Enclosure to
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Minimum Critical Numbers
of
LWBR Rods and Pellets

L. Laude

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Introduction

Previous limits for LWBR fuel rods and pellets (see Reference (1)) have been derived on the basis of data available for homogeneous systems of ^{233}U-Th-H_2O (References (2) and (3)) rather than explicit heterogeneous systems. It is important to prove that these limits are conservative and to determine how much of a safety margin is available. With this in mind, this study was undertaken to provide a consistent set of minimum critical numbers for rods and pellets with both nominal and double ^{233}U content. This was accomplished by determining the critical number in a hexagonal array of rods or pellets under optimum conditions of moderation and fully reflected by water. Note that the resulting minimum critical numbers are not to be used as approved piece count limits. The check on the homogeneous limits was provided by homogenizing near critical assemblies of pellets at the optimum spacing and by then comparing the resulting multiplication with the heterogeneous multiplication.

Description of Rods and Pellets

There are three types of LWBR binary fuel: seed, standard blanket and power flattening blanket. Since the two types of blanket fuels are quite similar, only the more reactive of the two, the power flattening blanket, was considered. The results are to be applied to both types of blanket fuel. The dimensions and loadings for the seed and blanket fuels considered in the the analysis are summarized in Table 1. The rods were assumed to be infinitely long to simplify the calculations.

RCP Program

The program chosen for the analysis was the RCP Monte Carlo program, since it provides both a versatile geometry and the most rigorous solution of neutron transport porblems available at Bettis. The cross-section library tapes required by RCP were generated with the MO481 program and were based on the same master tapes used by the LWBR project to generate their library tapes. The RCP program provides a statistical estimate of the multiplication factor. The precision of the estimate is dependent on the number of histories per iteration and on the number of iterations. For the problems run for this study, twenty iterations with 500 histories each were found to produce a sufficiently converged multiplication factor. The statistical uncertainties given in this report are probable errors as given in the RCP output.

Model

The rods were assumed to be infinitely long. The most reactive arrangement of these rods would be a cylinder. In RCP, this is best approximated by a hexagonal array of rods. With the use of symmetry boundary conditions, it was only necessary to depict one-third of a planar section of the array. In all problems run with rods, the cladding was omitted for simplication.

For an array of pellets the most reactive geometry is a sphere, since this provides minimum leakage. Due to the limitations of RCP, it was necessary to approximate the sphere with a hexagonal prism. In order to reduce the error introduced this way, the surface to volume ratio (and hence the relative leakage) of the prism was minimized. This required that the height be a factor of $\sqrt{3}$ greater than the length of a side of the hexagon.

For both the rod and pellet cases, a twelve inch reflector was provided. The moderator in all cases was also water. Also, the spacing of rods on pellets was defined as the surface-to-surface separation. It was felt that this provided the most meaningful indicator of the amount of moderator present.

Analysis

In order to determine the optimum spacing for a particular case (e.g., nominal w/o blanket rods), it was first necessary to develop a curve showing the variation of multiplication with spacing. RCP is a long running program. Consequently, it would have been impractical to run enough problems to define the multiplication curve in sufficient detail. Instead, three or four points on the curve were calculated using RCP , and then an interpolation scheme was used to fill in the detail at the maximum.

The initial spacing and core size were calculated on the basis of available homogeneous data on the critical mass at optimum H/U. The H/U ratio was used to calculate the spacing while the critical mass was used to calculate the number of rods/pellets and thus the core size. The resulting multiplication was always less than unity; that is, the critical mass of the heterogeneous system was always larger than the critical mass of the equivalent homogeneous system. The multiplication factors for heterogeneous vs homogeneous systems of double w/o seed and blanket pellets provide an example:

	Heterogeneous	Homogeneous
Seed	.801 ± .071	1.000 ± .065
Blanket	.875 ± .068	.987 ± .065

The core size was increased to bring the multiplication closer to unity, and the three or four problems were run with spacings that bracketed the estimated optimum spacing. The spacings chosen were 0.25", 0.40", 0.65" and 0.90" between rods. There were a total of 217 rods in a hexagonal array with 9 rods on a side. The RCP geometry overlay is shown in Figure 1, and the results for this step for nominal w/o blanket rods are shown in Figure 2. It can be seen that these four points in themselves would not have been sufficient to define the location of the maximum multiplication.

The interpolation between the four points was accomplished by separating the multiplication factors into their separate components until only monotonic functions of the spacing were obtained. The components of the multiplication factor (K_{eff}) were defined as follows:

$k_{eff} = k_1 + k_2$

$k_1 = (1 - L_1 - Q_{12})\, f_1 \eta_1$ (fast multiplication)

$k_2 = (Q_{12} + L_2)\, f_2\ {}_2 = A_2 f_2 \eta_2$ (thermal multiplication)

Q_{12} = slowing down into the thermal group in the core

L_1 = fast leakage out of the core

L_2 = thermal leakage into the core

f = utilization (fraction of core absorptions occuring in the fuel)

η = neutrons produced per absorption in the fuel

A_2 = thermal absorption in the fuel

The fast group is devoted by the subscript 1 and the thermal group by the subscript 2. For most problems, it was sufficient to interpolate only on the quantities k_1, A_2, and f_2; also, η_2 is constant over the range of spacings considered. The values needed were obtained from various data available from the RCP output, and are listed in Table 2 for the nominal w/o blanket rods. The quantities k_1, A_2 and f_2 are plotted in Figures 3, 4, and 5, respectively.

Additional points were obtained from these curves and were recombined in Table 3 to get additional points for the multiplication curve. These additional points were added to those shown in Figure 1 and are plotted in Figure 6. It is readily seen that the maximum multiplication occurs at a spacing of 0.5 inches.

A final problem was run with the same spacing but with fewer (169) rods. It is assumed here that the optimum spacing does not change significantly with small changes in core size. The multiplication for the 169 rods was 0.963 ± .007. A linear interpolation showed that the critical number at this separation would be 194 rods as indicated in Figure 4. This is then taken as the minimum critical number of nominal w/o blanket rods.

In order to show that omitting the clad in the rod calculations was conservative, the calculations for nominal w/o blanket rods were repeated using clad rods. The results of these calculations are shown in Figure 7. The curve for 217 rods is lower than the corresponding curve for bare rods in Figure 6, and the maximum occurs at slightly smaller separation. Although the separation is smaller, due to the increased rod diameter the H/U ratio remains about the same as the unclad rod - 379 vs 395. The minimum critical number of clads rods is 204 as compared to 194 unclad rods, showing that the unclad assumption is indeed conservative.

Results

Minimum critical numbers for the rods and pellets were obtained in a similar manner. The curves used to produce these values are shown in Figures 8, 9, 10 and 11 for nominal w/o pellets, double w/o pellets, nominal w/o rods and double w/o rods, respectively. The minimum critical values are tabulated in Table 4, and the corresponding spacing and H/U^{233} ratio in Table 5. The optimum spacing is approximately 1/3 inch for pellets and approximately 1/2 inch for rods. The double w/o blanket rods at 0.70 inch is an exception.

