 46) is done after everything else. If magma is found, there is no need to do anything else because everything else is affected by the outcome of Activity 3. A decision document to amend the study plan in the event of the presence of magma needs to be developed before any probability decisions are made. Perhaps, the investigators could combine Activities 1 & 3.

Review of Los Alamos National Laboratory Study Plan 8.3.1.8.1.1
Probability of Magmatic Disruption of the Repository

Prepared by Chih-Hsiang Ho, University of Nevada, Las Vegas

References:
The reference list is incomplete. Some texts on introductory probability and statistics should be adequately referenced to eliminate noted errors that undermine confidence in the validity of the study results.

APPENDIX V

V. "Prediction of Volcanic Eruptions: An Application to the Yucca Mountain Site, U.S.A.," abstract presented at the International Conference on Active Volcanoes and Risk Mitigation held in Napoli, Italy, 8/27/91 - 9/1/91.

Prediction of Volcanic Eruptions: An Application to the Yucca Mountain Site, U.S.A.

by

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Abstract

In the United States, spent fuel and high-level radioactive waste will be permanently disposed of in a geologic repository. Disposal of the spent fuel and high-level waste is scheduled to begin in the year 2010. The candidate site for the first U.S. geologic repository is located at Yucca Mountain, Nevada, approximately 100 miles, or about 160 kilometers, northwest of Las Vegas, Nevada. Comprehensive studies are underway on the potential host rock formation. These studies are called site characterization. An important element in assessing the suitability (or lack of suitability) of the Yucca Mountain site is an assessment of the potential for future volcanic activity. A potentially adverse condition with respect to volcanism is judged to be of concern at the Yucca Mountain site because of the presence of multiple basalt centers of Quaternary age.

The possible recurrence of volcanic activity near the proposed nuclear waste repository at Yucca Mountain, Nevada, U.S.A. is evaluated by estimating the instantaneous recurrence rate using a nonhomogeneous Poisson process with Weibull intensity and by using a homogeneous Poisson process to predict future eruptions. Based on the Quaternary data, the estimated instantaneous recurrence rate is about 5.5×10^{-6}/yr. An estimate of the mean time to the next eruption is about 1.8×10^{5} years from now, if it is assumed that the intensity remains constant during the projected time frame. Also, the risk (probability of at least one major eruption during the projected time frame) increases approximately linearly with the time frame chosen as the required interval for radioactive waste to decay to an acceptable level. Our study concludes that a 90% confidence interval for the risk for an isolation time of 10^{4} years is (0.02, 0.13).

Key words: Geologic repository; Volcanic risk; Weibull distribution.

Scientific session: Volcanic Hazard and Risk Mitigation

Preferred Type of Presentation: Oral

APPENDIX VI

VI. "Statistical Analysis of Basaltic Volcanism Near the Yucca Mountain Site," presented to the United States Nuclear Waste Technical Review Board in the March 1, 1991 meeting in Tucson on volcanism.

STATISTICAL ANALYSIS OF BASALTIC VOLCANISM NEAR THE YUCCA MOUNTAIN SITE

C. -H. Ho

Department of Mathematical Sciences

University of Nevada, Las Vegas

(This work is supported by the Nevada Nuclear Waste Project Office)

One of the site characterization studies is the

VOLCANISM

near the Yucca Mountain site (approximately 100 miles northwest of Las Vegas, Nevada), candidate site for the first U.S. geologic repository for spent fuel and high-level radioactive waste.

FACTS

1. An important element in assessing the suitability of the site is an assessment of the potential for future volcanic disruption of the repository.

2. The phenomenon is stochastic, the answer is necessarily probabilistic !

GOALS

To estimate

1. the recurrence rate

2. the waiting time of the next eruption

3. the probability of at least one eruption during the next 10,000 years

4. the probability of volcanic disruption of the repository during the next 10,000 years (in progress)

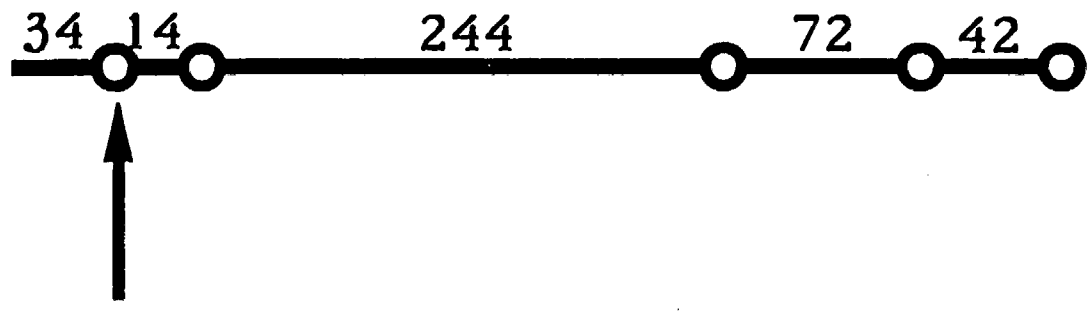
Need a model that captures the basic elements of the study :

