A CLASSIFICATION SCHEME FOR IMPORTANCE MEASURES

Leslie E. Lancaster *

Nuclear Regulatory Commission, Washington, D. C. 20555

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Leslie E. Lancaster

Nuclear Regulatory Commission

Mail Stop: T-10-E-50

Washington, D. C. 20555

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ABSTRACT

Importance measures are playing a valuable role in the transitioning strategy to risk-informed regulation. In this report the basic mathematical elements, the F(.) parameters, used in defining importance measures will be further analyzed to bring their properties into the open. The F(.) parameters are used throughout this paper to define the six importance measures used in the SAPHIRE code, to show relationships among these measures, to describe differences among these measures, to make comparisons among these measures, and to introduce and define a classification scheme for these measures.

Overlapping, related, and nearly equivalent information is prevalent among these six importance measures, thus some importance measures are nearly equal in value and in rank ordering. The Birnbaum Importance Measure [1, 2] can be uniquely expressed in all three possible pair-wise combinations of the three F(.) parameters, which means that the Birnbaum Importance Measure can be easily compared with the definitions of all of the other interval measures, making the Birnbaum Importance Measure a very important importance measure with an important structure.

A FORTRAN program was written to perform an inverse-engineering process on two examples in order to look at the values of the three F(.) parameters for learning more about the interactions between and among the importance measures. Besides the F(.) parameters, other tools such as a classification scheme and some inequality flags are developed to help select component importance.

KEY WORDS: Reliability, SAPHIRE Code, Importance Measures, Classification Scheme, Tools, Relationships.

1. INTRODUCTION

To manage nuclear plant safety the reliability analyst (the end-user/analyst) needs to suggest to the nuclear plant designer where to allocate resources for reliability improvements, e.g., give a suggestion on which components to improve from a reliability point of view. To do this the reliability analyst needs a measure that he can give to the designer, on the relative importance of each component with respect to system reliability.

To construct an importance measure that will be used, one needs to assess the relative contribution, or relative importance, of each component to the overall reliability of the system; to address the relationships of seemingly different measures of importance of individual components; and to examine the properties of these importance measures under specified conditions.

In this report the six measures used in the SAPHIRE code will be compared to show how these measures are related to each other and to extend this relationship to a classification scheme. The basic mathematical elements, the three F(.) parameters, used in defining these measures will be analyzed in order to bring their properties into the open so that some guidance can be made on which importance measures to use and some guidance can be made on choosing component importance.

The remainder of this paper is organized as follows. In Section 2 the notation for defining the importance measures and the unreliabilities will be expressed via the three F(.)

parameters. In Section 3 the three definitions of the Birnbaum Importance Measure will be expressed and for each definition, the relationships with the other importance measures will be displayed. In Section 4 several more relationships among and between the six importance measures will be displayed. An inverse-engineering procedure will be introduced in Section 5, which provides a vehicle for looking at the F(.) parameters. In Section 6 some differences between the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure will be given. A classification scheme made up of the Birnbaum Importance Measure, the Fussell-Vesely Importance Measure, the F(.) parameters, and some inequalities will be introduced in Section 7.

2. NOTATION AND DEFINITIONS OF THE SIX IMPORTANCE MEASURES

In setting up our notation we are motivated by our interest in examining the relative importance of the various components to the overall system. We study the contribution of the components to the system unreliability by defining the importance measures used in the SAPHIRE code. In the process we shall define some parameters that we shall call F(.) parameters.

Consider a system which can be represented by a reliability block diagram. Denote this system by a k-dimensional vector X whose elements are the component unreliabilities, where this reliability block diagram is made up of these k components. This vector is expressed as follows:

$$X = (x(1), x(2), \ldots , x(t), \ldots , x(k)) \tag{1}$$

In this paper the index moves from component t to component t+1 over the k components and these k components making up the vector X are assumed to be fixed. The system unreliability, which is a constant over the k components, is given as follows:

$$F(X) = F(x(1), x(2), \ldots, x(t), \ldots\ldots, x(k)) \tag{2}$$

Let F(a,t) denote a transformation of the system unreliability, where the component indexed by t has an unreliability value equal to a and for t = 1, 2, ..., k, define the following 2k parameters:

$$\begin{aligned} F(0,t) &= F(x(1), x(2), \ldots, x(t) = 0, \ldots, x(k)) \\ F(1,t) &= F(x(1), x(2), \ldots, x(t) = 1, \ldots, x(k)) \end{aligned} \tag{3}$$

F(0,t) is the system unreliability, if the component t never fails; and F(1,t) is the system unreliability, if the component t always fails. In this paper the three F(.) parameters are F(X), F(0,t), and F(1,t). As defined in the SAPHIRE code [1] these 2k parameters given in Equation (3) are used to compute the six importance measures as given in Figure I.

Figure I. The Six Importance Measures.

$$\begin{aligned} B(t) &= F(1,t) - F(0,t) \quad \text{(Birnbaum)} \\ FV(t) &= (F(X) - F(0,t)) / F(X) \quad \text{(Fussell-Vesely)} \\ RRR(t) &= F(X) / F(0,t) \quad \text{(Risk Reduction Worth Ratio)} \\ RRI(t) &= F(X) - F(0,t) \quad \text{(Risk Reduction Worth Interval)} \\ RIR(t) &= F(1,t) / F(X) \quad \text{(Risk Achievemant Worth Ratio)} \\ RII(t) &= F(1,t) - F(X) \quad \text{(Risk Achievement Worth Interval)} \end{aligned} \tag{4}$$

Note that each importance measure will have k values, one for each component. For each importance measure the k values are rank ordered whereby the maximum value corresponds to the most important component, and so on in the rank ordering to the least important component.