The near critical assemblies of pellets used in the determination of these minimum critical numbers were homogenized using the optimum spacings of Table 5. The corresponding multiplications were calculated and the results tabulated in Table 6. The heterogeneous results are also included and it may be seen that, with the exception of the nominal blanket pellets, the homogeneous cases are more reactive. The two nominal blanket pellet problems, given the uncertainties, have the same multiplication.

Conclusion

Minimum critical numbers have been derived for rods and pellets with the seed and power flattening blanket compositions at both nominal and double ^{233}U content. The minimization was performed by searching for the optimum moderation of rods and pellets in optimum geometry. The results are summarized in Table 4. The values in Table 4 should not be used as limits themselves. They should be reduced as required to provide the desired margin to criticality after any of the particular accidents being considered. It was also shown that the heterogeneous problems were never more reactive than the homogeneous problems. Thus, the limits of Reference (1) would not need to be reduced if re-derived on the basis of heterogeneous calculations.

References

(1) WAPD-CL(RA)L-32, "LWBR Binary Fuel and Fuel Rod Manufacturing Radiological Safety Operating Philosophy and Design Criteria," 4/70
(2) TID-7028, "Critical Dimensions of Systems Containing ^{235}U, ^{239}Pu and ^{233}U," 6/64
(3) WAPD-CL(RA)C-524, "Critical Parameters for H_2O Moderated and Reflected Systems of ThO_2 + $^{233}UO_2$ Mixtures," 2/26/70

TABLE 1

Physical Characteristics of Rods and Pellets

(All values for nominal w/o)

	Seed	Blanket
w/o UO_2	6.709	3.163
w/o U^{233}	5.899	2.781
Pellet diameter (inches)	0.252	.465
Pellet length (inches)	0.745	.890
Gm U^{233}/pellet	0.361	0.691
Gm U^{233}/inch	0.485	0.776
N (U^{233}) (atom/bn-cm)	0.153×10^{-2}	0.721×10^{-3}
N (Th^{232}) (atom/bn-cm)	0.214×10^{-1}	0.222×10^{-1}
N (Oxygen) (atom/bn-cm)	0.459×10^{-1}	0.457×10^{-1}
Clad ID (inches)	.257	.470
Clad OD (inches)	.304	.526

Table 2

Components of keff for Nominal w/o Blanket Rods

Spacing (inches)	0.25	0.40	0.65	0.90
*Multiplication Factor	0.9658	1.0188	1.0123	0.9189
*Thermal Production	0.7097	0.8268	0.8869	0.8305
Fast Production (k_1)	0.2561	0.1920	0.1254	0.0884
*Fuel Absorption	0.4319	0.5032	0.5399	0.5057
Neutrons/Absorption (η_2)	1.6432	1.6431	1.6427	1.6432(used 1.643)
*Moderator Absorption	0.0356	0.0775	0.1655	0.2723
Total Thermal Absorption (A_2)	0.4675	0.5807	0.7054	0.7780
Thermal Utilization (f_2)	0.9238	0.8665	0.7654	0.6500
Thermal Production (k_2)	0.7097	0.8268	0.8869	0.8305
Total Production (keff)	0.9658	1.0188	1.0123	0.9189

*Obtained directly from RCP

Table 3

Interpolated Values of keff for Nominal w/o Blanket Rods

Spacing (inches)	k_1	A_2	f_2	η_2	k_2	k_{eff}
0.25	0.256	0.468	0.924	1.643	0.710	0.966
0.30	0.233	0.508	0.905	"	0.755	0.988
0.35	0.211	0.545	0.886	"	0.793	1.004
0.40	0.192	0.581	0.866	"	0.827	1.019
0.45	0.175	0.613	0.847	"	0.853	1.028
0.50	0.161	0.642	0.827	"	0.872	1.033
0.55	0.148	0.667	0.807	"	0.884	1.032
0.60	0.136	0.687	0.786	"	0.887	1.023
0.65	0.125	0.705	0.765	"	0.886	1.011
0.70	0.117	0.721	0.742	"	0.879	0.996
0.75	0.109	0.736	0.720	"	0.871	0.980
0.80	0.101	0.751	0.697	"	0.860	0.961
0.85	0.095	0.764	0.673	"	0.845	0.940
0.90	0.088	0.778	0.650	"	0.831	0.919

Table 4

Minimum Critical Numbers of Pellets and Rods*

	Nominal w/o	Double w/o
Seed Pellets	4896	2001
Blanket Pellets	5744	1417
Seed Rods	174	84
Blanket Rods	194	74

* These are critical numbers under achievable optimum conditions; they should not under any circumstances be utilized as actual piece count criticality limits.

Table 5

Optimum Spacing and H/U^{233} for Arrays of Rods and Pellets*

	Nominal w/o	Double w/o
Seed Pellets	0.30 (280)	0.42 (244)
Blanket Pellets	0.29 (265)	0.45 (252)
Seed Rods	0.50 (389)	0.55 (193)
Blanket Rods	0.50 (395)	0.70 (274)

* Spacings are given in inches followed by H/U^{233} in parentheses.

Table 6

Comparison of Heterogeneous and Homogeneous Pellet Calculations

		Multiplication Factors	
Pellet	Loading	Heterogeneous	Homogeneous
Seed	Nominal	.996 ± .005	1.036 ± .003
Seed	Double	.993 ± .007	1.079 ± .006
Blanket	Nominal	1.008 ± .005	1.007 ± .005
Blanket	Double	1.008 .007	1.006 ± .004