1. **Objectivity**
2. **Trend**
3. **Predictability**
4. **Mathematical Simplicity**

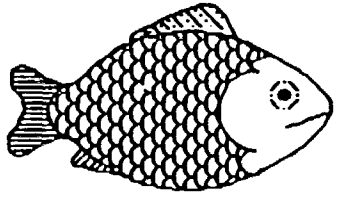
TIME SERIES

generated by stochastic phenomena (events)

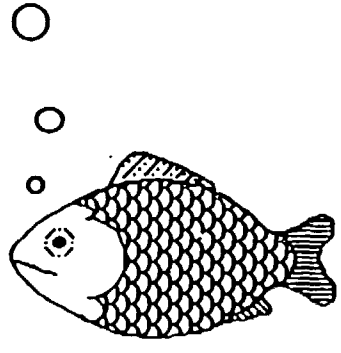
Data : 34, 14, 244, 72, 42 (inter-event times)

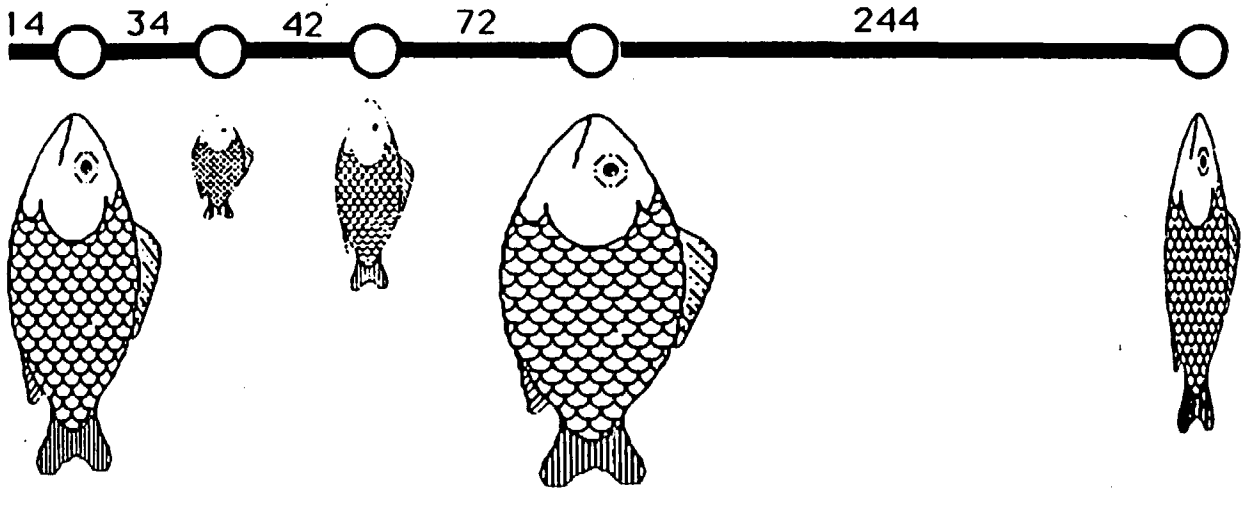


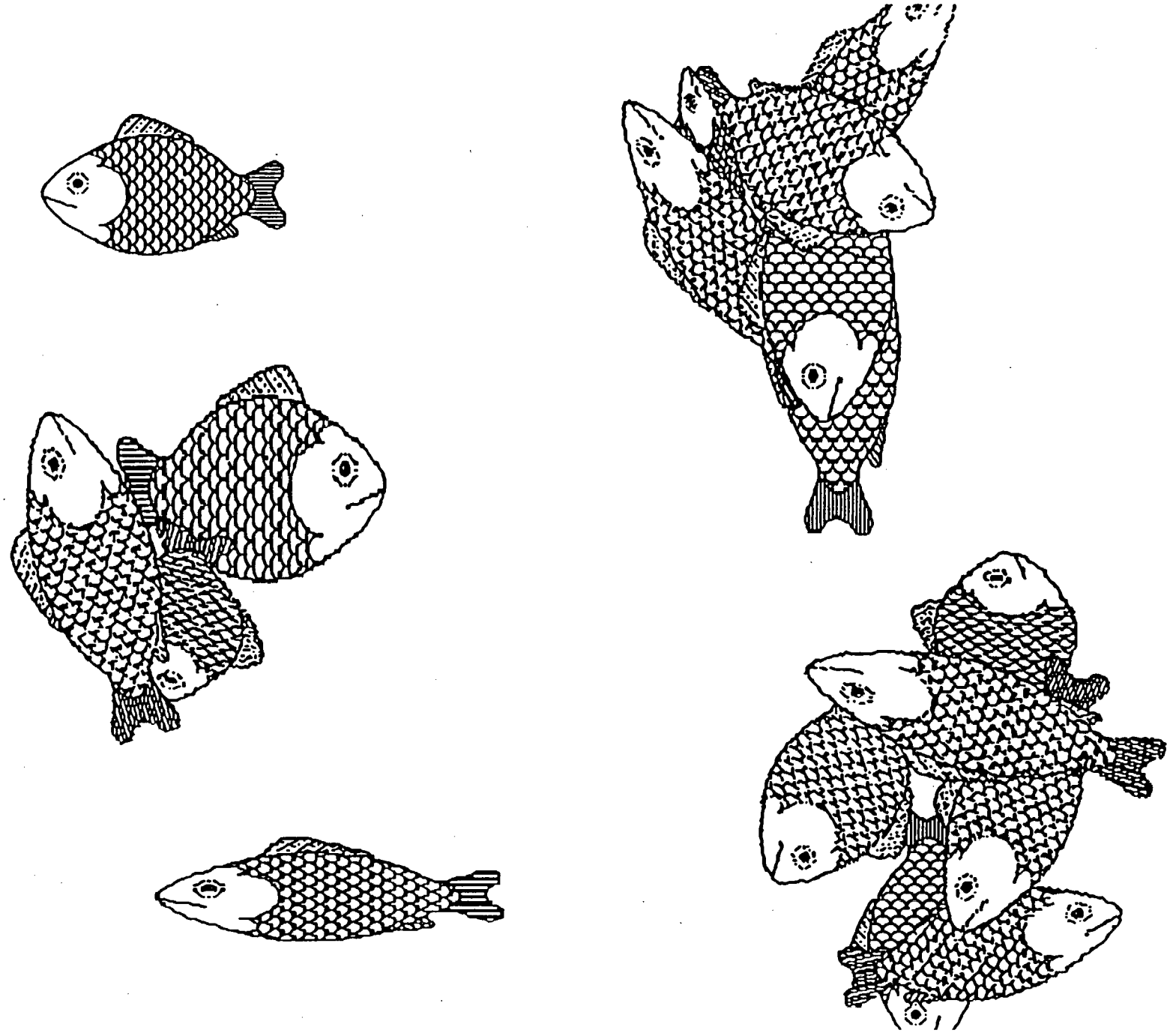
earthquake, volcanic eruption, etc.



And you should have seen
the one that got away!



