CENTER FOR NUCLEAR WASTE REGULATORY ANALYSES

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# TRIP REPORT

SUBJECT: Eighth International Conference on Computational Water Resources

DATE/PLACE OF TRIP: Venice, Italy; June 11-15, 1990

AUTHOR: Rachid Ababou

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TRIP REPORT

SUBJECT: Eighth International Conference on Computational Water Resources

DATE/PLACE OF TRIP: Venice, Italy; June 11-15, 1990

AUTHOR: Rachid Ababou

PERSON PRESENT:

CNWRA

Rachid Ababou

BACKGROUND:

This is the eighth of a series of conferences covering a wide array of topics on computational approaches to problems in environmental hydraulics and water resources. This conference was endorsed by AGU, ASCE, IAHR, National Science Foundation, National Society of Computational methods in Engineering, and Italian Universities (the chairman, Prof. Giuseppe Gambolati, being from the University of Padua).

CONFERENCE TOPICS:

Contributions at the conference were published in a two-volume proceedings by Computational Mechanics Publications and Springer-Verlag, under the titles: (1) "Computational Methods in Subsurface Hydrology", and (2) "Computational Methods in Surface Hydrology". Briefly the first volume covered saturated, unsaturated, multiphase flow, fracture flow, contaminant migration, chemical reactions in porous media, stochastic problems, and optimization for groundwater projects. The second volume dealt with shallow water, coastal, and estuarial models, sediment transport, advection-diffusion and mixing, boundary element methods, special computational techniques, and water management. These volumes can be ordered directly from the publisher, e.g. by contacting Prof. C.A. Brebbia at the Computational Mechanics Institute in Southampton, U.K.

SCIENTIFIC ACTIVITIES:

My own contribution at the conference was about the "Numerical Analysis of Nonlinear Unsaturated Flow Equations." This paper can be found in Volume 1 (Part A, Section 2: Unsaturated Groundwater Flow, pp. 151-160). I also chaired the session on Boundary Element Methods (Vol. 2, Part C, Section 2: Boundary Element Models), and contacted a number of participants with common research interests on applied numerical methods for advective-dispersive and nonlinear flow-transport problems. I am attaching a copy of my paper, as well as a list of sponsoring institutions, title pages, and table of contents of the two proceedings volumes.

PROBLEMS ENCOUNTERED:

None

PENDING ACTIONS:

None

RECOMMENDATIONS:

Attendance at such meetings plays an important role in publicizing NRC and CNWRA research efforts as well as obtaining up to date information on recent progress in key areas of research. It is recommended that such communication channels be kept open in the future, notably through research presentations by CNWRA staff at scientific conferences and workshops.

SIGNATURE:

Rachid Ababou — October 3, 1990

Rachid Ababou — Date

CONCURRENCE SIGNATURES:

for John L. Russell — October 4, 1990

John L. Russell — Date

Allen R. Whiting — October 4, 1990

Allen R. Whiting — Date

RA/yl

Attachments

R. RACHID ABABOU

VIII International Conference on Computational methods
in

# Water Resources

# PROGRAM

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Proceedings
T.O.C.

Venezia (Fondazione Cini), June 11th-15th, 1990

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## OBJECTIVES

The Conference is intended as a forum for the review of the advances so far achieved in the field of computational simulation of surface and subsurface water, presentation of new research ideas and exchange of experiences in the practical applications of computer methods in water resources. All aspects related to accuracy of approximation, efficiency of techniques, economy of application, improvement of existing methodologies and limitations vis à vis the scale effect as well as the quality and quantity of the available information should be discussed. The non-linear problems and the great potential offered by the supercomputer technology are especially within the scope of the Conference. The Organizing Committee welcomes and solicits contributions related to any of the following topics:

- modeling of groundwater flow in porous and fractured media
- modeling of estuary, river, lake, lagoon and ocean hydrodynamics
- modeling of surface and subsurface transport
- modeling of water quality
- modeling of sedimentation processes
- optimization of water resources systems
- parameter estimation techniques
- simulation of water resources on supercomputers
- basic principles and computational methods
- numerical mathematics and advances in software
- methods for non-linearities

This Conference is ideally connected to and represents the natural evolution of the previous conferences of the series which were held at Princeton University, U.S.A. (1976), Imperial College, London, U.K. (1978), University of Mississipi, Oxford, U.S.A. (1980), University of Hannover, Germany (1982), University of Vermont, Burlington, U.S.A. (1984), Laboratorio Nacional de Engenharia Civil, Lisbon, Portugal (1986) and MIT, Cambridge, U.S.A. (1988).