It follows from the definitions of the importance measures that, except for the Birnbaum Importance Measure, the five other importance measures are rank ordered according to either F(0,t) or F(1,t). Specifically,

<> FV, RRR, and RRI are all rank ordered according to the inverse of F(0,t), i.e., the most important component corresponds to the smallest value of F(0,t), etc. (5)

<> RIR and RII are both rank ordered according to F(1,t), i.e., the most important component corresponds to the largest value of F(1,t), etc.

In other words, the set {FV, RRR, RRI} is redundant in that each importance measure in the set ranks the components in exactly the same way. Similarily, the set {RIR, RII} is also redundant. Thus, the six importance measures defined by Equation (4) can be replaced by three measures: Birnbaum and one each from the sets {FV, RRR, RRI} and {RIR, RII}.

3. THREE DEFINITIONS OF THE BIRNBAUM IMPORTANCE MEASURE

An interesting perspective is to begin the discussion with the following linear relationship. For a fixed component, denoted by the index t, let x(t) be a variable so that the constant F(X) is transformed to the function F(x(t)) as follows:

$$F(x(t)) = (F(1,t) - F(0,t)) x(t) + F(0,t) \qquad (6)$$

where F(x(t)) is the ordinate or dependent variable, x(t) is the abscissa or independent variable, F(1,t) - F(0,t) is the slope, and F(0,t) is the intercept. It follows from the definition of system unreliability that F(x(t)) = F(X), where F(X) is defined by Equation (2).

The derivative of F(x(t)) [2] with respect to x(t) is the Birnbaum Importance Measure (see Figure II) expressed as follows:

$$F'(x(t)) = B(t) = F(1,t) - F(0,t) \tag{7}$$

Besides being the component unreliability, the parameter x(t) plays another role, it can be expressed in terms of the three F(.) parameters [1,2] as follows:

$$x(t) = \frac{F(X) - F(0,t)}{F(1,t) - F(0,t)} = \text{component unreliability}$$

$$1 - x(t) = \frac{F(1,t) - F(X)}{F(1,t) - F(0,t)} = \text{component reliability} \tag{8}$$

The Birnbaum Importance Measure [1, 2] can be uniquely expressed in all three possible pair-wise combinations of the three F(.) parameters, which means that the Birnbaum Importance Measure can be easily compared with the definitions of all of the other interval measures. These three definitions for the Birnbaum Importance Measure, as compared with the other importance measures, are given in Figures II, III, and IV.

Figure II. First Definition of the Birnbaum Importance Measure.

Definition 1: $B(t) = F(1,t) - F(0,t)$ (9)

From this definition the following are obtained:

$$B(t) = RRI(t) + RII(t)$$
$$RII(t) = (1-x(t))\, B(t) \tag{10}$$
$$RRI(t) = x(t)\, B(t)$$

and it follows that if we know B(t) and RII(t), then RRI(t) is determined and RII(t) is approximately equal to B(t) for small values of x(t) . However, the rankings of RII(t) and B(t) may not be exactly the same (see Table IIc).

Figure III. Second Definition of the Birnbaum Importance Measure.

Definition 2:

$$B(t) = \frac{F(1,t) - F(X)}{1 - x(t)} \qquad (11)$$

Using this definition one can compare B(t) with RII(t) as follows:

$$RII(t) = F(1,t) - F(X)$$

and $\qquad\qquad\qquad\qquad$ (12)

$$RII(t) = (1-x(t))\ B(t)$$

which means that RII(t) is nearly equal to B(t) for small values of x(t). However, the rankings of RII(t) and B(t) may not be exactly the same (see Table IIc).

Figure IV. Third Definition of the Birnbaum Importance Measure.

Definition 3: $B(t) = (\ F(X) - F(0,t)\ ) / x(t) \qquad (13)$

Under this definition one can easily compare B(t) with FV(t) and RRI(t) as follows:

$$FV(t) = (\ F(X) - F(0,t)\ ) / F(X) \qquad (14)$$

$$RRI(t) = (\ F(X) - F(0,t)\ )$$

or

$$FV(t) = x(t)\ B(t) / F(X)$$

$$RRI(t) = x(t)\ B(t) \qquad (15)$$

$$RRI(t) = F(X)\ FV(t)$$

Hence, FV(t) is proportional to x(t) B(t) and FV(t) is proportional to RRI(t). Further, since the x(t) values are usually small and the B(t) values are between zero and one, the RRI(t) values will be very small (see Table IIb)

These three definitions of the Birnbaum Importance Measure [1, 2, 3, 4], are based on a solid probabilistic structure. Birnbaum published his landmark paper in 1969. All of the other importance measures came later and have emulated the make-up of the Birnbaum Importance Measure by using two out of the three F(.) parameters that make up the various ways of expressing the Birnbaum Importance Measure. And as shown, the underlying definition of the Birnbaum Importance Measure can be uniquely expressed in all three possible pair-wise combinations of the three F(.) parameters, making the Birnbaum Importance Measure an important importance measure that engineers should use.

4. RELATIONSHIPS AMONG AND BETWEEN THE IMPORTANCE MEASURES

As we have seen, overlapping information and even related and equivalent information up to a constant is prevalent among these six importance measures. Mainly because there are six equations that use a mix of two parameters, F(0,t) and F(1,t), and one constant, F(X), between them for each fixed component. Some importance measures are nearly equal in value to another importance measure and some pairs of importance measures even have the same or very nearly the same rank ordering. In this section we shall study more of these relationships.