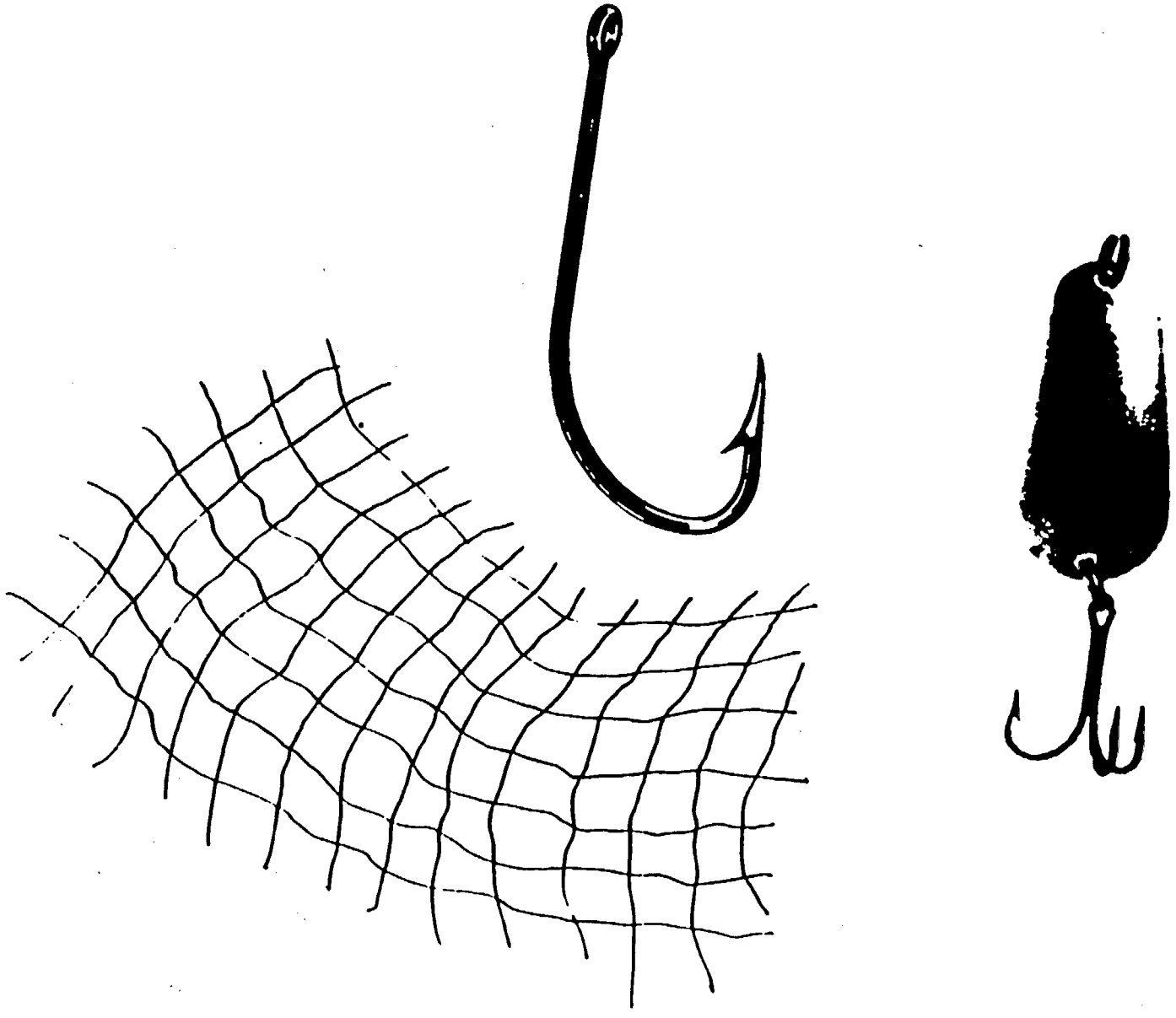




WHAT IS A SINGLE EVENT?

1. Need a clear definition

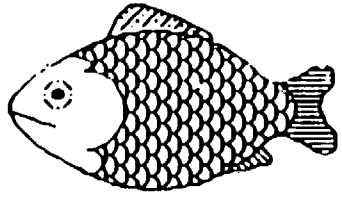
2. Based on the understanding of fishing techniques (or eruptive processes, etc.)



WHAT TO MEASURE?

- variables of interest

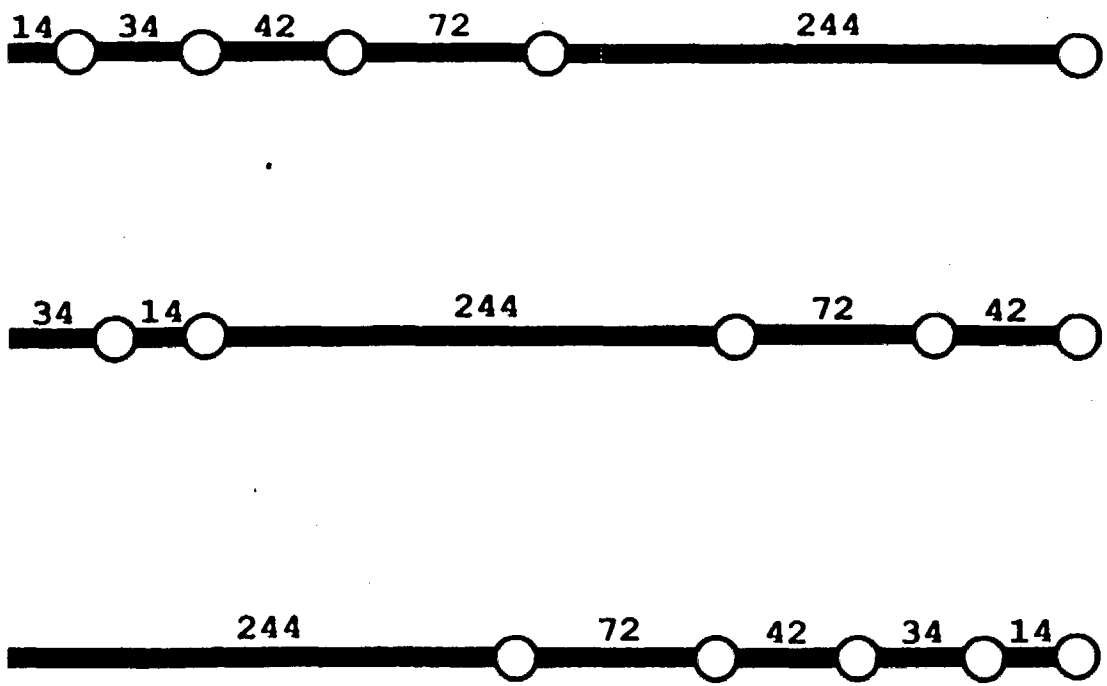
1. length
2. weight
3. volume
4. age
5. freshness

