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Supplement to Topical Report ER-80P:

Basis for a Power Uprate

With the LEFM✓™ or LEFM CheckPlus™ System

Revision 3

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**Supplement to Caldon Topical Report ER-80N:
Basis for Power Uprates
with an LEFM Check or an LEFM CheckPlus**

1. Background and Purpose

On May 3, 2000, the NRC approved a rule change amending 10CFR50 Appendix K to permit power increases based on improvements in accuracy of the instrumentation used to measure thermal power. These power increases, referred to as "Appendix K Uprates", are relatively small increases on the order of 1% to 1.7%, depending on the demonstrated instrument accuracy. A previously submitted Topical Report, ER 80P, described the principles and performance characteristics of a Caldon feedwater measurement system, the LEFM Check, that, because of its superior flow measurement accuracy, can be used for such uprates. The analyses documented in ER 80P were performed in 1997, well in advance of the rulemaking and also in advance of the delivery of a system. ER 80P used as a basis for its "typical" analysis a 2-loop PWR and a 2-feedwater line BWR. It also employed performance data for earlier Model LEFM Systems; in early 1997 when ER 80P was submitted, prototype hardware for LEFM Check was being built. The magnitude of the uprate postulated in ER 80P was conservatively set at 1%. NRC reviewed and accepted ER 80 P in connection with a 1% uprate of both units at a pressurized water reactor plant.¹

In May, 2000 Caldon issued a supplement to ER 80P², ER 160P, that assesses, for a 1.4% power uprate, the margin between the uprate power and the power at which the Appendix K safety analyses were performed. ER 160P was reviewed and accepted by NRC, in connection with a 1.4% power uprate of a single unit PWR plant.³ The information in ER 160P remains bounding for the performance of LEFM Check systems. Accordingly, licensees using the LEFM Check system may continue to reference this document when applying for uprates in the 1.4% range.

This document supplements ER 80P and ER 160P in several respects:

- It describes a second Caldon system, the LEFM CheckPlus, that is more accurate than the LEFM Check is. Licensees who wish to take advantage of the superior accuracy of the LEFM Check Plus in selecting the amount of a power uprate will reference this document.
- It characterizes the performance of both the LEFM Check and CheckPlus using measured data for systems in service. As a result, the performance of the LEFM Check system calculated herein is slightly better than that of ER 80P. (Time measurement uncertainties, for example, are slightly smaller). Licensees using an LEFM Check system and applying for an uprate greater than 1.4% may choose to reference this document.

¹ Reference 4 (section 5 of this report): NRC SER dated March 8, 1999

² Reference 5

³ Reference 6: NRC Safety Evaluation related to Amendment 31 Operating License NPF-390, Docket number 50-390

- It assesses the margin between the uprate power and the power at which safety analyses are performed for uprates of 1.4% and 1.7%. Uprates with LEFM Check and LEFM CheckPlus Systems will lie between these values.
- It updates, through 1999, LER data on overpower incidents previously included in ER 80P.

Much of the information contained in ER 80P remains applicable and is necessary to a full understanding of the material presented in this document. In particular, it may be necessary to consult Appendices A, B, C, D, and F* of ER 80 P for a complete understanding of the material herein. An updated version of Appendix E of ER 80P, a detailed accounting of the uncertainties in a power measurement using an LEFM Check or an LEFM CheckPlus, is provided as Appendix A to this document.

2. System Descriptions

The characteristics and design features of the LEFM Check are described in detail in ER 80P. A very brief recapitulation is provided here. Figure 1 is a diagram of an LEFM Check employing one flow measuring spool piece (for the measurement of total feedwater flow). The eight transducer assemblies that form the four acoustic paths of an LEFM Check are shown in the diagram. As described in Appendix B of ER 80P, the LEFM Check measures the transit times of pulses of ultrasonic energy traveling these paths, both with the flow and against it. From these measurements it forms four path length-fluid velocity products, which it numerically integrates to determine volumetric flow. It also measures sound velocity along the four acoustic paths, which it uses, along with a feedwater pressure input, to determine fluid temperature and density.

* These ER 80P Appendices cover the following subjects:

A Uncertainties in existing [conventional] calorimetric instrumentation (including thermodynamic sensitivity coefficients also applicable to the LEFM analyses of this document)

B The general principles of chordal LEFMs.

C The LEFM water temperature algorithm

D Identifying and Bounding the uncertainties in LEFM flow and temperature measurements

F A survey of hydraulic performance of chordal LEFMs.

