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UNITED STATES OF AMERICA
NUCLEAR REGULATORY COMMISSION

BEFORE THE ATOMIC SAFETY AND LICENSING BOARD

In the Matter of:	)	Docket No. 72-22-ISFSI
PRIVATE FUEL STORAGE, LLC	)	ASLBP No. 97-732-02-ISFSI
(Independent Spent Fuel Storage Installation)	)	January 21, 2000

STATE OF UTAH'S RESPONSE TO THE APPLICANT'S MOTION FOR SUMMARY DISPOSITION OF UTAH CONTENTION GG – FAILURE TO DEMONSTRATE CASK-PAD STABILITY DURING SEISMIC EVENT FOR TRANSTOR CASKS

On December 30, 1999 the Applicant filed a Motion for Summary Disposition of Utah Contention GG ("PFS Motion"), supported by the Declaration of Dr. Alan Soler. The State now files this Response to the Applicant's Motion, supported by a Statement of Material and Disputed Facts and the Declaration of Dr. Farhang Ostadan ("Ostadan Dec.").

BACKGROUND

As admitted, Utah Contention GG states:

> The Applicant has failed to demonstrate that the TranStor storage casks and the pads will remain stable during a seismic event, and thus, the application does not satisfy 10 C.F.R. §§ 72.122(b)(2) and 72.128(a), in that Sierra Nuclear's consultant, Advent Engineering Services, Inc., used a nonconservative "nonsliding cask" tipover analysis that did not consider that the coefficient of friction may vary over the surface of the pad and did not consider the shift from the static case to the kinetic case when considering momentum of the moving casks.

Private Fuel Storage, L.L.C. (Independent Spent Fuel Storage Installation), LBP-98-7, 47 NRC 142, 257 (1998). This contention was filed in response to the "TranStor Storage

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Cask Seismic Stability Analysis for PFS Site," July 24, 1997 ("Private Utility Fuel Storage Project Cask Seismic Tipover Analysis," prepared for Sierra Nuclear Corporation by Advent Engineering Services, Inc. (hereinafter "Advent Report")). PFS admits that the Advent Report "analytically 'pinned' the cask to the pad, thereby failing to consider potential cask sliding during seismic activity." PFS Motion at 2. In other words, the Advent Report simply analyzed whether the TranStor cask would tip over during seismic activity. *See* Advent Report.

PFS has now submitted a revised analysis of the stability of the TranStor cask which considers potential sliding at different coefficients of friction. *See* PFS Motion at 3, and "PFSF Site-Specific Cask Stability Analysis for the TranStor Storage Casks," (September 23, 1999), attached as Exhibit 2 to Declaration of Dr. Alan Soler, (hereinafter "Revised Analysis"). However, the Revised Analysis still fails to consider coefficients of friction reasonably expected to occur during seismic activity and does not demonstrate that the casks will remain stable under those conditions.

STANDARD OF REVIEW

Pursuant to 10 CFR § 2.740, a party is entitled to summary disposition if "there is no genuine issue as to any material fact" and the party "is entitled to a decision as a matter of law." The burden of proving entitlement to summary disposition is on the movant. Advanced Medical Systems, Inc. (One Factory Row, Geneva, Ohio 44041), CLI-93-22, 38 NRC 98, 102 (1993). Moreover, "the evidence submitted must be construed in favor of the party in opposition thereto, who receives the benefit of any favorable inferences that can be drawn." Sequoyah Fuels Corp. and General Atomics

Corp. (Gore, Oklahoma Site Decontamination and Decommissioning Funding), LBP-94-17, 39 NRC 359, 361, *aff'd* CLI-94-11, 40 NRC 55 (1994). If there is any possibility that a litigable issue of fact exists or any doubt as to whether the parties should be permitted or required to proceed further, the motion must be denied. *See* General Electric Co. (GE Morris Operation Spent Fuel Storage Facility), LBP-82-14, 15 NRC 530, 532 (1982).