Invited keynote lectures will be delivered in the areas of major interest with emphasis to be placed on the state of the art and the development of new research directions and applications.

| Session # | Monday | Tuesday | Wednesday | Thursday | Friday |
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| | opening session: G. Gambolati | key-note lecture: G. Gambolati | key-note lecture: G. Gambolati | key-note lecture: G. Gambolati | key-note lecture: G. Gambolati |
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| 2 | | CEL1: M. Fanelli | CT1: W.A. Mulder | RCF2: J.C.J. Nihoul | WMO1: J. Carrera |
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| 2 | SWM2: V. Casulli | CEL3: A. Quarteroni | CT3: R. Ababou | | WMO3: I. Bogardi |
| 3 | | GTP1: S.P. Neuman | STP: K.P. Holz | | |
| 1 | SGF3: U Meissner | UMF2: J.F. Sykes | GTP4: J.P. Laible | | closing session: G. Gambolati |
| 2 | SWM3: G. Di Silvio | CEL4: E. Marchi | SC: A. Peters | | |
| 3 | | GTP2: W. Kinzelbach | RCF1: S.S.Y. Wang | | |

RACHID FBABOU

# Computational Methods in Subsurface Hydrology (Vol. 1) = (Part A)

Proceedings of the Eighth International Conference on Computational Methods in Water Resources, held in Venice, Italy, June 11-15 1990.

Editors: G. Gambolati
A. Rinaldo
C.A. Brebbia
W.G. Gray
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# PREFACE

The choice of Venice as the site of the VIII International Conference on Computational Methods in Water Resources is both timely and meaningful. Its timeliness and significance stem from the visibility and the complexity of the water resources problem affecting the environment of the city: the lagoon and its contributing mainland. It is not therefore by chance that among the various sponsorships, gratefully acknowledged, we particularly appreciate the major contribution from the Consorzio Venezia Nuova, i.e. the institution in charge of the rescue of Venice, its lagoon and its mainland. It is the organizers' hope that this book will also contribute to the professional background and the scientific advances needed to address the complex issues related to the use and preservation of the Venetian water resources system.

This book results from the edited proceedings of the VIII International Conference on Computational Methods in Water Resources (originally Finite Elements in Water Resources) held at the Giorgio Cini Foundation, Venice, Italy, June 1990. The Conference series was started in 1976 to serve as an internationally acknowledged forum for researchers in the - at that time - novel and emerging field of finite element methods applied to water resources. The name and the peculiar role of the ongoing Conference series were later (1986) modified to host contributions based on the increasingly diverse computational techniques being applied in water resources research. The previous meetings were held at: Princeton University, USA, 1976; the Imperial College, UK, 1978; the University of Mississippi, USA, 1984; the Laboratorio Nacional de Engenharia Civil, Portugal, 1986; and the Massachussets Institute of Technology, USA, 1988.

The 1990 Proceedings cover a wide spectrum of computational methods encompassing both theory and applications. Contaminant transport in surface and subsurface hydrology has attracted most of the researchers' interest, in a way reflecting the trends observed in the referred literature in this field and plays an important role in this book. It is significant that several papers edited in this book concern the increasingly studied field of computational stochastic hydrology .

The organizing committee of the Venice Conference wishes to express deep appreciation to the key-note invited lecturers J. Glimm, S.P. Neumann,

J.C.J. Nihoul, Y. Pomeau and I. Rodriguez-Iturbe. We are also indebted to the invited lecturers J. Carrera, V. Casulli, M.A. Celia, G. Dagan, R.E. Ewing, I. Herrera, M. Kawahara, U. Meissner, A. Quarteroni, W.M. Schestakow, A.J. Valocchi, S.S.Y. Wang, and M.F. Wheeler. A significant contribution to the scientific fallout of the meeting came from the organizers of the Wave Propagation in Shallow Waters Forum (A. Adami and A. Noli) and the Supercomputing in Water Resources Forum (A. Peters). It is also a pleasure to acknowledge the continuing efforts and support offered by C.A. Brebbia, W.G. Gray and G.F. Pinder, of the permanent organizing committee.