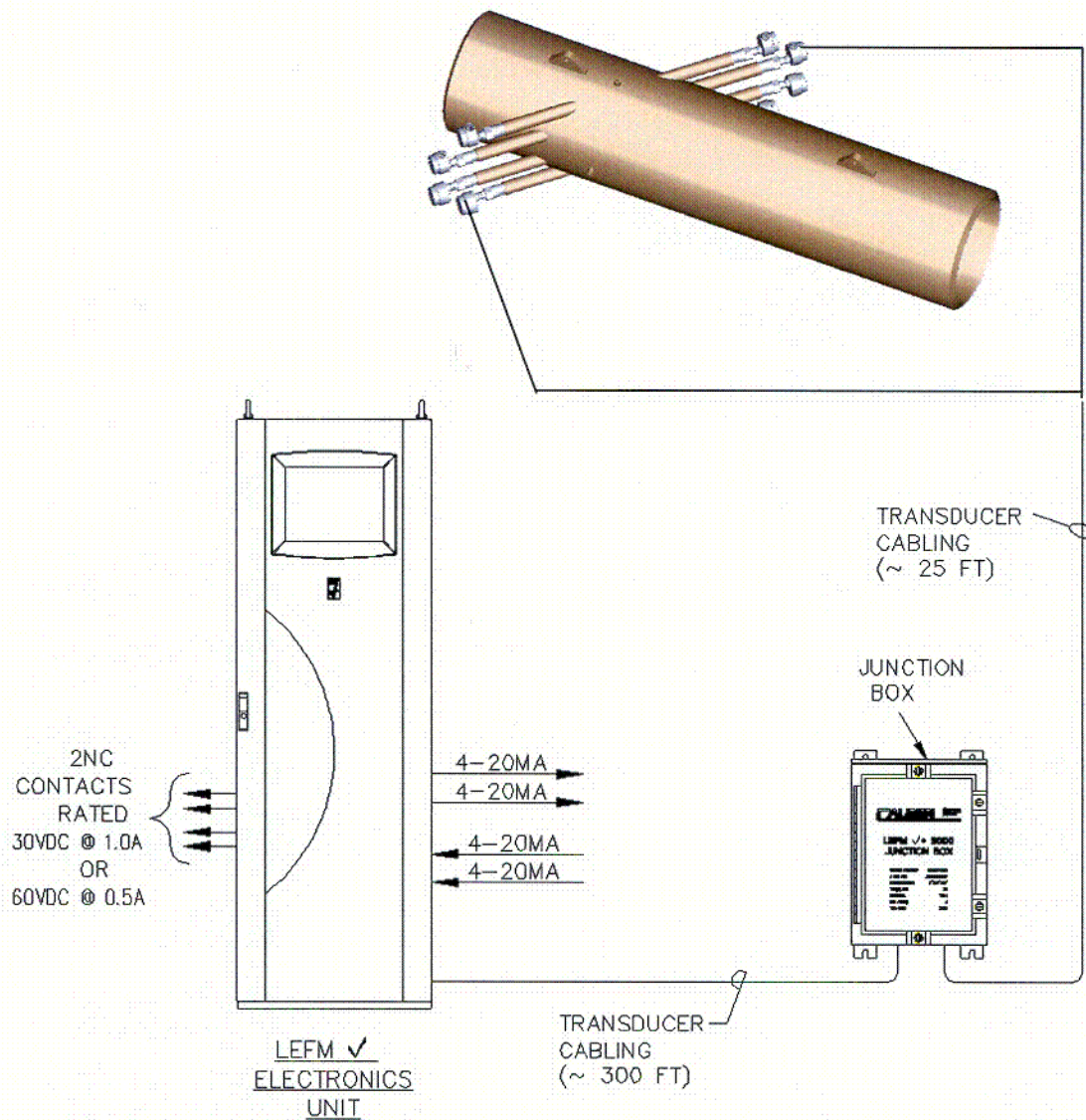


Figure 1: Diagram of a One Loop LEFM Check System

An LEFM CheckPlus System, illustrated in Figure 2, consists essentially of two LEFM Check systems, both hydraulically and electronically. The electronics for the two subsystems, while electrically separated, are housed in a single cabinet, as shown in Figure 3 (the LEFM Check cabinet is similar).