ARGUMENT

PFS argues that Utah Contention GG is now moot because the Revised Analysis considers a range of coefficients of friction. *See* PFS Motion at 3. First, the State disputes that the Revised Analysis considers a relevant range of coefficients of friction. Moreover, the Revised Analysis still fails to demonstrate the stability of the TranStor casks during seismic activity because the Analysis erroneously assumes that the pads are rigid under dynamic loading conditions and that dynamic forces will not affect the coefficient of friction. *See* Ostadan Dec. ¶¶ 7-8. Furthermore, the Revised Analysis does not consider the effects of cold bonding and its effect on the coefficient of friction. Thus, the Revised Analysis fails to satisfy the concerns raised by Utah Contention GG. Second, Dr. Ostadan's Declaration raises several issues of material fact, which renders Summary Disposition procedures inapplicable. PFS's Motion, therefore, should be denied, and Utah Contention GG should proceed to hearing.

I. Utah Contention GG Is Not Moot Because the Applicant's Revised Analysis Is Based on Erroneous Assumptions and It Fails to Consider a Relevant Range of Coefficients of Friction

An applicant for an ISFSI license must demonstrate that structures, systems, and components important to safety are designed to withstand natural phenomena, including earthquakes, as well as accident conditions. *See* 10 C.F.R. §§ 72.122(b)(2) and 72.128(a). In Utah Contention GG, the State asserted that the analysis performed by Advent Engineering Services, Inc. inadequately analyzed the stability of the TranStor storage cask system during seismic activity. *See* 47 NRC 142, 257 (Utah Contention GG as admitted). PFS claims that Utah Contention GG only "concerns the coefficient of friction." *See* PFS Motion at 2 n.1. The State disputes this point. However, even under the assumption that Utah Contention GG only concerns the coefficient of friction and its effects on the stability of TranStor storage casks, the Revised Analysis fails to correctly consider variable coefficients of friction. *See* Ostadan Dec. ¶¶ 9, 10.

The Revised Analysis assumes that the contact surface between the bottom of the cask and top of the foundation will remain intact after loading the casks on the pad and during seismic excitation which effectively implies that the concrete pad is rigid under both static and dynamic loading. *See* id. ¶ 7. Therefore, the Revised Analysis used a uniform coefficient of friction at 0.2 and then again at 0.8 to analyze the stability of the TranStor cask. *See* PFS Motion at 6-7. The Revised Analysis is inaccurate in several aspects. First, the Revised Analysis assumes that the pad will remain rigid under cask loading. *See* id. ¶ 7. Because the Revised Analysis assumed the pad was rigid, the analysis failed to consider the effects of frictional forces which cause the coefficient of friction to vary across the surface of the pad. Under this assumption, the Revised Analysis employs simple frictional elements at the contact points between the casks and the pad. *See* id. However, the pad will deform when subjected to cask loading. *See* id. ¶ 8. Consequently, the use of simple frictional elements at the contact points in the Revised

Analysis will not accurately predict the stability of the casks under dynamic conditions because the coefficient of friction will depend on frictional forces. *See* id. The high and low coefficients of friction employed in the Revised Analysis will not bound the actual frictional behavior because the Revised Analysis is based on the assumption that frictional forces will not affect the coefficient of friction. *See* id. ¶ 9.

Second, the Revised Analysis did not take into account the effects of cold bonding. *See* id. ¶ 10. This condition between the cask and the pad is not covered by the highest coefficient of friction (0.8). *See* id.; *see also* Revised Analysis. If a cold bond were broken during seismic activity, it could cause the contact points between the casks and pad to be nonuniform. *See* Ostadan Dec. ¶ 10. Since the Revised Analysis only considered simple friction elements, *see* id. ¶ 7, the actual frictional forces may fall outside the high bound of the coefficients of friction used in the Revised Analysis. In other words, if cold bonding were to occur, the coefficient of friction would be higher than 0.8 (the highest value analyzed in the Revised Analysis). *See* Ostadan Dec. ¶ 10.

