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U.S. Nuclear Regulatory Commission  
ATTN: Mrs. Deborah A. DeMarco  
Two White Flint North  
11545 Rockville Pike  
Mail Stop T8A23  
Washington, DC 20555

Subject: Programmatic Review of Abstract

Dear Mrs. DeMarco:

The enclosed abstract is being submitted for programmatic review. This abstract will be submitted for presentation at the Unsaturated Zone Interest Group 2001 to be held July 30–August 2, 2001, in Idaho Falls, Idaho. The title of this abstract is:

“Improved Nonlinear Iteration Schemes for Solution of Richards Equation” by R. Fedors

Please advise me of the results of your programmatic review. Your cooperation in this matter is appreciated.

Sincerely,

  
Budhi Sagar  
Technical Director

/ph  
Enclosures

|     |             |             |                          |
|-----|-------------|-------------|--------------------------|
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## Improved Nonlinear Iteration Schemes for Solution of Richards Equation

Computational effort in solving the nonlinear partial differential equation of variably saturated flow inhibits basin-wide studies. The nonlinear solver algorithm used to solve the Richards equation is investigated to reduce the computational effort. The outer-loop iteration scheme tied to the nonlinear stepping is the focus here; the presence of an inner-loop iteration that would be tied to a matrix solver is not addressed. Because it balances mass, the mixed form of the Richards equation improves the convergence rate over the pressure-head based form of the Richards equation. The mixed form of the Richards equation uses both pressure head and water content as dependent variables. The Picard iteration scheme is typically implemented for nonlinear stepping due to its simplicity and wide convergence domain. The Newton scheme can have fast convergence, however, it diverges for poor initial guesses and is computationally intensive. Three schemes to improve the nonlinear iteration stepping allow for better usage of the Newton iteration method. One, a line search and backtrack scheme is used to widen the convergence domain for the Newton iteration scheme. Two, a quasi-Newton scheme is implemented to reduce the computational effort of each Newton step. A rank one update scheme known as the Broyden Method is used in conjunction with a finite-difference scheme for calculation of the Jacobian terms. Three, a Picard scheme can initially be used to speed convergence for extremely poor initial guesses before switching to the Broyden approximation or the classic Newton method. For highly nonlinear problems, the combination of these improvements in the nonlinear stepping algorithm led to (i) convergence when the classic Newton method would fail; (ii) significant reductions in computer CPU time (75% reduction over Picard scheme); (iii) significant reduction in floating point operations per time step; and (iv) allowed the use of larger time steps. [Work presented here does not necessarily reflect views or positions of NRC].

UZIG 2001, General Topic: UZ Simulation and Modeling

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