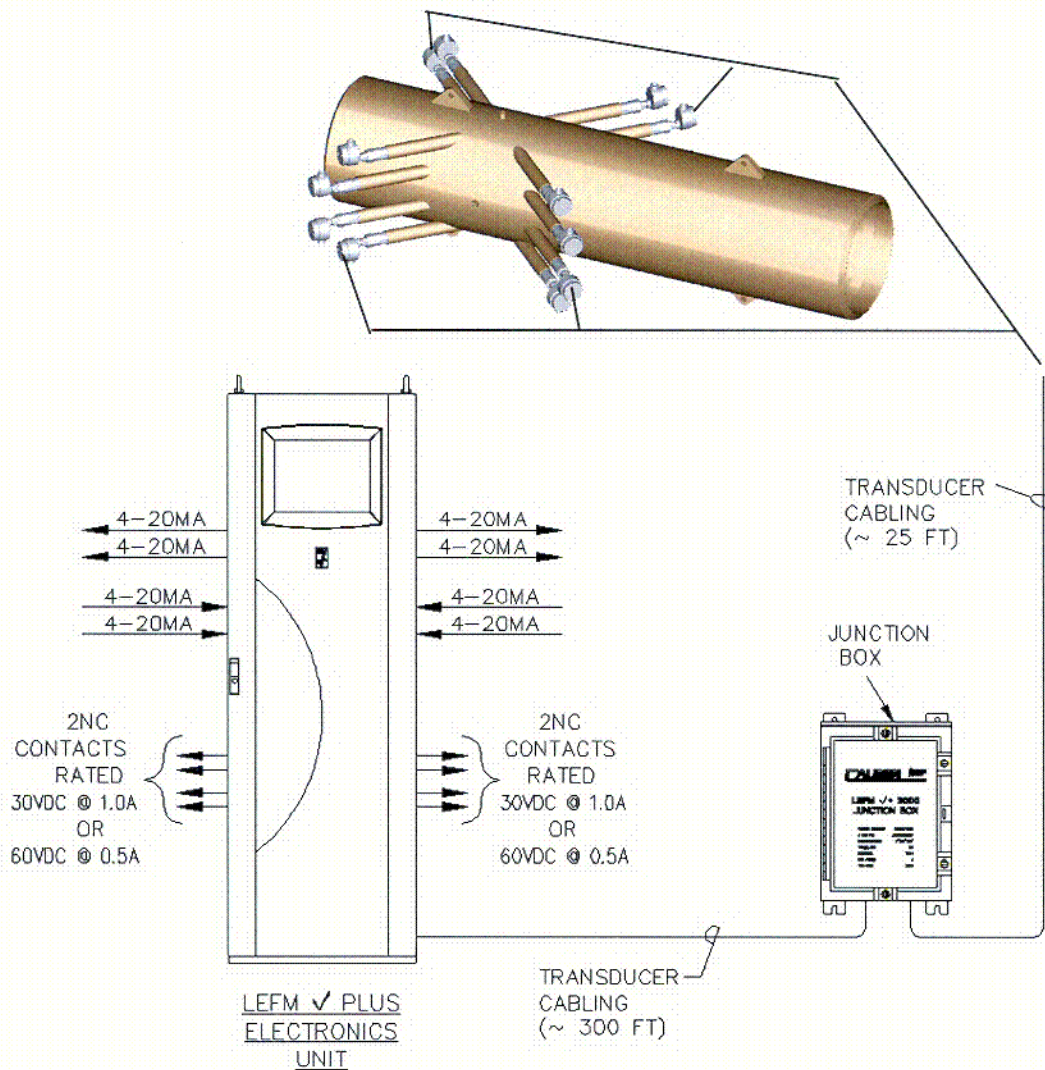


Figure 2: Diagram of a One Loop LEFM CheckPlus System

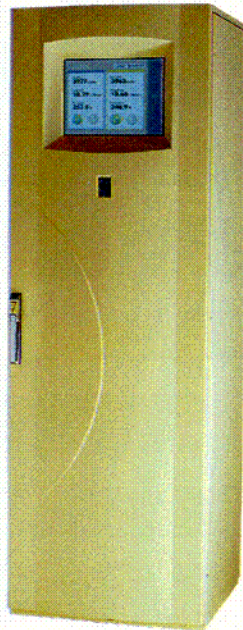


Figure 3: LEFM CheckPlus Electronics

An LEFM CheckPlus will provide a more accurate measurement of feedwater flow than will an LEFM Check system. This advantage arises from two features of the CheckPlus System:

- As can be seen in Figure 2, the 8 acoustic paths of the CheckPlus form two measurement planes, 90° apart. The velocity measured by an acoustic path in one of these planes consists of the vector sum of the axial fluid velocity as projected onto the path and any transverse component of fluid velocity, also as projected onto the path. When the net velocity measured by this acoustic path is averaged with the velocity measured by its companion path in the second plane, the transverse components of the fluid velocity will cancel, so that the average reflects the contribution of the axial velocity only. Thus the numerical integration of the 4 pairs of averaged axial velocities measured by an LEFM CheckPlus is inherently a more accurate computation of the volumetric flow than can be obtained from a single plane of 4 acoustic paths⁴
- Because there are twice as many paths, errors due to uncertainties in transit time measurements and path length are reduced. [To ensure independence, the two measurement planes of a CheckPlus employ independent clocks].

The redundancy inherent in the two measurement planes of an LEFM CheckPlus also makes this system more resistant to component failures. Continued operation at the uprate power can be justified with a CheckPlus system for any single component failure, whereas, with an LEFM Check, a failure will generally require a reduction in power to a level consistent with the accuracy of the system following the failure.

⁴ It should be noted that in hydraulic configurations where transverse flow components are small, an LEFM Check can produce equally accurate results. However, the calibration of a CheckPlus is in general less sensitive to the specifics of a hydraulic configuration than is that of an LEFM Check.

A power uprate can be obtained based on improved accuracy of the instrumentation used to measure thermal power, in accordance with the 10CFR50 Appendix K rule change described in 1 above. The LEFM Check and LEFM CheckPlus are instrument systems that will support such uprates. Both provide measurements of feedwater mass flow and temperature leading to an uncertainty in thermal power significantly better than current instrumentation⁵. Table 1 tabulates the uncertainties in the thermal power computation for a representative BWR or PWR using feedwater flow and temperature measurements by a single meter, either an LEFM Check or an LEFM CheckPlus. The basis for all of the entries in this table and how these entries are combined is found in Appendix A of this document. As shown in Table 1, a thermal power measurement using the LEFM Check can provide an overall accuracy of better than $\pm 0.6\%$ ($\pm 0.47\%$ to 0.51% ⁶ for the representative system of the Table). The LEFM CheckPlus system can support power measurement accuracies approaching $\pm 0.3\%$ ($\pm 0.33\%$ to 0.39% for the system of the Table).

Table 1. Thermal Power Uncertainties With Single Meter Chordal LEFM Systems in a Typical PWR or BWR

The performance figures of Table 1 are based on the detailed accounting given in Appendix A of this document. As described in Appendix A, the data in the table bound the actual performance

⁵ Appendix A of ER 80P calculates an uncertainty of $\pm 1.4\%$ (95% confidence level) for a 2 loop installation using typical conventional flow and temperature instruments.

⁶ The range is due to differences in the accuracy with which steam enthalpy is determined. See Table 1 and Appendix A.

of LEFM Check and CheckPlus hardware. For example, the photograph of the 33 inch LEFM Check spool in Figure 4 was taken during calibration testing at a certified hydraulic laboratory. The uncertainty in the profile factor for this spool demonstrated during this testing is $\pm 0.20\%$, significantly better than the budgeted uncertainty of $\pm 0.4\%$. In this case, the certified facility accuracy was substantially better than the $\pm 0.25\%$ budgeted for facility accuracy. Also, the hydraulic configuration was relatively benign (a long straight pipe upstream) so that the uncertainties associated with modeling and Reynolds Number extrapolation were very small.



Figure 4: Calibration Test of an LEFM Check Spool (Looking Upstream)

Similarly, the uncertainty in the profile factor of a recently calibrated LEFM CheckPlus spool was shown to be $\pm 0.22\%$, 0.03% better than the 0.25% budget, in an extremely complex hydraulic configuration. That configuration, as tested, is shown in Figure 5 (a full scale model of the section of the feedwater system in which the spool is now installed). The testing demonstrated that the profile factor was insensitive to variations in the hydraulic setup (for example, changes in the flow split between the two feeds to the header upstream of the spool).



Figure 5: Calibration Test of an LEFM CheckPlus Spool (Looking Downstream)

It should be emphasized that the figures of Table 1 are bounding values. Each licensee will commit to meeting the bounding values in the Table, or will submit, as part of his uprate package, an accounting of the uncertainties applicable to his plant. In either case, the licensee will maintain, as part of his design basis, LEFM calibration and other data justifying the amount of the uprate requested in his license amendment application.

Some installations will employ multiple LEFM Check or CheckPlus systems to measure flow in the individual feeds to steam generators (for PWRs) or to each reactor vessel feedwater nozzle (for BWRs). Such systems may have slightly better performance than the systems of Table 1. When individual loop flow measurement errors are randomly combined, the net measurement error will likely be smaller.