While the Revised Analysis attempts to account for coefficients of friction over the surface of the pad, the Revised Analysis fails to account for the actual variations in coefficients of friction. Moreover, the use of 0.8 does not bound the highest coefficient of friction, such that may occur during cold bonding. Therefore, the Revised Analysis does not render Utah Contention GG moot.

II. PFS Is Not Entitled to Summary Disposition Because There Are Genuine Issues of Material Facts

A movant is only entitled to summary disposition when there are no genuine issues of material facts. *See* Advanced Medical Systems, Inc. (One Factory Row, Geneva, Ohio 44041), CLI-93-22, 38 NRC 98, 102 (1993). PFS claims that there are no genuine issues of material fact. *See* PFS Motion at 5. However, Dr. Ostadan's Declaration raises several issues regarding the accuracy of the Revised Analysis, thereby rendering summary disposition inapplicable.

Dr. Ostadan, after reviewing the Revised Analysis, has concluded that the Analysis is incorrect and fails to consider the actual coefficients of friction that would occur during seismic activity. *See* Ostadan Dec. ¶¶ 7-10. Specifically, there is a genuine issue as to whether the pad will remain rigid under cask loading. *See* *id.* ¶¶ 7-8 . Moreover, there is a dispute as to whether simple frictional elements applied at coefficients of friction of 0.2 and 0.8 bound the actual behavior of the casks under dynamic loading. *Compare* PFS Motion at 6-7 *with* Ostadan's Dec. ¶¶ 9-10. Apparently, Dr. Soler relies on the assumption that the coefficient of friction is independent of frictional forces. *See* Soler Declaration in Support of Applicant's December 27, 1999 Response to Utah's Motion to Compel Applicant to Respond to State's Fifth Set of Discovery Requests, ¶ 10. The State disputes the fact that the coefficient of friction is independent of frictional force and dynamic loading. *See* Ostadan's Dec. ¶ 8. Since the coefficient of friction is dependent on dynamic forces, under the circumstances, it is material to the issue of whether the Revised Analysis considers the variability of coefficients of friction over the pad.

All of these disputes directly relate to the accuracy of the Revised Analysis and the variability of the coefficient of friction over the pad. These factual disputes, taken in the light most favorable to the State, entitle the State to go forward with Contention GG. Therefore, the PFS Motion must be denied.

DATED this 21st day of January, 2000.

Respectfully submitted,

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CERTIFICATE OF SERVICE

I hereby certify that a copy of STATE OF UTAH'S RESPONSE TO THE APPLICANT'S MOTION FOR SUMMARY DISPOSITION OF UTAH CONTENTION GG – FAILURE TO DEMONSTRATE CASK-PAD STABILITY DURING SEISMIC EVENT FOR TRANSTOR CASKS was served on the persons listed below by electronic mail (unless otherwise noted) with conforming copies by United States mail first class, this 21st day of January, 2000:

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UNITED STATES OF AMERICA
NUCLEAR REGULATORY COMMISSION

BEFORE THE ATOMIC SAFETY AND LICENSING BOARD

In the Matter of:	)	Docket No. 72-22-ISFSI
	)	
PRIVATE FUEL STORAGE, LLC	)	ASLBP No. 97-732-02-ISFSI
(Independent Spent Fuel	)	
Storage Installation)	)	January 21, 2000

STATE OF UTAH'S STATEMENT OF DISPUTED AND RELEVANT MATERIAL FACTS FOR UTAH CONTENTION GG

1. The State disputes PFS Material Fact No. 10 in that 0.8 does not bound the highest coefficient of friction expected to occur. Ostadan Dec. at ¶ 10.

2. The cask and the concrete pad may develop cold bonding. Ostadan Dec. ¶ 10. The cold bonding causes a contact condition between the cask and the pad that is not covered by the coefficient of friction 0.8. Ostadan Dec. at ¶ 10.

3. The cold bonding may break during seismic shaking in a nonuniform pattern causing a nonuniform contact condition between the cask and the pad. Ostadan Dec. at ¶ 10. The coefficient of friction would not remain constant.