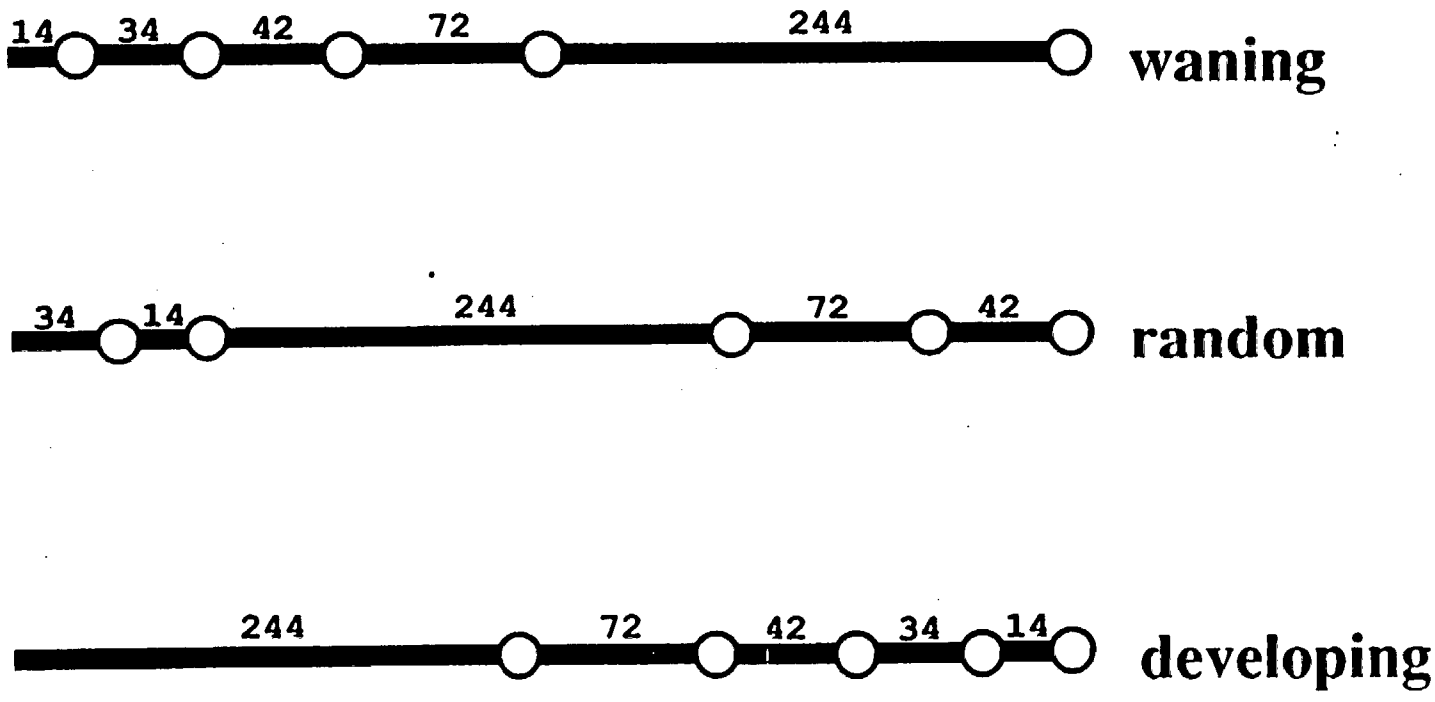


1. Define a single event
2. Measure each event
3. Count them all

→ generate a TIME SERIES

TIME TREND




14
34
42
72
244
waning
34
14
244
72
42
random
244
72
42
34
14
developing

A Simple Poisson Model

(a homogeneous Poisson Process, HPP)

ignores the time trend, and assumes

A CONSTANT RATE OF OCCURRENCES (λ).