RCP Geometry Overlay for 217 Blanket Rods

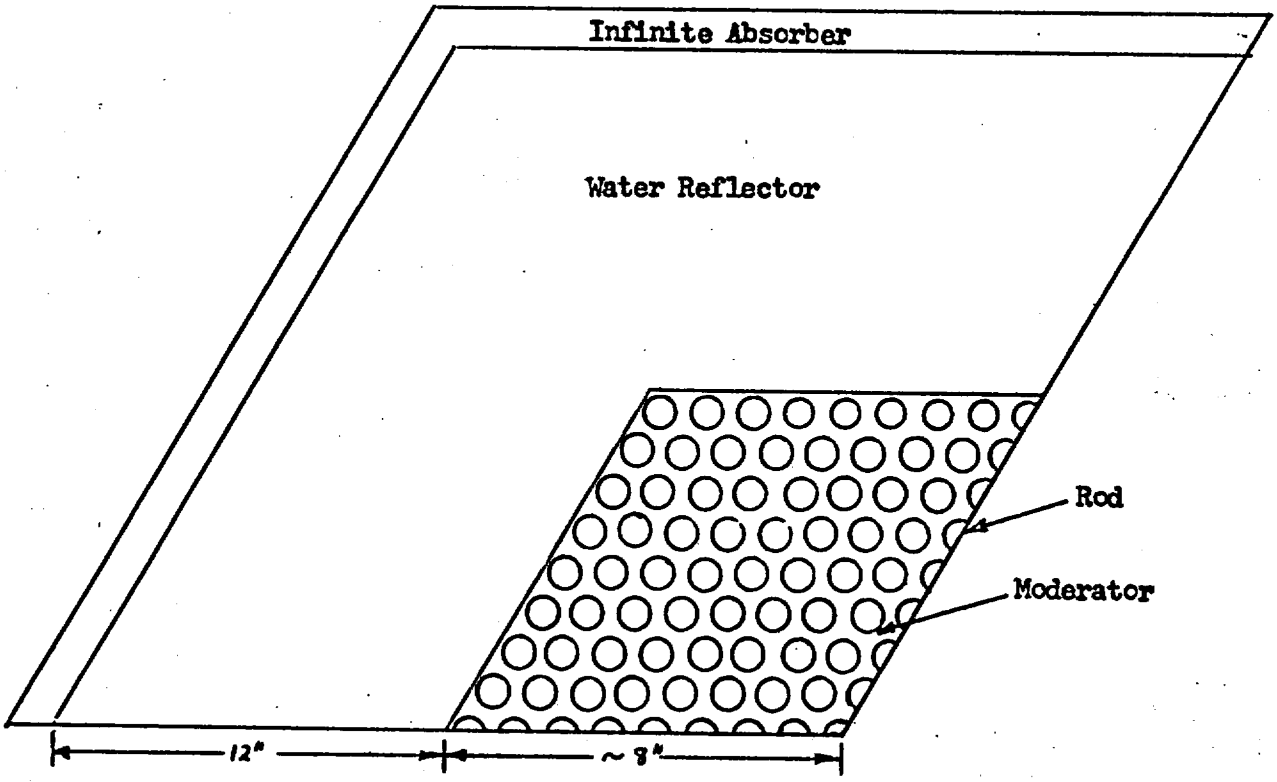


Figure 1

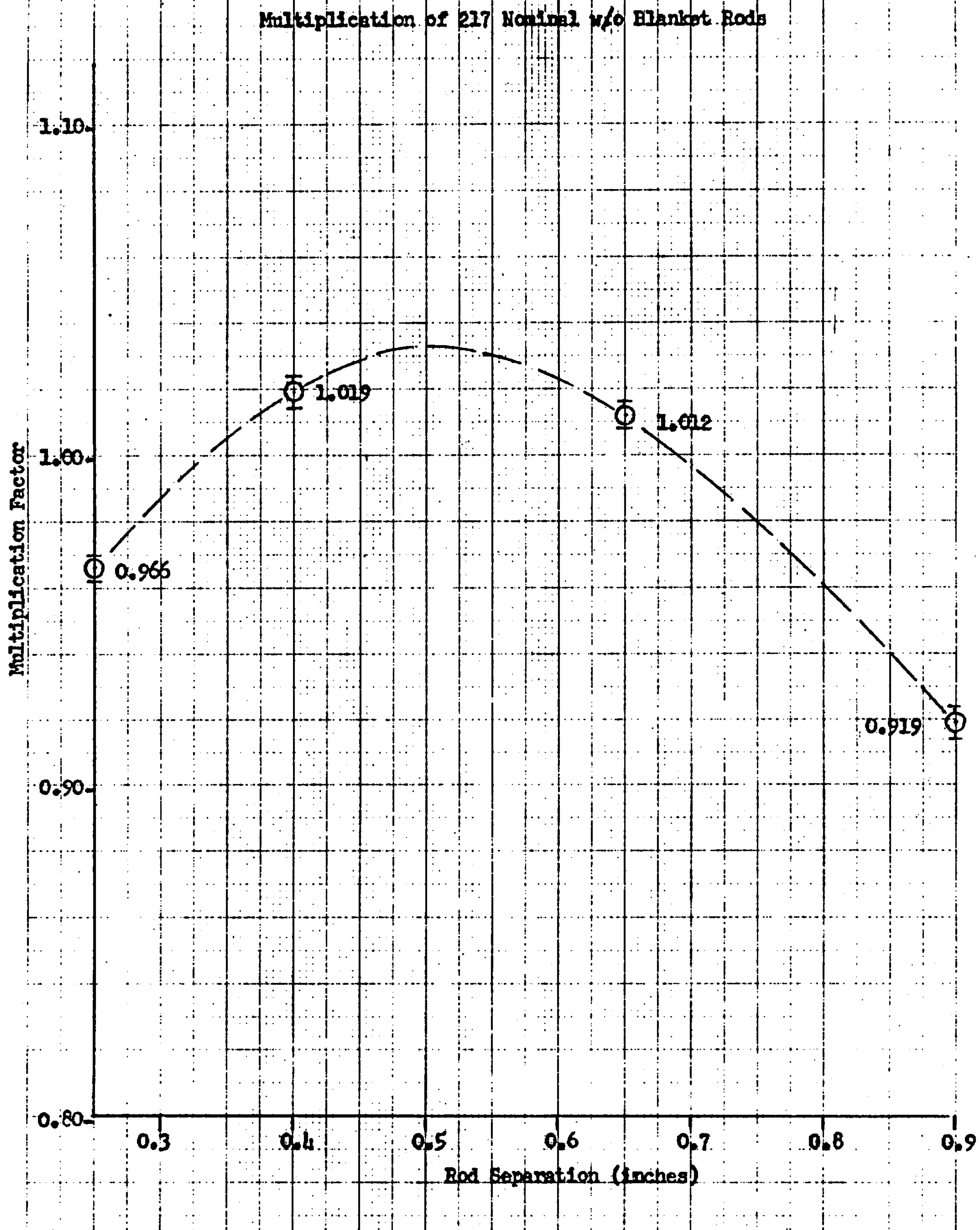

Multiplication of 217 Nominal w/o Blanket Rods
Multiplication Factor
1.10
1.00
0.90
0.80
Rod Separation (inches)
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.966
1.019
1.012
0.919

