



Department of Energy
Washington, DC 20585

MAY 27 1993

Mr. Joseph J. Holonich, Director
Repository Licensing & Quality Assurance
Project Directorate
Division of High-Level Waste Management
Office of Nuclear Material Safety
and Safeguards
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

Dear Mr. Holonich:

Enclosed are the Department of Energy's (DOE) responses to two U.S. Nuclear Regulatory Commission's (NRC) comments and one reference requested in NRC's letter dated January 28, 1993, Phase I review of the subject study plan (enclosure 1). Enclosure 2 contains the response to these comments. Enclosure 3 is the reference that was required.

The NRC: (1) identified an internal inconsistency in the study plan with respect to technical procedures, and (2) inquired how work will be coordinated between the subject study plan and Study Plan 8.3.1.2.2.9 (Site Unsaturated-Zone Modeling and Synthesis). Both inquiries are responded to in Enclosure 2.

If you have any questions, please contact Ms. Sheila Long at 202-586-1447 or Mr. Chris Einberg of my office at 202-586-8869.

Sincerely,

Dwight E. Shelor
Associate Director for
Systems and Compliance
Office of Civilian Radioactive
Waste Management

Enclosures:

1. Ltr, 1/28/93, Holonich to Roberts,
w/encl
2. Response to NRC Comments
3. Reference

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102.8
WM-11

cc w/enclosures:

C. Gertz, YMPO

T. J. Hickey, Nevada Legislative Committee

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B. Mettam, Inyo County, CA

C. Abrams, NRC



UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

JAN 28 1993

Mr. John P. Roberts, Acting Associate Director
for Systems and Compliance
Office of Civilian Radioactive Waste Management
U. S. Department of Energy
1000 Independence Avenue, SW
Washington, D.C. 20585

Dear Mr. Roberts:

SUBJECT: PHASE I REVIEW OF U.S. DEPARTMENT OF ENERGY (DOE) STUDY PLAN "FLUID FLOW IN UNSATURATED FRACTURED ROCK"

On September 15, 1992, DOE transmitted the study plan, "Fluid Flow in Unsaturated Rock" (Study Plan 8.3.1.2.2.8) to the U.S. Nuclear Regulatory Commission for review and comment. NRC has completed its Phase I Review of this document using the Review Plan for NRC Staff Review of DOE Study Plans, Revision 1 (December 6, 1990).

I-107156
4)

The material submitted in the study plan was considered to be consistent, to the extent possible at this time, with the NRC-DOE agreement on content of study plans made at the May 7-8, 1986, meeting on Level of Detail for Site Characterization Plans and Study Plans. The study plan states (Section 7.1, number 5) that the technical procedures are listed in Section 3 of the study plan. The staff found no such list in Section 3. The staff did not consider that the absence of such information compromised its ability to conduct its Phase I Review of the material provided. However, the NRC staff requests that a list of applicable technical procedures and their status be provided to NRC.

Comment 1

Among the references listed for this study plan the staff has identified one that is not readily available in the public domain. We therefore request that DOE provide the NRC with the reference listed in the Enclosure.

Reference Request

A major purpose of the Phase I Review is to identify concerns with studies, tests, or analyses that, if started, could cause significant and irreparable adverse effects on the site, the site characterization program, or the eventual usability of the data for licensing. Such concerns would constitute objections, as that term has been used in earlier NRC staff reviews of DOE's documents related to site characterization (Consultation Draft Site Characterization Plan and the Site Characterization Plan for the Yucca Mountain site). No field tests will be conducted under this study; therefore, it does not appear that the conduct of the activities described in this study plan will have adverse impacts on repository performance and the Phase I Review of this study plan identified no objections with any of the activities proposed.

Enclosure 1

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After completion of the Phase I Review, selected study plans are to receive a second level of review, called a Detailed Technical Review, based on the relationship of a given study plan to key site-specific issues or NRC open items, or its reliance on unique, state-of-the-art test or analysis methods. Based on these criteria, we have decided not to proceed with a Detailed Technical Review of this study plan at this time. The NRC staff will reevaluate this decision after it receives and reviews the closely related Study Plan 8.3.1.2.2.9, "Site Unsaturated-Zone Modeling and Synthesis." The subject study involves the development and validation of conceptual and numerical flow models of the unsaturated zone over various scales. The SCP describes study 8.3.1.2.2.9 as developing models for site-scale analyses. It also refers to code testing and code verification. Both studies refer to the development of conceptual and numerical models. It is not clear how work will be coordinated between these studies in the development of conceptual models, code development, and verification, and the development, application, and validation of numerical models.

Comment 2

If you have any questions concerning this letter, please contact Charlotte Abrams (301) 504-3403 of my staff.

Sincerely,



Joseph J. Holonich, Director
Repository Licensing and Quality Assurance
Project Directorate
Division of High-Level Waste Management
Office of Nuclear Material Safety
and Safeguards

Enclosure: As stated

cc: R. Loux, State of Nevada
T. J. Hickey, Nevada Legislative Committee
C. Gertz, DOE/NV
M. Murphy, Nye County, NV
M. Baughman, Lincoln County, NV
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E. Holstein, Nye County, NV

NOT READILY REFERENCE FOR STUDY PLAN 8.3.1.2.2.8

Voss, R. F., 1985, Random fractals: Characterization and measurement, Proceedings NATO A.S.I. Scaling Properties of Disordered Media, Geilo, Norway, April 1985.