λ = # events / obs. time

= reciprocal of average inter-event time

1. **GENERALIZE a constant λ with $\lambda(t)$, a function of time**

2. **Model $X(t)$ = number of events in $[0,t]$**

   $X(t)$ follows a nonhomogeneous Poisson process (NHPP) with parameter $\mu(t)$

$$\mu(t) = \int_0^t \lambda(s)\, ds$$

(Parzen, 1962, p. 138)

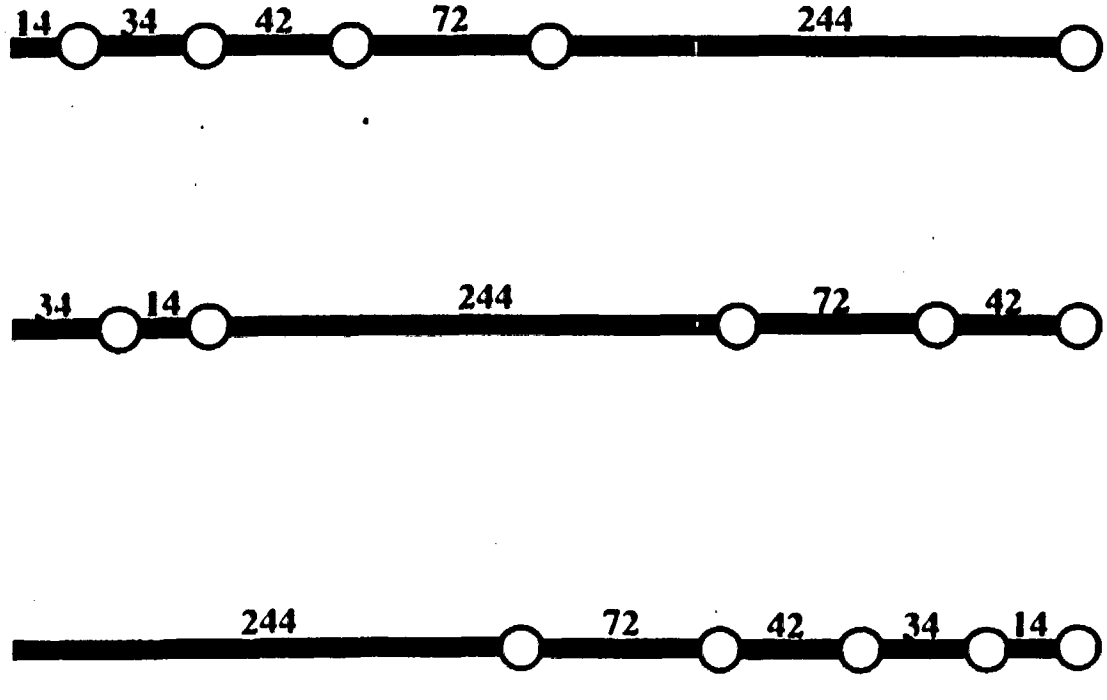
- Choice of $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$
- yields $\mu(t) = (t/\theta)^{\beta}$
- implies a Weibull (θ, β)

$$\beta \begin{cases} > 1 & \text{increasing} \\ = 1 & \text{simple Poisson} \\ < 1 & \text{decreasing} \end{cases}$$

Let $t_1, t_2, \ldots, t_n$ be the first n successive times of events in $[0,t]$: $t_1 < t_2 < \ldots < t_n$

- $\hat{\beta} = n / \sum_{i=1}^{n} \ln(t/t_i)$
- $\hat{\theta} = t/n^{1/\hat{\beta}}$
- $\hat{\lambda} = (\hat{\beta}/\hat{\theta})(t/\hat{\theta})^{\hat{\beta}-1}$

(Crow 1974, 1982)