4. The State disputes PFS Material Fact No. 11 in that the two coefficients of friction, 0.2 and 0.8, do not effectively bracket any variation in the coefficient of friction over the surface of the pad. Ostadan Dec. at ¶¶ 7, 8, 9, 11.

5. The coefficient of friction between the cask and the concrete pad is not uniform across the surface of the cask. Ostadan Dec. at ¶ 9.

6. The concrete pad will not behave as a rigid surface under either static or dynamic loading. Ostadan Dec. at ¶ 8.

7. Due to the flexible nature of the pad, the coefficient of friction will be dependent upon frictional forces. Ostadan Dec. at ¶ 8.

8. The contact surface between the bottom of the cask and the top of the foundation pad will not remain intact during cask loading or seismic movement. Ostadan Dec. at ¶ 8.

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In the Matter of:	)	Docket No. 72-22-ISFSI
PRIVATE FUEL STORAGE, LLC	)	ASLBP No. 97-732-02-ISFSI
(Independent Spent Fuel Storage Installation)	)	January 21, 2000

DECLARATION OF FARHANG OSTADAN, PH. D.

I, FARHANG OSTADAN, hereby declare under penalty of perjury and pursuant to 28 U.S.C. § 1746, that:

1. I hold a Ph.D. in civil engineering from the University of California at Berkeley. My curriculum vitae listing my qualifications, experience, training, and publications has already been filed in this proceeding. *See*, Exhibit No. 2 of the "State's Motion to Compel Applicant to Respond to State's Fifth Set of Discovery Requests" dated December 20, 1999.

2. I have fifteen years experience in dynamic analysis and seismic safety evaluation of above and underground structures and subsurface materials. I co-developed and implemented SASSI, a system for seismic soil-structure interaction analysis currently in use by the industry worldwide. I also developed a method for liquefaction hazard analysis currently in use for critical facilities in the United States.

3. I have participated in seismic studies and review of numerous

nuclear structures, including Diablo Canyon Nuclear Station and the NRC/EPRI large scale seismic experiment in Lotung, Taiwan. I have published numerous papers in the area of soil structure interaction and seismic design.

4. I have read the materials filed by PFS in support of its Motion for Summary Disposition of Contention GG, including the "Safety Analysis Report for the TranStor Storage Cask System," rev. B; the "TranStor Storage Cask Seismic Stability Analysis for PFS Site," July 24, 1997 ("Private Utility Fuel Storage Project Cask Seismic Tipover Analysis," prepared for Sierra Nuclear Corporation by Advent Engineering Services, Inc. (hereinafter "Advent Report")); the "PFSF Site Specific Cask Stability Analysis for the TranStor Storage Cask," September 23, 1999; and the "TranStor Dynamic Response to 2000 Year Return Seismic Event, Holtec Report No. HI-992295." I am familiar with the circumstances and materials in this case as they relate to Contention GG, including PFS's Safety Analysis Report. I am also familiar with and have reviewed the documents that PFS has provided to the State of Utah concerning Utah Contention GG, PFS's responses to Discovery Requests submitted by the State, and PFS's responses to the NRC Staff's Requests for Additional Information.

5. The Applicant has performed a series of simple nonlinear time history analyses in which the interaction between the cask and the foundation pad is modeled by frictional elements. The coefficient of friction was changed in successive analyses from 0.20 to 0.80. Soler Dec., Sum. Disp. at ¶ 9.

6. Dr. Alan Soler states that the "coefficient of friction" is a property

associated with a contact point between two surfaces and the value of the coefficient is dependent on the characteristics of the two materials at the interface contact point. Soler Dec., Sum. Disp. at ¶ 7. In a declaration supporting the Applicant's Response to State of Utah's Motion to Compel Applicant to Respond to State's Fifth Set of Discovery Requests, Dr. Solar claims that the coefficient of friction is independent of the friction forces. Solar Dec., Resp. Mo. Compel at ¶ 10. However, the coefficient of friction is only independent of friction forces under certain circumstances.