ENCLOSURE

DOE RESPONSES TO NRC PHASE 1 COMMENTS ON STUDY PLAN 8.3.1.2.2.8

NRC Comment No. 1

The study plan states (Section 7.1, number 5) that the technical procedures are listed in Section 3 of the study plan. The staff found no such list in Section 3. The staff did not consider that the absence of such information compromised its ability to conduct its Phase I Review of the material provided. However, the NRC staff requests that a list of applicable technical procedures and their status be provided to NRC.

DOE Response

Study 8.3.1.2.2.8 is a modeling and interpretive study which relies on data collected from other studies, most notably Study 8.3.1.2.2.4, but which collects no data in itself. No technical procedures, as they have been applied to study plans concerned with data gathering in the field or the laboratory, will be used for modeling and interpretive activities, although this was not clear at the time the SCP description for Study 8.3.1.2.2.8 was written. The recent renegotiation of the DOE/NRC Study Plan level-of-detail and review process agreement was undertaken primarily to construct a study plan format for describing the conduct of modeling synthesis or interpretive activities that is more appropriate for this type of work. Numerical modeling work does have specific software quality assurance and recordkeeping procedural requirements, but these procedures are not technical procedures.

DOE notes the error identified by NRC in the comment and will delete the cross reference between Section 7.1 and Section 3.0 if a revision to the study plan is warranted for other reasons. Such a minor administrative change alone does not warrant initiating a revision.

NRC Comment No. 2

The subject study involves the development and validation of conceptual and numerical flow models of the unsaturated zone over various scales. The SCP describes study 8.3.1.2.2.9 as developing models for site-scale analyses. It also refers to code testing and code verification. Both studies refer to the development of conceptual and numerical models. It is not clear how work will be coordinated between these studies in the development of conceptual models, code development, and verification, and the development, application and validation of numerical models.

DOE Response

The subject study plan (8.3.1.2.2.8) will develop and test the specific models that permit estimation of the unsaturated hydrologic properties of fractured porous materials. The evaluation of the accuracy and limitations of continuum models as applied to unsaturated fractured rock to further elucidate modeling alternatives for the hydrologic behavior of such media is also involved. Although such fracture flow results are important, they are but a part of the entire spectrum of features and processes involved in the overall site analyses. The necessary overall site analysis models involving the whole spectrum of features, events, and processes are described in Study Plan 8.3.1.2.2.9 (Site Unsaturated-Zone Modeling and Synthesis). This study plan is now in the final approval stage within DOE. Study Plan 8.3.1.2.2.9 is the overall modeling study that will use results from the fracture flow study (8.3.1.4.2.2, Characterization of the Structural Features within the Site Area) and from several other studies to model the entire unsaturated zone. Specifically, Study Plan 8.3.1.2.2.9 will provide the necessary analyses through combined hypothesis testing and sensitivity analysis to enhance confidence in modeling results through ongoing testing, identify alternative conceptual models, and guide code selection and verification.

Reference for Study Plan 8.3.1.2.2.8

Enclosure 3

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RANDOM FRACTALS: characterization and measurement

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ABSTRACT

Mandelbrot's fractal geometry provides both a description and a mathematical model for many of the seemingly complex shapes found in nature. Such shapes often possess a remarkable invariance under changes of magnification. This statistical self-similarity may be characterized by a fractal dimension D , a number that agrees with our intuitive notion of dimension but need not be an integer. A brief mathematical characterization of random fractals is presented with emphasis on variations of Mandelbrot's fractional Brownian motion. The important concepts of fractal dimension and exact and statistical self-similarity and self-affinity will be reviewed. The various methods and difficulties of estimating the fractal dimension and lacunarity from experimental images or point sets are summarized.

random fractals: an introduction

Mandelbrot's fractal geometry [1] has revolutionized the application of geometrical constructs to the natural sciences. Fractals provide the proper mathematical framework for a treatment of the irregular, seemingly-complex shapes found in nature from the small scale structure of disordered systems and percolation clusters to coastlines, mountain ranges, clouds, and the distribution of stars in the night sky. Some of the building blocks of fractal geometry originated in the exactly self-similar mathematical "monsters" (such as the Koch curve and Seirpinski gasket) of the early 1900's. Although such exact deterministic constructs serve as useful tools in building understanding and intuition about scaling properties, the fractal shapes found in nature possess a statistical rather than exact self-similarity. The following sections present an expository summary of the major mathematical definitions and relations used in the characterization and measurement of random or statistical fractals as condensed from Mandelbrot[1]. A detailed discussion of the algorithmic considerations in generating or simulating such random fractals is found in [2].

fractional Brownian motion

One of the most useful mathematical models for the random fractals found in nature (such as mountainous terrain and clouds) has been the *fractional Brownian motion* (fBm) of Mandelbrot and Wallis[1,3]. It is an extension of the central concept of *Brownian motion* that has played an important role in both physics and mathematics. Sample *traces* of fBm are shown in Fig. 1. Almost all computer graphics fractal simulations[2] are based on an extension of fBm to higher dimensions such as the fractional Brownian landscape of Fig. 2. Fractional Brownian motion is also a good starting point for understanding anomalous diffusion and random walks on fractals.

A fractional Brownian motion, $V_H(t)$, is a single valued function of one variable, t (usually time). Its increments $V_H(t_2) - V_H(t_1)$ have a Gaussian distribution with variance

$$\langle |V_H(t_2) - V_H(t_1)|^2 \rangle \propto |t_2 - t_1|^{2H}, \quad (1)$$

where the brackets $\langle \rangle$ and $\langle \rangle$ denote averages over many samples of $V_H(t)$ and the parameter H has a value $0 < H < 1$. Such a function is both stationary and isotropic. Its mean square increments depend only on the time difference $t_2 - t_1$ and all t 's are statistically equivalent. The special value $H = 1/2$ gives the familiar Brownian motion with $\Delta V^2 \propto \Delta t$.