To see how certain importance measures are linearly related, look at the following exapression:

$$RRI(t) = F(X)FV(t) \tag{16}$$

noting that RRI(t) and FV(t) are proportional. Importance measures are defined by a rank ordering from most important to least important but here, the RRI(t) values will have the same rank ordering as the FV(t) values and this one-to-one correspondence is equivalent up to a constant and identically equivalent when normalized (see Tables Ic and IIc). In fact, the rank ordering of the FV(t) values and the RRI(t) values will be the same as if we rank ordered the '1 - F(0,t)' values.

To show further dependence among the importance measures, RRR(t) and FV(t) are related by the following transformation:

$$RRR(t) = \frac{1}{1 - FV(t)} \qquad (17)$$

and, RRI(t) and RRR(t) are related by the following expression:

$$RRI(t) = F(X)\left(1 - \frac{1}{RRR(t)}\right) \qquad (18)$$

On checking for more pair-wise relationships, note that RII(t), RIR(t), and B(t) are related as follows:

$$\begin{aligned} RII(t) &= F(X)\ (RIR(t) - 1) \\ B(t) &= F(X)\ (RIR(t) - 1)\ /\ (1-x(t)) \end{aligned} \qquad (19)$$

As we have illustrated throughout this report, all of these interval measures are related, and here are more relationships:

$$\begin{aligned} x(t) &= RRI(t) / B(t) \\ 1 - x(t) &= RII(t) / B(t) \\ x(t) / (1-x(t)) &= RRI(t) / RII(t) \\ x(t) / F(X) &= FV(t) / B(t) \end{aligned} \qquad (20)$$

Figure V summarizes the relationships with the Fussell-Vesely Importance Measure and the Birnbaum Importance Measure between the other four importance measures. Thereby stressing the fact

Figure V. FV and B expressed in terms of the other four measures.

Risk Reduction Worth Ratio:

$$FV(t) = 1 - \frac{1}{RRR(t)}$$

$$B(t) = \frac{F(X)}{x(t)} \left(1 - \frac{1}{RRR(t)} \right)$$

Risk Reduction Worth Interval:

$$FV(t) = RRI(t) / F(X)$$

$$B(t) = RRI(t) + RII(t)$$

$$B(t) = RRI(t) / x(t)$$

Risk Achievemant Worth Ratio:

$$FV(t) = RIR(t) \frac{F(X) - F(0,t)}{F(1,t)}$$

$$B(t) = F(X) (RIR(t) - 1) / (1-x(t))$$

Risk Achievement Worth Interval:

$$FV(t) = \frac{F(X) - F(0,t)}{F(1,t) - RII(t)}$$

$$B(t) = RRI(t) + RII(t)$$

$$B(t) = RII(t) / (1 - x(t))$$

that the original set of six importance measures can be reduced, especially for small values of x(t). For example, as seen from Figure V, FV(t) and RRR(t) are in one-to-one correspondence, FV(t) and RRI(t) are proportional, B(t) and RIR(t) are conversely proportional for small values of x(t), and B(t) and RII(t) are equal for small values of x(t). Hence, together, FV(t) and B(t) cover the other four importance measures, except as shown before, it may turn out that the rankings may change slightly.

5. DATA ANALYSIS OF THE SAPHIRE OUTPUT

In any computer package that generates importance measures, Figure VI shows the information that should be provided.

Figure VI. Output Information From An Ideal Computer Program.

<> for each component:

1 - F(0,t)

F(0,t)

x(t) (the x(t) values are input)

F(1,t)

and the importance measures (21)

<> for the system:

F(X)

Since the SAPHIRE output did not provide enough information to do a proper exploratory analysis using the values of the F(.) parameters, an inverse-engineering process was performed on two examples taken from SAPHIRE runs [1] in order to look at the data values of the F(.) parameters so we can learn more about the relationships among the importance measures. Results of these two examples are given in Tables I and II. The FORTRAN program was written to perform the inverse-engineering process in four steps which are listed in Figure VII.

Figure VII. Steps To Be Taken in the Inverse-Engineering Process.

Step 1. Read in the Input Data from the SAPHIRE output. The only data that are needed are the k values for each of the following importance measures:

$$
\begin{array}{l}
FV(t) \\
B(t) \\
RRI(t)
\end{array}
\qquad (22)
$$

Step 2. Compute the folowing values for each component (note that F(X) is also computed for each t as a check on the round-off error):

$$
\begin{aligned}
x(t) &= RRI(t) \;/\; B(t) \\
F(X) &= B(t)\; x(t) \;/\; FV(t) \\
F(0,t) &= F(X) - B(t)\; x(t) \\
F(1,t) &= F(X) + B(t)\; (1-x(t))
\end{aligned}
\qquad (23)
$$

Step 3. For each component compute the importance measures as a check on the computations:

$$
\begin{aligned}
B(t) &= F(1,t) - F(0,t) \\
FV(t) &= (\, F(X) - F(0,t) \,) \;/\; F(X) \\
RRR(t) &= F(X) \;/\; F(0,t) \\
RRI(t) &= F(X) - F(0,t) \\
RIR(t) &= F(1,t) \;/\; F(X) \\
RII(t) &= F(1,t) - F(X)
\end{aligned}
\qquad (24)
$$

Step 4. Output Tables I and II.

From this data analysis the following observations were gleaned:

Observation 1: Note that this inverse-engineering technique works by reading and operating on only information from a single component at each step, where the process is ignited in Step 1 with the three importance measures for a particular component.