β
0.63
0.99
5.4

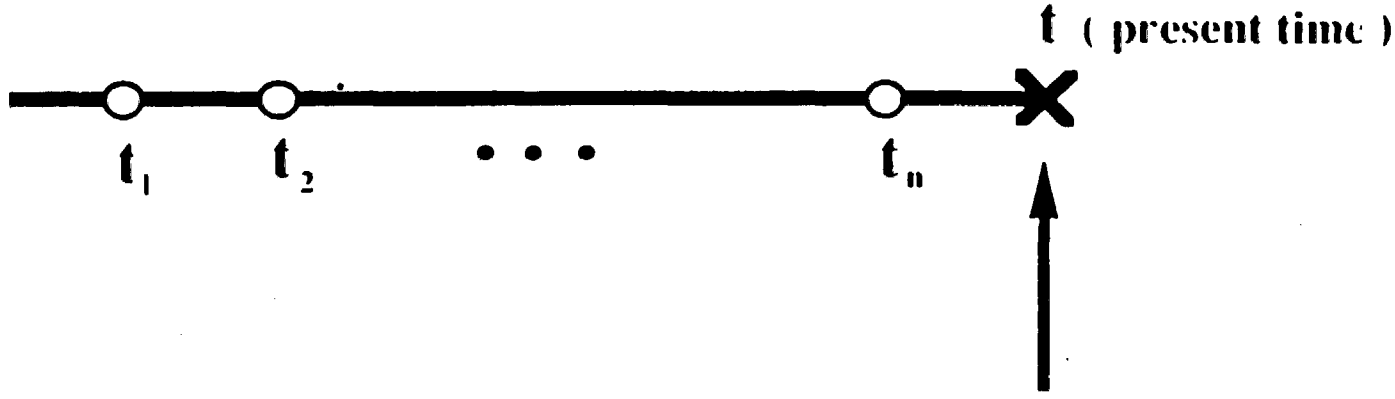
Goodness-of-fit test

$$H_0 : \beta = 1$$

$$H_A : \beta \neq 1$$
$$> 1$$
$$< 1$$

$$X^2 = 2n/\hat{\beta} \sim \chi^2(2n)$$

Instantaneous Recurrence Rate



$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\Theta})(t/\hat{\Theta})^{\hat{\beta} - 1}$$

Volcanism Near the Yucca Mountain Site

t = ?

- Post-6 Ma (Pliocene and younger)
- Quaternary (< 1.6 Ma)

(Crowe et al. 1982, Smith et al. 1990, Wells et al. 1990)

GOALS

To estimate

1. the recurrence rate

2. the waiting time of the next eruption

3. the probability of at least one eruption during the next 10,000 years

4. the probability of volcanic disruption of the repository during the next 10,000 years (in progress)

Identify a single event (eruption)

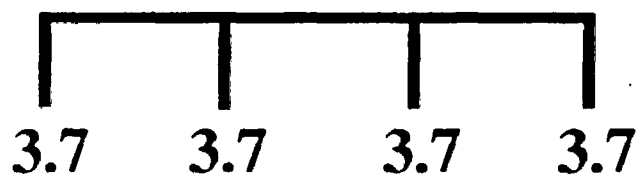
- cluster of centers (volcanic belt) ?
- a volcanic center ?
- a main cone ?
- a small vent ?

A main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone

(Crowe et al. 1983)

Count each widely recognized main cone as a single event, but do not require that the main cones in each center be of separate ages.

A. 3.7 Ma basalts



Daniel Feuerbach (personal communication 1990)

B. Buckboard Mesa

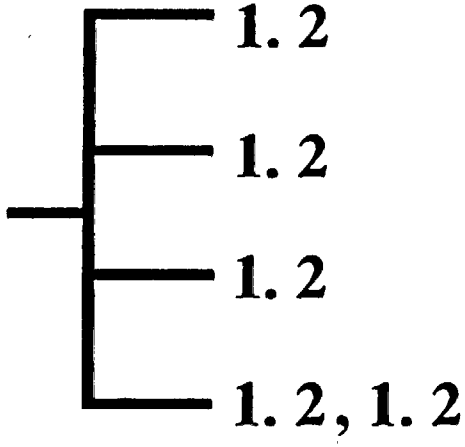
2.8

C. Red Cone,

Northern Cone,

Black Cone,

Little Cones (2)



(Vaniman et al. 1982)

D. Sleeping Butte Cones (2)

0. 28 0. 28

E. Lathrop Wells Cone

0. 01

(Crowe and Perry 1989, Wells et al. 1990)

Preliminary Data Set

3.7, 3.7, 3.7, 3.7, 2.8, 1. 2, 1. 2, 1. 2, 1. 2, 1. 2, 0. 28, 0. 28, 0. 01

(B) Quaternary

(A) Post-6 Ma

(A)
- $\hat{\beta}$ = 2.29 (one-sided p-value $\doteq$ 0.005)
- $\hat{\lambda}$ = 5×10^{-6} /yr