As with the usual Brownian motion, although $V_H(t)$ is continuous, it is nowhere differentiable. Nevertheless, many constructs have been developed (and are relevant to the problem of light scattering from fractals) to give meaning to "derivative of fractional Brownian motion" as *fractional Gaussian noises*[1,3]. Such constructs are usually based on averages of $V_H(t)$ over decreasing scales. The derivative of normal Brownian motion, $H = 1/2$, corresponds to uncorrelated *Gaussian white noise*, and Brownian motion is said to have *independent increments*. Formally, for any three times such that $t_1 < t < t_2$, $V_H(t) - V_H(t_1)$ is statistically independent of $V_H(t_2) - V_H(t)$ for $H = 1/2$. For $H > 1/2$ there is a positive correlation both for the increments of $V_H(t)$ and its derivative fractional Gaussian noise. For $H < 1/2$ the increments are negatively correlated. Such correlations extend to arbitrarily long time scales and have a large effect on the visual appearance of the fBm traces as shown in Fig. 1.

$V_H(t)$ shows a statistical scaling behavior. If the time scale t is changed by the factor r , then the increments ΔV_H change by a factor r^H . Formally,

$$\langle \Delta V_H(rt)^2 \rangle \propto r^{2H} \langle \Delta V_H(t)^2 \rangle. \quad (2)$$

Unlike statistically self-similar curves (such as the coastlines in Fig. 2), a $V_H(t)$ trace requires *different* scaling factors in the two coordinates (r for t but r^H for V_H) reflecting the special status of the t coordinate. Each t can correspond to only one value of V_H but any specific V_H may occur at multiple t 's. Such non-uniform scaling is known as *self-affinity* rather than self-similarity.

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Figure 2, a $V_H(t)$ trace
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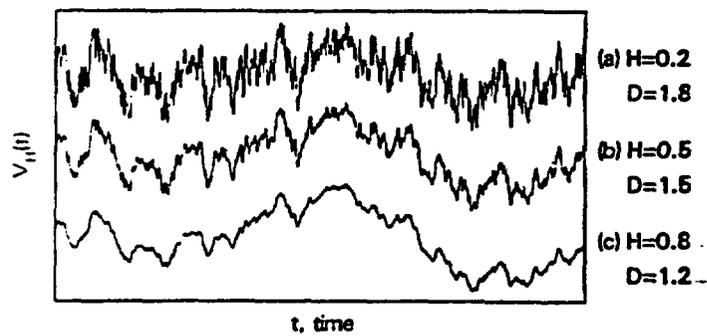


Figure 1. Sample traces of statistically self-affine fractional Brownian motion. (a) Increments are negatively correlated. (b) Normal Brownian motion with uncorrelated increments. (c) Increments are positively correlated.

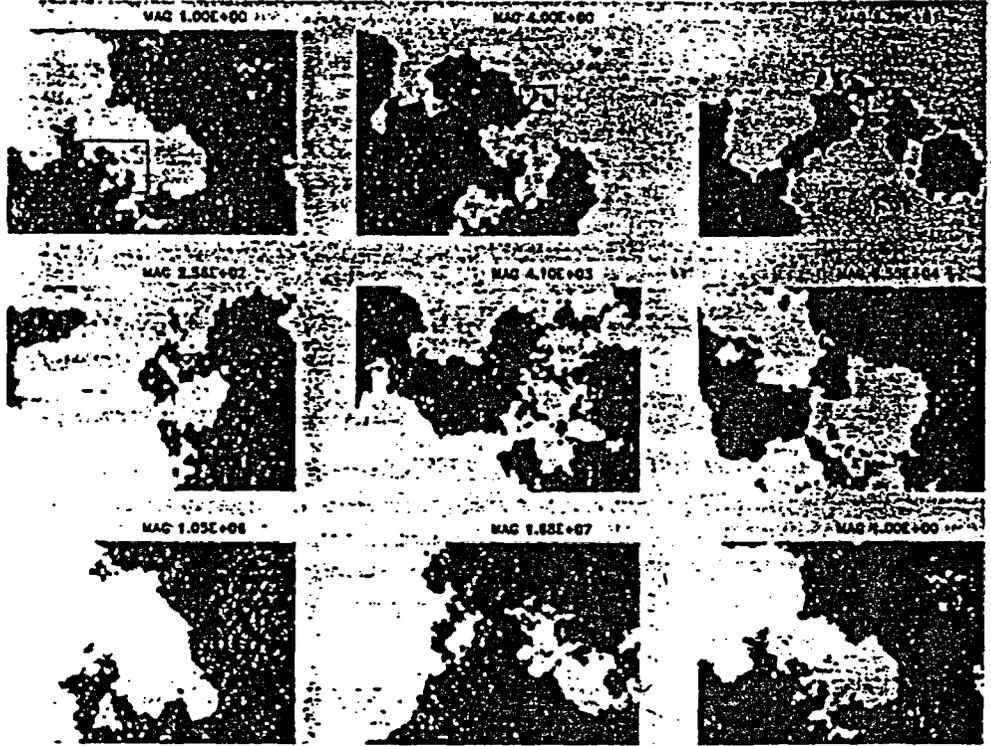


Figure 2. Computer realization of a statistically self-affine fractional Brownian landscape with $H=0.8$, $D=2.2$ at various magnifications. The altitude variations along any straight line path give a fBm trace such as Fig. 1(c). Successive frames show magnified view of the black outline from the previous frame and demonstrate that the landscape's coastline is statistically self-similar.

self-similar vs self-affine fractals.