Figure 2

Fast Multiplication (k_1) for Nominal w/o Blanket Rods

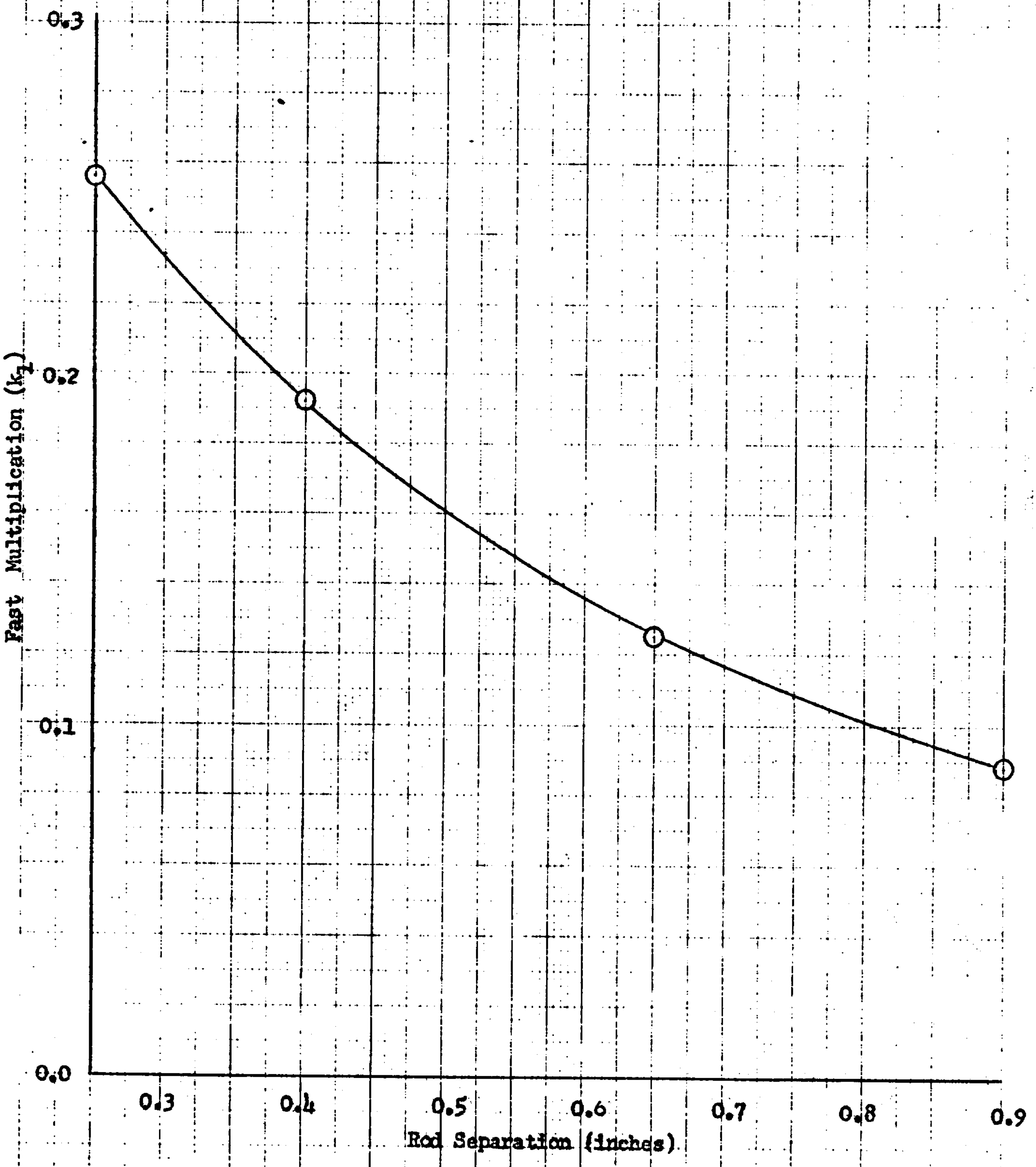
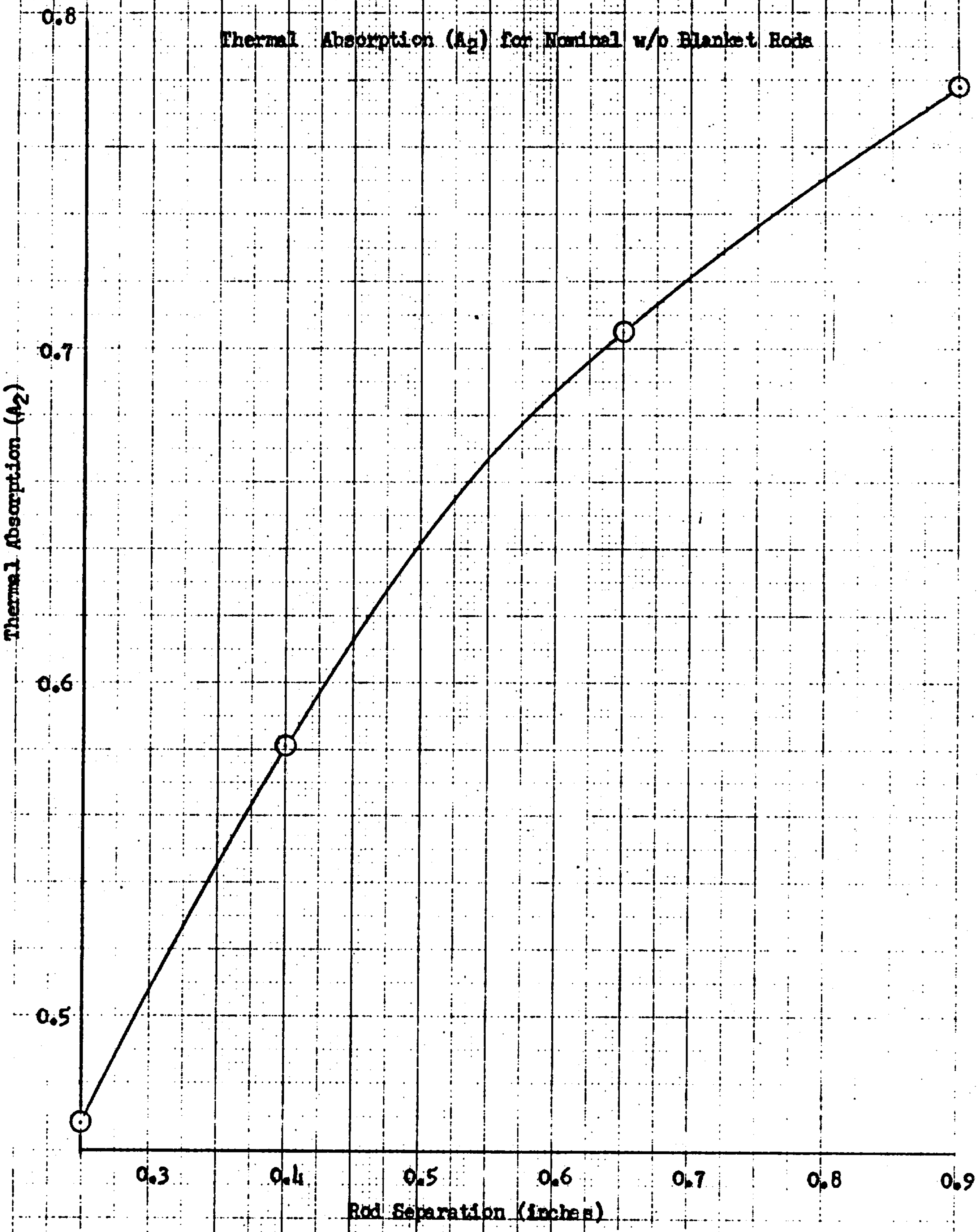


