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COMBINING MODAL RESPONSES AND SPATIAL COMPONENTS IN SEISMIC RESPONSE ANALYSIS

A. INTRODUCTION

Criterion 2, "Design Bases for Protection Against Natural Phenomena," of Appendix A, "General Design Criteria for Nuclear Power Plants," to 10 CFR Part 50, "Licensing of Production and Utilization Facilities," requires, in part, that nuclear power plant structures, systems, and components important to safety be designed to withstand the effects of earthquakes without loss of capability to perform their safety functions. Paragraph (a)(1) of Section VI, "Application to Engineering Design," of Appendix A, "Seismic and Geologic Siting Criteria for Nuclear Power Plants," to 10 CFR Part 100, "Reactor Site Criteria," requires, in part, that structures, systems, and components important to safety remain functional in the event of a Safe Shutdown Earthquake (SSE). It specifies the use of a suitable dynamic analysis as one method of ensuring that the structures, systems, and components can withstand the seismic loads. Similarly, paragraph (a)(2) of Section VI of the same appendix requires, in part, that the structures, systems, and components necessary for continued operation without undue risk to the health and safety of the public remain functional in an Operating Basis Earthquake (OBE). Again, the use of a suitable dynamic analysis is specified as one method of ensuring that the structures, systems, and components can withstand the seismic loads.

This guide describes methods acceptable to the NRC staff for:

1. Combining the values of the response of individual modes in a response spectrum modal dynamic analysis to find the representative maximum value of a particular response of interest for the design of a given element of a nuclear power plant structure, system, or component.

2. Combining the maximum values (in the case of time-history dynamic analysis) or the representative maximum values (in the case of spectrum dynamic analysis) of the response of a given element of a structure, system, or component, when such values are calculated independently for each of the three orthogonal spatial components (two horizontal and one vertical) of an earthquake. The combined value will be the representative maximum value of the combined response of that element of the structure, system, or component to simultaneous action of the three spatial components.

The Advisory Committee on Reactor Safeguards has been consulted concerning this guide and has concurred in the regulatory position. *

B. DISCUSSION

1. Combining Modal Responses

To find the values of the response of different elements of a nuclear power plant structure, system, or component to a prescribed response spectrum, it is first necessary to calculate the mode shapes and frequencies of the structure, system, or component. This is done by solving the following equation for the eigenvectors and eigenvalues:

$$\left[[K] - \omega_n^2\ [M]\right] \left\{\phi_n\right\} = 0 \qquad (1)$$

where [K] is the stiffness matrix, ω_n is the natural frequency for the nth mode, [M] is the mass matrix, and $\{\phi_n\}$ is the eigenvector for the nth mode.

*Lines indicate substantive changes from previous issue.

Note that it may not be necessary to solve Equation 1 for all modes. In many cases, determination of only those modes that are significant should be sufficient.

The next step is to determine the maximum modal displacement relative to the supports. This is done as follows:

$$\left\{q_n\right\}_{max} = \Gamma n \left\{\phi_n\right\} \frac{S_{an}}{\omega_n^2} \qquad (2)$$

where $\left\{q_n\right\}_{max}$ is the maximum displacement vector for the nth mode, Γn is the modal participation factor for the nth mode and is expressed by

$$\left\{\phi_n\right\}^T [M] \left\{1\right\} / \left\{\phi_n\right\}^T [M] \left\{\phi_n\right\},$$

S_{an} is the value of acceleration in the specified response spectrum corresponding to ω_n and design damping, and superscript T designates the transpose. Other maximum values of the responses per mode such as stress, strain, moment, or shear can be computed from the appropriate $\left\{q\right\}_{max}$ by using the stiffness properties of the elements of the structure, system, or component. Newmark (Ref. 1) has shown that the representative maximum value of a particular response of interest for design (such as components in given directions of stress, strain, moment, shear, or displacement) of a given element can be obtained from the corresponding maximum values of the response of individual modes as computed above by taking the square root of the sum of the squares (SRSS) of the maximum values of the response of these individual modes of the structure, system, or component. The Newmark study, however, does not address the problem of closely spaced modes. Other studies (see References 2 and 3) have shown that SRSS procedure can significantly underestimate the true response in certain cases in which some of the modal frequencies of a structural system are closely spaced (see regulatory position 1.1 for definition of closely spaced modes). The nuclear industry has used many different methods to combine the response when closely spaced modes exist. Some of these methods can be found in References 2, 4, and 5. A recent unpublished study has shown that the resulting combined response of nuclear plant facilities using any of the methods delineated in regulatory position 1.2, which covers a broad range of methods currently being used by the industry, is in good agreement with time-history response. Therefore, any of the methods given in regulatory position 1.2 is acceptable for combining the modal responses when closely spaced modes exist.