Observation 2: From the FORTRAN program some interesting results are printed and displayed as Tables I and II in order to learn more about these measures. For example, in Table Ia and Id, note that one component has F(0,t) < x(t) , while the other nine components have F(0,t) > x(t) (see the section on classification), and in Tables IIa and IId, thirteen components have F(0,t) < x(t) , while seven components have F(0,t) > x(t) .

Observation 3: As a check on the round-off error, note that the F(X) values for each component should all be equal. In Table Ia they range from 0.02119 to 0.02121 and in Table IIa they range from 0.550756E-5 to 0.551696E-5 -- which is comparatively good.

Obvervation 4: In Table IIa note that the component having the unreliability x(t) = 0.5 is the most important via the Fussell-Vesely Importance Measure. But, in Table IId, when ranked via the Birnbaum Importance Measure, this component is ranked as being seventeenth in importance. Hence, this component should be studied further using the classification scheme (see Section 7).

Observation 5: One should compare the FV(t) rank ordering with the round-off error in the F(0,t) values. For example, in Table Ia and Id, as F(0,t) varies from 0.0012 to 0.0212, FV(t) varies from 0.9421 to 0.000004656. To illustrate another phenomenon, look at just the two largest FV(t) values and the corresponding

'1 - F(0,t)' values given as follows:

DG-B: FV(t) = 0.9421 and 1 - F(0,t) = 0.9987727

C-MOV-1: FV(t) = 0.0462 and 1 - F(0,t) = 0.9797786

and note how this transformation re-calibrates a big jump in FV(t) for a small difference in '1 - F(0,t)'.

Observation 6: To compare measures from another tactic, in Tables Ic and IIc the measures are normalized (sum to one), and importance measures with the same percentages and the same rank ordering are:

<> B(t) and RII(t) (see Equation (12))

<> FV(t) and RRI(t) (see Equation (15))

Observation 7: After looking at the importance measures and the values of the F(.) parameters in Tables I and II, the end-user/ analyst needs to make his own decision about which importance measures he feels are most reliable and the ones he wants to use.

6. DIFFERENCES BETWEEN THE BIRNBAUM IMPORTANCE MEASURE AND THE FUSSELL-VESELY IMPORTANCE MEASURE

In order to see some complex differences between the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure, one needs to first look at the following simple systems [5]:

<> strictly series system

<> strictly parallel system

<> two-out-of-three system

and noting that importance measures may not flag the component that the end-user/analyst judges is the most important. Instead, importance measures take the system structure into account as well as the values of the F(.) parameters in order to choose the component that has the highest imfluence on overall system reliability as the most important.

For a system made up of components that are in a strictly series arrangement [5] the Birnbaum Importance Measure takes the component with the largest unreliability to be the most important, since under this arrangement the Birnbaum Importance Measure implies that one would want to improve this weakest link. For a system made up of components that are in a strictly parallel arrangement [5] the Birnbaum Importance Measure takes the component with the smallest unreliability to be the most important, since under this redundant arrangement, the Birnbaum Importance Measure implies that one only needs to improve one component, the best one.

The Fussell-Vesely Importance Measure scores the components of the strictly series system in the same way as the Birnbaum Importance Measure.

For the parallel system and the two-out-of-three system the two measures differ significantly. The Fussell-Vesely Importance Measure scores the components of the strictly parallel system as all equally important and all at values of one. For the two-out-of-three system the Birnbaum Importance Measure scores the component with the highest reliability as the most important, while the Fussell-Vesely Importance Measure scores the component with the lowest reliability as the most important.

Usually, a system is made up of a mix of series and parallel components and as the system configuration emerges to more complicated configurations, the interpretation of importance measures become more and more complex. Therefore, in order to get more information for analyzing importance measures, it would be helpful if importance measure codes such as SAPHIRE, would flag or assign the components with respect to the occurrence of the following inequalities:

$$x(t) > F(X)$$
$$FV(t) > B(t) \qquad (25)$$
$$x(t) > F(0,t)$$

Thus alerting users with some extra information, so they can learn more about their systems and how the components interact via the F(.) parameters and these flagged inequalities (see Observation 2 and Tables I and II where a * next to the values denotes one of these inequalities). When these three inequalities are true, then the component is a candidate for a Class II or a Class IV component (see the next Section for class definitions).

As seen in Tables I and II, the F(0,t) values dominate the Fussell-Vesely Importance Measure and the F(1,t) values dominate the Birnbaum Importance Measure, (see Tables Ia, Id, IIa, IId). As can be seen there is more variability in the F(1,t) values than in the F(0,t) values. For example, in Tables IIa and IId, F(0,t) varies from .0000029 to .0000055 while F(1,t) varies from .000008 to 1.00000. It should be cautioned that these two examples given by Tables I and II can not be used as a guide, instead, one should expect something new with each example.

7. CLASSIFICATION SCHEME FOR THE IMPORTANCE MEASURES

Using the results presented in this paper, a classification scheme will be defined, which is based on the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure, plus, it will be based on the inequalities derived from the first expression given in Equation (15). These inequalities can be expressed as follows:

$$\begin{array}{ll} 0 < FV(t) < B(t) < 1 & \text{when } x(t) < F(X) \\ 0 < B(t) < FV(t) < 1 & \text{when } x(t) > F(X) \end{array} \qquad (26)$$

Using these inequalities the classification scheme can be partitioned and defined as given in Figure VIII.

Figure VIII. The Classification Scheme.