The committee gratefully acknowledges the sponsorship of: AGIP, Alitalia, Aquater, Banca Popolare Veneta, Bonifica, Camera di Commercio I.A.A. di Venezia, Cassa di Risparmio di Padova e Rovigo, CISE, Comune di Venezia, Consiglio Nazionale delle Ricerche, Consorzio Venezia Nuova, Centro Sperimentale per l'Idrologia e la Meteorologia della Regione del Veneto, Dagh Watson, Digital Equipment Italia, ENEL, FIATIMPRESIT, Gruppo Acqua, Hydrodata, IBM Italia, Idroser, INC - il nuovo castoro, ISMES, Istituto di Credito Fondiario delle Venezie, Istituto Federale delle Casse di Risparmio delle Venezie, Lotti & Associati, Provincia di Venezia, Rodio, SIP, Studio Geotecnico Italiano, Technital, Tecnomare, ZF-MPM, Zollet Ingegneria. The endorsement of the following organizations is also acknowledged: AGU, AIMETA, ASCE, IAHR, Istituto Veneto di Scienze Lettere ed Arti, National Science Foundation, National Society of Computational Methods in Engineering, University of Padua - School of Engineering, University of Trent - School of Engineering, (ISME) the International Society for Computational Methods in Engineering, (ISBE) the International Society for Boundary Elements

May we finally add that the final version of the accepted papers appearing in this volume is reproduced directly from the material submitted by the authors who are therefore responsible for their content.

The Editors
June 1990

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# Computational Methods in Surface Hydrology (Vol. 2) = (Part B + Part C)

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# PREFACE

The choice of Venice as the site of the VIII International Conference on Computational Methods in Water Resources is both timely and meaningful. Its timeliness and significance stem from the visibility and the complexity of the water resources problem affecting the environment of the city: the lagoon and its contributing mainland. It is not therefore by chance that among the various sponsorships, gratefully acknowledged, we particularly appreciate the major contribution from the Consorzio Venezia Nuova, i.e. the institution in charge of the rescue of Venice, its lagoon and its mainland. It is the organizers' hope that this book will also contribute to the professional background and the scientific advances needed to address the complex issues related to the use and preservation of the Venetian water resources system.

This book results from the edited proceedings of the VIII International Conference on Computational Methods in Water Resources (originally Finite Elements in Water Resources) held at the Giorgio Cini Foundation, Venice, Italy, June 1990. The Conference series was started in 1976 to serve as an internationally acknowledged forum for researchers in the - at that time - novel and emerging field of finite element methods applied to water resources. The name and the peculiar role of the ongoing Conference series were later (1986) modified to host contributions based on the increasingly diverse computational techniques being applied in water resources research. The previous meetings were held at: Princeton University, USA, 1976; the Imperial College, UK, 1978; the University of Mississippi, USA, 1984; the Laboratorio Nacional de Engenharia Civil, Portugal, 1986; and the Massachussets Institute of Technology, USA, 1988.

The 1990 Proceedings cover a wide spectrum of computational methods encompassing both theory and applications. Contaminant transport in surface and subsurface hydrology has attracted most of the researchers' interest, in a way reflecting the trends observed in the referred literature in this field and plays an important role in this book. It is significant that several papers edited in this book concern the increasingly studied field of computational stochastic hydrology .

The organizing committee of the Venice Conference wishes to express deep appreciation to the key-note invited lecturers J. Glimm, S.P. Neumann,

J.C.J. Nihoul, Y. Pomeau and I. Rodriguez-Iturbe. We are also indebted to the invited lecturers J. Carrera, V. Casulli, M.A. Celia, G. Dagan, R.E. Ewing, I. Herrera, M. Kawahara, U. Meissner, A. Quarteroni, W.M. Schestakow, A.J. Valocchi, S.S.Y. Wang, and M.F. Wheeler. A significant contribution to the scientific fallout of the meeting came from the organizers of the Wave Propagation in Shallow Waters Forum (A. Adami and A. Noli) and the Supercomputing in Water Resources Forum (A. Peters). It is also a pleasure to acknowledge the continuing efforts and support offered by C.A. Brebbia, W.G. Gray and G.F. Pinder, of the permanent organizing committee.

The committee gratefully acknowledges the sponsorship of: AGIP, Alitalia, Aquater, Banca Popolare Veneta, Bonifica, Camera di Commercio I.A.A. di Venezia, Cassa di Risparmio di Padova e Rovigo, CISE, Comune di Venezia, Consiglio Nazionale delle Ricerche, Consorzio Venezia Nuova, Centro Sperimentale per l'Idrologia e la Meteorologia della Regione del Veneto, Dagh Watson, Digital Equipment Italia, ENEL, FIATIMPRESIT, Gruppo Acqua, Hydrodata, IBM Italia, Idroser, INC - il nuovo castoro, ISMES, Istituto di Credito Fondiario delle Venezie, Istituto Federale delle Casse di Risparmio delle Venezie, Lotti & Associati, Provincia di Venezia, Rodio, SIP, Studio Geotecnico Italiano, Technital, Tecnomare, ZF-MPM, Zollet Ingegneria. The endorsement of the following organizations is also acknowledged: AGU, AIMETA, ASCE, IAHR, Istituto Veneto di Scienze Lettere ed Arti, National Science Foundation, National Society of Computational Methods in Engineering, University of Padua - School of Engineering, University of Trent - School of Engineering, (ISME) the International Society for Computational Methods in Engineering, (ISBE) the International Society for Boundary Elements

May we finally add that the final version of the accepted papers appearing in this volume is reproduced directly from the material submitted by the authors who are therefore responsible for their content.