Figure 3



Thermal Absorption (A_2) for Nominal w/o Blanket Rods

Figure 4

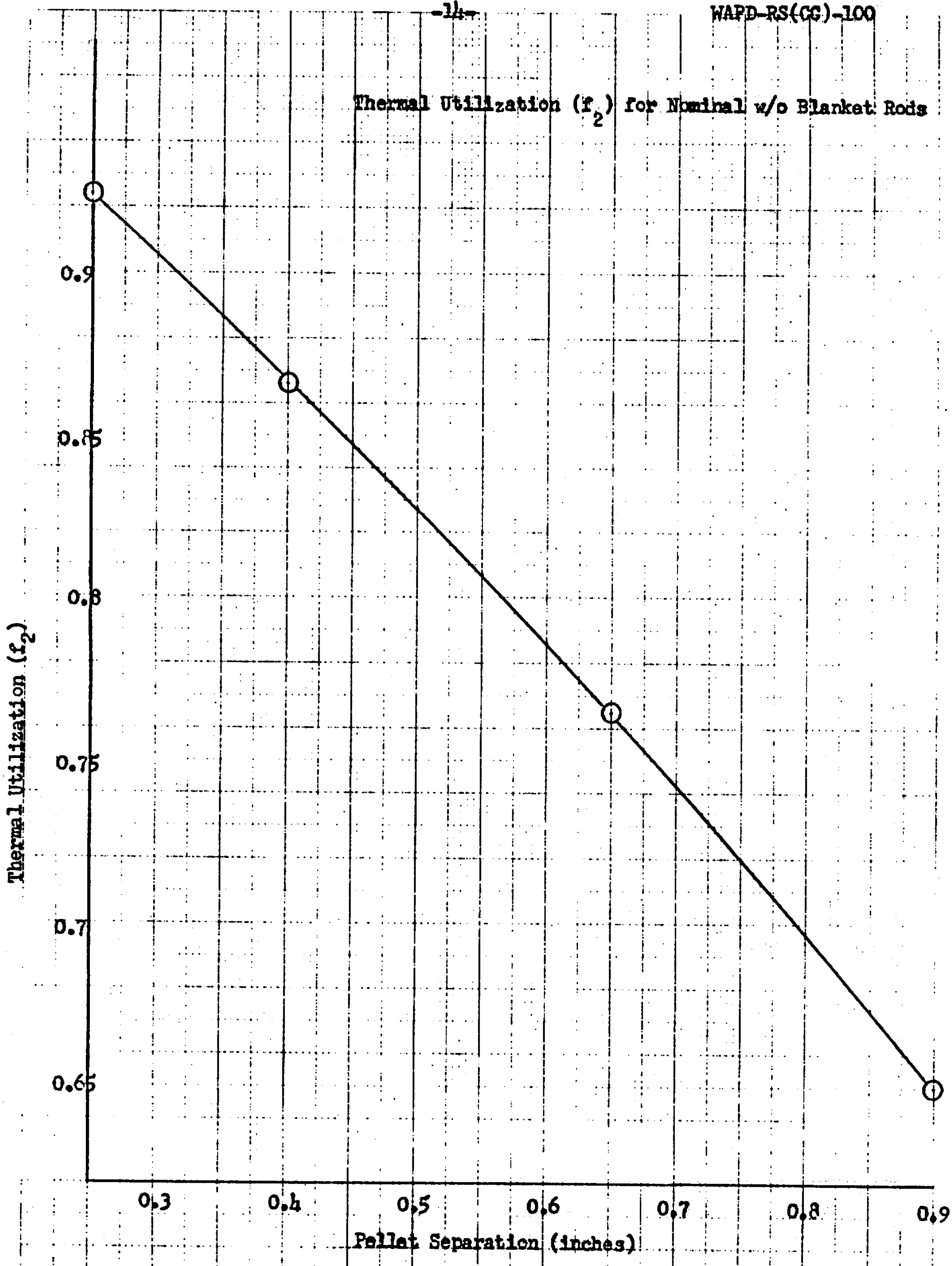

Thermal Utilization (f_2) for Nominal w/o Blanket Rods
Thermal Utilization (f_2)
0.9
0.85
0.8
0.75
0.7
0.65
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Pellet Separation (inches)

