Technical Letter Report

TLR-RES/DE/CIB-2021-08

Calculational Methods for Reactor Pressure Vessel Fluence in Extended Beltline Locations

Date:

April 2021

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OAK RIDGE NATIONAL LABORATORY
MANAGED BY UT-BATTELLE FOR THE US DEPARTMENT OF ENERGY

ORNL/SPR-2020/1882

Nuclear Energy and Fuel Cycle Division

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Date Published

April 2021

Prepared by
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, TN 37831-6283
managed by
UT-BATTELLE, LLC
for the
US DEPARTMENT OF ENERGY
under contract DE-AC05-00OR2272

ABSTRACT

It has become increasingly challenging to accurately predict neutron fluence in reactor pressure vessels (RPVs) as plant life extensions and power uprates expand the area of concern, causing neutron damage to locations in the so-called *extended beltline region*. At this writing, the only available guidance on RPV fluence calculations is from analyses that only address the traditional beltline region. This study evaluated the impact of multiple physical parameters on fast fluence ($E > 1$ MeV) estimates to ascertain the degree to which extended beltline fluence evaluations are more sensitive to those parameters compared with traditional beltline evaluations. In addition, quadrature sensitivity in the widely used discrete ordinates method was evaluated to determine its impact on traditional and extended beltline fluence estimates. Hybrid radiation transport calculations, which employ the current state of the art in radiation transport simulations, were used as benchmark solutions in the absence of measured data in extended beltline locations. These hybrid calculations utilize continuous-energy Monte Carlo calculations and eliminate the discretizations in space, energy, and angle that impose accuracy limitations on discrete ordinates calculations. This report details the results of the physical and calculational parameter studies and provides insights into where modifications in analysis methodology may be necessary to obtain calculational uncertainty in the extended beltline region comparable to that specified for traditional beltline fluence analyses.

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EXECUTIVE SUMMARY

Regulatory Guide (RG) 1.190, "Calculational and Dosimetry Methods for Determining Pressure Vessel Neutron Fluence," describes the application and qualification of a methodology acceptable to the U.S. Nuclear Regulatory Commission (NRC) for determining the best-estimate neutron fluence experienced by materials in the beltline region of light water reactor (LWR) reactor pressure vessels (RPVs). Although the beltline region is not explicitly defined in RG 1.190, NUREG/CR-1511, "Reactor Pressure Vessel Status Report," states that materials with a projected neutron fluence greater than 1.0×10^{17} neutrons per square centimeter (n/cm^2) at end of license experience sufficient neutron damage to be included in the beltline.

Subsequent to the issuance of RG 1.190, the continuing trend of plant life extension and power uprates for both pressurized water reactors (PWRs) and boiling water reactors (BWRs) has led to growing concern about lifetime fluence levels in materials outside the traditional beltline region and in the RPV internals. The regions of the RPV that lie outside the traditional beltline are referred to as the *extended beltline region*.

Although the fundamental radiation transport phenomena for fluence levels in the extended beltline region are the same as those for the traditional beltline region, the characteristics and limitations of the numerical methods used to solve the transport equation, as well as the different transport paths from the core to the reactor vessel, result in additional considerations when determining fluence outside the beltline region relative to calculations within the beltline region. In addition, calculation of other neutron responses of interest—including a variety of dosimetry reactions that serve as measured data for use in benchmarking transport methods—may be more sensitive to the selection of transport methods and parameters in the extended beltline.

The primary objectives of this report are to identify transport phenomena that are important in calculation of RPV fluence levels in the extended beltline region and to evaluate radiation transport methodologies to determine which ones are best suited to such analyses. PWR and BWR reference models were used with discrete ordinates calculations, which represent the most widely used technique for RPV fluence evaluations. PWR and BWR reference models were also used with hybrid radiation transport calculations, which represent the current state of the art.

Sensitivity of extended beltline neutron transport calculations to physical parameters

A variety of physical parameters, including coolant density, neutron fission spectra, changes in the geometry of the cavity gap region, changes in the composition of the bioshield concrete, and the presence of a bioshield liner and reflective metallic thermal insulation were evaluated using high-fidelity hybrid radiation transport calculations. These studies provide key insights into how these parameters often affect fast neutron flux levels in the extended beltline region more significantly than in the traditional beltline region. Some factors, such as changes in concrete composition, changes in cavity gap width, and the presence or absence of a steel bioshield liner, have little to no effect on flux levels within the traditional beltline region, but they do have significant effects in extended beltline locations. Of particular concern are the effects that physical parameter variations have on neutron flux in the PWR model’s vessel supports.