The distinction between similarity and affinity is important. By way of summary, a *self-similar* object is composed of N copies of itself (with possible translations and rotations) each of which is scaled down by the ratio r in all E coordinates from the whole. More formally, consider a set S of points at positions $\bar{x} = (x_1, \dots, x_E)$ in Euclidean space of dimension E . Under a *similarity* transform with real scaling ratio $0 < r < 1$, the set S becomes rS with points at $r\bar{x} = (rx_1, \dots, rx_E)$. A bounded set S is *self-similar* when S is the union of N distinct (non-overlapping) subsets each of which is congruent to rS . *Congruent* means identical under translations and rotations. The fractal or *similarity dimension* of S is then given by

$$1 = Nr^D \quad \text{or} \quad D = \frac{\log N}{\log 1/r}. \quad (3)$$

This relation leads to several important methods of *estimating* D for a given set S .

For topologically one-dimensional fractal "curves", the apparent "length", varies with the measuring ruler size. If the entire self-similar curve is of maximum size L_{\max} , then, on a smaller scale $L = rL_{\max}$, the curve consists of $N = 1/r^D$ segments of length L . Thus, for $D \geq 1$

$$\text{LENGTH} = L \times N = L \times (L_{\max}/L)^D \propto 1/L^{D-1}. \quad (4)$$

The fractal dimension D also characterizes the covering of the set S by E -dimensional "boxes" of linear size L . If the entire S is contained within one box of size L_{\max} , then each of the $N = 1/r^D$ subsets will fall within one box of size $L = rL_{\max}$. Thus, the number of boxes of size L , $N_{\text{box}}(L)$, needed to cover S is given by

$$N_{\text{box}}(L) = (L_{\max}/L)^D \quad \text{or} \quad N_{\text{box}}(L) \propto 1/L^D. \quad (5)$$

This definition of *box dimension* is one of the most useful methods for estimating the fractal dimension of a given set. The box dimension can be conveniently estimated by dividing the E -dimensional Euclidean space containing the set into a grid of boxes of size L^E and counting the number of such boxes $N_{\text{box}}(L)$ that are non-empty. It is useful to examine as large a range of L as possible and to average over various origins for the boxes.

One can also estimate the "volume" or "mass" of the set S by a covering with boxes of linear size L . If one considers only distances of order L about a given point in S , one finds a single box of size L with E -dimensional volume L^E . If the distance scale about the same point is increased to $L_{\max} = L/r$, one now finds a total of $N = 1/r^D = (L_{\max}/L)^D$ boxes of mass L^E covering the set. Thus, the mass within a distance L_{\max} of some point in S , $M(L_{\max}) = N \times M(L) = M(L) \times (L_{\max}/L)^D$ or

$$M(L) \propto L^D. \quad (6)$$

The fractal dimension D , thus, also corresponds to the commonly used *mass dimension* in physics. Mass dimension also has a strong connection with intuitive notions of dimension. The amount of material within a distance L of a point in a one-dimensional

object increases as L^1 . For an E -dimensional object it varies as L^E . The mass dimension is another extremely useful method for estimating the fractal dimension of a given object.

The set S is also *self-similar* if each of the N subsets is scaled down from the whole by a different similarity ratio r_n . In this case, D is given implicitly by

$$1 = \sum_{n=1}^N r_n^D, \quad (7)$$

which reduces to the familiar result in Eq. (3) when all of the r_n are equal.

The set S is *statistically self-similar* if it is composed of N distinct subsets each of which is scaled down by the ratio r from the original and is identical in all statistical respects to rS . The similarity dimension is again given by Eq. (3). In practice, it is impossible to verify that all moments of the distributions are identical, and claims of statistical self-similarity are usually based on only a few moments. Moreover, a sample of a random set (such as a coastline) is often statistically self-similar for all scaling ratios r . Its fractal dimension is usually estimated from the dependence of box coverings $N_{\text{box}}(L)$ or mass $M(L)$ on varying L as in Eqs. (5) and (6).

Under an *affine* transform, on the other hand, each of the E coordinates of \bar{x} may be scaled by a different ratio (r_1, \dots, r_E). Thus, the set S is transformed to $r(S)$ with points at $r(\bar{x}) = (r_1 x_1, \dots, r_E x_E)$. A bounded set S is *self-affine* when S is the union of N distinct (non-overlapping) subsets each of which is congruent to $r(S)$. Similarly, S is *statistically self-affine* when S is the union of N distinct subsets each of which is congruent in distribution to $r(S)$. The fractal dimension D , however, is not as easily defined as with self-similarity.

the relation of D to H for self-affine fractional Brownian motion

The assignment of a fractal dimension D to a self-affine set can be illustrated with a trace of fractional Brownian motion $V_H(t)$ from above. Consider, for convenience, a trace of $V_H(t)$ covering a time span $\Delta t=1$ and a vertical range $\Delta V_H=1$. $V_H(t)$ is statistically self-affine when t is scaled by r and V_H is scaled by r^H . Suppose the time span is divided into N equal intervals each with $\Delta t=1/N$. Each of these intervals will contain one portion of $V_H(t)$ with vertical range $\Delta V_H = \Delta t^H = 1/N^H$. Since $0 < H < 1$ each of these new sections will have a large vertical to horizontal size ratio and the occupied portion of each interval will be covered by $\Delta V_H / \Delta t = (1/N^H) / (1/N) = N/N^H$ square boxes of linear scale $L=1/N$. In terms of box dimension, as t is scaled down by a ratio $r=1/N$ the number of square boxes covering the trace goes from 1 to $N(L) = \text{number of intervals} \times \text{boxes per interval}$ or

$$N(L) = N \times N/N^H = N^{2-H} = 1/L^{2-H}. \quad (8)$$

Thus, by comparison with Eq. (5),

$$D = 2 - H \quad \text{for a trace of } V_H(t). \quad (9)$$

Consequently, the trace of normal Brownian motion has $D = 1.5$.

