SAAD, 1967

2784

CONSEIL INTERNATIONAL DES UNIONS SCIENTIFIQUES
INTERNATIONAL COUNCIL OF SCIENTIFIC UNIONS

UNION GÉODÉSIQUE ET GÉOPHYSIQUE INTERNATIONALE
INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS

Bulletin of the International Association of Scientific Hydrology

Bulletin de l'Association Internationale d'Hydrologie Scientifique

XII[e] Année N° 1

MARS 1967
MARCH 1967

Published on behalf of
THE INTERNATIONAL ASSOCIATION OF SCIENTIFIC HYDROLOGY

DETERMINATION OF THE VERTICAL AND HORIZONTAL PERMEABILITIES OF FRACTURED WATER BEARING FORMATIO

Dr. Eng. Kamal F. SAAD *

Desert Institute, Matarla, Cairo, UAR

Résumé

On peut considérer les formations aquifères fracturées comme des aquifères anisotropes. Lorsq s'agit de fractures horizontales, la perméabilité horizontale correspond aux propriétés de transmiss des fractures et la perméabilité verticale est celle de la roche mère. Au contraire, lorsqu'il s'agit fractures verticales, la perméabilité verticale correspond aux propriétés de transmissivité des fractu et la perméabilité horizontale caractérise la roche mère. Autrement dit, on considère que la roche m est isotrope. Le présent travail vise à déterminer les perméabilités verticale et horizontale des roc fracturées, aussi bien que leur capacité de réserve, par l'étude des données des essais de pompage. procédé employé est fondé sur la méthode de la double pente (Saad *et al.*, 1964). De plus, la magnitu des perméabilités verticale et horizontale peut fournir des indices sur les réseaux de fractures si l connaît la facilité de l'écoulement dans chaque direction.

Abstract

Fractured water bearing formation may be considered as an anisotropic aquifer. That is the perm ability through the fractures will be different from that of the original parent formation. In the pres paper, the values of both the vertical and horizontal permeabilities as well as the storage coefficient h been determined through analysis of the pumping test data. The permeability in one of these two directi represents that of the fractures having the same direction, while the other characterizes the transmitt property of the parent formation. The procedure of analysis is based on the double slope meth Saad *et al.* (1965) through analysis of the modified solution of the nonsteady flow toward a well partia penetrating the fractured water bearing formation. It can also be concluded that knowledge of magnitude of the permeabilities in both directions may indicate the pattern and trend of the fractu

Introduction

Anisotropic permeability of water bearing formation is a result of many reasons. Among th are the presence of fractures, with a certain pattern, in previous aquifer such as limesto Consequently the permeability through the fractures is different and usually higher from that the parent formation. The fractures may have either a horizontal or a vertical trend (Fig. 1) a thus they are represented hydraulically by the horizontal or the vertical permeability respective The permeability in the other direction represents the transmitting property of the parent wa bearing formation.

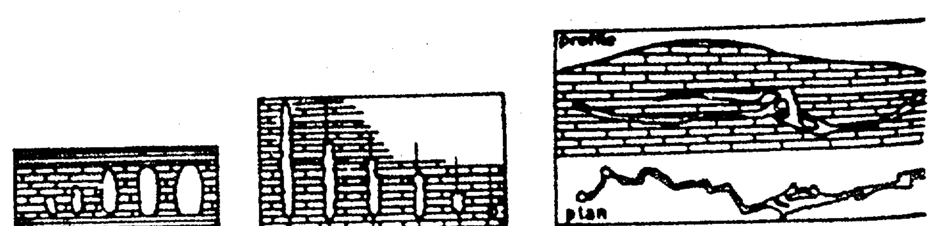

Fig. 1 — Cross section of fractured rock. A and B: Vertical fracture. C: Horizontal fracture.

* UNESCO Expert to the Republic of Mali, "Direction de l'Hydraulique", Bamako.

In practice, it frequently happens that the producing wells do not penetrate completely the fractured formation from which they are pumping. This is rather due to many technical reasons, among these are the large thickness of the fractured bed or the wide fracture openings. In both cases, drilling operation may be either expensive or impractical for large depths for excessive loss of mud circulation and other difficulties. For this, producing wells drilled in fractured water bearing formation, are usually partially penetrating the equifer.

The purpose of the present paper is to determine the permeability of both the fractures and the parent formation. These parameters as well as the storage coefficient can be determined through analysis of the pumping test data, recordes from a partially penetrating observation well where the pumped well itself does not reach the bottom of the fractured formation. It is assumed that the fractures are either horizontal or vertical and that the storage coefficient remains constant in the whole region.

The procedure of analysis is based on the double slope method, Saad *et al*, (1965), and on the modified solution of the nonsteady flow toward a partially penetrating well, Hantush, 1961, to account for anisotropy, Muskat, 1937.