It should be noted that, if the frequencies of a system are all widely separated, all the terms in the second summation sign in Equations 4 and 5 of regulatory position 1.2 will vanish, and these equations will degenerate to the SRSS method (Equation 3).

2. Combining Spatial Components

2.1 Response to Three Spatial Components Calculated Separately

Regulatory Guide 1.60, "Design Response Spectra for Seismic Design of Nuclear Power Plants," indicates that design of all Seismic Category I structures, systems, or components should be based on three orthogonal component motions (two horizontal and one vertical) of a prescribed design earthquake. Chu, Amin, and Singh (Ref. 3) have concluded that the representative maximum value of a particular response of interest for design (e.g., stress, strain, moment, shear, or displacement) of a given element of a structure, system, or component subjected to the simultaneous action of the three components of the earthquake can be satisfactorily obtained by taking the square root of the sum of the squares of corresponding representative maximum values of the spectrum response, or the maximum response values from time-history dynamic analysis, to each of the three components calculated independently.

The SRSS procedure used by Newmark (Ref. 1) and Chu, Amin, and Singh (Ref. 3) for combining the values of the response to three components of an earthquake is based on the consideration that it is very unlikely that peak values of a response of a given element would occur at the same time during an earthquake.

2.2 Response to Three Spatial Components Calculated Simultaneously

The maximum value of a particular response of interest for design of a given element can be obtained through a step-by-step method. The time-history responses from each of the three components of the earthquake motions can be obtained and then combined algebraically at each time step or the response at each time step can be calculated directly owing to the simultaneous action of three components. The maximum response is determined by scanning the combined time-history solution. When this method is used, the earthquake motions specified in the three different directions should be statistically independent. For a discussion of statistical independence, see Reference 6.

C. REGULATORY POSITION

The following procedures for combining the values of the response of individual modes and the response to the three independent spatial components of an earthquake in a seismic dynamic analysis of a nuclear power plant structure, system, or component are acceptable to the NRC staff:

1. Combination of Modal Responses

1.1 With No Closely Spaced Modes

In a response spectrum modal dynamic analysis, if the modes are not closely spaced (two consecutive modes are defined as closely spaced if their frequencies differ from each other by 10 percent or less of the lower frequency), the representative maximum value of a particular response of interest for design (e.g., components of stress, strain, moment, shear, or displacement) of a given element of a nuclear power plant structure, system, or component subjected to a single independent spatial component (response spectrum) of a three-component earthquake should be obtained by taking the square root of the sum of the squares (SRSS) of corresponding maximum values of the response of the element attributed to individual significant modes of the structure, system, or component. Mathematically, this can be expressed as follows:

$$R = \left[\sum_{k=1}^{N} R_k^2 \right]^{1/2} \qquad (3)$$

where R is the representative maximum value of a particular response of a given element to a given component of an earthquake, R_k is the peak value of the response of the element due to the kth mode, and N is the number of significant modes considered in the modal response combination.

1.2 With Closely Spaced Modes

In a response spectrum modal dynamic analysis, if some or all of the modes are closely spaced, any of the following regulatory positions (i.e., 1.2.1, 1.2.2, or 1.2.3) may be used as a method acceptable to the NRC staff to combine the modal responses.

1.2.1 Grouping Method

Closely spaced modes should be divided into groups that include all modes having frequencies lying between the lowest frequency in the group and a frequency 10 percent higher.[1] The representative maximum value of a particular response of interest for the design of a given element of a nuclear power plant structure, system, or component attributed to each such group of modes should first be obtained by taking the sum of the absolute values of the corresponding peak values of the response of the element attributed to individual modes in that group. The representative maximum value of this particular response attributed to all the significant modes of the structure, system, or component should then be obtained by taking the square root of the sum of the squares of corresponding representative maximum values of the response of the element attributed to each closely spaced group of modes and the remaining modal responses for the modes that are not closely spaced.

Mathematically, this can be expressed as follows:

$$R = \left[\sum_{k=1}^{N} R_k^2 + \sum_{q=1}^{P} \sum_{\ell=i}^{j} \sum_{m=i}^{j} \left| R_{\ell q} R_{mq} \right| \right]^{1/2} \quad \ell \neq m \qquad (4)$$

where $R_{\ell q}$ and R_{mq} are modal responses, R_ℓ and R_m within the qth group, respectively; i is the number of the mode where a group starts; j is the number of the mode where a group ends; R, R_k, and N are as defined previously in regulatory position 1.1 of this guide; and P is the number of groups of closely spaced modes, excluding individual separated modes.