It is important to note that the association of a similarity dimension D with a self-affine fractal such as fBm is implicitly fixing a scaling between the (otherwise independent) coordinates. The result depends strongly on whether one is looking at scales large or small compared to this (artificially introduced) characteristic length. The difference is particularly clear when one attempts to estimate D for a trace of fBm from Eq. (4). As above, one can divide the t axis into N segments of size $\Delta t = 1/N$. For each of these segments the typical V variation will be $\Delta V = \Delta t^H$. The length *along* each segment is typically $l = (\Delta t^2 + \Delta V^2)^{1/2}$ and the total

$$\text{LENGTH} = N \times l \propto (1 + \Delta V^2/\Delta t^2)^{1/2} \propto (1 + 1/\Delta t^{2-2H})^{1/2}. \quad (12)$$

On small scales with $\Delta t \ll 1$, the second term dominates and $\text{LENGTH} \propto 1/\Delta t^{1-H}$ so $D = 2-H$ by comparison with Eq. (4) and in agreement with Eq. (9). On the other hand, on large scales with $\Delta t \gg 1$, LENGTH is independent of Δt and $D = 1$.

The *zeroset* of fBm is the the intersection of the trace of $V_H(t)$ with the t axis, the set of all points such that $V_H(t) = 0$. The zeroset is a disconnected set of points with topological dimension zero and a fractal dimension $D_0 = D-1 = 1-H$ that is less than 1 but greater than 0. Although the trace of $V_H(t)$ is self-affine, its zeroset is self-similar.

trails of fBm

Consider a particle undergoing a fractional Brownian motion or random walk in E dimensions where each of the coordinates is tracing out an independent fBm in time. Over an interval Δt each coordinate will vary by typically $\Delta L = \Delta t^H$. If overlap can be neglected, the "mass" of the trail $M \propto \Delta t \propto L^{1/H}$. In comparison with Eq. (6), the trail of fBm has a fractal dimension

$$D = 1/H \quad (13)$$

provided $1/H < E$. Normal Brownian motion with $H=0.5$ has $D=2$. $1/H$ is known as the *latent* fractal dimension[4,5] of a *trail* of fBm. When $1/H > E$ overlap cannot be neglected and the actual $D = E$. When $1/H = E$ the trail is *critical* and most quantities will have important logarithmic corrections to the usual fractal power laws. Note that although the *trail* of a fBm in E dimensions is self-similar, each of the E coordinates has a self-affine *trace* vs time.

self-affinity in higher dimensions: Mandelbrot landscapes and clouds

The traces of fBm, particularly Fig. 1(c) with $H=0.8$, bear a striking resemblance to a mountainous horizon. The modelling of the irregular Earth's surface as a generalization of traces of fBm was first proposed by Mandelbrot. The single variable t can be replaced by coordinates x and y in the plane to give $V_H(x,y)$ as the surface altitude at position x,y as shown in Fig. 2. In this case, the altitude variations of a hiker following any straight line path at constant speed in the xy plane is a fractional Brownian motion. In analogy with Eq. (1),

$$\langle |V_H(x_2, y_2) - V_H(x_1, y_1)|^2 \rangle \propto [(x_2 - x_1)^2 + (y_2 - y_1)^2]^H. \quad (10)$$

ion D with a self-affine (otherwise independent) looking at scales large or length. The difference is of fBm from Eq. (4). $1/N$. For each of these along each segment is

$$\Delta t^{2-2H})^{1/2} \quad (12)$$

LENGTH $\propto 1/\Delta t^{1-H}$ Eq. (9). On the other of Δt and $D = 1$.

with the t axis, the set of points with $D = 1-H$ that is less than one, its zero set is self-

random walk in E dependent fBm in time. Δt^H . If overlap can be with Eq. (6), the trail

$$(13)$$

$D=2$. $1/H$ is known $H > E$ overlap cannot critical and most quantitative power laws. Note each of the E coordi-

ds

resembling to a face as a generalization of a variable t can be re- the surface altitude at ns of a hiker following onal Brownian motion.

$$[2 - y_1)^2]^{1/2} \quad (10)$$

Once again, the fractal dimension D must be greater than the topological dimension 2 of the surface. Here,

$$D = 3 - H \quad \text{for a fractal landscape, } V_H(x,y). \quad (11)$$

The intersection of a vertical plane with the surface $V_H(x,y)$ is a self-affine fBm trace with $D=2-H$, smaller by one than the value of Eq. (11). Similarly, the zero set of $V_H(x,y)$, its intersection with a horizontal plane, also has a fractal dimension $D_0=2-H$. This intersection, which produces a family of (possibly disconnected) curves, forms the coastlines of the $V_H(x,y)$ landscape. Since the two coordinates x and y are, however, equivalent, the coastlines of $V_H(x,y)$ are self-similar, not self-affine. Figure 2 demonstrates how the coastline remains statistically invariant under changes of magnification.

