HOLTEC INTERNATIONAL
CALCULATION SHEET

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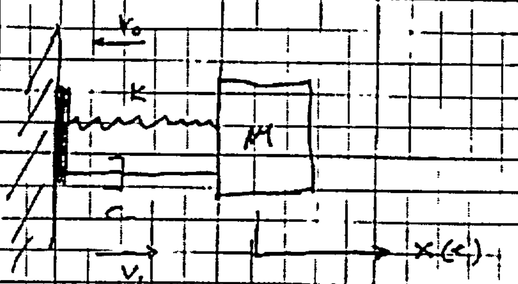
Project No ____ Report No ____ Other ____

Prepared By Alan Soler Date ____ Reviewed By ____ Date ____

COEFFICIENT OF RESTITUTION AND LINEAR VISCOUS DAMPING

Consider a 1-DOF mass-spring-damper after impact with a fixed target



The equation of motion is:

$$M\ddot{X} + C\dot{X} + KX = 0$$

$$X(0) = 0 \qquad \dot{X}(0) = -V_0$$

If we define ω_0, ξ by the relation

$$\omega_0 = \sqrt{K/M} \qquad 2\xi\omega_0 = \frac{C}{M}$$

then the equation of motion is

$$\ddot{X} + 2\xi\omega_0\dot{X} + \omega_0^2 X = 0 \qquad (1)$$

Per C. Smith, Applied Mechanics - More Dynamics, John Wiley, 1976, pp 187-189, the solution to eq (1), subject to the prescribed initial conditions is:

NUCLEAR REGULATORY COMMISSION

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Applicant ✓ RECEIVED ✓

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DATE 4-30-02 Witness ______

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$$x(t) = -\frac{V_0}{\omega} e^{-\xi\omega_0 t} \sin \omega t \quad (2)$$

The mass leaves the target at time, t_f, when

$$x(t_f) = 0 = -\frac{V_0}{\omega} e^{-\xi\omega_0 t_f} \sin \omega t_f$$

or, when

$$\omega t_f = \pi = \omega_0 (1-\xi^2)^{1/2} t_f \quad (3)$$

During the time $0 \le t \le t_f$ when the mass impacts the target, compresses the spring and damper, and reverses direction to return to the starting point, the velocity of the mass is

$$\dot{x}(t) = -\frac{V_0}{\omega}\left[-\xi\omega_0 e^{-\xi\omega_0 t} \sin\omega t + \omega e^{-\xi\omega_0 t} \cos\omega t \right]$$

so that at time, t_f, when $\omega t_f = \pi$

$$\dot{x}(t_f) = V_1 = V_0 e^{-\xi\omega_0 t_f} \quad (4)$$

Since $\frac{V_1}{V_0} = e$ = coefficient of restitution, and

$$\omega_0 = \frac{\omega}{\sqrt{1-\xi^2}}$$

Then

$$\boxed{e = e^{-\frac{\xi\pi}{(1-\xi^2)^{1/2}}}} \quad (5)$$

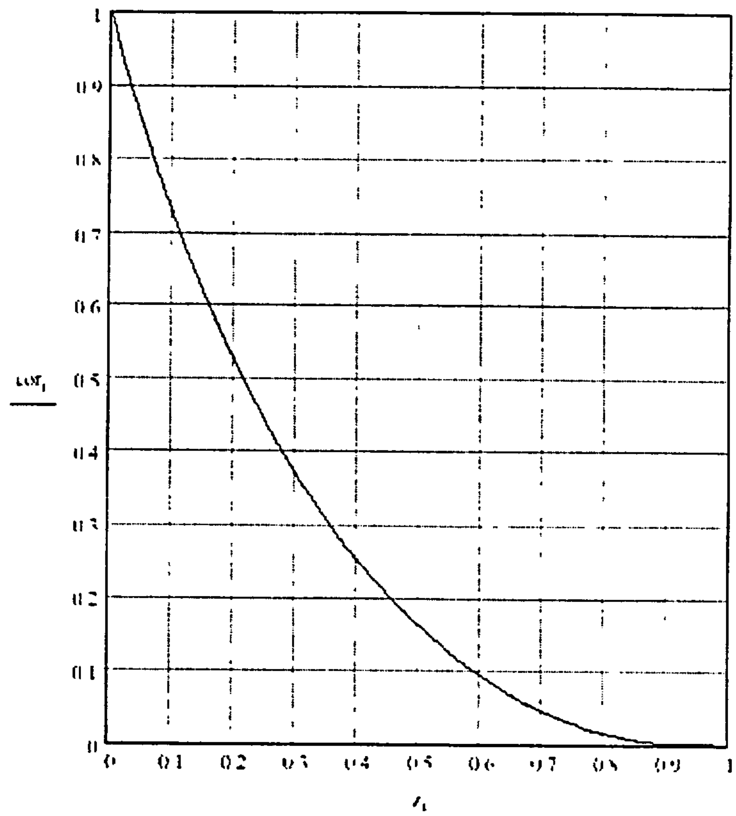
The above equation is plotted in the following

$i := 1..40$

$z_i := \frac{(i - 1)}{40}$

$g_i := \frac{z_i}{\left[1 - (z_i)^2\right]^{.5}} \cdot \pi \qquad cor_i := e^{g_i}$

i	cor_i	z_i
1	1	0
2	0.924	0.025
3	0.854	0.05
4	0.79	0.075
5	0.729	0.1
6	0.673	0.125
7	0.621	0.15
8	0.572	0.175
9	0.527	0.2
10	0.484	0.225
11	0.444	0.25
12	0.407	0.275
13	0.372	0.3
14	0.34	0.325
15	0.309	0.35
16	0.281	0.375
17	0.254	0.4
18	0.229	0.425
19	0.205	0.45
20	0.183	0.475
21	0.163	0.5



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1