The Editors
June 1990

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# Numerical Analysis of Nonlinear Unsaturated Flow Equations

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ABSTRACT:

The numerical behavior of the nonlinear unsaturated flow equation is examined analytically for an implicit finite difference scheme. The governing equation combines nonlinear diffusion and convection operators, and is characterized by a simple Peclet number. Numerical errors are investigated using truncation error analysis, frozen stability analysis, and functional analysis of the nonlinear mapping associated with Picard iterations. These approaches shed light on different but complementary aspects of the same numerical problem.

INTRODUCTION:

Flow in unsaturated porous media is governed by a strongly nonlinear diffusion type equation with a nonlinear, forced convection term due to gravity. These features make it particularly difficult to solve by any means. Analytical solutions have been and are still providing valuable insights for certain classes of flows, but most realistic problems have to be solved numerically: see [1] and [2] for high-resolution supercomputer simulations of 3-dimensional, transient unsaturated flow in randomly heterogeneous and stratified media.

Our experience is that the strong nonlinearity of unsaturated flow usually causes considerable numerical difficulties and requires trial-and-error adjustments of mesh size, time step, relaxation parameters, tolerance criteria, and adaptive controls. To improve the efficiency of future numerical algorithms will require some understanding of how the specific features of the unsaturated flow equation contribute to numerical errors. Exploring this question constitutes the main purpose of this paper.

**UNSATURATED FLOW EQUATIONS:**

For transient flow in variably saturated porous media, a mixed variable formulation of the governing equation is obtained by combining the mass conservation equation ($\partial\theta/\partial t=-\nabla.Q$) with the Darcy-Buckingham equation ($Q=-K\nabla H$):

$$\partial\theta(h,x)/\partial t = \nabla(\ K(h,x)\ (\nabla h + g)\ ) \qquad (1)$$

where $H=h+g.x$ is the hydraulic potential, h is pressure head, and g is the cosine vector aligned with the acceleration of gravity and equal to (0,0,-1) if the third axis is vertical pointing downwards. Defining the specific moisture capacity $C=\partial\theta/\partial h$ yields the pressure-based Richards equation:

$$C(h,x)\ \partial h/\partial t = \nabla(\ K(h,x)\ (\nabla h + g)\ ) \qquad (2)$$

Introducing a nonlinear moisture diffusivity $D=K/C$, assuming a spatially homogeneous moisture retention curve $\theta(h)$, and $h \leq 0$ everywhere, yields the moisture-based version of eq.(1):

$$\partial\theta/\partial t = \nabla(\ D(\theta,x)\ \nabla\theta + g\ K(\theta,x)\ ) \qquad (3)$$

In the detailed 3-dimensional simulations of [1] and [2], the finite difference method was applied to the mixed form (1), which is more mass conservative than eq.(2) and is not limited to negative pressures as eq.(3). For convenience, however, we will use the standard Richards equation (2) for the numerical analyses to be developed in this paper.

The nonlinear gravity term containing (g) acts as forced convection, in competition with the diffusion term represented by the elliptic operator $\nabla(K\nabla h)$. For homogeneous media, eqs.(2) and (3) can be reformulated as follows, respectively:

$$C\ (\partial h/\partial t + V.\nabla h) = \nabla(K\nabla h) \qquad (2)'$$

and:

$$\partial\theta/\partial t + V.\nabla\theta = \nabla(D\nabla\theta) \qquad (3)'$$

where:

$$V = -\ (\partial K/\partial\theta)\ g \qquad (4)$$

The vector V represents the velocity of pressure or moisture disturbances, in the absence of the right-hand side diffusive terms. Note that moisture and pressure waves propagate at the same speed in homogeneous media. The convective-diffusive form taken by equations (2-3) suggests that a Peclet number might be used to characterize convection versus diffusion effects.