Class I: $0 < FV(t) < B(t) < 1$ or $x(t) < F(X)$ and $F(0,t) < x(t)$

- Case 1: High B(t), Low FV(t)
- Case 2: High B(t), High FV(t)
- Case 3: Low B(t), Low FV(t)

Class II: $0 < B(t) < FV(t) < 1$ or $x(t) > F(X)$ and $F(0,t) < x(t)$

- Case 1: High FV(t), Low B(t)
- Case 2: High FV(t), High B(t)
- Case 3: Low FV(t), Low B(t)

Class III: $0 < FV(t) < B(t) < 1$ or $x(t) < F(X)$ and $F(0,t) > x(t)$

- Case 1: High B(t), Low FV(t)
- Case 2: High B(t), High FV(t)
- Case 3: Low B(t), Low FV(t)

Class IV: $0 < B(t) < FV(t) < 1$ or $x(t) > F(X)$ and $F(0,t) > x(t)$

- Case 1: High FV(t), Low B(t)
- Case 2: High FV(t), High B(t)
- Case 3: Low FV(t), Low B(t)

Note that in this partition the Case 3 values are tabled for bookkeeping purposes in order for the end-user/analyst to look at the whole distribution of values.

In this scheme Class II members and Class IV members have components that are highly redundant, since x(t) > F(X), and when choosing an importance measure, more thought needs to be given to these two classes. In order to see the complexities involved with these two classes, consider the discussion in the previous section where differences were shown with different measures when looking at the following simple systems [5]:

<> strictly series system

<> strictly parallel system

<> two-out-of-three system

Hence, the end-user/analyst must especially study the importance measures of components belonging to Class II and Class IV. For these components the end-user/analyst needs to pay attention to the values of the following parameters:

<> F(0,t) for each component

<> x(t) for each component

<> F(X) for the system

<> F(1,t) for each component

For comparative studies between importance measures and for selecting component importance the end-user/analyst should use these tools such as the classification scheme, the inequality flags, and the values of the F(.) parameters to make decisions on his reliability problems. Classification studies will be useful to the analyst in interpretating differences between the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure.

To illustrate the classification scheme, the frequency in each Class/Case for the two examples given by Table I and Table II is arrayed and displayed in Figure IX.

Figure IX. Example Results of the Classification Scheme.

CLASS/CASE	Frequency TABLE I	Frequency TABLE II
Class I: 0 < FV(t) < B(t) < 1 or x(t) < F(X) and F(0,t) < x(t)		
Case 1: High B(t), Low FV(t)		
Case 2: High B(t), High FV(t),	1 *	
Case 3: Low B(t), Low FV(t)		
Class II: 0 < B(t) < FV(t) < 1 or x(t) > F(X) and F(0,t) < x(t)		
Case 1: High FV(t), Low B(t)		1 *
Case 2: High FV(t), High B(t)		
Case 3: Low FV(t), Low B(t)		12
Class III: 0 < FV(t) < B(t) < 1 or x(t) < F(X) and F(0,t) > x(t)		
Case 1: High B(t), Low FV(t)	2 *	4 *
Case 2: High B(t), High FV(t)		
Case 3: Low B(t), Low FV(t)	7	3
Class IV: 0 < B(t) < FV(t) < 1 or x(t) > F(X) and F(0,t) > x(t)		
Case 1: High FV(t), Low B(t)		
Case 2: High FV(t), High B(t)		
Case 3: Low FV(t), Low B(t)		

Looking at this table for the two example results, at least one component belonging to the subset of frequencies indicated by a * should be considered as being classified as most important.

8. CONCLUSION

On studying the relationships among the six importance measures in this paper, the end-user/analyst should have a better notion of which measures to use when analyzing the components in his system. Operationally, the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure are interwoven into the classification scheme which is described in Section 7. The end-user/analyst should use this combination of the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure and set up this classification scheme. After classifying the importance measure values into this classification scheme, and after studying the frequencies in this scheme, he should have some good ideas on which components are really important.

As shown in this paper the set of six importance measures can be reduced to a smaller set (see Figure V). Guidance learned from reading this paper would be to remember that the Birnbaum Importance Measure and the Fussell-Vesely Importance Measure are the only important measures that need to be used, if they are used in conjunction with the classification scheme.

The classification scheme, as developed in this paper, is a natural tool for getting more information from the component unreliabilities. By looking at the values of the F(.) parameters and their inequalities and how his components are classified, the end-user/analyst should have a clearer picture for making decisions on applying importance measures and for suggesting to the nuclear plant designer where to allocate his resources for improving reliability.

As shown in this paper, the three F(.) parameters are very useful parameters and should be studied individually. Until codes, such as the SAPHIRE code, are modified to print out the values of these three F(.) parameters, end-user/analysts should use the inverse-engineering procedure developed in Section 5 to look at the values of these F(.) parameters.

9. REFERENCES

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2. Z. W. Birnbaum, On the Importance of Different Components in a Multicomponent System, Multivariate Analysis--II, (ed. P. R. Krishnaiah), 1969, Academic Press, New York.
3. W. E. Vesely, T. C. Davis, R. S. Denning, and N. Saltos, Measures of Risk Importance and Their Applications, NUREG/CR-3385, March 1983.
4. Richard Engelbrecht-Wiggans and David R. Strip, On the Relation of Various Reliability Measures to Each Other and to Game Theoretic Values, NUREG/CR-1860, SAND80-2624, January 1981.
5. Richard E. Barlow and Frank Proschan, Statistical Theory of Reliability and Life Testing, Probability Models, Holt, Rinehart and Winston, New York, 1975.
6. S. A. Eide, et al, Reliability Study: Westinghouse Reactor Protection System, 1984-1995, NUREG/CR-5500, Vol.2, INEEL/EXT-97-00740, April 1999.

