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U.S. ATOMIC ENERGY COMMISSION

REGULATORY GUIDE

DIRECTORATE OF REGULATORY STANDARDS

REGULATORY GUIDE 5.33

STATISTICAL EVALUATION OF MATERIAL UNACCOUNTED FOR

A. INTRODUCTION

Paragraph 70.51(e) of 10 CFR Part 70 requires certain AEC licensees authorized to possess special nuclear material to calculate material unaccounted for (MUF) quantities and their associated statistical limits of error (LEMUF) as part of their material control and accounting procedures for use in assuring that special nuclear material in their possession is accounted for. Paragraph 70.53(b)(1) requires that a report be made to the Commission if any single MUF exceeds its associated LEMUF and certain specified quantities and that the report include a statement of the probable reasons for the MUF and actions taken or planned. This guide identifies methods acceptable to the Regulatory staff for evaluating the statistical significance[1] of observed MUF values.

B. DISCUSSION

While there may be mechanisms of process control involving identification of process anomalies that can provide an indication of possible missing material, the only positive means for assuring that the material is not missing is to measure all of the material and establish a measured material balance. Records are maintained of the measured quantities received into a plant and the measured quantities removed from a plant. The difference between these quantities should be on inventory in the plant. A measured physical inventory will either confirm that this quantity is present or indicate that some material is missing. Assuming that there is no inventory at the beginning of the time interval, the balance can be expressed by the equation:

Receipts – Removals = Inventory

If there was material on hand at the beginning of the time interval for which a balance is being taken, a beginning inventory component would have to be added to the equation to give:

$$\text{Beginning Inventory} + \text{Receipts} - \text{Removals} = \text{Ending Inventory} \quad (1)$$

Because of uncertainties[1] in measurements or unknown removals such as losses or thefts, this equation seldom balances in practice, indicating some observed material unaccounted for (MUF). Equation (1) can be recast as:

$$(\text{Beginning Inventory} + \text{Receipts}) - (\text{Removals} + \text{Ending Inventory}) = \text{MUF} \quad (2)$$

If equation (1) balances, the MUF of equation (2) is zero. When equation (1) does not balance, the MUF of equation (2) represents some finite quantity of SNM. The significance of this quantity could represent only the uncertainties of the measurements, or it could include an unknown loss or theft. The first step in determining the significance of the MUF is to determine what value might be attributable to uncertainties of the measurement system.

Each of the measured quantities in the material balance will have some uncertainty associated with it. Combination of these individual uncertainties by appropriate statistical methodology will result in limits in terms of SNM quantities by which equation (1) could be expected to be out of balance due only to the measurement system uncertainties, i.e., the MUF in equation (2) that could be expected. Since measurement uncertainties may be either positive or negative, a

[1] As defined in Regulatory Guide 5.3, "Statistical Terminology and Notation for Special Nuclear Materials Control and Accountability," February 2, 1973.

confidence interval[1] is established within which the MUF could be expected due only to measurement uncertainty. This confidence interval is defined by the "limits of error of the material unaccounted for" (LEMUF). It is the magnitude of this LEMUF value that determines, at a 95% confidence level, how well the total material balance can confirm that all SNM is present or detect whether some is missing. When the MUF quantity is equal to or smaller than the LEMUF, the indication is that MUF could have occurred by chance due to measurement variation. If the MUF is greater than the LEMUF in either a positive or negative direction, the indication is that the MUF could not have occurred by chance due to measurement variation but that some other mechanism has had an effect. This could be a loss or theft of material (positive MUF) or an error in the system causing an unaccounted for gain (negative MUF).

If MUF is due solely to random variations in the measurement system, MUF values taken over time from a series of balances around the same process or plant should tend toward zero. If this is not the case, and MUF values tend to show a consistent difference from zero, factors other than random measurement variations are indicated. Consistently positive MUF values would indicate that there may be some biased measurements, consistent measurement or recording mistakes, unknown or unrecorded inventory, or some continual small losses or thefts of material. Consistently negative MUF values would indicate that there may be some biased measurements or consistent measurement or recording mistakes.

MUF values that are not consistent with measurement system variations, i.e., are in excess of such variations within stated statistical probabilities, are considered statistically significant. The purpose of this guide is to provide guidance in assessing the significance of MUF values but not in the investigation of MUF values that are found to be statistically significant.

C. REGULATORY POSITION

The concepts, principles, and methods discussed and referenced below are generally acceptable to the Regulatory staff for evaluating the significance of MUF values resulting from measured material balances as specified in paragraph 70.51(e) of 10 CFR Part 70. Individual MUF values (short-term MUF) that are statistically significant (i.e., those that exceed the LEMUF values specified in paragraph 70.51(e)(5) and that exceed the minimum quantities specified in paragraph 70.51(e)(5)) are required to be investigated and reported to the AEC. In addition, combinations or sequences of MUF values (long-term MUF) which show trends significantly different from zero should be investigated to determine and correct the causes.

[1]As defined in Regulatory Guide 5.3, "Statistical Terminology and Notation for Special Nuclear Materials Control and Accountability," February 2, 1973.

The evaluation of either short-term or long-term MUF should consist of testing whether the MUF quantity observed over a single material balance period or a cumulative period exceeds the LEMUF for that period. Specifically, the test criterion should be whether MUF ± LEMUF contains zero, i.e.,

$$0 \in (\text{MUF} \pm \text{LEMUF}) \tag{3}$$

Thus, an appropriate decision rule is that if observed MUF is greater than LEMUF, i.e.:

$$\text{MUF} > \text{LEMUF} \tag{4}$$

then MUF should be declared significant and investigated. This 95% confidence interval test is equivalent to a two-sided hypothesis[1] test with a null hypothesis that MUF is zero at a 5% level of significance.

Regulatory Guide 5.18, "Limit of Error Concepts and Principles of Calculation in Nuclear Materials Control," describes acceptable methods for calculation of limits of error.

An example of how single, isolated observed MUF values should be evaluated is presented by John L. Jaech, *Statistical Methods in Nuclear Material Control*, TID-26298, December 1973, Section 7.1, specifically answers 7.A and 7.B of Section 7.1.3. However, the following specifications should be applied:

1. The value of M_0 should be set to zero.
2. The limit of error is $c_\alpha \sigma$.
3. Absolute signs should be placed around x, the observed MUF, in order to indicate a two-sided test of significance.
4. A significance level of 5% should be assigned.
5. If the method of Section 7.1.3, question 7.B, is used, a confidence coefficient of 95% should be chosen.

An example of how combinations and sequences of MUF values should be evaluated is given in Section 7.2 of the above reference, specifically answer 7.E of Section 7.2.2 with the following conditions specified:

1. Any related M_0 should be zero.
2. Any test of significance should be two-sided.
3. Control charts in answer 7.F would be more appropriate as the correlation between successive MUF values increases.
4. A 5% level of significance should be chosen.
5. The term denoted as a systematic-error variance in equation (7.9) and throughout TID-26298 is clearly defined on an applications basis in that reference. It should be noted in current practice that it may be determined by a mean square error or the variance of the estimated bias. Caution must be used in selecting which is appropriate for the process in question.