3. Probabilistic Basis for Power Uprate

To assess the increase in thermal power rating appropriate to the use of LEFM systems, this discussion will interpret the data of Table 1 on a probabilistic basis. When the ASME developed standards for the measurement of steam turbine heat rate in power plants, they performed a series of Monte Carlo analyses which demonstrated that, if the uncertainty elements of a measurement system are calculated on a 2 standard deviation basis, the uncertainty in the overall measurement is characterized by a normal distribution with 2 standard deviations equal to the root sum square of appropriately weighted individual elements (Reference 1). This result held even when the uncertainties of individual elements were not normally distributed.

For example, a particular element might be characterized by a "roulette wheel" distribution (a flat or uniform distribution) between defined uncertainty bounds. It was subject only to one condition: that no single element dominate the calculation of the overall uncertainty.

The individual entries of Table 1 meet this condition. The profile factor uncertainties of the LEFM Check and LEFM CheckPlus appear dominant, but are, in fact, made up of several elements, none of which is dominant (see the discussion in Appendix A). Therefore, the overall uncertainties described in Table 1 will be normally distributed. Furthermore, the sensitivity of the results to the nature of the elemental uncertainty distribution has been investigated in ER 80P, Appendix G. This investigation also shows that the distribution of the total uncertainty is likely to be normal whether the contributors are each normally distributed or distributed uniformly between limits.

Table 1 implies an uncertainty probability distribution wherein one standard deviation of the LEFM Check uncertainty is likely to be less than $\pm 0.3\%$ full power. Table 2, below, shows that, with a distribution characterized by this standard deviation, there is essentially no chance (less than one in 3 million) that an operator using the LEFM Check to determine thermal power will exceed a power level 1.5% above that to which he is controlling. For the LEFM CheckPlus distribution, one standard deviation is likely to be a little greater than $\pm 0.15\%$ full power. From Table 2, there is essentially no chance that an operator using the LEFM CheckPlus will exceed a power level 0.75% above that to which he is controlling. For comparison purposes Table 2 tabulates odds for a conventional (nozzle based) power determination having an uncertainty distribution with one standard deviation of 1% in accordance with the original Appendix K allowance of 2% for instrument uncertainty. As compared to the Appendix K base case, the reduction in the probability of overpower incidents with LEFM Check and CheckPlus systems, even at uprated power levels, is obvious from the Table.

Table 2.
Probabilities and Odds Associated With Assumed Nozzle and LEFM Uncertainty Bounds⁹

Number of Standard Deviations	Venturi Nozzle Bounds ($\pm 1.0\%$)	LEFM Check Bounds ($\pm 0.3\%$)	LEFM CheckPlus Bounds ($\pm 0.15\%$)	Probability of Operation Within Bounds	Odds of Exceeding Bounds on the High Side
1	1.0%	0.3%	0.15%	68%	1/6.3
2	2.0%	0.6%	0.3%	95.4%	1/44
3	3.0%	0.9%	0.45%	99.7%	1/741
4	4.0%	1.2%	0.6%	99.994%	1/32,300
5	5.0%	1.5%	0.75%	99.99994%	1/3.3 million

⁹ The probabilities and odds of Table 2 are computed in Appendix B to this document.

To clarify the basis for a power increase with use of the LEFM Check or LEFM CheckPlus, the results of Table 2 are shown graphically in Figures 6, 7, and 8. All three figures show power level (as a percent of the pre-uprate 100% power) along the “x” axis, and probability data along the “y” axis. All three figures illustrate three cases:

1. Operation at the current 100% power level with the current instrumentation providing a 2% power determination uncertainty,
2. Operation at a 1.4% power increase with an LEFM Check providing a $\pm 0.6\%$ power determination uncertainty, and
3. Operation at a 1.7% power increase with an LEFM CheckPlus providing a $\pm 0.3\%$ power determination uncertainty.

Figure 6 shows the probable operating ranges for each of the three cases. As expected, the curves peak at the power level where operation is intended, and fall off symmetrically on either side of the peak. Of greater interest from the standpoint of operating safety is the probability that any given power level will be exceeded, as shown in Figure 7. As Figure 7 shows, the probability of exceeding a given power level is 100%, or a sure thing, just prior to the intended power level. The probability for each of the three cases equalizes at 102% power, which is the power level at which most plants’ safety systems are analyzed for proper performance.

Figure 8 presents the same data as Figure 7, but focuses in the vicinity of 102% power where the probability curves for the LEFM Check, the LEFM CheckPlus, and current instrumentation intersect. Though the intended operating point is higher for both the LEFM Check and LEFM CheckPlus systems due to the power increase, the probability of exceeding 102% power is the same for all three instruments. In other words, the probability of exceeding the analyzed power level of 102% is the same for the current instrumentation operating at 100%, for the LEFM Check operating at 101.4% and for the LEFM CheckPlus operating at 101.7%.

Figure 8 also shows another advantage of more accurate power measurements. As power measurement precision increases, the chance of a *significant* overpower incident decreases. For example, a plant equipped with flow nozzles, intending to operate at 100% of its licensed power, has about a 1 in 100 chance of exceeding 102.3%. On the other hand, the same plant, equipped with the LEFM CheckPlus, and intending to operate at 101.7% of its (previous) licensed power, has less than a 1 in 30,000 chance of exceeding 102.3%. (These odds are based on Table 2. It is not possible to read a probability this low on Figure 8).

There are two assumptions critical to the preceding discussion of thermal power margin. The first is the necessity of an uncertainty distribution that is normal, which has been discussed and, based on the ASME studies and Appendix G of ER 80P, is satisfied. The second is that Tables 1 and 2 *actually describe* the performance of the instruments in service. Verification that the LEFM systems are operating within their design bounds is provided continuously, on-line, as discussed in detail in ER 80P. But there is no comparable on-line assurance that current nozzle-based instrumentation is operating within its design bounds. This is the basis for the conclusion that power increases with LEFM systems increase safety.