Sensitivity of extended beltline discrete ordinates calculations to angular quadrature

Regulatory Guide 1.190 provides guidance on the selection of discrete ordinates angular quadrature sets for RPV fluence calculations in the traditional beltline region but does not provide guidance for locations in the extended beltline region. In this work, an extensive set of calculational parameter studies was performed to assess typical discrete ordinates parameter selections for cases in which calculations extend beyond the traditional beltline region, particularly for locations in the vicinity of nozzles and vessel supports.

Quadrature sensitivity studies confirmed that the commonly used S8 quadrature set is not suitable for extended beltline fluence calculations. Furthermore, localized deficiencies in S8 solutions in the traditional beltline region may be significant, particularly for vessel dosimetry calculations. Even though the higher-order S16 quadrature provides improvements in the accuracy of discrete ordinates solutions, it is still likely inadequate for extended beltline applications in the nozzle and vessel support locations.

More recently developed quadruple range (QR) quadratures were also evaluated. These quadrature sets were developed specifically to improve solution accuracy with models that have material discontinuities parallel to the coordinate axes in areas such as the edges of fuel assemblies and/or the streaming paths along a coordinate axis. Of particular importance is the ability of a quadrature set to accurately model neutron streaming paths in the cavity gap between an RPV and the concrete bioshield.

Quadrature sets that are best suited for use in extended beltline applications were selected by comparing Denovo discrete ordinates calculations having various quadrature selections to Shift Monte Carlo solutions that used the same multigroup (MG) cross-section library that was used by the Denovo calculations. These studies demonstrated that the QR quadratures provide superior solution accuracy. However, even the use of high-order QR quadratures can still produce solutions with localized differences of 10% or more in the extended beltline region.

Additional studies

The sensitivity of transport calculations to multigroup cross-section energy structures and scattering expansion order will be addressed in a companion report. That report will also evaluate the relationship between fast flux and dpa rate in extended beltline regions.

ACKNOWLEDGMENTS

The work described in this report was sponsored by the U.S. Nuclear Regulatory Commission (NRC), Office of Nuclear Regulatory Research (RES). The authors thank Jay Wallace for his support and guidance as the NRC project manager. His insights, questions, and encouragement have been invaluable to the success of this project and report. A number of NRC staff contributed constructive comments on draft versions of this report, on earlier documents, and in public meetings held during the course of this project. Among those contributors, special thanks are due to Ben Parks, Nate Hudson, Dan Widrevitz, and Amrit Patel. The authors also thank Oak Ridge National Laboratory (ORNL) staff members Tara Pandya and Katherine Royston for their reviews of this work, and Rose Raney and Kathy Jones for final document preparation.

ABBREVIATIONS

1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
ADAMS	Agencywide Documents Access and Management System
ANS	American Nuclear Society
ANSI	American National Standards Institute
ASTM	American Society for Testing and Materials
B&W	Babcock and Wilcox
BAF	bottom of active fuel
BOL	beginning of life
BWR	boiling water reactor
CE	continuous energy
C/M	calculated-to-measured ratio
CPU	central processing unit
dpa	displacements per atom
DOM	the dominant axial zone in a GE14 BWR fuel assembly
DSA	diffusion synthetic acceleration
DTW	directional theta weighted
EFPY	effective full-power year
ENDF	Evaluated Nuclear Data File
EOL	end of life
EVND	ex-vessel neutron dosimetry
IAEA	International Atomic Energy Agency
IRDF	International Reactor Dosimetry File
LD	linear discontinuous
LWR	light water reactor
MCNP	Monte Carlo N-Particle
MG	multigroup
NAT	the natural U axial zone at the bottom of each rod in a GE14 BWR fuel assembly
NPP	nuclear power plant
NRC	United States Nuclear Regulatory Commission
N-T	the natural U axial zone at the top of each rod in a GE14 BWR fuel assembly
N-V	the natural uranium vanished rod axial zone in a GE14 BWR fuel assembly
ORNL	Oak Ridge National Laboratory
PNNL	Pacific Northwest National Laboratory
PCA	Pool Critical Assembly [Pressure Vessel Facility Benchmark]
PCR	partial-current rebalance
PDF	probability distribution function
PLE	the plenum axial zone in a GE14 BWR fuel assembly
PSZ	the power shaping axial zone in a GE14 BWR fuel assembly
PWR	pressurized water reactor
PWROG	PWR Owners Group
QR	quadruple range
RPV	reactor pressure vessel
RVI	reactor vessel internals
SC	step characteristic
SI	source iteration