This generalization of fBm can continue to still higher dimensions to produce, for example, a self-affine fractal temperature or density distribution $V_H(x,y,z)$. Here, the variations of an observer moving at constant speed along any straight line path in space generate a fBm and the fractal dimension

$$D = 4 - H \quad \text{for a fractal cloud } V_H(x,y,z). \quad (12)$$

The zero set $V_H(x,y,z) = \text{constant}$ now gives a self-similar fractal with $D_0=3-H$.

To summarize, a statistically self-affine fractional Brownian function, V_H of $\vec{x} = (x_1, \dots, x_E)$ in E Euclidean dimensions satisfies

$$\langle |V_H(\vec{x}_2) - V_H(\vec{x}_1)|^2 \rangle \propto |\vec{x}_2 - \vec{x}_1|^{2H} \quad (13)$$

and has a fractal dimension

$$D = E + 1 - H. \quad (14)$$

The zero sets of $V_H(\vec{x})$ form a statistically self-similar fractal with dimension $D_0 = E-H$.

perimeter vs area scaling

The fractal dimension of the coastlines of $V_H(x,y)$ can also be estimated from the perimeter area scaling of the islands. Eq. (4) gives the LENGTH or perimeter of a fractal object of size L_{max} when "measured" with a ruler of size L . Consider a population of such islands with the same coastline D and all measured with the same ruler L . For a given island, perimeter $P \propto L_{\text{max}}^D$ from Eq. (4). Provided the coastline $D < 2$, each island will have a well-defined area $A \propto L_{\text{max}}^2$ and

$$P \propto A^{D/2}. \quad (15)$$

A study of P vs A scaling is a useful method of estimating a coastline D for a population of fractal objects.

spectral densities for fBm and the spectral exponent β

Random functions in time $V(t)$ are often characterized [6,7] by their spectral densities $S_V(f)$. If $V(t)$ is the input to a narrow bandpass filter at frequency f and bandwidth

Δf , then $S_V(f)$ is the mean square output $V(f)$ divided by Δf , $S_V(f) = |V(f)|^2/\Delta f$. $S_V(f)$ gives information about the time correlations of $V(t)$. When $S_V(f)$ increases steeply at low f , $V(t)$ varies more slowly. If one defines $V(f,T)$ as the Fourier transform of a specific sample of $V(t)$ for $0 < t < T$,

$$V(f,T) = \frac{1}{T} \int_0^T V(t) e^{2\pi i f t} dt, \text{ then } S_V(f) \propto T |V(f,T)|^2 \text{ as } T \rightarrow \infty. \quad (16)$$

An alternate characterization of the time correlations of $V(t)$ is given by the *2 point autocorrelation function*

$$G_V(\tau) = \langle V(t)V(t+\tau) \rangle - \langle V(t) \rangle^2.$$

$G_V(\tau)$ provides a measure of how the fluctuations at two times separated by τ are related. $G_V(\tau)$ and $S_V(f)$ are not independent. In many cases they are related by the Wiener-Khintchine relation[6,7]

$$G_V(\tau) = \int_0^\infty S_V(f) \cos(2\pi f \tau) df. \quad (17)$$

For a Gaussian white noise $S_V(f) = \text{constant}$ and $G_V(\tau) = \Delta V^2 \delta(\tau)$ is completely uncorrelated. For certain simple power laws for $S_V(f)$, $G_V(\tau)$ can be calculated exactly. Thus, for

$$S_V(f) \propto 1/f^\beta, \text{ with } 0 < \beta < 1, \quad G_V(\tau) \propto \tau^{\beta-1}. \quad (18)$$

Moreover, $G_V(\tau)$ is directly related to the mean square increments of fBm,

$$\langle |V(t+\tau) - V(t)|^2 \rangle = 2[\langle V^2 \rangle - G_V(\tau)]. \quad (19)$$

Roughly speaking, $S_V(f) \propto 1/f^\beta$ corresponds to $G_V(\tau) \propto \tau^{1-\beta}$ and a fBm with $2H = \beta - 1$ from Eqs. (1) and (19). Thus, the statistically self-affine fractional Brownian function, $V_H(\bar{x})$, with \bar{x} in an E -dimensional Euclidean space, has a fractal dimension D and spectral density $S_V(f) \propto 1/f^\beta$, for the fluctuations along a straight line path in any direction in E -space with

$$D = E + 1 - H = E + \frac{3 - \beta}{2} \quad (20)$$

This result agrees with other "extensions" of the concepts of *spectral density* and *Wiener-Khintchine relation* to *non-stationary* noises where some moments may be undefined. Moreover, it provides an extremely useful connection between D , H and β for finite simulations. For H in the range $0 < H < 1$, $E < D < E + 1$, and $1 < \beta < 3$. The value $H \approx 0.8$ is a good choice for many natural phenomena.