7. In justifying that the coefficient of friction is independent of friction forces, Dr. Soler must assume that the contact surface between the bottom of the cask and top of the foundation will remain intact after loading the casks on the pad and during the seismic excitation which effectively implies that the concrete pad is rigid under both static and dynamic loading. This assumption led the Applicant to the simplifying assumption for the dynamic analysis of the cask by using simple frictional elements at the contact points.

8. However, using the Applicant's parameters, including the coefficient of the subgrade reaction of 2.75 kips/ft3 (SAR, Rev. 8 at 2.6-35) and the pad dimensions (SAR, Rev. 8, at 2.6-87), and the relationship described in "Foundation Analysis and Design," Fourth edition, Joseph E. Bowels, McGraw Hill Company, 1988, Section 9.7, hereto attached as Exhibit A, to distinguish a flexible versus a rigid mat, I have calculated that the pad will not be rigid and, in fact, will deform when subjected to cask loading. Thus, Dr. Soler's assumption that the cask pad is rigid is incorrect.

Moreover, because the pad is flexible, the coefficient of friction is dependent on friction forces and will be affected at the contact points between the cask and the pad.

9. Under dynamic loading, the dynamic properties of a flexible pad are different from those of a rigid pad.[1] The flexible behavior of the foundation pad will amplify under the inertia of the casks on the pad. Thus, the coefficient of friction will not be constant across the pad and the Applicant's analysis of uniform coefficient of friction will not bound the actual behavior of the casks and the pad.

10. It is also possible that the casks on the pad could develop a cold bonding over time. The cold bonding causes a contact condition between the cask and the pad that is not covered by the highest coefficient of friction used by the Applicant. Additionally, the effect of the cold bonding is not necessarily the same as the hinge condition that the Applicant assumed in the previous Advent analysis.[2] The bonding may break during seismic shaking in a nonuniform pattern depending on the contact stresses causing a nonuniform contact condition between the cask and the pad.

11. Within the context of Contention GG and the modeling technique used by the Applicant and considering the realistic and flexible behavior of the pad under

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[1] An excellent comparison of the dynamic properties of rigid versus flexible foundation is presented by Iguchi and Luco in "Dynamic response of Flexible Rectangular Foundations on an Elastic Halfspace," Journal of Earthquake Engineering and Structural dynamics, 1981, Vol. 9.

[2] The Advent Report assumed that the cask was analytically pinned at one edge and did not consider the coefficients of friction. Soler Dec., Sum. Disp. at ¶ 4.

both the static and dynamic loading, it is my opinion that the Holtec 2000 analysis relied upon by the Applicant still fails to consider variation of coefficient of friction over the surface of the pad and the shift from static case to kinetic case.

12. This Declaration has been prepared in support of the State of Utah's Response to Applicant's Motion for Summary Disposition of Contention Utah GG, and the State's accompanying Statement of Material Facts, and is true and correct to the best of my knowledge and belief.

DATED this January 21, 2000.

Farhang Ostadan, Ph.D.

ATTACHMENT A

FOUNDATION ANALYSIS AND DESIGN

Fourth Edition

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9-7 CLASSICAL SOLUTION OF BEAM ON ELASTIC FOUNDATION

When flexural rigidity of the footing is taken into account, a solution is used that is based on some form of a beam on an elastic foundation. This may be of the classical Winkler solution of about 1867 in which the foundation is considered as a bed of springs ("Winkler foundation") or a finite-element procedure of the next section.