SLR	subsequent license renewal
SPEO	subsequent period of extended operation
TAF	top of active fuel
TLD	trilinear discontinuous
TSA	transport synthetic acceleration
TW	theta weighted
Voxel	volume element
VAN	the vanished rod axial zone in a GE14 BWR fuel assembly
VF	void fraction
VR	variance reduction
WBN1	Watts Bar Nuclear Plant Unit 1
WDD	weighted diamond difference

1 INTRODUCTION

For the past several decades, the main region of concern for reactor pressure vessel (RPV) fluence calculations has been the portion of the RPV referred to as the *beltline region,* which can be defined [1] as “the region adjacent to the reactor core that must be evaluated to account for the effects of radiation on fracture toughness.” With the continuing trend of plant life extension and power uprates for both pressurized water reactors (PWRs) and boiling water reactors (BWRs) throughout the United States, there is growing concern about lifetime fluence levels in regions above and below what has historically been considered the beltline region and in reactor vessel internals (RVI).

Regulatory Guide 1.190 [2] describes the application and qualification of a methodology acceptable to the U.S. Nuclear Regulatory Commission (NRC) for determining the best-estimate neutron fluence experienced by materials in the beltline region of light water reactor (LWR) RPVs. This methodology is also acceptable for determining the overall uncertainty associated with those best-estimate values. However, Regulatory Guide 1.190 does not specifically define the beltline region.

In Title 10 of the Code of Federal Regulations (CFR) Part 50 [3], Section II of Appendix G defines the beltline region as “The region of the reactor vessel (shell material including welds, heat affected zones, and plates or forgings) that directly surrounds the effective height of the active core and adjacent regions of the reactor vessel that are predicted to experience sufficient radiation damage to be considered in the selection of the most limiting material with regard to radiation damage.” 10 CFR Part 50, Section III of Appendix H [3] requires that reactor vessels for which the peak neutron fluence at the end of the design life of the vessel exceeds 10^{17} cm^{-2} ($E > 1$ MeV) must have their beltline materials monitored by a surveillance program complying with American Society for Testing and Materials (ASTM) E185-82 [4], as modified by Appendix H.

Section 2.3 of NUREG/CR-1511 [5] states that “The NRC staff considered materials with a projected neutron fluence of greater than 1.0E17 neutrons per square centimeter (n/cm^2) at end of license to experience sufficient neutron damage to be included in the beltline.”

An International Atomic Energy Agency (IAEA) Nuclear Energy Series report on the integrity of RPVs in nuclear power plants (NPPs) [6] refers to the beltline as “the region of shell material directly surrounding the effective height of the fuel element assemblies, plus an additional volume of shell material both below and above the active core, with an [end-of-life] fluence of more than 10^{21} m^{-2} ($E > 1$ MeV) (10^{17} cm^{-2}).” This definition is consistent with that given in NUREG/CR-1511.

Chapter 12 of *Nuclear Power – Control, Reliability, and Human Factors* [7] states that typical end-of-life design neutron fluences are on the order of 10^{18} n/cm^2 for BWRs and on the order of 10^{19} n/cm^2 for PWRs. Values of 4×10^{18} n/cm^2 for BWRs, 4×10^{19} n/cm^2 for Westinghouse PWRs, and 1.2×10^{19} n/cm^2 for Babcock and Wilcox (B&W) PWRs are provided in the IAEA assessment [6]. The PWR fluence values are noted as corresponding to a lifetime of 32 effective full-power years (EFPYs). Lifetime is not noted for BWRs.

In the context of the current report, the portion of the RPV where the end-of license fluence would be expected to exceed 10^{17} n/cm^2 for plant operations consistent with those in the original operating license is referred to as the *traditional beltline region*, or simply the *beltline region*.

Locations above and below the traditional beltline region are referred to as the *extended beltline region*.

While the fundamental radiation transport phenomena for fluence levels in the extended beltline region are the same as those for the traditional beltline region, the characteristics and limitations of the numerical methods used to solve the transport equation, as well as the different transport paths from the core to the reactor vessel, result in additional considerations for the determination of fluence outside the beltline region relative to calculations within the beltline region.

This report discusses the use of PWR and BWR reference models for the evaluation of RPV fast (E > 1 MeV) neutron fluence and describes studies that were performed to evaluate transport phenomena that must be specifically addressed for such calculations in the extended beltline region. Parameter sensitivity studies are performed to assess the accuracy that can be expected for extended beltline fluence calculations using discrete ordinates transport codes, which are the most common method currently used for RPV fluence analyses. Modern hybrid radiation transport methods that combine both deterministic and Monte Carlo calculations are discussed and contrasted with the discrete ordinates method.