A Peclet number emerges quite naturally from the Kirchhoff transform formulation [1]. This transform is valid for the class of heterogeneous media possessing a homogeneous relative conductivity curve Kr(h), i.e. with a separable conductivity curve $K(h,x)=Ks(x)Kr(h)$. But we focus here on the more restricted case where both the saturated and relative

conductivities are homogeneous, i.e. with a homogeneous conductivity curve $K(h)=KsKr(h)$. The Kirchhoff transform is then defined by:

$$\phi(h) = \int_{-\infty}^{h} K(h')\, dh' \tag{6}$$

Substituting eq.(6) in eq.(2) and using chain rules yields:

$$\partial\phi/\partial t + V.\nabla\phi = D\, \nabla^2\phi \tag{7}$$

Here again, the wave velocity V is given by equation (4), or equivalently by $V = -\alpha Dg$, where $\alpha=\partial \ell nK/\partial h$ is the slope of the log-conductivity/pressure curve.

Given the form of the Kirchoff equation (7), the Peclet vector $Pe=V\ell/D$ emerges as a relative measure of convective versus diffusive moisture transport over the chosen length scale ($\ell$). Furthermore, substituting the above expression for V gives a very simple expression for the Peclet vector:

$$Pe = -\alpha \ell g \tag{8}$$

which reveals the special role played by the $\alpha$-parameter. Since transforms like $\theta(h)$ or $\phi(h)$ do not fundamentally alter the ratio of convection versus diffusion coefficients, this Peclet vector characterizes the transport of pressure as well as moisture, Kirchoff potential, or any other quantity that can be related to pressure in a one-to-one fashion.

## TRUNCATION ERROR ANALYSIS:

In this section, we evaluate truncation error as a function of mesh size and time step for a nonlinear, implicit finite difference discretization of the Richards equation. Note that truncation analysis compares the discrete and differential operators, but is not concerned with the numerical errors incurred by the dependent variable itself, or with the space-time propagation of such errors (stability), or with the additional errors incurred while solving the nonlinear discretized system (linearization). Our purpose here is to identify potential sources of inaccuracies by looking at the leading order terms of truncation error.

Consider the Richards equation (2), to be solved for a 1-dimensional homogeneous medium using a fully implicit nonlinear finite difference discretization (Euler backwards in time, 2-point centered in space). The differential and discretized equations are, respectively:

$$\mathcal{L}(h) = -C(h)\, h_t + (\, K(h)\, (h_x + g)\,)_x = 0 \tag{9.a}$$

$$L(h_i^{n+1}) = -C_i^{n+1}\, \frac{h_i^{n+1}-h_i^n}{\Delta t} + \frac{K_{i+1/2}^{n+1}}{\Delta x}\left[\frac{h_{i+1}^{n+1}-h_i^{n+1}}{\Delta x} + g\right] - \frac{K_{i-1/2}^{n+1}}{\Delta x}\left[\frac{h_i^{n+1}-h_{i-1}^{n+1}}{\Delta x} + g\right] = 0 \tag{9.b}$$

where g is a cosine representing gravity, with g=0 if x is horizontal and g=-1 if x is vertical downwards. The coefficients of the discrete operator are fully nonlinear, being expressed at the current time step (n+1). The mid-nodal conductivities are approximated by a geometric weighting:

$$K_{i+1/2} = \left[\, K(h_i)\, K(h_{i+1})\,\right]^{1/2} \tag{10}$$

In the case of an exponential K(h) curve, this scheme weights pressures arithmetically, and it yields the exact midnodal conductivity in zones of spatially constant pressure gradient.

The truncation error $E(h)=L(h)-\mathcal{L}(h)$ at the nodes of the space-time mesh was calculated in [1] using intermediate results from [3]. We choose here to express the final result in terms of both flux (Q) and pressure (h) as follows:

$$E(h) \approx \Delta t \left[\frac{\partial}{\partial x}\left[\, K \frac{\partial}{\partial t}\left(\frac{\partial h}{\partial x}\right)\right] - \frac{C}{2}\frac{\partial^2 h}{\partial t^2}\right] + \frac{\Delta x^2}{24}\left[\frac{\partial}{\partial x}\left(K \frac{\partial^3 h}{\partial x^3}\right) - \frac{\partial^3 Q}{\partial x^3}\right] - \frac{\Delta x^2}{8}\left[\alpha \frac{\partial}{\partial x}\left(Q \frac{\partial^2 h}{\partial x^2}\right)\right] \tag{11}$$

where $Q=-K(\partial h/\partial x+g)$, and $\alpha=\partial \ln K/\partial h$. It is interesting to note that one recovers the linear heat equation by letting K=1, C=1, and $\alpha=0$ in (9-11). Inserting in eq.(11) the identities:

$$h_t = h_{xx} \quad \text{and} \quad h_{tt} = -Q_{xxx} = h_{xxxx}\,,$$

leads to the verification of a well known result [4]: the order of accuracy of the linear heat equation increases from $O(\Delta t)+O(\Delta x^2)$ to $O(\Delta t^2)+O(\Delta x^4)$ with the choice $\Delta t/\Delta x^2 = 1/6$.