3. Benefits of On-Line Verification

To illustrate the benefits of on-line verification, Figure 9 shows the results of a survey of sustained overpower events reported in Licensee Event Reports from 1981 through 1999 (Reference 3). The 61 identified events have been categorized by cause in order to examine whether they would have been preventable with the on-line verification capabilities of LEFM systems.

Figure 9 shows that the LEFM systems with on-line verification would have prevented all significant sustained overpower events. Looking at the extremes, five cases have been reported in Licensee Event Reports where steady state overpower has occurred in an amount not consistent with the probability predictions implied by Table 3; i.e., operation at 2% or more beyond the licensed power level. The causes for these events are summarized in Table 3.

Table 3. Sustained Overpower Events Above 102% and Their Causes

LER Number	Reported Power Excursion	Reported Duration	Reported Cause of Event
82-002	2.7%	46 days	Differential pressure transmitter found out of tolerance.
87-069	2.1%	2 days	Procedural - nuclear instruments interval and deadband error allowed beyond limit.
88-035	2%-3%	10 days	Hole in venturi pressure tap.
91-012	2.09%	5 years	Core power calculation error; improper density compensation.
94-002	2.6%	8 months	Perimeter bypass flow of venturi feed nozzles.

In three of these cases, the sustained overpower event was the result of the instrumentation system (transmitters or nozzles) failing to operate as designed. The other two cases were due to procedural errors and improper density compensation. The common link in all of these cases is that there was no indication of a problem until an independent means of measurement or calculation was employed. There is currently no indication available to the operators for the accuracy of the thermal power measurement. All of these cases would have been prevented by use of LEFM systems, because LEFM systems incorporate on-line verification features and real-time control room displays that prevent occurrences of subtle failure by providing operators continuous information about the measurement, and about the accuracy of the measurement.

It is the LEFM's ability to confirm on-line that it is performing within its accuracy bounds, as well as its high accuracy, that justifies a power uprate with its use. In addition to providing for a power uprate, LEFM systems will assure that the probability of exceeding the analyzed power level (i.e., 1.02 times the current licensed rating) by as little as 0.5% is negligibly small.

4. Using the LEFM to Control Thermal Power

With the existing instrumentation, for each feedwater flow measurement, the differential pressure transmitters provide an output proportional to the differential pressure across the flow nozzle. Resistance thermometers (or thermocouples) measure the feedwater temperature. Typically, these outputs are supplied to the plant computer where the density and enthalpy are calculated with the aid of synthesized ASME steam tables. The thermal power is then calculated, also by the plant computer.

It is anticipated that a licensee will make use of LEFM mass flow and temperature measurements by directly substituting the LEFM indications for the nozzle-based mass flow indication and the RTD temperature indications in the plant computer. The plant computer would then calculate feedwater enthalpy and thermal power as it does now. As an alternative, the calorimetric power can be manually calculated, using LEFM indications and following a prescribed procedure.

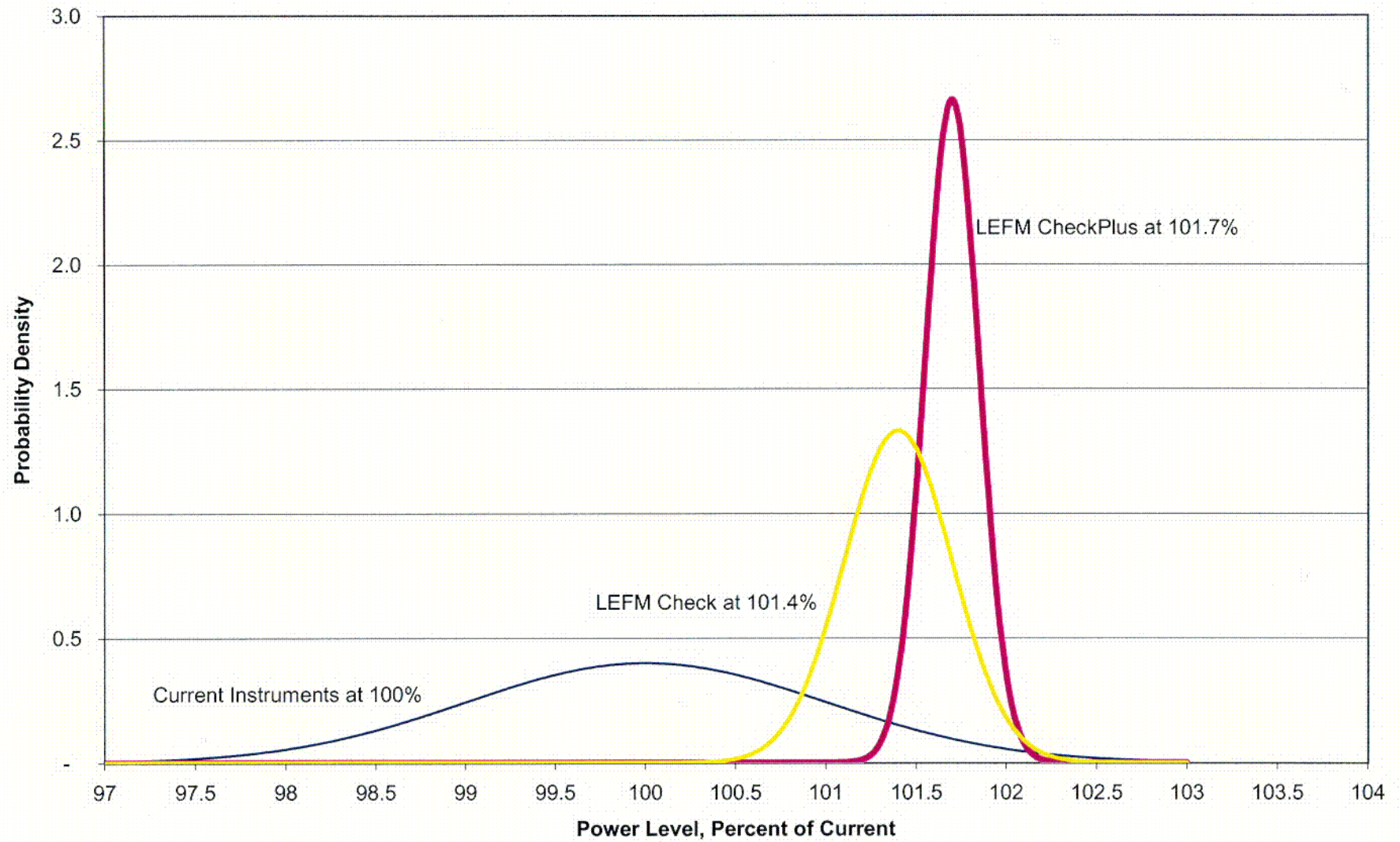
While this discussion is focused on operation at full power, it should be noted that LEFM systems provide accurate flow and temperature indications from synchronization to full power. The LEFM Check or LEFM CheckPlus may be used for thermal power determinations following synchronization at 10% to 15% power (when feedwater heating commences) and up to full power, with an accuracy better than the present instrumentation. [The LEFM provides a valid and accurate volumetric flow indication down to zero flow.]