2 AN OVERVIEW OF RADIATION TRANSPORT CALCULATIONAL METHODS

The neutral particle radiation transport calculations performed for reactor physics and radiation shielding analyses are typically based on obtaining solutions to the steady-state Boltzmann transport equation. Detailed discussions of the derivation and application of the Boltzmann equation for nuclear reactor analyses can be found in the literature [8], [9], and [10].

The steady-state Boltzmann transport equation for fixed-source shielding calculations can be written as[1]

$$\begin{aligned}\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi(\boldsymbol{r},E,\boldsymbol{\Omega}) + \sigma(\boldsymbol{r},E)\phi(\boldsymbol{r},E,\boldsymbol{\Omega}) \\ = \int_0^\infty\int_{4\pi}\sigma_s\,(\boldsymbol{r},E' \longrightarrow E,\boldsymbol{\Omega}' \longrightarrow \boldsymbol{\Omega})\phi(\boldsymbol{r},E',\boldsymbol{\Omega}')d\boldsymbol{\Omega}'dE' + q_e(\boldsymbol{r},E,\boldsymbol{\Omega}),\end{aligned} \tag{1}$$

where

Ω = a unit vector in the direction of particle travel,
ϕ = the particle flux,
$\boldsymbol{r}$ = the particle's position,
E = the particle's energy,
σ = the macroscopic total cross section,
σ_S= the macroscopic scattering cross section, and
q_e = external (flux-independent) sources

For the problems encountered in reactor physics and shielding analyses, the angular distribution of scattered particles depends only on the cosine of the scattering angle between the incoming and exiting particles, and Eq. (1) can be written as

$$\begin{aligned}\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi(\boldsymbol{r},E,\boldsymbol{\Omega}) + \sigma(\boldsymbol{r},E)\phi(\boldsymbol{r},E,\boldsymbol{\Omega}) \\ = \int_0^\infty\int_{4\pi}\sigma_s\,(\boldsymbol{r},E' \longrightarrow E,\omega)\phi(\boldsymbol{r},E',\boldsymbol{\Omega}')d\boldsymbol{\Omega}'dE' + q_e(\boldsymbol{r},E,\boldsymbol{\Omega}),\end{aligned} \tag{2}$$

where

$$\omega = \boldsymbol{\Omega}'\cdot\boldsymbol{\Omega}.$$

Exact analytical solutions for the transport equation exist only for very simple cases (e.g., monoenergetic particles in a one-dimensional system). For realistic shielding problems, a variety of methods have been developed to solve the transport equation using numerical techniques. The primary computational methods used to solve neutral particle transport theory problems have traditionally been either *deterministic* or *stochastic*. Deterministic methods were pioneered by Carlson and Lathrop in the 1950s and 1960s and are described in numerous Los Alamos Scientific Laboratory Reports, conference proceedings, and journal articles. A thorough description of the early development of the method, including an extensive list of references, can

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[1] Throughout this section, **boldface** symbols are used to represent vector quantities.

be found in Chapter 3 of the book by Greenspan et al. [11]. Stochastic methods are based on the Monte Carlo technique pioneered by Ulam, Von Neumann, et al. [12]. Current state-of-the-art radiation transport calculations utilize both deterministic and stochastic codes in the hybrid radiation transport methodology. A paper by Mosher et al. [13] provides a more current discussion of the development and implementation of hybrid methods for radiation shielding calculations. The computer codes used in this report are briefly described in Section 2.4.

2.1 Deterministic calculations

Modern computer codes based on deterministic methods are generally referred to as *discrete ordinates* or S_N codes. Although there are differences among the major discrete ordinates transport codes currently in use, all of them are based on discretizing the spatial, energy, and angular variables and solving the resulting set of equations using numerical methods. The techniques used to discretize energy, space, and angle are discussed in this section.