Let us now discuss the implications of (11) in the fully nonlinear case. The $O(\Delta t)$ term, due to temporal discretization errors, appears to be controlled by the rate of change of the pressure gradient and by the second order time-derivative of pressure. The first of the two $O(\Delta x\Delta x)$ terms is due to spatial discretization errors other than midnodal conductivity weighting. The second $O(\Delta x\Delta x)$ term is due solely to errors in evaluating midnodal conductivities by the geometric rule (10), and vanishes in regions of spatially constant pressure gradient, as expected in the case of exponential K(h).

Equation (11) simplifies considerably in the steady state case, since the 1-dimensional flux Q becomes constant in both space and time. The result suggests that, even in the transient case, spatial errors are dominated by the rate of change of pressure curvature with depth, which can become quite large near sharp wetting fronts above and below the inflexion point. This type of information may be used for designing optimal adaptive grid procedures.

STABILITY ANALYSIS:

To complement the previous truncation error analysis, we now examine how numerical errors propagate as a function of time. In addition, we hope to capture at least some of the additional error amplification effects due to inexact treatment of nonlinearity. Our approach is to analyze the stability of a linearized version of the finite difference problem, such that all nonlinear coefficients are evaluated from the solution at the previous time step (no iterations). The unstable effect of linearization is partially taken into account by unfreezing the nonlinear convective coefficient, while other coefficients remain frozen.

We focus once more on the case of 1-dimensional homogeneous media as in eqs.(9). Consider the following linearized form of the finite difference system:

$$C_i^n \frac{h_i^{n+1}-h_i^n}{\Delta t_n} = \frac{1}{\Delta x}\left[K_{i+\frac{1}{2}}^n \left[\frac{h_{i+1}^{n+1}-h_i^{n+1}}{\Delta x}+g\right]-K_{i-\frac{1}{2}}^n\left[\frac{h_i^{n+1}-h_{i-1}^{n+1}}{\Delta x}+g\right]\right] \qquad (12)$$

where the superscript (n+1) indicates the current time level. The form of this finite difference system suggests that, while the nonlinear diffusion operator is treated implicitly, the nonlinear gravity term $g(K[i+1/2]-K[i-1/2])/\Delta x$ is treated explicitly since it is entirely evaluated at the previous time level. Based on this remark, we will now examine how this discrepancy affects the numerical stability of the solution.

The proposed method is to develop a Fourier stability analysis of equation (12) with partially frozen coefficients. This is analogous to the usual frozen coefficients analysis as described in [4], except that the nonlinearity of the gravity term is taken into account via the quasilinear approximation:

$$g\,\frac{K_{i+\frac{1}{2}}^n - K_{i-\frac{1}{2}}^n}{\Delta x} = g\,\frac{K_i^n}{\Delta x}\,\frac{\alpha}{2}\,(h_{i+1}^n - h_{i-1}^n) + O(\Delta x^2) \qquad (13)$$

where again $\alpha = d\ell nK/dh$. The leading term on the right-hand side is expected to be a reasonable approximation of the left-hand side if the quantity $\alpha|h[i+1]-h[i]| \approx \alpha\Delta x|\partial h/\partial x|$ is on the order of unity or less. At any rate, even rough indications on

the numerical stability of the nonlinear unsaturated flow system will be useful given the lack of theoretical results in this area. With this provision, substituting (13) into (12) yields the following mixed implicit/explicit scheme:

$$-\tilde{D}_{i-\frac{1}{2}}\, h_{i-1}^{n+1} + (1+\tilde{D}_{i-\frac{1}{2}}+\tilde{D}_{i+\frac{1}{2}})\, h_i^{n+1} - \tilde{D}_{i+\frac{1}{2}}\, h_{i+1}^{n+1} \simeq$$
$$-\frac{1}{2}\, g\, \alpha\Delta x\, \tilde{D}_i\, h_{i-1}^n + h_i^n + \frac{1}{2}\, g\, \alpha\Delta x\, \tilde{D}_i\, h_{i+1}^n \qquad (14.a)$$

where $\tilde{D}$ is the dimensionless diffusion coefficient:

$$\tilde{D}_{i(\pm\frac{1}{2})} = \frac{K_{i(\pm\frac{1}{2})}}{C_i}\,\frac{\Delta t}{\Delta x^2} \qquad (14.b)$$

The stability of equation (14) with frozen diffusion coefficients can be studied in the standard way using Fourier stability analysis [4]. This leads to a complex amplification factor, $\rho$, characterizing the growth rate of numerical errors in time:

$$\rho \simeq \frac{1 + j\, \alpha g\Delta x\, \tilde{D}_i\, \sin(k\Delta x)}{1+(\tilde{D}_{i+\frac{1}{2}}+\tilde{D}_{i-\frac{1}{2}})(1-\cos(k\Delta x)) - j(\tilde{D}_{i+\frac{1}{2}}-\tilde{D}_{i-\frac{1}{2}})\sin(k\Delta x)} \qquad (15)$$

where j is the square-root of -1, and k is a Fourier mode or wavenumber taking discrete values: $k \in \{\pi/L, \cdots, n\pi/L\}$.