Figure 5

Optimum Moderation for Nominal w/o Blanket Rods

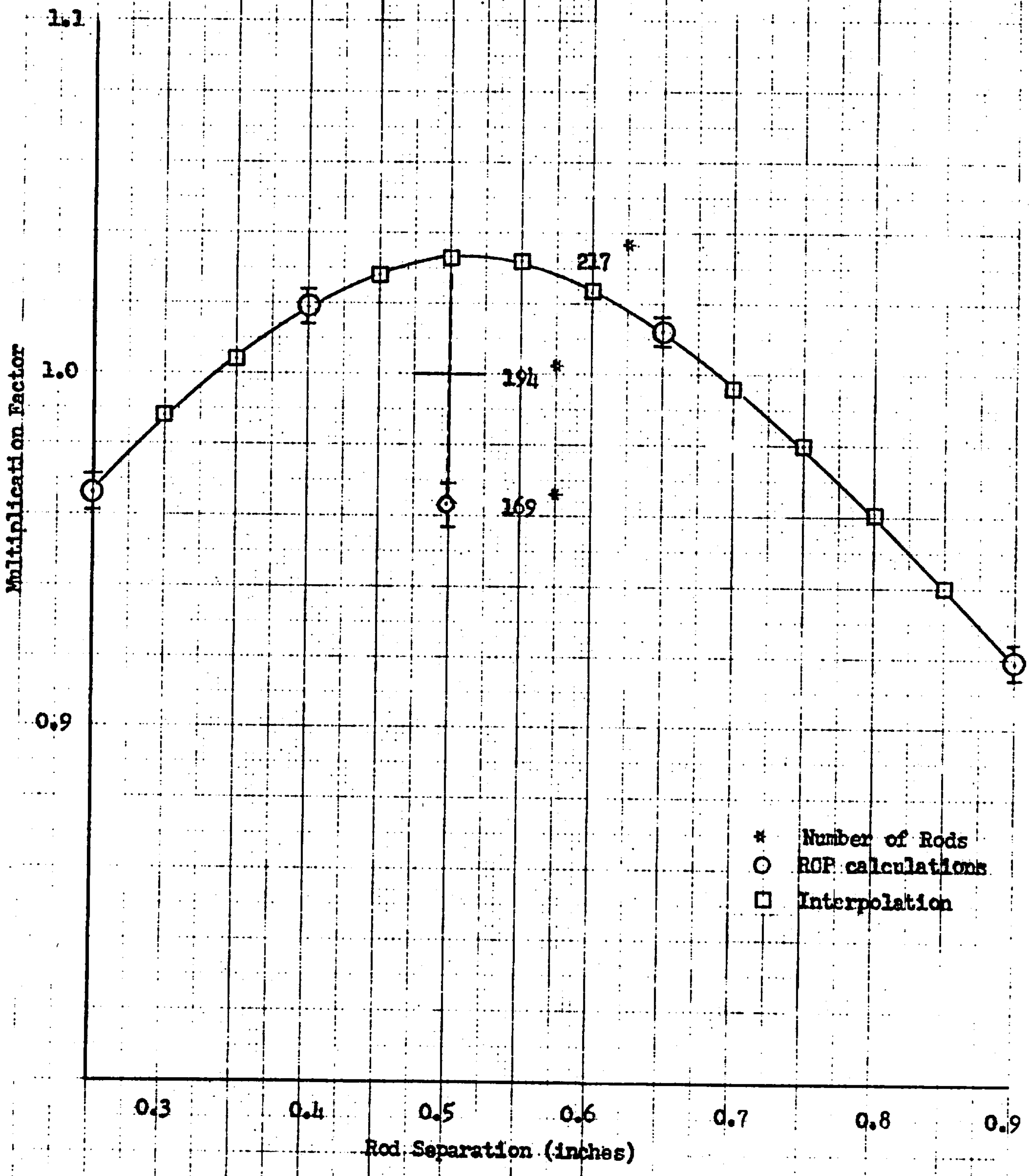


Figure 6

Optimum Moderation for Clad Nominal w/o Blanket Rods

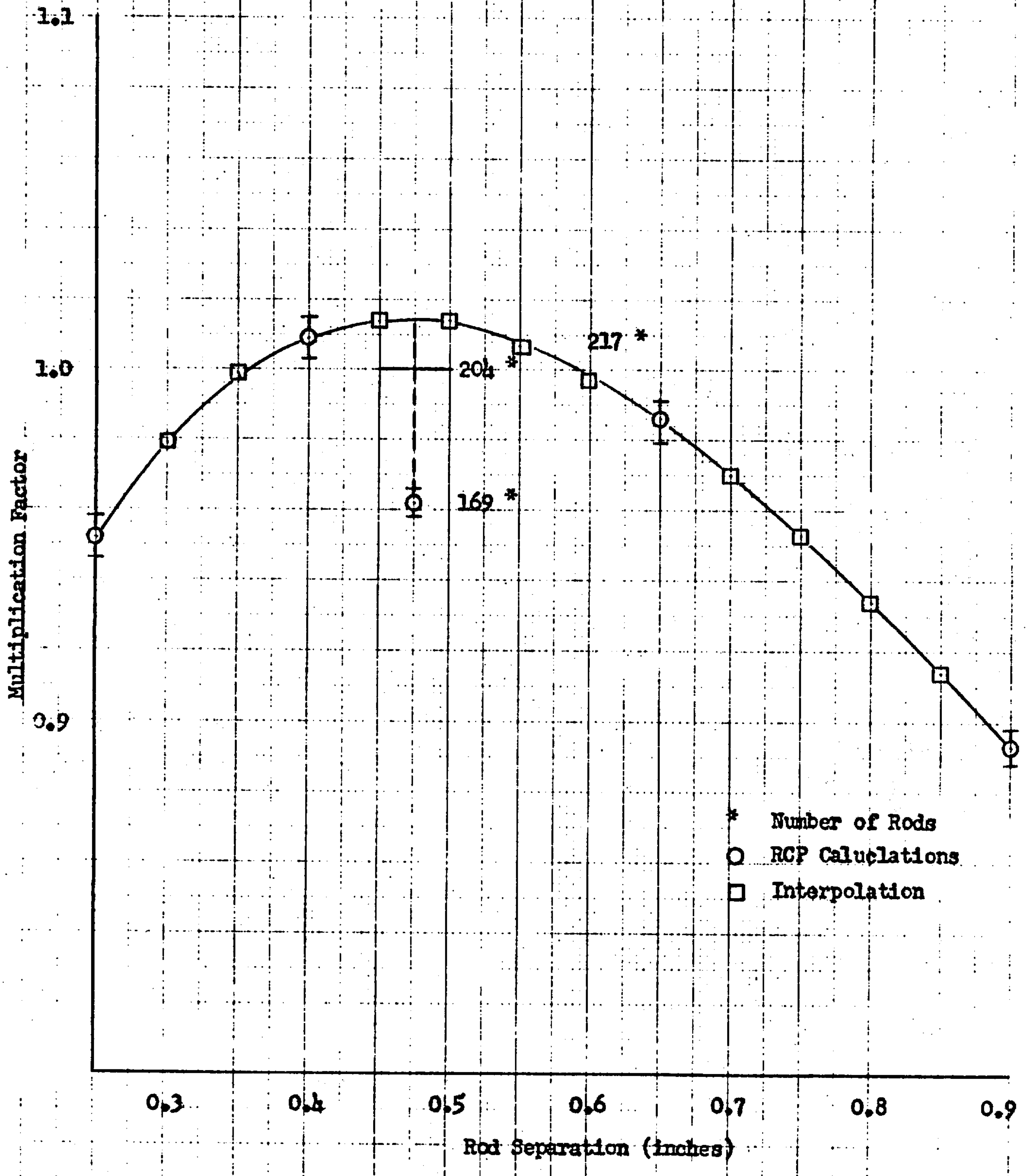
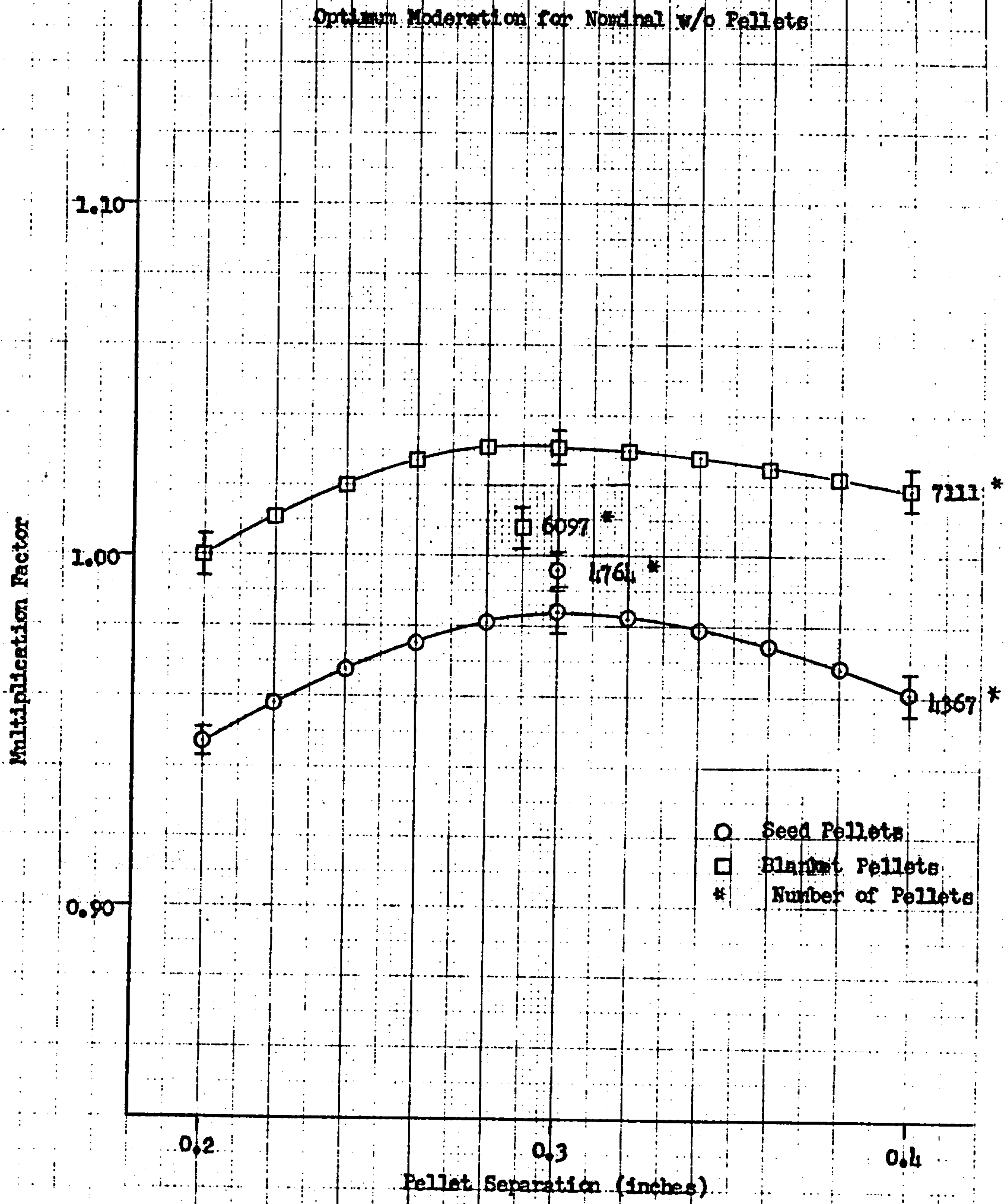