2.1.1 Energy discretization

Energy discretization in the discrete ordinates method is accomplished through the multigroup (MG) approximation. In this method, the energy range of interest $[E_{min}, E_{max}]$ is divided into G intervals or groups. The standard convention is to establish group one as the highest energy group, with an upper group boundary E_0 equal to E_{max}, and a lower group boundary of E_1. The group numbers increase as energy decreases. The energy E lies within group *g* if $E_g < E \leq E_{g-1}$.[2]

For a given energy group *g*, the group flux and external source at position **r** with direction **Ω** are defined as

$$\boldsymbol{\phi}_{\boldsymbol{g}}(\mathbf{r},\boldsymbol{\Omega}) = \int_{\boldsymbol{E}_{\boldsymbol{g}}}^{\boldsymbol{E}_{\boldsymbol{g}-\mathbf{1}}} \boldsymbol{\phi}_{\boldsymbol{g}}(\mathbf{r},E,\boldsymbol{\Omega})\ \boldsymbol{dE} \tag{3}$$

and

$$q_{\boldsymbol{g}}(\mathbf{r},\boldsymbol{\Omega}) = \int_{\boldsymbol{E}_{\boldsymbol{g}}}^{\boldsymbol{E}_{\boldsymbol{g}-\mathbf{1}}} q_{\boldsymbol{g}}(\mathbf{r},E,\boldsymbol{\Omega})\ \boldsymbol{dE}. \tag{4}$$

A set of MG cross sections is generated by averaging the continuous-energy (CE) cross-section data over the energy groups using an appropriate weighting function.

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[2] While most deterministic codes (e.g. DORT, TORT, PARTISN) follow this convention, Denovo uses group zero for the highest energy group. In this case the energy E lies within the group g if $E_{g+1} < E \leq E_g$. The notation used throughout this section follows the standard convention, with group numbers beginning with one.

Applying the MG approximation to Eq. (2) yields the MG transport equation for the flux in group g:

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla}\phi_g(\boldsymbol{r}, \boldsymbol{\Omega}) + \sigma_g(\boldsymbol{r})\phi(\boldsymbol{r}, \boldsymbol{\Omega}) = \sum_{g'=1}^{G} \int_{4\pi} \sigma_{s,gg'}(\mathbf{r}, \omega)\phi_g(\boldsymbol{r}, \boldsymbol{\Omega}')d\boldsymbol{\Omega}' + q_g(\boldsymbol{r}, \boldsymbol{\Omega}). \tag{5}$$

The standard approach in discrete ordinates calculations is to represent the angular dependence of the scattering cross section using a Legendre polynomial expansion:

$$\sigma_{s,gg'}(\mathbf{r}, \omega) = \sum_{n=0}^{N} \frac{2n+1}{4\pi} P_n(\omega)\sigma_{sn,gg'}(\mathbf{r}), \tag{6}$$

where P_n is the Legendre polynomial of order n, and the $\sigma_{sn,gg'}$ values are referred to as the *moments of the scattering cross section*. The zeroth moment, $\sigma_{sn,gg'}$, is the total cross section for scattering from group g' to group g. MG cross-section libraries used for LWR shielding analyses typically represent the angular distribution of scattering cross sections using a maximum order N in Eq. (6) ranging from three to seven.

For the hypothetical scenario of particle transport in a medium in which all scattering is isotropic, a P_0 expansion would be adequate. In actual practice, scattering is rarely isotropic. The degree of anisotropy in a scattering cross section is dependent on the isotope and the neutron energy. In general, neutron scattering becomes increasingly anisotropic as the neutron energy increases and/or the mass of the scattering nucleus decreases [14].

2.1.2 Spatial discretization

In discrete ordinates transport calculations, the problem geometry is discretized into cells or voxels. Depending on the size and complexity of the system being modeled, the number of cells can range from thousands to tens or even hundreds of millions. For the majority of discrete ordinates codes, a regular structured mesh is employed. Typical mesh geometries for structured mesh codes are two-dimensional (2D) and three-dimensional (3D) Cartesian geometry and one-dimensional (1D), 2D, or 3D cylindrical geometry. In most shielding analyses, including the calculation of fluxes in a reactor system, either a 3D cylindrical or 3D Cartesian geometry is applied. While some unstructured mesh discrete ordinates codes are available, the discussion in this report is limited to the more widely used structured mesh codes.

Most shielding calculations are performed using models that include a wide variety of components, not all of which are best represented by a single coordinate system. For example, cylindrical geometry is well suited to modeling the cylindrical portions of an RPV, but not the lower hemispherical RPV head, fuel assemblies, or many of the core internals. In addition, a cylindrical coordinate system with its Z-axis oriented with the vertical axis of an RPV is not well suited to modeling the cylindrical nozzles whose axes are orthogonal to the Z-axis of the coordinate system.