THEORY

The average drawdown (s) in an observation well, where the screens in both the pumped and the observation wells extend for the whole length of each well, and that both are partially penetrating a water bearing formation, Figure 2, has been found by Hantush, (1961) as follows:

$$s = Q/4\pi K_r b\left[W(u)+(4b^2/\pi^2 dd')\sum_{n=1}^{\infty}(1/n^2)K_0(n\pi r/b)(\sin n\pi d'/b)(\sin n\pi d'/b)\right] \quad (1)$$

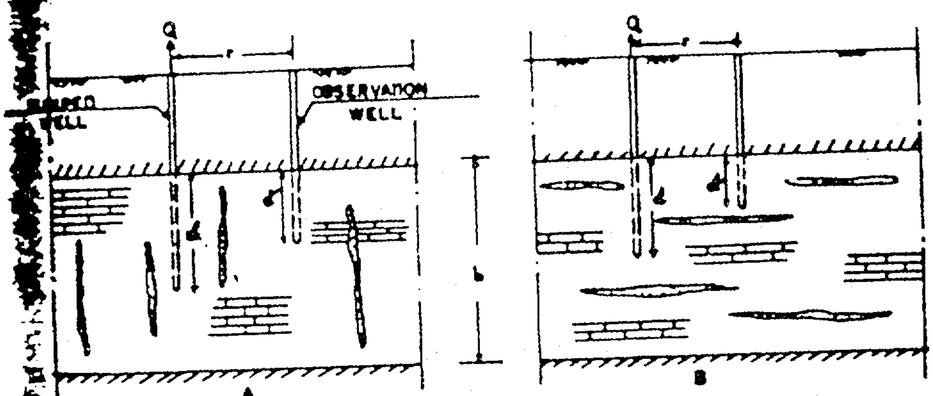

Fig. 2 — Diagrammatic representation of a well tapping fractured rock.
A: Vertical fracture. B: Horizontal fracture.

Equation (1), can be modified, to account for anisotropic permeability resulting from the presence of fractures, by multiplying the term (r/b) by $(K_d/K_r)^{1/2}$, Muskat, (1937). Thus Equation (1), will reduce to:

$$s = Q/4\pi K_r b\left[W(u)+(4b^2/\pi^2 dd')\sum_{n=1}^{\infty}1/n^2 K_0\{(n\pi r/b)(K_d/K_r)^{1/2}\}(\sin n\pi d/b)(\sin n\pi d'/b)\right] \quad (2)$$

Equation (2) shows that the rate of changes of the average drawdown behaves as of radial toward a well fully penetrating an aquifer.

For the determination of the hydraulic coefficients K_r and K_z, the duoble slope, Saad *et al.* (1965), can be used. The procedure of the mathematical analysis is outlined as follows:

i) differantiating (s) in Equation (2) with respect to $\log_e t$

$$ds/d(\ln t) = \frac{2.30\,Q}{4\pi K_r b} e^{-u} = m \qquad (3)$$

ii) differantiating (m) in Equation (3) with respect to $\ln t$

$$dm/d(\ln t) = \frac{(2.30)^2 Q}{4\pi K_r b} u\, e^{-u} = m' \qquad (4)$$

iii) the double slope function $f(u) = m'/m$, can be found

$$f(u) = m'/m = 2.30u \qquad (5)$$

Equations (5), (3), and the relationship ($u = r^2 S/4K_r bt$) and the data of pumping test, enable determining K_r and S, as will be shown later. The value of (K_z), can also be found if the terms in the summation form appearing in Equation (2), can be evaluated. For this, Equation (2) can be put in the following forms:

$$A = \sum_{n=1}^{\infty} (1/n^2) K_0\{(n\pi r/b)(K_z/K_r)^{1/2}\} (\sin n\pi d/b)(\sin n\pi d'/b) \qquad (6)$$

Where

$$A = \left[\frac{s}{Q/4\pi K_r b} - W(u)\right](\pi^2 dd'/4b^2) \qquad (7)$$

Equation (6), can be further reduced to the form of Fourrier series by multiplying both sides by ($\sin(x)dx$), where ($x = \pi d'/b$), and integrating between the limits 0 and π

$$\int_0^{\pi} A \sin(x)dx = \int_0^{\pi} \sum_{n=1}^{\infty} (1/n^2) K_0\{(n\pi r/b)(K_z/K_r)^{1/2}\}(\sin nx)(\sin x)dx \qquad (8)$$