In order to maintain control of thermal power at 100 percent power, a real-time display of thermal power as calculated using the LEFM will be available in the main control room for the reactor operator's use. The operator will use this display to maintain reactor power at or below the licensed thermal rating, with a tolerance in accordance with current plant practice. The thermal power display will also be present, in the same location as the thermal power value, a clear indication of the validity of the thermal power measurement, as determined by LEFM diagnostics. This indication will be provided by the LEFM's on-line verification system, which is discussed in detail in ER 80P.

5. References

1. ANSI/ASME Power Test Code PTC 19.1 – 1985, Part 1 Measurement Uncertainty, Reaffirmed 1990.
2. Caldon Topical Report ER-80P, "Improving Thermal Power Accuracy and Plant Safety While Increasing Operating Power Level Using the LEFM Check System", Rev. 0.
3. Regan, J., "Operation Near 100% Rated Thermal Power: Historical Licensee Event Reports", Proceedings of the 1999 ANS Winter Meeting, November 1999.
4. NRC SER dated March 8 1999, "Safety Evaluation by the Office of Nuclear Reactor Regulation Topical Report ER-80P, 'Improving Thermal Power Accuracy and Plant Safety While Increasing Operating Power Level Using the LEFM System', Comanche Peak Steam Electric Station, Units 1 and 2 Docket Nos. 50-445 and 50-446".
5. Caldon Engineering Report ER 160P, "Supplement to Topical Report ER 80P: Basis for a Power Uprate with the LEFM System", May 2000
6. NRC Safety Evaluation related to Amendment 31 Operating License NPF-390, Docket number 50-390, January 2001

Figure 6. Probable Operating Ranges For LEFM Systems at Increased Power Levels



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Figure 7. Probability of Exceeding Power Levels With LEFM Systems And Increased Power