Each spatial cell in a deterministic transport model is filled with a single material. A material may be an isotope, an element, a compound such as water, an engineering material such as stainless steel or concrete, or a mixture of any of these, such as a region represented by a homogenization of steel and water. In typical discrete ordinates calculations, the boundaries where material changes occur will not necessarily coincide perfectly with voxel boundaries, so some voxels will overlap two or more distinct materials. Early discrete ordinates codes such as DORT [15], [16] and TORT [15], [17], [18] assign the material with the maximum volume fraction in such a voxel. Some modern discrete ordinates codes like Denovo [19] and PARTISN [20] perform volume weighting of the materials in such cells and create a mixed material to better model radiation transport through that cell. An example of the effect of material mixing is seen in Figure 2-1 and Figure 2-2. In Figure 2-1 the voxels that include the RPV clad (a stainless steel layer with a thickness of 0.56 cm) are modeled as either coolant, stainless steel (RPV clad), or carbon steel (RPV base metal). With material mixing (Figure 2-2), the same voxels are modeled as mixtures of two or three of those materials. While the use of material mixing provides a more accurate transport solution, it also increases the memory requirements for a given calculation. Codes like Denovo allow the user to specify a tolerance level for the creation of unique mixed materials.

In addition to the approximation introduced by discretizing a transport model into cells, discrete ordinates solutions are also affected by the differencing scheme that is used to relate the directional flux at the center of a spatial cell center to the flux at the cell boundaries. Differencing schemes that are used in current discrete ordinates codes include the following:

- weighted diamond difference (WDD)
- theta weighted (TW)
- directional theta weighted (DTW)
- linear discontinuous (LD)
- trilinear discontinuous (TLD)
- step characteristic (SC)

Various publications [18], [19], [21], [22] provide useful information regarding the characteristics and application of these schemes. The most commonly used differencing schemes in RPV fluence calculations are TW and DTW. The LD scheme is used for the majority of the Denovo calculations performed in this analysis. The SC scheme is often used in the discrete ordinates step(s) of a hybrid radiation transport sequence (Section 2.3).

The accuracy that can be obtained in a discrete ordinates solution is dependent on the differencing scheme and the spatial mesh intervals. Furthermore, the mesh and differencing scheme can be interdependent. For example, the LD scheme tends to give very accurate results on well-refined meshes, but it also tends to be sensitive to the aspect ratio of the mesh cells. There are no universal rules that can be applied to ensure that a spatial mesh and differencing scheme are appropriate for a discrete ordinates calculation.

Determination that a solution has converged with respect to the spatial discretization is typically obtained by parameter studies for a given model. Petrovic and Haghighat [23] present meshing parameter studies for 2D cylindrical Rθ vessel fluence calculations using DORT. Davidson and Burre [24] present meshing studies for 2D cylindrical RZ and 3D XYZ discrete ordinates calculations of gamma transport using DORT, TORT, and PARTISN models of spent fuel sources in a fuel cask. Regulatory Guide 1.190 provides general guidelines for spatial mesh in 2D cylindrical (Rθ and RZ) geometries, but it has no recommendations for Cartesian geometries.