TABLE I: FORTRAN Output for Example 1. Reference: Page 36 of the SAPHIRE Basics Workshop Manual [1].

Table Ia: The Parameters Ranked by FV(t)

Name	1- F(0,t)	F(0,t)	x(t)	F(X)	F(1,t)
DG-B	.9987727E+0	.1227324E-2*	.2000E-1	.211973E-1	.9999E+0
C-MOV-1	.9797786E+0	.2022134E-1	.1000E-2	.212010E-1	.9999E+0
DG-A	.9789658E+0	.2103417E-1	.2003E-1	.211927E-1	.2895E-1
C-MOV-B	.9789380E+0	.2106201E-1	.5031E-2	.211995E-1	.4839E-1
C-PUMP-B	.9788772E+0	.2112284E-1	.3019E-2	.212053E-1	.4845E-1
C-MOV-A	.9788305E+0	.2116956E-1	.5009E-2	.212092E-1	.2908E-1
C-PUMP-A	.9788105E+0	.2118942E-1	.3005E-2	.212132E-1	.2910E-1
C-CV-B	.9788000E+0	.2120003E-1	.1006E-3	.212028E-1	.4852E-1
C-CV-A	.9787973E+0	.2120274E-1	.1002E-3	.212035E-1	.2912E-1
TANK	.9787956E+0	.2120441E-1	.1000E-6	.212045E-1	.1000E+1

Table Ib: The Importance Measures Rank Ordered on FV(t)

Name	B(t)	FV(t)	RRR(t)	RRI(t)	RIR(t)	RII(t)
DG-B	.9987E+0	.9421E+0	.1727E+2	.1997E-1	.4717E+2	.9787E+0
C-MOV-1	.9797E+0	.4621E-1	.1048E+1	.9797E-3	.4716E+2	.9787E+0
DG-A	.7914E-2	.7479E-2	.1008E+1	.1585E-3	.1366E+1	.7755E-2
C-MOV-B	.2733E-1	.6486E-2	.1007E+1	.1375E-3	.2283E+1	.2719E-1
C-PUMP-B	.2733E-1	.3891E-2	.1004E+1	.8251E-4	.2285E+1	.2725E-1
C-MOV-A	.7913E-2	.1869E-2	.1002E+1	.3964E-4	.1371E+1	.7873E-2
C-PUMP-A	.7913E-2	.1121E-2	.1001E+1	.2378E-4	.1372E+1	.7889E-2
C-CV-B	.2732E-1	.1297E-3	.1000E+1	.2749E-5	.2289E+1	.2732E-1
C-CV-A	.7913E-2	.3742E-4	.1000E+1	.7935E-6	.1373E+1	.7912E-2
TANK	.9788E+0	.4656E-5	.1000E+1	.9872E-7	.4716E+2	.9788E+0
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SUM	.3071E+1	.1009E+1		.2140E-1		

Table Ic: The Importance Measures Normalized and Ordered on B(t)

Name	B(t)	FV(t)	RRR(t)	RRI(t)	RIR(t)	RII(t)
DG-B	.3252E+0	.9334E+0	.6556E+0	.9334E+0	.3066E+0	.3209E+0
C-MOV-1	.3190E+0	.4578E-1	.3979E-1	.4579E-1	.3066E+0	.3209E+0
TANK	.3187E+0	.4613E-5	.3796E-1	.4614E-5	.3066E+0	.3210E+0
C-MOV-B	.8900E-2	.6426E-2	.3823E-1	.6427E-2	.1484E-1	.8916E-2
C-PUMP-B	.8900E-2	.3855E-2	.3812E-1	.3856E-2	.1485E-1	.8936E-2
C-CV-B	.8897E-2	.1285E-3	.3796E-1	.1285E-3	.1488E-1	.8959E-2
DG-A	.2577E-2	.7410E-2	.3827E-1	.7408E-2	.8880E-2	.2543E-2
C-MOV-A	.2577E-2	.1852E-2	.3804E-1	.1853E-2	.8912E-2	.2582E-2
C-PUMP-A	.2577E-2	.1111E-2	.3800E-1	.1111E-2	.8919E-2	.2587E-2
C-CV-A	.2577E-2	.3707E-4	.3796E-1	.3709E-4	.8925E-2	.2595E-2

Table

Table Id: Measures B(t) & FV(t) with Parameters Rank Ordered on B(t)

Name	B(t)	FV(t)	F(0,t)	x(t)	F(X)	F(1,t)
DG-B	.9987E+0	.9421E+0	.1227E-2*	.2000E-1	.2120E-1	.9999E+0
C-MOV-1	.9797E+0	.4621E-1	.2022E-1	.1000E-2	.2120E-1	.9999E+0
TANK	.9788E+0	.4656E-5	.2120E-1	.1000E-6	.2120E-1	.1000E+1
C-MOV-B	.2733E-1	.6486E-2	.2106E-1	.5031E-2	.2120E-1	.4839E-1
C-PUMP-B	.2733E-1	.3891E-2	.2112E-1	.3019E-2	.2121E-1	.4845E-1
C-CV-B	.2732E-1	.1297E-3	.2120E-1	.1006E-3	.2120E-1	.4852E-1
DG-A	.7914E-2	.7479E-2	.2103E-1	.2003E-1	.2119E-1	.2895E-1
C-MOV-A	.7913E-2	.1869E-2	.2117E-1	.5009E-2	.2121E-1	.2908E-1
C-PUMP-A	.7913E-2	.1121E-2	.2119E-1	.3005E-2	.2121E-1	.2910E-1
C-CV-A	.7913E-2	.3742E-4	.2120E-1	.1002E-3	.2120E-1	.2912E-1

TABLE II: FORTRAN Output for Example 2. Reference: Appendix F, Page F-38, Table F-10 [6].