Requiring $|\rho| \leq 1$ in equation (15) finally leads to the necessary and sufficient stability condition:

$$Pe = |\alpha g\Delta x| \leq 2\sqrt{1+(2\,\frac{K_i}{C_i}\,\frac{\Delta t}{\Delta x^2})^{-1}} \qquad (16)$$

where Pe represents the grid Peclet number [see eq.(8)]. If the Peclet number is less than 2, then the stability condition is always satisfied irrespective of the time step size. On the other hand, if the Peclet number is greater than 2, stability requires a stringent constraint on the time step size. To summarize, the stability condition is:

$$\text{either:} \quad Pe = |\alpha g\Delta x| \leq 2,$$
$$\text{else:} \quad Pe = |\alpha g\Delta x| \geq 2 \quad \text{and} \quad \frac{K_i}{C_i}\,\frac{\Delta t}{\Delta z^2} \leq \frac{2}{(Pe-2)(Pe+2)} \qquad (17)$$

Recall that the Peclet number was defined as a convection to diffusion ratio [see discussion above eq.(8)]. When the grid Peclet number of eqs.(16-17) is much smaller than unity, pressure disturbances appearing at any node are smeared out by diffusion before reaching the next node (stable case).

Finally, the effects of heterogeneity can be analyzed in a qualitative manner as follows. Assume for instance that the $\alpha$-parameter of the exponential conductivity curve is spatially variable. The local Peclet number is therefore also spatially variable. Assuming (roughly) that the previous stability analysis still holds locally, we see from equation (17) that instabilities must be triggered in zones of coarse porosity where $\alpha$ takes large values. Equation (15) shows that the most unstable Fourier modes are those with largest wavenumbers, having fluctuation scales comparable to mesh size. And, equation (16) indicates that such instabilities will grow faster where moisture diffusivity is high, e.g. in wet zones.

In order to minimize the chances of explosive error amplification, it seems reasonable to require that the vertical mesh size be a fraction of the average length scale $1/\alpha$, which typically lies in the range 10-100 cm for sandy to clayey soils. This guideline was used to design large scale numerical experiments of unsaturated flow in [1] and [2].

## CONVERGENCE ANALYSIS OF NONLINEAR PICARD ITERATIONS:

In practice, an iterative scheme such as Picard or Newton must be used to iteratively linearize and solve the nonlinear algebraic system at each time step. For instance, a modified Picard scheme that preserves the symmetry of the system was used in [1,2]. In this section, we show how the convergence of the Picard scheme can be investigated by applying functional analysis methods [5,6,7] to the nonlinear mapping associated with the iteration scheme. The proposed approach is to apply the Picard method directly to the partial differential equation of unsaturated flow, and to examine the convergence properties of the resulting iteration scheme, a priori independent of discretization.

For illustration here, we will restrict our analysis to the special case of steady unsaturated flow in a spatially homogeneous 1-dimensional medium, for which an exact solution can be derived by direct integration. Assume that K(h) is exponential with exponent $\alpha=\partial \ln K/\partial h$ and that the x-axis is vertical downwards. Define the dimensionless variables:

$$\xi = x/L,\ \Psi = -h/L,\ a = \alpha/L,\ q = Q/Ks,\ \text{and}\ k = K/Ks,$$

where $\Psi$ is the dimensionless suction head, always positive in unsaturated media. Our model problem is steady infiltration or evaporation in a vertical column extending, say, from soil surface at z=0 ($\xi$=0) to a water table or other boundary at z=L ($\xi$=1). This can be formulated as the boundary value problem:

$$\mathcal{L}(\Psi) = ( k(\Psi) (\Psi_\xi + 1) )_\xi = 0$$
$$\Psi(0)=\Psi_0 \; ; \quad \Psi(1)=\Psi_1 \tag{18}$$