1.2.2 Ten Percent Method

$$R = \left[\sum_{k=1}^{N} R_k^2 + 2 \sum \left| R_i R_j \right| \right]^{1/2} \quad i \neq j \qquad (5)$$

where R, R_k, and N are as defined previously in regulatory position 1.1 of this guide. The second summation is to be done on all i and j modes whose frequencies are closely spaced to each other. Let ω_i and ω_j be the frequencies of the ith and jth mode. In order to verify which of the modes are closely spaced, the following equation will apply:

$$\frac{\omega_j - \omega_i}{\omega_i} \leq 0.1 \qquad (6)$$

$$\text{also } 1 \leq i < j \leq N \qquad (7)$$

1.2.3 Double Sum Method

$$R = \left[\sum_{k=1}^{N} \sum_{s=1}^{N} \left| R_k R_s \right| \epsilon_{ks} \right]^{1/2} \qquad (8)$$

where R, R_k, and N are as defined previously in regulatory position 1.1 of this guide. R_s is the peak value of the response of the element attributed to sth mode.

$$\epsilon_{ks} = \left[1 + \left\{ \frac{(\omega_k' - \omega_s')}{(\beta_k' \omega_k + \beta_s' \omega_s)} \right\}^2 \right]^{-1} \qquad (9)$$

in which

$$\omega_k' = \omega_k \left[1 - \beta_k^2 \right]^{1/2} \qquad (10)$$

[1] Groups should be formed starting from the lowest frequency and working towards successively higher frequencies. No one frequency is to be in more than one group.

$$\beta_k' = \beta_k + \frac{2}{t_d \omega_k} \quad (11)$$

where ω_k and β_k are the modal frequency and the damping ratio in the kth mode, respectively, and t_d is the duration of the earthquake.

2. Combination of Effects Due to Three Spatial Components of an Earthquake

Depending on which basic method is used in the seismic analysis, i.e., response spectra or time-history method, the following two approaches are considered acceptable for the combination of three-dimensional earthquake effects.

2.1 Response Spectra Method

When the response spectra method is adopted for seismic analysis, the representative maximum values of the structural responses to each of the three components of earthquake motion should be combined by taking the square root of the sum of the squares of the maximum representative values of the codirectional responses caused by each of the three components of earthquake motion at a particular point of the structure or of the mathematical model.

2.2 Time-History Analysis Method

When the time-history analysis method is employed for seismic analysis, two types of analysis are generally performed depending on the complexity of the problem:

a. When the maximum responses due to each of the three components of the earthquake motion are calculated separately, the method for combining the three-dimensional effects is identical to that described in regulatory position 2.1 except that the maximum responses are calculated using the time-history method instead of the spectrum method.

b. When the time-history responses from each of the three components of the earthquake motion are calculated by the step-by-step method and combined algebraically at each time step, the maximum response can be obtained from the combined time solution.[2]

3. If the applicant has used the methods described in this guide, the Preliminary Safety Analysis Report (PSAR) should indicate in each applicable section which of the alternative acceptable methods were used for the structures, systems, or components covered by that section.

D. IMPLEMENTATION

The purpose of this section is to provide information to applicants and licensees regarding the NRC staff's plans for utilizing this regulatory guide.

Except in those cases in which the applicant proposes an alternative method for complying with specified portions of the Commission's regulations, the methods described herein will be used by the staff in the evaluation of submittals for construction permit applications docketed after the date of issue of this guide.

If an applicant wishes to use this regulatory guide in developing submittals for applications docketed on or before the date of issue of this guide, the pertinent portions of the application will be evaluated on the basis of this guide.

[2]When this method is used, the earthquake motions specified in the three different directions should be statistically independent. For a discussion of statistical independence, see Reference 6.

REFERENCES

1. R. L. Wiegel, editor, *Earthquake Engineering*, Englewood Cliffs, N.J., Prentice-Hall, Inc., 1970, chapter by N. M. Newmark, p. 403.

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3. S. L. Chu, M. Amin, and S. Singh, "Spectral Treatment of Actions of Three Earthquake Components on Structures," *Nuclear Engineering and Design*, 1972, Vol. 21, No. 1, pp. 126-136.

4. E. Rosenblueth and J. Elorduy, "Response of Linear Systems to Certain Transient Disturbances," *Proceedings, Fourth World Conference on Earthquake Engineering*, Vol. 1, Santiago, Chile, 1969.

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6. C. Chen, "Definition of Statistically Independent Time Histories," *Journal of the Structural Division, ASCE*, February 1975.