Figure 7



Optimum Moderation for Nominal w/o Pellets

Figure 8

Optimum Moderation for Double w/O Pellets

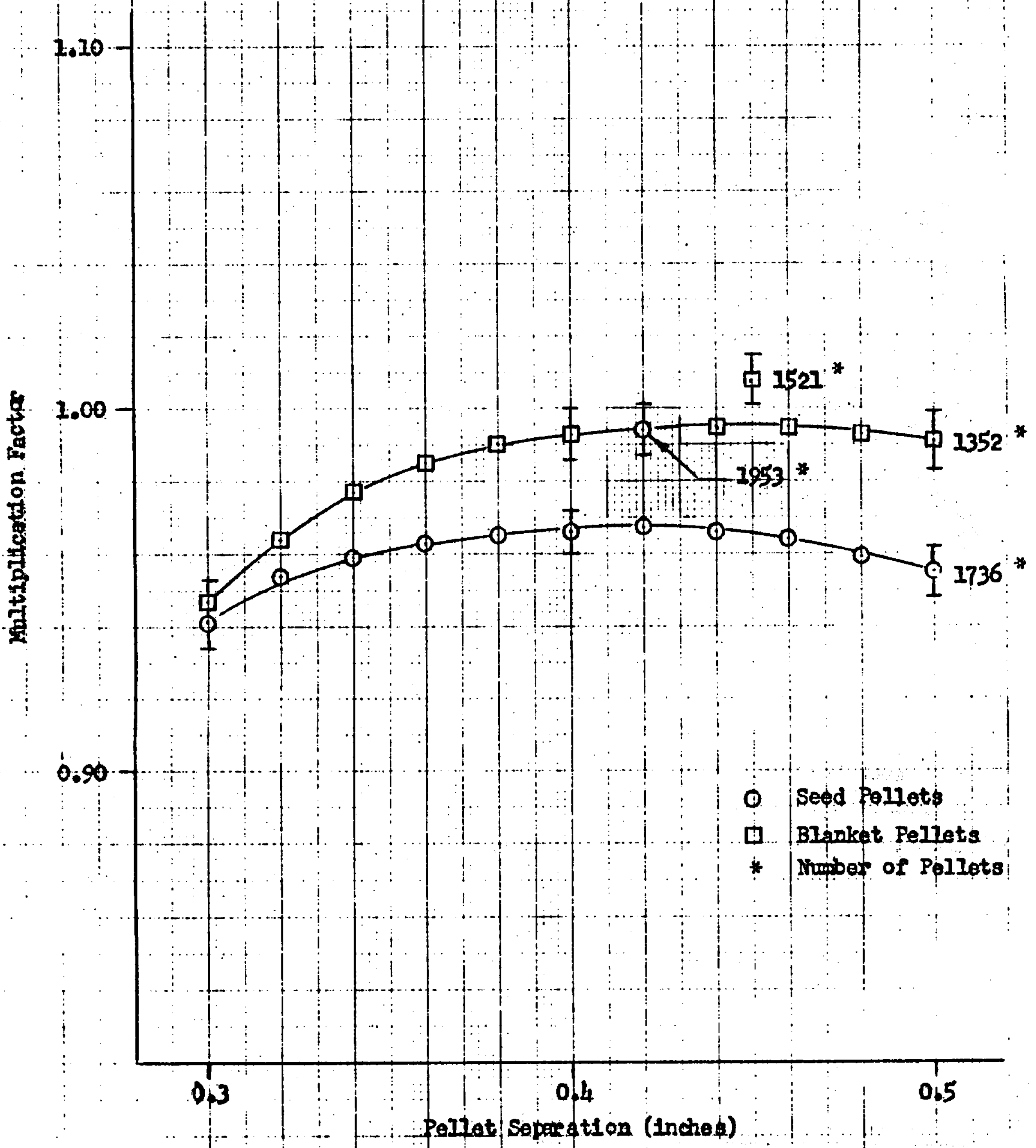


Figure 9