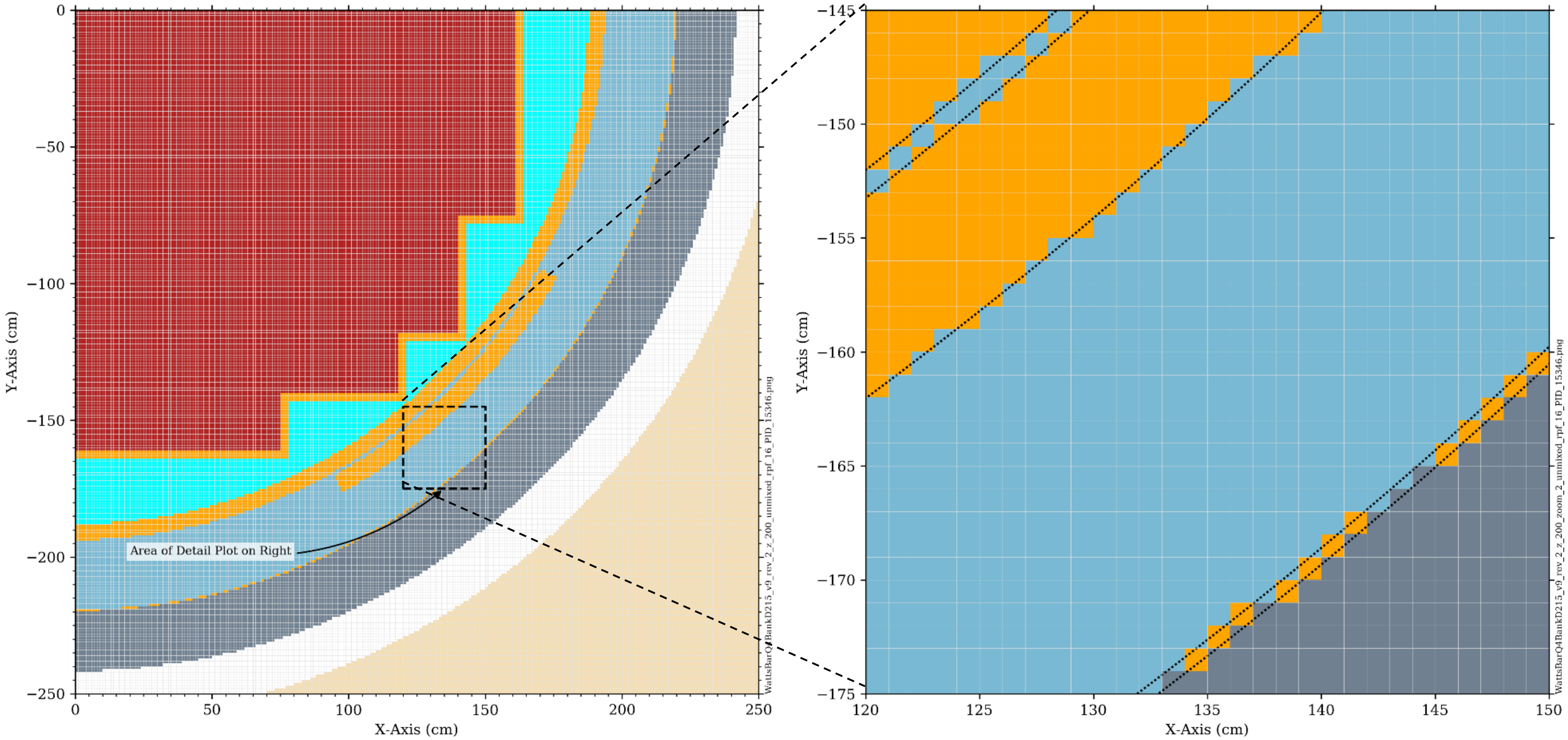

Figure 2-1 **Two-dimensional slice of the PWR reference model at Z = 200 cm for a Denovo calculation with uniform 1 cm mesh intervals in X, Y, and Z. Material mixing is suppressed in the Denovo spatial discretization. The detail plot on the right shows a close-up of a region that includes the core barrel, neutron pad, coolant, RPV clad, and RPV base metal. The light gray lines are the Denovo spatial mesh, and the dotted black lines in the right-hand view represent the radial boundaries of the core barrel, neutron pad, RPV clad, and RPV base metal**