Table IIa: The Parameters Ranked by FV(t)

Name	1- F(0,t)	F(0,t)	x(t)	F(X)	F(1,t)
XHE-XE-NS	.9999970E+0	.2958531E-5*	.5000E+0*	.551553E-5	.8073E-5
BME-CF-RT	.9999961E+0	.3909768E-5	.1610E-5	.551526E-5	.9972E+0
CBI-CF-60	.9999958E+0	.4237639E-5	.2700E-5	.550772E-5	.4704E+0
ROD-CF-RC	.9999957E+0	.4305041E-5	.1210E-5	.551504E-5	.1000E+1
CCX-CF-60	.9999954E+0	.4646731E-5	.1830E-5	.550756E-5	.4704E+0
CCP-TM-CH	.9999948E+0	.5224291E-5*	.5800E-1*	.551528E-5	.1024E-4
CBI-CF-40	.9999947E+0	.5277082E-5*	.8210E-5*	.551517E-5	.2901E-1
CCX-CF-40	.9999947E+0	.5332355E-5*	.6330E-5*	.551593E-5	.2901E-1
XHE-XE-SI	.9999946E+0	.5376756E-5*	.1000E-1*	.551406E-5	.1911E-4
UVL-CF-UV	.9999946E+0	.5410317E-5*	.1040E-4*	.551432E-5	.1001E-1
TLC-CF-SS	.9999945E+0	.5495155E-5	.2100E-5	.551615E-5	.1001E-1
UVL-FF-UA	.9999945E+0	.5509918E-5*	.3370E-3*	.551595E-5	.2340E-4
BME-TM-RA	.9999945E+0	.5511628E-5*	.1400E-2*	.551657E-5	.9043E-5
BME-CF-RA	.9999945E+0	.5513015E-5	.1610E-5	.551527E-5	.1404E-2
BME-FO-RA	.9999945E+0	.5515142E-5*	.3690E-4*	.551664E-5	.4608E-4
TLR-CF-12	.9999945E+0	.5515887E-5	.8070E-7	.551665E-5	.9426E-2
CBI-CF-P3	.9999945E+0	.5514240E-5*	.1190E-4*	.551483E-5	.5468E-4
CDT-CF-T3	.9999945E+0	.5515223E-5*	.5550E-4*	.551571E-5	.1420E-4
CDT-CF-T2	.9999945E+0	.5516489E-5*	.2500E-3*	.551696E-5	.7390E-5
CBI-CF-P2	.9999945E+0	.5515990E-5*	.4190E-4*	.551644E-5	.1615E-4

Table IIb: The Importance Measures Rank Ordered on FV(t)

Name	B(t)	FV(t)	RRR(t)	RRI(t)	RIR(t)	RII(t)
XHE-XE-NS	.5114E-5*	.4636E+0	.1864E+01	.2557E-5	.1464E+1	.2557E-5
BME-CF-RT	.9972E+0	.2911E+0	.1411E+01	.1605E-5	.1808E+6	.9972E+0
CBI-CF-60	.4704E+0	.2306E+0	.1300E+01	.1270E-5	.8541E+5	.4704E+0
ROD-CF-RC	.1000E+1	.2194E+0	.1281E+01	.1210E-5	.1813E+6	.1000E+1
CCX-CF-60	.4704E+0	.1563E+0	.1185E+01	.8608E-6	.8541E+5	.4704E+0
CCP-TM-CH	.5017E-5*	.5276E-1	.1056E+01	.2910E-6	.1857E+1	.4726E-5
CBI-CF-40	.2900E-1*	.4317E-1	.1045E+01	.2381E-6	.5259E+4	.2900E-1
CCX-CF-40	.2900E-1*	.3328E-1	.1034E+01	.1836E-6	.5258E+4	.2900E-1
XHE-XE-SI	.1373E-4*	.2490E-1	.1026E+01	.1373E-6	.3465E+1	.1359E-4
UVL-CF-UV	.1000E-1*	.1886E-1	.1019E+01	.1040E-6	.1814E+4	.1000E-1
TLC-CF-SS	.1000E-1	.3807E-2	.1004E+01	.2100E-7	.1814E+4	.1000E-1
UVL-FF-UA	.1789E-4*	.1093E-2	.1001E+01	.6029E-8	.4242E+1	.1788E-4
BME-TM-RA	.3531E-5*	.8961E-3	.1001E+01	.4944E-8	.1639E+1	.3526E-5
BME-CF-RA	.1398E-2	.4081E-3	.1000E+01	.2251E-8	.2545E+3	.1398E-2
BME-FO-RA	.4056E-4*	.2713E-3	.1000E+01	.1497E-8	.8352E+1	.4056E-4
TLR-CF-12	.9420E-2	.1378E-3	.1000E+01	.7603E-9	.1709E+4	.9420E-2
CBI-CF-P3	.4917E-4*	.1061E-3	.1000E+01	.5853E-9	.9916E+1	.4917E-4
CDT-CF-T3	.8685E-5*	.8739E-4	.1000E+01	.4820E-9	.2575E+1	.8685E-5
CDT-CF-T2	.1874E-5*	.8490E-4	.1000E+01	.4684E-9	.1340E+1	.1874E-5
CBI-CF-P2	.1063E-4*	.8070E-4	.1000E+01	.4452E-9	.2927E+1	.1063E-4
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SUM	.3027E+1	.1541E+1		.8496E-5		