The classical solutions, being of closed form, are not as general in application as the finite-element method. The basic differential equation is (see Fig. 9-10)

$$EI\,\frac{d^4y}{dx^4} = q = -k_s' y \qquad (9\text{-}11)$$

TABLE 9-2 Closed-form solutions of infinite beam on elastic foundation (Fig. 9-10*a*)

Concentrated load at end	Moment at end
$y = \dfrac{2V_1\lambda}{k'_s} D_{\lambda x}$	$y = \dfrac{-2M_1\lambda^2}{k'_s} C_{\lambda x}$
$\theta = \dfrac{-2V_1\lambda^2}{k'_s} A_{\lambda x}$	$\theta = \dfrac{4M_1\lambda^3}{k'_s} D_{\lambda x}$
$M = \dfrac{-V_1}{\lambda} B_{\lambda x}$	$M = M_1 A_{\lambda x}$
$Q = -V_1 C_{\lambda x}$	$Q = -2M_1\lambda B_{\lambda x}$
Concentrated load at center	**Moment at center**
$y = \dfrac{P\lambda}{2k'_s} A_{\lambda x}$	$y = \dfrac{M_0\lambda^2}{k'_s} B_{\lambda x}$ deflection
$\theta = \dfrac{-P\lambda^2}{k'_s} B_{\lambda x}$	$\theta = \dfrac{M_0\lambda^3}{k'_s} C_{\lambda x}$ slope
$M = \dfrac{P}{4\lambda} C_{\lambda x}$	$M = \dfrac{M_0}{2} D_{\lambda x}$ moment
$Q = \dfrac{-P}{2} D_{\lambda x}$	$Q = \dfrac{-M_0}{2} A_{\lambda x}$ shear

The A, B, C, and D coefficients are:

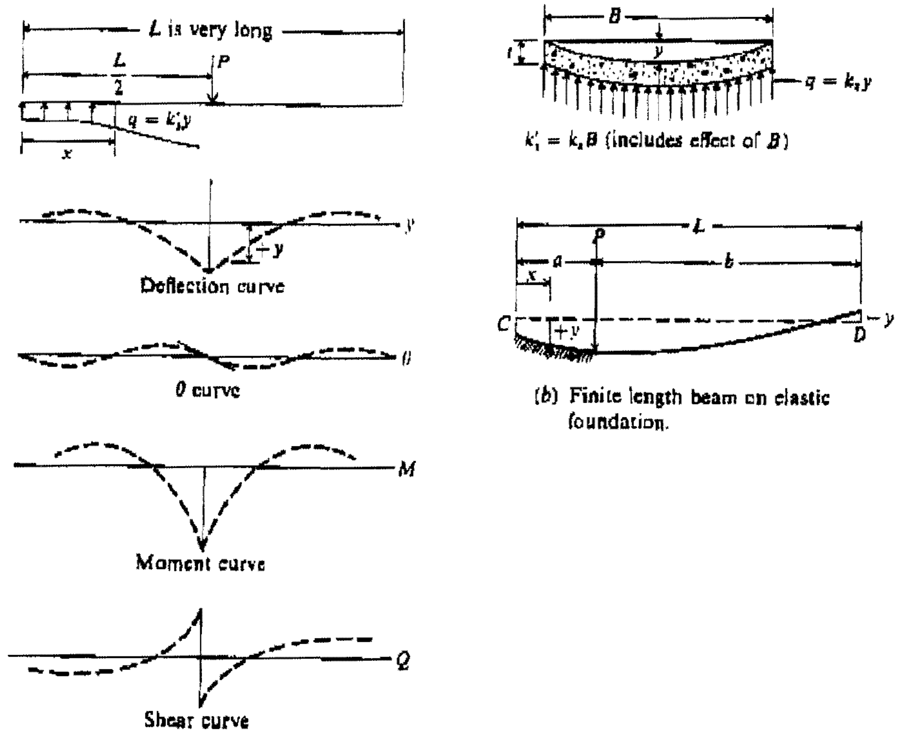
$$A_{\lambda x} = e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$$
$$B_{\lambda x} = e^{-\lambda x} \sin \lambda x$$
$$C_{\lambda x} = e^{-\lambda x}(\cos \lambda x - \sin \lambda x)$$
$$D_{\lambda x} = e^{-\lambda x} \cos \lambda x$$

where $k'_s = k_s B$. In solving the equations, a variable is introduced:

$$\lambda = \sqrt[4]{\frac{k'_s}{4EI}} \qquad \text{or} \qquad \lambda L = \sqrt[4]{\frac{k'_s L^4}{4EI}}$$

Table 9-2 gives the closed-form solution of the basic differential equations for several loadings shown in Fig. 9-10 utilizing the Winkler concept. It is convenient to express the trigonometric portion of the solutions separately as in the bottom of Table 9-2.