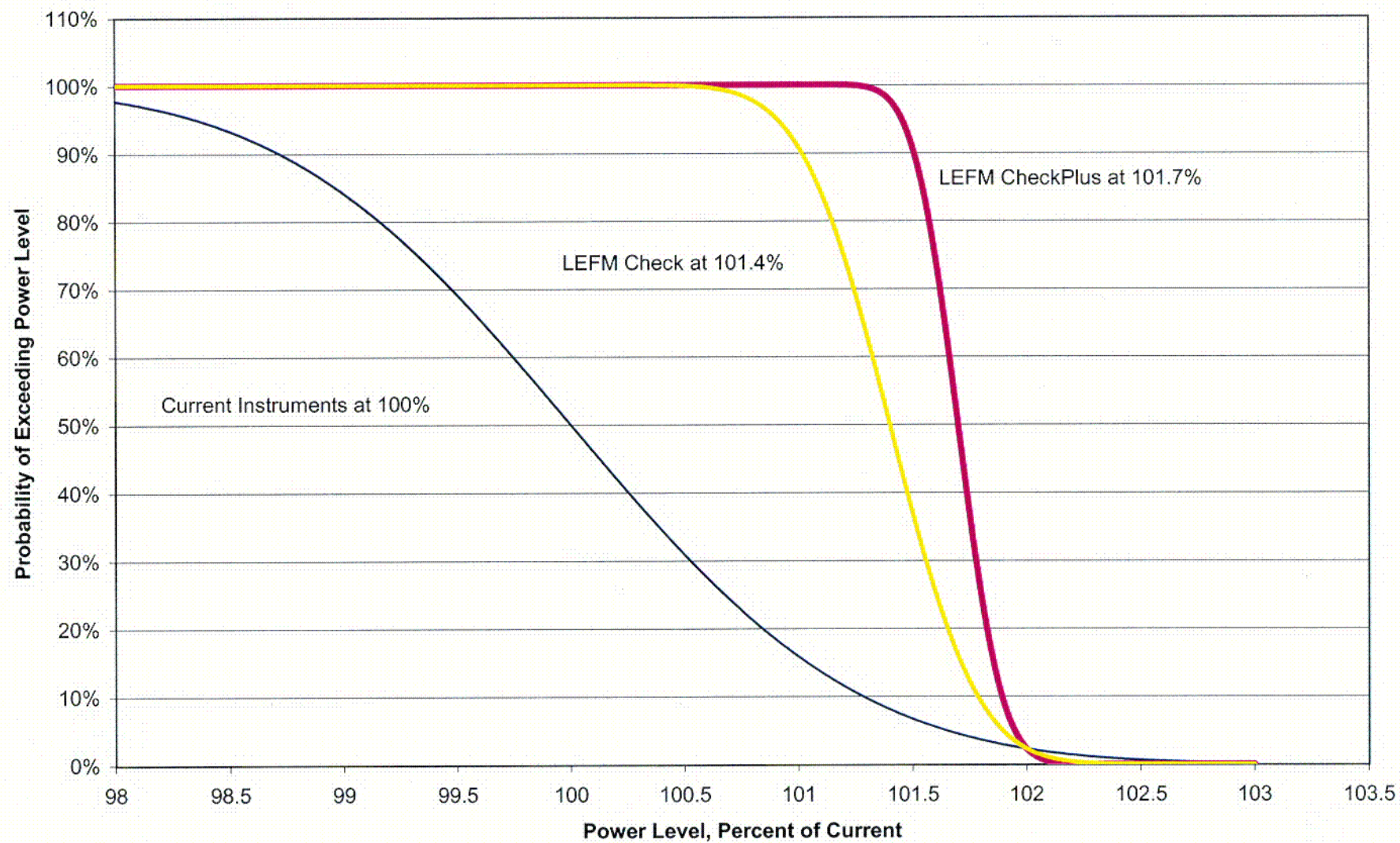
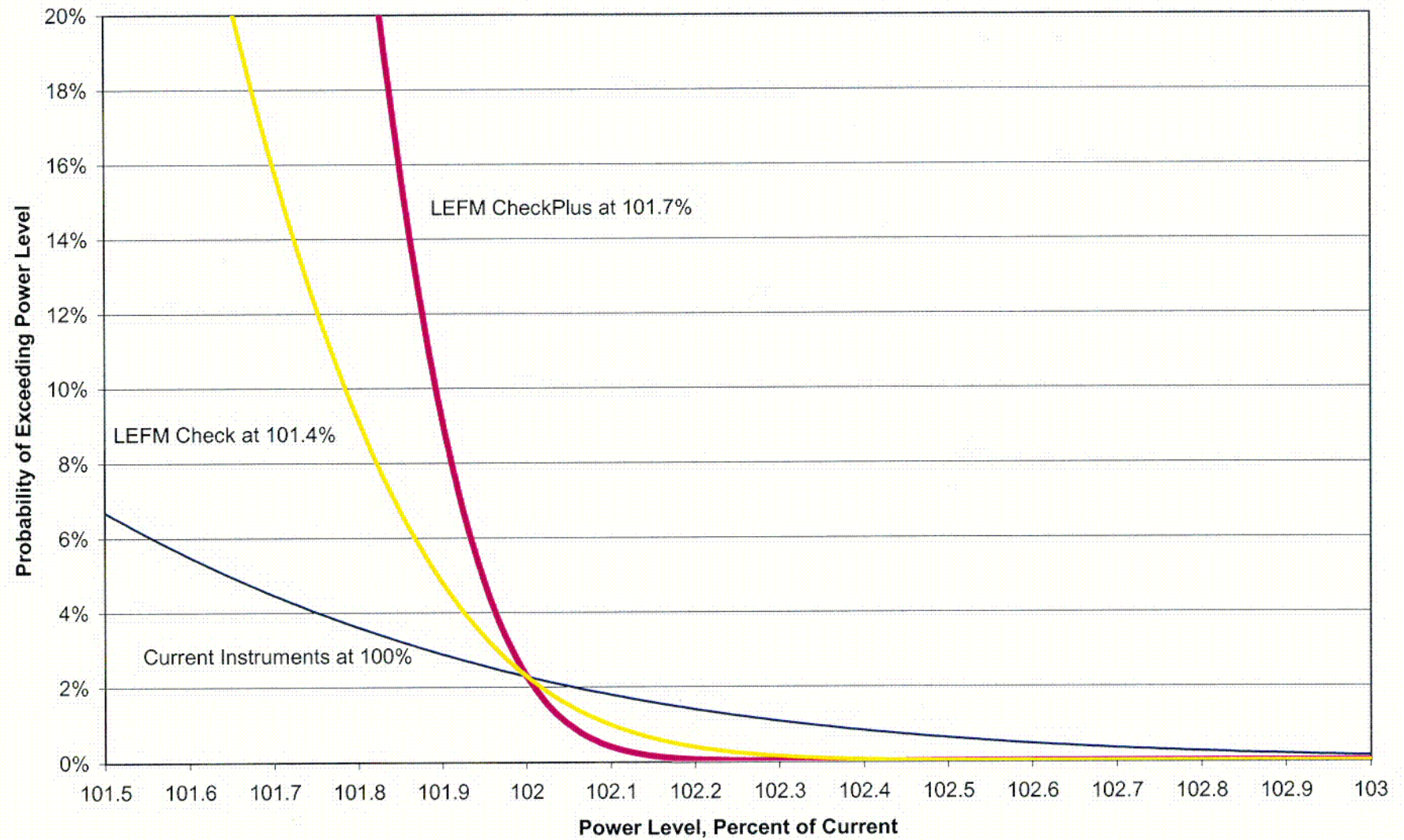


Figure 8. Probability of Exceeding Power Level in the Vicinity of 102% Power



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Appendix A is Proprietary to Caldon in its Entirety

APPENDIX B to Caldon ER-157N**ESTIMATES OF THE PROBABILITY OF EXCEEDING POWER MARGINS
OF UP TO FIVE STANDARD DEVIATIONS****Purpose:**

The purposes of this appendix are to:

- a) Quantify the effect of turbulence on the two standard deviation uncertainty bands of current calorimetric instrumentation as well as calorimetric instrumentation utilizing the chordal LEFM Check or LEFM CheckPlus system and,
- b) Estimate the probabilities of exceeding power levels one, two, three, four and five standard deviations above the power levels measured by instruments characterized in (a) above.

Assumptions:

1. The impact of turbulence on existing instrumentation will be assumed comparable to that calculated for the LEFM Check in Appendix A under Item 6 subsection B, "Setting the Sample Size". In that subsection, the observational uncertainty due to turbulence was calculated to be $\pm 0.05\%$ (2σ).
2. The 95% confidence limits for a set of instruments are assumed to define an error probability distributed normally about the true value of the measured variable with two standard deviations equal to the 95% confidence limits. The possibility of error distributions other than normal and the implications of such distributions on overall error probability are discussed in Reference 2.
3. The aggregate power measurement uncertainties of current instrumentation and the LEFM Check and LEFM CheckPlus are made up of numerous small contributing elements. Errors characterizing these elements may not always be normally distributed; however, for initial probability estimates, it will be assumed that, when the error elements aggregate, the resulting error distribution is normally distributed.