Although the formal definition of fBm restricts H to the range $0 < H < 1$, it is often useful to consider integration and an appropriate definition of "derivative" as extending the range of H . Thus, integration of a fBm produces a new fBm with H increased by 1, while "differentiation" reduces H by 1. When $H \rightarrow 1$, the derivative of fBm looks like a fBm with $H \rightarrow 0$. In terms of spectral density, if $V(t)$ has $S_V(f) \propto 1/f^\beta$ then its

$S(f) = |V(f)|^2 / \Delta f$.
 When $S_V(f)$ increases
 the Fourier transform

as $T \rightarrow \infty$. (16)

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variated by τ are re-
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$<H < 1$, it is often
 "invariant" as extend-
 with H increased
 derivative of fBm looks
 $S(f) \propto 1/f^\beta$ then its

derivative dV/dt has spectral density $f^2/f^\beta = 1/f^{\beta-2}$. In terms of Eq. (20), differen-
 tiation decreases β by 2 and decreases H by 1.

measuring fractal dimensions: Mandelbrot measures

Numerical simulation or experimental image analysis often produces a geometrical ob-
 ject defined by a set S of points at positions $\vec{x} = (x_1, \dots, x_E)$ in an E -dimensional
 Euclidean space. Lacking other information, all of the points may be assumed to be
 equivalent and all points are equally probable origins for analysis. The spatial ar-
 rangement of the points determines $P(m,L)$. $P(m,L)$ is the probability that there are m
 points within an E -cube (or sphere) of size L centered about an arbitrary point in S .
 $P(m,L)$ is normalized

$$\sum_{m=1}^N P(m,L) = 1 \text{ for all } L \quad (21)$$

$P(m,L)$ is directly related to other probability measures as used by Mandelbrot[1,8],
 Hentschel and Procaccia[9], and others. This particular definition of $P(m,L)$ is, how-
 ever, particularly efficient to implement on a computer.

The usual quantities of interest are derived from the moments of $P(m,L)$. The mass di-
 mension,

$$M(L) = \sum_{m=1}^N m P(m,L). \quad (22)$$

The number of boxes of size L needed to cover S ,

$$N_{\text{box}}(L) = \sum_{m=1}^N \frac{1}{m} P(m,L). \quad (23)$$

The configurational entropy when space is divided into cubes of size L

$$S(L) = \sum_{m=1}^N \log m P(m,L). \quad (24)$$

For a fractal set $M(L) \propto L^D$, $N_{\text{box}}(L) \propto 1/L^D$, and $e^{S(L)} \propto L^D$.

In fact, one can define all moments

$$M^q(L) = \sum_{m=1}^N m^q P(m,L). \quad (25)$$

for $q \neq 0$ (the $q=0$ case is given by Eq.(24) above) and one can then estimate D from
 the logarithmic derivatives

$$D = \frac{1}{q} \left\langle \frac{\partial \log M_q(L)}{\partial \log L} \right\rangle \text{ for } q \neq 0. \quad (26)$$

and

$$D = \frac{1}{q} \left\langle \frac{\partial S(L)}{\partial \log L} \right\rangle \quad \text{for } q = 0. \quad (27)$$

A double logarithmic plot of $M^q(L)$ vs L is an essential tool in verifying whether a fractal interpretation is valid for S . One expects a fractal to have power-law behavior of $M^q(L)$ over a wide range of L .

For a uniform fractal (fractal set) as the number of points examined, $N \rightarrow \infty$ the distribution is expected to take the scaling form $P(m,L) \rightarrow f(m/L^D)$ and all moments give the same D . For a non-uniform fractal (a fractal measure such as a Poincare map) the moments satisfy the relation [9] that $D_{q_1} \leq D_{q_2}$ for $q_1 < q_2$.

lacunarity

It is obvious from the above discussion that the fractal dimension D characterizes only part of the information in the distribution $P(m,L)$. Different fractal sets may share the same D but have different appearances or *textures* corresponding to different $P(m,L)$. As an initial step toward quantifying texture, Mandelbrot [1, chapter 34] has introduced the parameter *lacunarity*, Λ (*lacuna* is Latin for gap). Although the qualitative visual effect of changing lacunarity at fixed D is quite striking [1], to date there have been no quantitative measurements of the lacunarity of various random fractals and Mandelbrot [1] offers several alternative definitions. The most useful derives from the width of the distribution $P(m,L)$ at fixed L . Given $M^q(L)$ as defined by Eq. (25),

$$\Lambda(L) = \frac{\langle M^2(L) \rangle - \langle M(L) \rangle^2}{\langle M(L) \rangle^2}. \quad (28)$$

When $P(m,L)$ takes the scaling form $f(m/L^D)$, Λ is just the relative mean square width of the distribution f . $\Lambda = \langle \Delta f^2 \rangle / \langle f \rangle^2$.

acknowledgement

This work was made possible by the help, encouragement, and inspiration of Benoit Mandelbrot.

(27)

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(28)



UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D.C. 20555-0001

APR 27 1993

MEMORANDUM FOR: Joseph J. Holonich, Director
Repository Licensing and Quality Assurance
Project Directorate
Division of High-Level Waste Management

FROM: Charlotte Abrams, Senior Project Manager
Repository Licensing and Quality Assurance
Project Directorate
Division of High-Level Waste Management

SUBJECT: FORTHCOMING NUCLEAR REGULATORY COMMISSION/U.S. DEPARTMENT OF
ENERGY (DOE) YUCCA MOUNTAIN SITE VISIT *

DATE: May 25-26, 1993

TIME: May 25 - 8:30 a.m. - 5:30 p.m.
May 26 - 7:00 a.m. - 5:30 p.m.