Hetenyi (1946) developed equations for a load at any point along a beam (see Fig. 9-10*b*) measured from the left end as follows:



(a) Infinite length beam on an elastic foundation.

(b) Finite length beam on elastic foundation.

FIGURE 9-10 Beam on elastic foundation.

$$y = \frac{P\lambda}{k'_s(\sinh^2 \lambda L - \sin^2 \lambda L)}\{2 \cosh \lambda x \cos \lambda x (\sinh \lambda L \cos \lambda a \cosh \lambda b$$
$$- \sin \lambda L \cosh \lambda a \cos \lambda b) + (\cosh \lambda x \sin \lambda x$$
$$+ \sinh \lambda x \cos \lambda x) [\sinh \lambda L (\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b)$$
$$+ \sin \lambda L (\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b)]\} \quad (9\text{-}12)$$

$$M = \frac{P}{2\lambda(\sinh^2 \lambda L - \sin^2 \lambda L)}\{2 \sin \lambda x \sin \lambda x (\sinh \lambda L \cos \lambda a \cosh \lambda b$$
$$- \sin \lambda L \cosh \lambda a \cos \lambda b) + (\cosh \lambda x \sin \lambda x - \sinh \lambda x \cos \lambda x)$$
$$\times [\sinh \lambda L (\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b)$$
$$+ \sin \lambda L (\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b)]\} \quad (9\text{-}13)$$

$$Q = \frac{P}{\sinh^2 \lambda L - \sin^2 \lambda L} \{(\cosh \lambda x \sin \lambda x + \sinh \lambda x \cos \lambda x)$$
$$\times (\sinh \lambda L \cos \lambda a \cosh \lambda b - \sin \lambda L \cosh \lambda a \cos \lambda b)$$
$$+ \sinh \lambda x \sin \lambda x [\sinh \lambda L (\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b)$$
$$+ \sin \lambda L (\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b)]\}$$

The equation for the slope θ of the beam at any point is not presented is of little value in the design of a footing. The value of x to use in the equa from the end of the beam to the point for which the deflection, moment, or desired. If x is less than the distance a, use the equations as given, and measure x from C. If x is larger than a, replace a with b in the equations, and measure x from D (Fig. 9-10b). These equations may be rewritten as

$$y = \frac{P\lambda}{k_s'} A' \qquad M = \frac{P}{2\lambda} B' \qquad \text{and} \quad Q = PC'$$

where the coefficients A', B', and C' are the values for the hyperbolic and trigonometric remainder of Eqs. (9-12) to (9-14).

It has been proposed that one could use λL previously defined to determine if a foundation should be analyzed on the basis of the conventional rigid procedure or as a beam on an elastic foundation.

Rigid members:	$\lambda L < \frac{\pi}{4}$	(bending not influenced much by k_s)
Flexible members:	$\lambda L > \pi$	(bending heavily localized)

The author has found the above criteria of limited application because of the influence of number of loads and their locations on the member.

The classical solution presented here has several distinct disadvantages over the finite-element solution presented in the next section, such as:

1. Assumes weightless beam (but weight will be a factor when footing tends to separate from the soil).
2. Difficult to remove soil effect when footing tends to separate from soil.
3. Difficult to account for boundary conditions of known rotation or deflection at selected points.
4. Difficult to apply multiple types of loads to a footing.
5. Difficult to change footing properties of I, D, and B.
6. Difficult to allow for change in subgrade reaction along footing.

Although the disadvantages are substantial some engineers prefer the classical beam-on-elastic-foundation approach over discrete element analyses. Rarely, the classical approach may be a better model than a discrete element analysis so it is worthwhile to have access to this method of solution.