Using the following two identities:

$$\int \sin(nx)\,.\,\sin(mx)dx = \frac{\sin(n-m)x}{2(n-m)} - \frac{\sin(n+m)x}{2(n+m)} \qquad (9)$$

and

$$\int \{\sin(nx)\}^2 dx = (x/2) - \frac{\sin(2nx)}{4n} \qquad (10)$$

it can be shown easily that all the terms in the summation form in Equation (7) will tend to zero except when $n = 1$. Therefore Equation (7) will reduce finally to:

$$\int_0^{\pi} A \sin(x)dx = (\pi/2) K_0[(\pi r/b) K_z/K_r)^{1/2}] \sin(\pi d/b) = 2A \qquad (11)$$

$$K_0[(\pi r/b(K_z/K_r)^{1/2}] = (4\,\lambda/\pi)\ \mathrm{cosec}\,(\pi d/b)) \quad (11)$$

APPLICATION

For determining the hydraulic properties of both the fractured and parent formation, i.e., the vertical and horizontal permeability and the storage coefficient from the data of the pumping test, the following procedure can be applied:

i) Draw the drawdown-time curve on semilog. paper, with the time on the log scale ($S - \log t$ curve).

ii) Choose several points on the curve and measure the slope (m), per cycle at each point. It may be more accurate if the chosen points comprise the latest portion of the original curve, or in other words when the time is large.

iii) Plot on the same semi-log paper the measured slopes (m) versus log t.

iv) Select few points on the ($m - \log t$) curve and measure the slope (m') of the tangent at each point, per cycle.

v) Knowing (m) and (m') at each time, find the double slope function $f(u)$, using Equation (5) at each point, and find the corresponding value of (u).

vi) Using Equation (3) with known values of (u), (Q), (b) and (m), determine the value of K_r.

vii) Using the relationship ($u = r_r S/4 K_r bt$), compute the value of S.

viii) Steps from v to vii, may be repeated for other values of (m'), at different values of time. However the computed values of (K_r) and (S), should be the same at each point. Any deviation is a result of improper measurements of (m) or (m').

ix) Choose any point on the ($s - \log t$) curve and record its coordinates.

x) Compute the value of (u) at that particular point and find its corresponding value of $W(u)$, from tables of the well function, Wenzel, (1937).

xi) Using Equation (11), the value of $K_0[(r/b)(K_z/K_r)^{1/2}]$, can be calculated. From which the value of (K_r/K_z), can be found, using the tables of the modified Bessel function $K_0(x)$.

xii) Knowing (K_r), determine the value of (K_z).

APPENDIX – NOTATION

b aquifer thickness (L).

d depth of penetration of the pumped well (L).

l depth of penetration of the observation well (L).

$K_0(x)$ the zero-order modified Bessel function of the second kind, tabulated, Watson (1944) or Dweight (1958).

K_r horizontal permeability radially from the well (L/T).

K_z vertical permeability (L^3T).

m slope of the tangent at any point on the drawdown-time curve ($s - \log t$) (L/cycle).

m' slope of the tangent at any point on the drawdonw-slope-time curve ($m - \log t$) (L/cycle).

Q constant well discharge (L^3/T).

r distance between the pumped and the observation wells (L).

s average drawdown in an observation well partially penetrating the aquifer (L).

S storage coefficient or storativity.

t time since pumping started (T).

u relation $r^2 S/4 K_r bt$.

$W(u) = \int_u^{\infty} (e^{-x}/x)\, dx$ = well function of (u), tabulated by Wenzel (1942).

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L'HYDRO
DANS LES

Laboratoire d'hy

SOMMAIRE

Dans les dunes modernes du lit
Celui-ci enfonce ses racines dans la
période historique. Ils sont essentie
sans parler un sol. Aucun horizon p
sont inexistants. La porosité et la p
et intense comme le montre l'étude

I. INTRODUCTION

A) *Objet de l'étude*

Dans les bas-fonds placés
relativement proche de la surface
que dans d'autres zones non encai
La répartition de ces « lettes maigr
représente un réel danger pour l'
les causes. Ceci nous a amené à a
L'humidité du sol, ses réserves e
sur les racines, la transpiration du

B) *Généralités*

1. *Ensemble des mesures et exp*

L'étude a eu lieu le long du cor
de Forge à 10 km au Nord du
hydrogéologique complète.
De nombreuses analyses chimi
tués. Certaines d'entre elles port
Des appareils enregistreurs Ric
phréatique et de la température du
La balance de torsion, de préci
l'étude de la transpiration du pin
De nombreux forages et sonda
L'ensemble de ce travail a été eff
Les moyens mis en œuvre et le ter
d'aboutir à la précision la plus sati

2. *Conditions écologiques généra*

Les dunes sont de formation trè
sur en place. Il y a eu triage mécan
La méthode de séparation par le bro
la profondeur, que le sable contien
essentiellement quartzeux.
Le climat est de type océanique
La température moyenne annuel
annuelle de 749 mm.

8.3.1.2.3.1.7

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