Optimum Moderation of Nominal w/o Rods

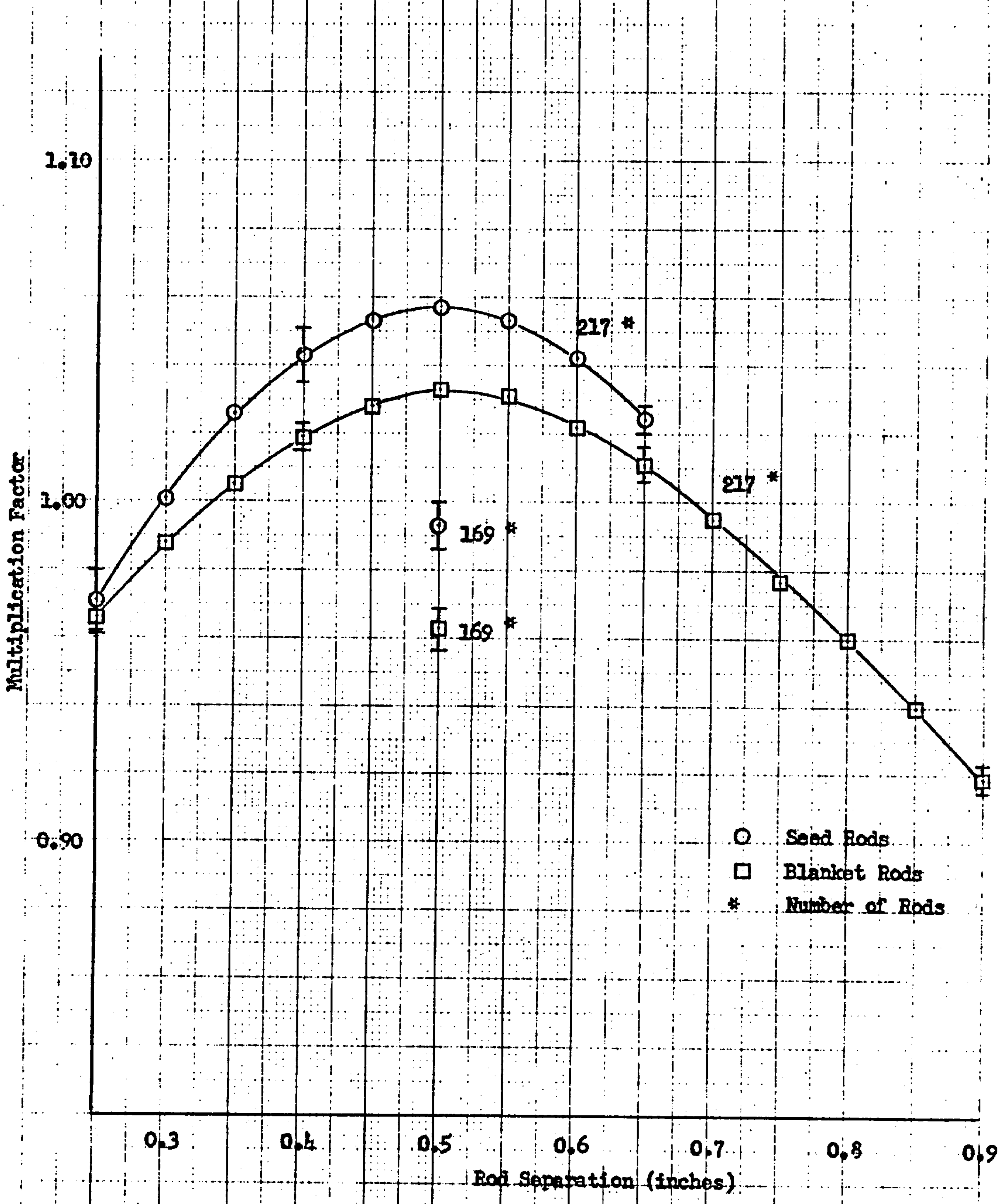


Figure 10

Optimum Moderation of Double w/o Rods

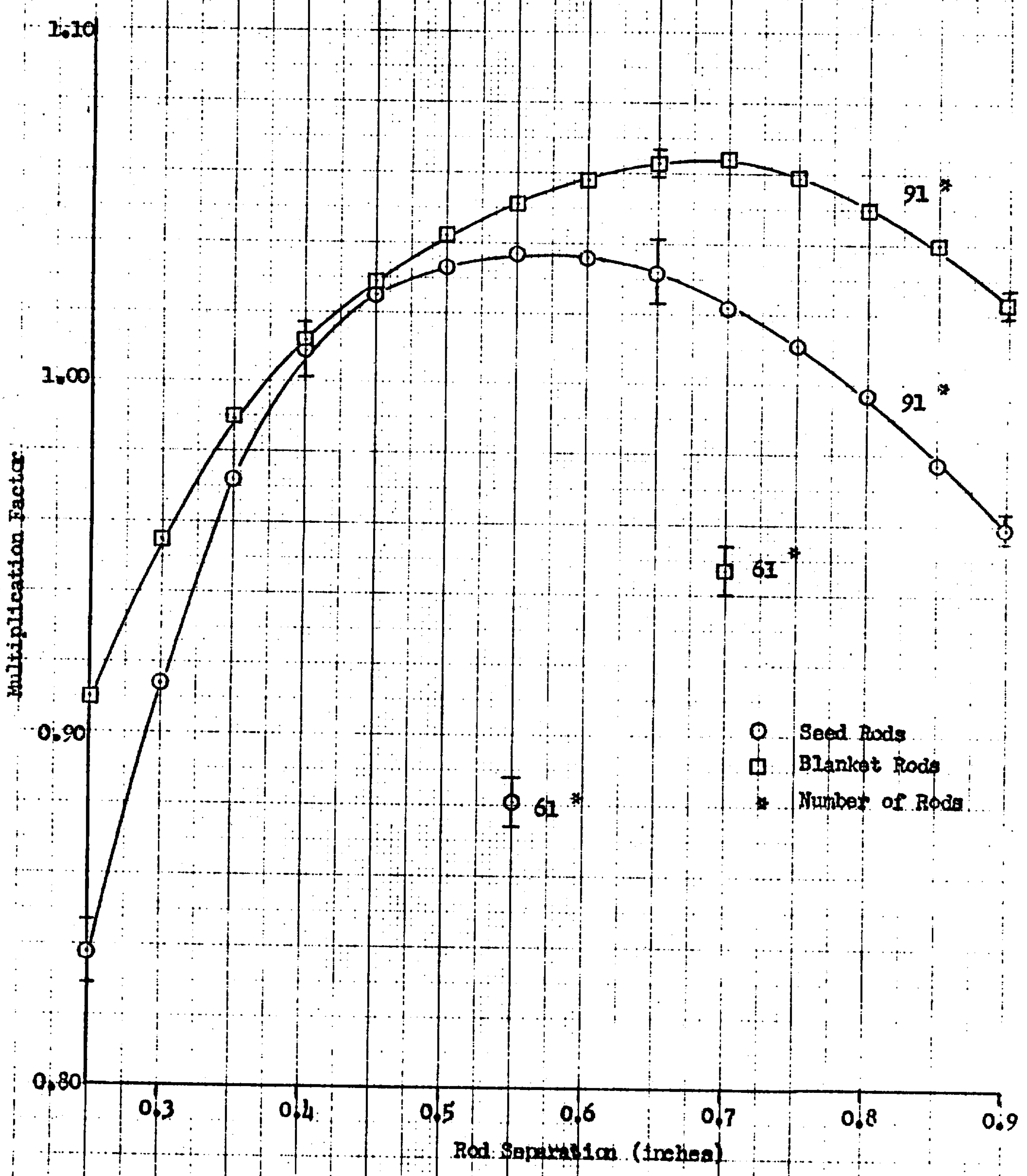


Figure 11