A straightforward integration of (18) yields the conductivity profile $k(\xi)$, which itself can be used to obtain the suction profile $\Psi(\xi)$. The complete solution is given by:

$$k(\Psi(\xi)) = q + (k_1-q) \exp\{a(\xi-1)\} \tag{19.a}$$

$$q = \{k_1-k_0\exp(a)\}/\{1-\exp(a)\} \tag{19.b}$$

$$k(\Psi)=\exp(-a\Psi), \; k_0=k(\Psi_0), \; k_1=k(\Psi_1), \tag{19.c}$$

$$q \leq \{1-k_1\exp(-a)\}/\{1-\exp(-a)\} \tag{19.d}$$

where the constant dimensionless flux q is either positive (downwards) or negative (upwards). Note that the solution is only valid for boundary conditions such that $\Psi(\xi) \geq 0$ on the [0,1] interval. This requires satisfying $\Psi_0 \geq 0$, $\Psi_1 \geq 0$, and the inequality (19.d) which boils down to $q \leq 1$ if $a \gg 1$.

Let us now apply a Picard scheme with relaxation parameter $\omega$ to iteratively solve (18). The solution is constructed by way of an iterated mapping:

$$( k(\Psi^n) (\Psi^{n+1}-\Psi^n)_\xi )_\xi = - \omega \mathcal{L}(\Psi^n)$$
$$\Psi^{n+1}(\xi) - \Psi^n(\xi) = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad \xi = 1 \tag{20}$$

The residual operator $\mathcal{L}(\Psi)$ is the same as the one defined in equation (18), and n is the iteration counter. The Dirichlet conditions are implemented exactly at each iteration, since they are linear.

At each iteration, the iterated mapping of equation (20) is a boundary value problem that is directly integrable in terms of the incremental suction $\Psi[n+1]-\Psi[n]$. One obtains after some manipulations:

$$\Psi^{n+1} = \{ (1-\omega) \mathcal{I}(.) + \omega \mathcal{P}(.) \} \Psi^n \tag{21.a}$$

where $\mathcal{I}$ is the identity operator satisfying $\mathcal{I}(\Psi)=\Psi$, and $\mathcal{P}$ is the Picard iteration operator defined by:

$$\mathcal{P}(\Psi^n) = \{ (\Psi_1-\Psi_0+1) \mathcal{F}(\Psi^n) + (\Psi_0- \xi) \} \tag{21.b}$$

where $\mathcal{F}$ is the ratio of two integral operators:

$$F(\Psi^n) = \left[\int_0^{\xi} k(\Psi^n(s))^{-1} ds\right] / \left[\int_0^1 k(\Psi^n(s))^{-1} ds\right] \qquad (21.c)$$

Each operator $\mathcal{U}(.)$, $\mathcal{P}(.)$, and $\mathcal{F}(.)$ maps onto itself the space of continuous functions $\Psi(\xi)$ defined on the interval [0,1], and (21.a) yields $\Psi[n+1](\xi) = \Psi[n](\xi)$ at $\xi=0$ and $\xi=1$.

The convergence properties of the Picard scheme are most directly related to the properties of the iteration operator (21.a) which maps the old solution $\Psi[n]$ into the new solution $\Psi[n+1]$. Without going into details, let us point out that the contraction mapping theorem and Ostrowski's local convergence theorem [7] can be used to test the conditions under which convergence occurs, and to estimate convergence rate in some function space norm. Work along these lines is ongoing. The case of transient flow will be developed by applying the same principles to the semi-discretized differential flow equation.

REFERENCES:


1. Ababou R. (1988): Three-Dimensional Flow in Random Porous Media, Ph.D. thesis, M.I.T., Cambridge, MA 02139, U.S.A., pp. 833, [Chapters 5 & 7].

2. Ababou R., and Gelhar L. W. (1988): A High-Resolution Finite Difference Simulator for 3D Unsaturated Flow in Heterogeneous Media, in: Computational Methods in Water Resources (VIIth Internat. Conf.), M. Celia et. al. ed., Elsevier & Comput. Mech. Publi., Vol. 1, 173-178.

3. Vauclin M., Haverkamp R., and Vachaud G. (1979): Résolution Numérique d'une Equation de Diffusion Nonlinéaire, P.U.G., Grenoble, France, pp. 183.

4. Ames W. F. (1977): Numerical Methods for Partial Differential Equations,. Academic Press, New York, pp. 365, [Chapter 2].

5. Curtain R. F. and Pritchard A. J. (1977): Functional Analysis in Modern Applied Mathematics, Academic Press, New York, pp. 339.

6. Milne R. D. (1980): Applied Functional Analysis: An Introductory Treatment, Pitman, Boston.

7. Ortega J. M. and Rheinboldt W. C. (1971): Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, New York, pp. 572, [5.1,7.1,10.1,12.1].

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