Discussion

1. As noted in Reference 4 Appendix K of 10CFR50 allocates a 2% margin to cover the uncertainties in thermal power measurements with conventional instrumentation. For purposes of comparison with LEFM performances, this figure will be taken as the 95% confidence limit for such instruments. One standard deviation of the error distribution for this instrument set is therefore 1%.
2. The effect of turbulence on the power measurement, in the case of the LEFM Check or LEFM CheckPlus, is to add an uncertainty of $\pm 0.05\%$ as the root sum square to the instrument uncertainty. The uncertainty caused by turbulence is controlled, in the case of an LEFM, by

selecting a sample size such that two standard deviations of the mean flow measurement do not exceed $\pm 0.05\%$ (See Appendix A Item 6B). A similar procedure can be followed in processing the analog output of a nozzle instrument; a sample period can be selected to limit the uncertainty in the mean of the flow data to $\pm 0.05\%$. Assuming that this procedure is followed, the effective value of 2 standard deviations of the power determination with current instrumentation, including the effects of turbulence, is

$$2 \text{ SD}_{\text{eff}} = \sqrt{[(2)^2 + (0.05)^2]} \cong \pm 2\%$$

3. Based on the calculations of Appendix A, an upper bound for the 95% confidence limit band for a power determination a single LEFM Check installation is $\pm 0.51\%$. For the LEFM CheckPlus system, a lower bound is $\pm 0.33\%$. Accordingly, the effective values for two standard deviations of the LEFM Check and LEFM CheckPlus power determinations including turbulence are

$$\text{LEFM Check} \quad 2 \text{ SD}_{\text{eff}} = \sqrt{[(0.51)^2 + (0.05)^2]} = \pm 0.51\%$$

$$\text{LEFM CheckPlus} \quad 2 \text{ SD}_{\text{eff}} = \sqrt{[(0.33)^2 + (0.05)^2]} = \pm 0.33\%$$

For the comparisons of this appendix, rounded values that bracket the uncertainties of power determination of both system will be employed. For the LEFM Check System, an upper bound of $\pm 0.6\%$ for two standard deviation will be used; for the LEFM CheckPlus a lower bound of $\pm 0.3\%$ will be used.

4. Values for the error probabilities for various uncertainty bands expressed in terms of standard deviations are tabulated in Table B-1, below. The Table is based on data interpolated or extrapolated from Eshbach's *Handbook of Engineering Fundamentals*, referenced in the Table. For example, from Table B-1, the probability that the true value of the measured variable will lie within 2 standard deviations of the true value is 0.95446. Accordingly, the probability of the true power level exceeding that indicated by a set of instruments by two standard deviations, $P(+2\text{SD})$, is 1/2 of the difference between 1.00000 and 0.95446 (one half, since the probability of exceeding 2 standard deviations only is being calculated).

$$P(+2 \text{ SD}) = 1/2 (1.00000 - 0.95446) = 0.02277 \cong 2.3\% .$$

The probabilities of exceeding the indicated power level by one, three, four and five standard deviations are also tabulated in Table B-1 for current power level instrumentation, as characterized in column 2 ("Error Band Nozzle or Venturi") and for power determinations using the LEFM Check (column 3) and LEFM CheckPlus (column 4). The probabilities of exceeding the various bands are expressed both in decimal and reciprocal (odds) formats.

Table B-1
Probability Tabulations

Based on Table I, Eshbach, *Handbook of Engineering Fundamentals*, Part 2, Section 61, "The Probable Error", First Edition, John Wiley and Sons

Number of Standard Deviations σ	Error Band Nozzle or Venturi	Error Band LEFM Check	Error Band LEFM CheckPlus	$h a^* =$ $\sigma / \sqrt{2}$	P^*	$P_{exc} =$ $1/2$ $(1.00000-P)$	$1/P_{exc}$
1.0	0.7%	0.3%	0.15%	0.707	0.68261	0.1587	1/6.3
2.0	1.4%	0.6%	0.3%	1.414	0.95446	0.0228	1/44
3.0	2.1%	0.9%	0.45%	2.121	0.99730	1.35×10^{-3}	1/741
4.0	2.8%	1.2%	0.6%	2.828	0.999938	3.1×10^{-5}	1/32,300
5.0	3.5%	1.5%	0.75%	3.536	0.9999994	3×10^{-7}	1/3.3 million

* P is tabulated in Table I of the *Eshbach* as a function of "h a", where "h a" is as defined in the Table above. Values of P for 1 through 4 sigma have been obtained by interpolating the values of probability given as a function of "h a" in the referenced table. The value for five sigma is based on a straight line log extrapolation of $(1 - P_{exc})$.

5. The probability computations above rest on two key assumptions:

- (a) that the computed uncertainty bands characterize the actual performance of the instruments in service, and
- (b) that the elemental uncertainty bands which, together, determine the overall uncertainty band are themselves normally distributed.

Assumption (a) generates a requirement to be able to confirm instrument performance on-line. The capability of an LEFM Check or LEFM CheckPlus system to fulfill this requirement is discussed in Reference 2.

As for assumption (b), many LEFM elemental uncertainties are normally distributed about the measured value for that element, since the contributors to the uncertainty tend to be random in nature, and any known biases are corrected. However, some elemental errors are likely to be uniformly distributed. To analyze the potential impact of uniform error distributions on overall LEFM Check uncertainty, results of exploratory *Monte Carlo* analyses of synthetic measurement systems whose elements are characterized by random and uniform error distributions are presented in Reference 2. The key conclusion from this work is that the probabilities tabulated in Table B-1 are valid, regardless of the specifics of the error distributions of the individual elements of the measurement systems.