(B)
- $\hat{\beta}$ = 1.09 (one-sided p-value $\doteq$ 0.45)
- $\hat{\lambda}$ = 5.5×10^{-6} /yr

$$\lambda = 5.5 \times 10^{-6}/yr$$

- The estimated instantaneous recurrrence rate

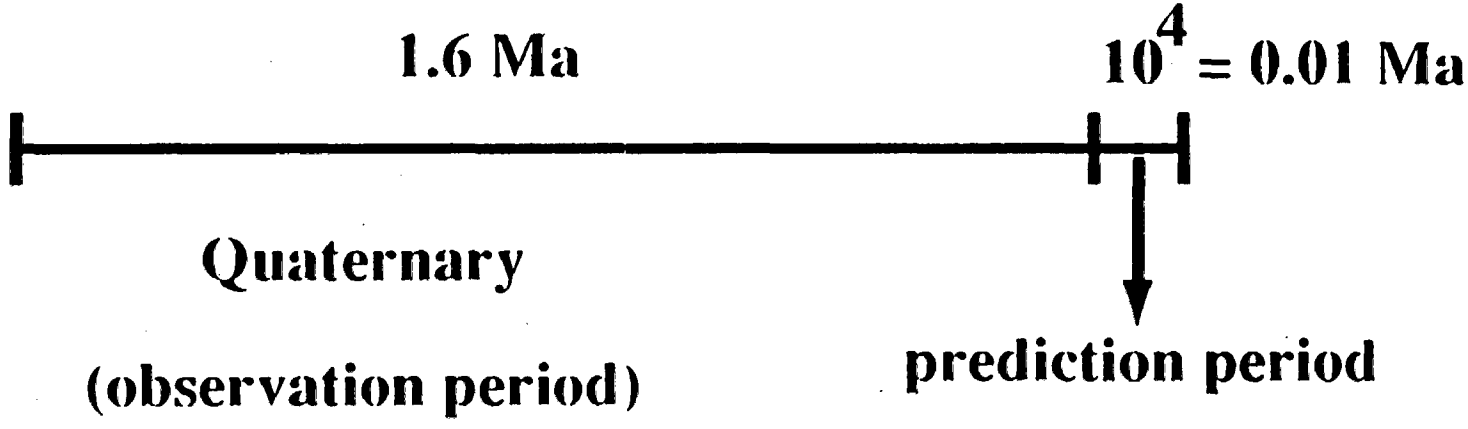
- It represents the instantaneous eruptive status of the volcanism at the end of the observation time t (present)

Interval estimate of $\lambda(t)$

A 90% confidence interval for $\lambda(t)$ is

$(\hat{\lambda}_1, \hat{\lambda}_2) = (1.85 \times 10^{-6}, 1.26 \times 10^{-5})$, which

is more informative than $\hat{\lambda} = 5.5 \times 10^{-6}$ / yr



1. The projected time frame is about 0.6% of the OP

2. It is only 5% of the average repose time

⇓

Suggests switching from a NHPP to a predictive HPP model

It is further justified on the basis of

1. **mathematical simplicity**
2. **objectivity (given the uncertainty of future geophysical phenomena)**
3. **a slight increasing trend ($\bar{\beta} = 1.09$ for the Quaternary volcanism**

Model $X(t_0)$ = # of eruptions during the next t_0 years.

$$X(t_0) \sim \text{Poisson}\,(\hat{\lambda} t_0)$$

Predictions

1. Average waiting time to the next eruption is $\hat{\lambda}^{-1}$

   (a confidence interval is possible)

2. Pr(at least one eruption during the next t_0 years)

   $= 1 - \exp\{\hat{\lambda} t_0\}$

Observation period	Empirical Results				
	$\hat{\beta}$ (p-value)	$1/\hat{\lambda}$ (Ma) (90% C.I.)	Probability Isolation Period (yr)		
			1	10^4	10^5
6.0 Ma-	2.29 (0.005)	0.20 (0.11, 0.45)	5×10^{-6}	0.05	0.39
1.6 Ma-	1.09 (0.45)	0.18 (0.08, 0.55)	5.5×10^{-6}	0.05	0.42

Polycyclic Volcanism

Lathrop Wells volcano is a polycyclic volcano

(Crowe et al. 1989, Wells et al. 1990)

One Step further: assuming there are 3 additional eruptions,

1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01, 0.01, 0.01, 0.01

Then 1. $\hat{\beta} = 1.50$ (p-value = 0.125)

2. $\hat{\lambda} = 10^{-5}$/yr (doubled)

3. $\hat{\lambda}^{-1} = 9.7 \times 10^{4}$ years (50% sooner)