Table IIc: The Importance Measures Normalized and Ordered on B(t)

Name	B(t)	FV(t)	RRR(t)	RRI(t)	RIR(t)	RII(t)
ROD-CF-RC	.3304E+0	.1424E+0	.5763E-1	.1424E+0	.3302E+0	.3304E+0
BME-CF-RT	.3294E+0	.1889E+0	.6348E-1	.1889E+0	.3293E+0	.3294E+0
CBI-CF-60	.1554E+0	.1496E+0	.5848E-1	.1495E+0	.1555E+0	.1554E+0
CCX-CF-60	.1554E+0	.1014E+0	.5331E-1	.1013E+0	.1555E+0	.1554E+0
CCX-CF-40	.9581E-2	.2160E-1	.4652E-1	.2161E-1	.9576E-2	.9581E-2
CBI-CF-40	.9581E-2	.2802E-1	.4701E-1	.2803E-1	.9578E-2	.9581E-2
UVL-CF-UV	.3304E-2	.1224E-1	.4584E-1	.1224E-1	.3304E-2	.3304E-2
TLC-CF-SS	.3304E-2	.2471E-2	.4517E-1	.2472E-2	.3304E-2	.3304E-2
TLR-CF-12	.3112E-2	.8943E-4	.4499E-1	.8949E-4	.3112E-2	.3112E-2
BME-CF-RA	.4618E-3	.2648E-3	.4499E-1	.2650E-3	.4635E-3	.4618E-3
CBI-CF-P3	.1624E-4	.6885E-4	.4499E-1	.6889E-4	.1806E-4	.1624E-4
BME-FO-RA	.1340E-4	.1761E-3	.4499E-1	.1762E-3	.1521E-4	.1340E-4
UVL-FF-UA	.5910E-5	.7093E-3	.4503E-1	.7096E-3	.7725E-5	.5907E-5
XHE-XE-SI	.4536E-5	.1616E-1	.4616E-1	.1616E-1	.6310E-5	.4490E-5
CBI-CF-P2	.3512E-5	.5237E-4	.4499E-1	.5240E-4	.5331E-5	.3512E-5
CDT-CF-T3	.2869E-5	.5671E-4	.4499E-1	.5673E-4	.4690E-5	.2869E-5
XHE-XE-NS	.1689E-5	.3009E+0	.8386E-1	.3010E+0	.2666E-5	.8447E-6
CCP-TM-CH	.1657E-5	.3424E-1	.4751E-1	.3425E-1	.3382E-5	.1561E-5
BME-TM-RA	.1167E-5	.5815E-3	.4503E-1	.5819E-3	.2985E-5	.1165E-5
CDT-CF-T2	.6191E-6	.5510E-4	.4499E-1	.5513E-4	.2440E-5	.6191E-6

Table IId: Measures B(t) & FV(t) with Parameters Rank Ordered on B(t)

Name	B(t)	FV(t)	F(0,t)	x(t)	F(X)	F(1,t)
ROD-CF-RC	.1000E+1	.2194E+0	.4305E-5	.1210E-5	.5515E-5	.1000E+1
BME-CF-RT	.9972E+0	.2911E+0	.3910E-5	.1610E-5	.5515E-5	.9972E+0
CBI-CF-60	.4704E+0	.2306E+0	.4238E-5	.2700E-5	.5508E-5	.4704E+0
CCX-CF-60	.4704E+0	.1563E+0	.4647E-5	.1830E-5	.5508E-5	.4704E+0
CCX-CF-40	.2900E-1*	.3328E-1	.5332E-5*	.6330E-5*	.5516E-5	.2901E-1
CBI-CF-40	.2900E-1*	.4317E-1	.5277E-5*	.8210E-5*	.5515E-5	.2901E-1
UVL-CF-UV	.1000E-1*	.1886E-1	.5410E-5*	.1040E-4*	.5514E-5	.1001E-1
TLC-CF-SS	.1000E-1	.3807E-2	.5495E-5	.2100E-5	.5516E-5	.1001E-1
TLR-CF-12	.9420E-2	.1378E-3	.5516E-5	.8070E-7	.5517E-5	.9426E-2
BME-CF-RA	.1398E-2	.4081E-3	.5513E-5	.1610E-5	.5515E-5	.1404E-2
CBI-CF-P3	.4917E-4*	.1061E-3	.5514E-5*	.1190E-4*	.5515E-5	.5468E-4
BME-FO-RA	.4056E-4*	.2713E-3	.5515E-5*	.3690E-4*	.5517E-5	.4608E-4
UVL-FF-UA	.1789E-4*	.1093E-2	.5510E-5*	.3370E-3*	.5516E-5	.2340E-4
XHE-XE-SI	.1373E-4*	.2490E-1	.5377E-5*	.1000E-1*	.5514E-5	.1911E-4
CBI-CF-P2	.1063E-4*	.8070E-4	.5516E-5*	.4190E-4*	.5516E-5	.1615E-4
CDT-CF-T3	.8685E-5*	.8739E-4	.5515E-5*	.5550E-4*	.5516E-5	.1420E-4
XHE-XE-NS	.5114E-5*	.4636E+0	.2959E-5*	.5000E+0*	.5516E-5	.8073E-5
CCP-TM-CH	.5017E-5*	.5276E-1	.5224E-5*	.5800E-1*	.5515E-5	.1024E-4
BME-TM-RA	.3531E-5*	.8961E-3	.5512E-5*	.1400E-2*	.5517E-5	.9043E-5
CDT-CF-T2	.1874E-5*	.8490E-4	.5516E-5*	.2500E-3*	.5517E-5	.7390E-5