LOCATION:** Yucca Mountain Project Office Field Operations Center and
Yucca Mountain area

PURPOSE: To hold discussions on DOE progress on Quaternary fault
studies, exploratory studies facility mapping, and the
seismic initiative. Discussions will take place at the site
of data gathering activities and will include preliminary
results. Participants will be allotted time to view fault
trenches and trench logs.***

PARTICIPANTS:

NRC

C.Abrams
J.Trapp
K.McConnell

DOE

T.Bjerstedt
S.Jones
S.Leroy

State of Nevada

C.Johnson
J.Bell
T.Hickey, NV Legislative
Committee

I-341174
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4-27-93

~~9304300257~~ SPP

Affected Local Governments

L. Bradshaw, Nye County, NV
D. Bechtel, Clark County, NV
V. Poe, Mineral County, NV
C. Schank, Churchill County, NV
R. Williams, Lander County, NV
B. Mettam, Inyo County, CA
M. Murphy, Nye County, NV

M. Baughman, Lincoln County, NV
F. Sperry, White Pine County, NV
P. Niedzielski-Eichner, Nye County, NV
L. Fiorenza, Eureka County, NV
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Charlotte E. Abrams

Charlotte E. Abrams, Senior Project
Manager
Repository Licensing and Quality
Assurance Directorate
Division of High-Level Waste Management

cc: S. Goldberg, OMB
D. Weigel, GAO
P. Meyer, NAS
W. Barnard, NWTRB
A. Kadak, ACORN

Enclosures: Site Visit itinerary
Site access form

- * Interactions between NRC and DOE are open to members of the public, Petitioners, intervenors, or other interested parties wishing to attend as observers pursuant to the spirit of "Open Meeting Statement of NRC Staff Policy," 43 Federal Register 28058, dated June 28, 1978, which details the open meeting policy for applicants and licensees.
- ** Permission to gain access to the facilities at Yucca Mountain must be obtained from DOE, Yucca Mountain Project Office (YMP). Those persons wishing to attend should contact Thomas Bjerstedt (DOE/YMP) at (702) 794-7590 or Steve Leroy (702) 794-7836 at least 14 days prior to the site visit and mail or fax the information requested on the enclosed form (Enclosure 2) to Ms. Carlene Hill, SAIC, 101 Convention Center Dr., Las Vegas, NV 89193, Fax (702) 794-5348, Verify (702) 794-7375.
- *** Only those persons wearing safety glasses, hard hats, and steel-toed boots will be allowed to enter pits, trenches, and construction areas.

ITINERARY
YUCCA MOUNTAIN SITE VISIT
MAY 25 -26, 1993
FOR NRC STAFF

Tuesday, May 25, 1993

8:30 a.m. Meet at Nevada Test Site (NTS) Gate 510

8:30 - 9:00 Badging (Participants need a photo ID)

9:00 - 9:30 Travel to DOE Field Operations Center (FOC)

9:30 - 9:45 Break

9:45 - 10:00 Greeting and opening remarks - DOE, NRC, State of Nevada,
and Affected Counties

10:00 - 10:30 Required safety training

10:30 - 12:00 DOE explanation of progress on Quaternary fault studies,
seismic initiative, and ESF mapping

12:00 - 12:30 Lunch #

12:30 - 1:00 Travel from FOC to NTS Gate 25-4P

1:00 - 1:30 Travel to Busted Butte exposures

1:30 - 2:45 Discussion of additional mapping and preliminary
observations of Paintbrush Canyon fault exposures

2:45 - 3:15 Travel to ESF North Portal construction site

3:15 - 5:00 Discussion of ESF mapping
- Activities to-date
- Presentation of stratigraphy, structures and mapping logs

5:00 - 5:30 Travel to NTS Gate 510

5:30 p.m. Depart for lodging in Beatty, NV

Participants will be expected to provide their own lunches and beverages. Water will be available.

**ITINERARY
YUCCA MOUNTAIN SITE VISIT
MAY 25-26, 1993
FOR NRC STAFF**

Wednesday, May 26, 1993

7:00 a.m.	Meet at Steve's Pass in Crater Flat
7:00 - 8:00	Travel to Trench 8 (Solitario Canyon fault)
8:00 - 9:00	Discussion at Trench 8 Discussions at each trench to include: - stratigraphy - structure
9:00 - 9:10	Travel to Crater Flats (CF) Trench 1
9:10 - 9:50	Discussion at CF 1
9:50 - 10:20	Travel to Solitario Canyon fault Trench (SCFT) 1
10:20 - 11:00	Discussions at SCFT 1
11:00 - 12:00	Travel to Stagecoach Road Trench (SCRT) 3
12:00 - 1:30	Discussions at SCRT 3 and LUNCH #
1:30 - 1:45	Travel to SCRT 1 and 2
1:45 - 2:45	Discussions at SCRT 1 and 2
2:45 - 4:30	Travel to Bare Mountain Trench (BMT) 2
4:30 - 5:15	Discussion at BMT 2
5:15 p.m.	Travel to Las Vegas via Steve's Pass

Participants will be expected to provide their own lunches and beverages. Water will be available.

BADGING INFORMATION REQUIRED FOR ACCESS TO THE NEVADA TEST SITE FOR U.S. CITIZENS ONLY

NAME OF GROUP _____ DATE OF VISIT _____

LAST NAME _____ FIRST NAME _____ MIDDLE INITIAL (MI)
(IF NO MI WRITE NMI)

SOCIAL SECURITY NO. _____

DATE OF BIRTH _____ PLACE OF BIRTH _____

NATURALIZATION CERTIFICATE NUMBER _____

JOB TITLE _____

COMPANY NAME _____

COMPANY ADDRESS _____

PHONE # _____

HOME ADDRESS _____

PHONE # _____

CITIZENSHIP _____

NOTE: ORIGINAL NATURALIZATION CERTIFICATE IS REQUIRED AT TIME OF BADGING