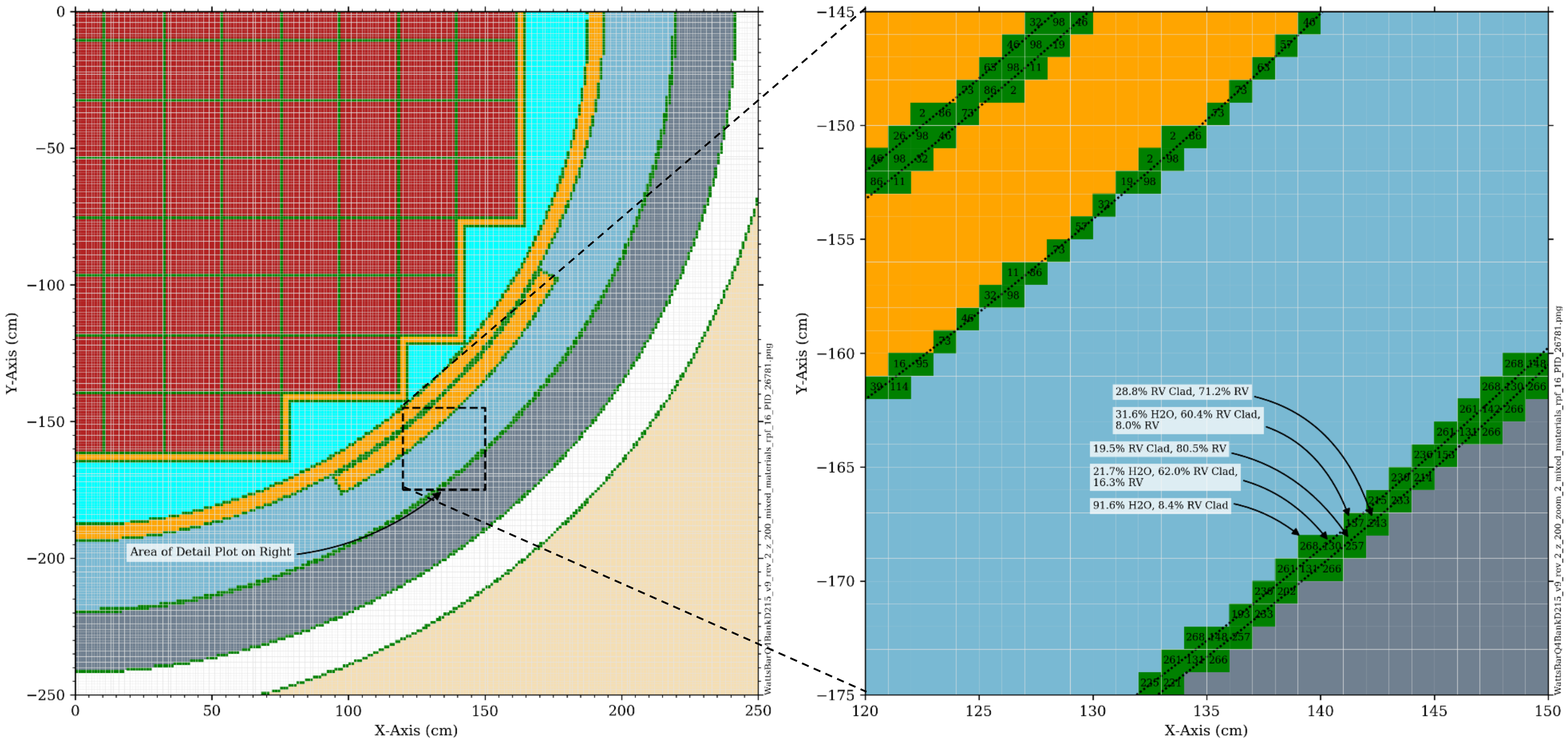

Figure 2-2 Two-dimensional slice of the PWR reference model at Z = 200 cm for a Denovo calculation with uniform 1 cm mesh intervals in X, Y, and Z. Material mixing is applied in the Denovo spatial discretization. Voxels with mixed materials are shaded green. The detail plot on the right shows a close-up of a region that includes the core barrel, neutron pad, coolant, RPV clad, and RPV base metal. The light gray lines are the Denovo spatial mesh, and the dotted black lines represent the radial boundaries of the core barrel, neutron pad, RPV clad, and RPV base metal. The integers in each of the mixed material voxels are mixed material numbers. The volume fractions for several voxels along the RPV clad and RPV base metal are shown

2.1.3 Angular discretization

In discrete ordinates calculations, particle travel is allowed only in a finite set of discrete directions. The angular flux in each of these directions is calculated using transport sweeps to solve the discrete ordinates equations. The scalar flux is then formed by integrating the angular fluxes using numerical quadrature, with each quadrature direction having a specified quadrature weight. The accuracy that can be obtained in solving the transport equation using the discrete ordinates method is dependent on several factors, including the set of quadrature weights and ordinates that is used. While there is no standard procedure for choosing an adequate set, Duderstadt and Martin [8] suggest that the following criteria should be considered:

1. **Projection invariance.** In cases where there is no *a priori* knowledge concerning the angular flux in the solution space, it is reasonable to select a quadrature set that is invariant with respect to allowable orientations of the physical domain. For 3D Cartesian geometry, this means quadrature ordinates should be invariant under arbitrary 90° rotations about the coordinate axes, and 180° reflections about the XY, XZ, or YZ planes. These conditions are met by the widely used level-symmetric S_N quadrature sets[3]. Regulatory Guide 1.190 states that an S8 fully symmetric angular quadrature must be used as a minimum for determining the fluence in the vessel, with the potential need for higher-order quadratures (i.e., quadratures with a greater number of ordinates) in reactor cavity fluence calculations.

2. **Positivity of the scalar flux.** The scalar flux should always be positive. Choosing a quadrature set in which all the weights are positive will ensure integration of a positive scalar flux provided the angular fluxes are positive. Level-symmetric S_N quadratures have negative weights for orders exceeding S20, which limits their ability to use increasing quadrature orders as a means of confirming that a solution has converged with respect to quadrature.

3. **Accurate evaluation of angular integrals.** The flux moments and the source should be integrated accurately with a minimum number of directions and weights.

While projection invariance is desirable in general, in some cases noninvariant quadratures are better suited to a particular application. Abu-Shumays [25], [26] developed quadruple range (QR) quadratures to accurately integrate functions that are discontinuous across octant boundaries of the unit sphere. Because the QR sets have directions closer to the coordinate axes than level-symmetric S_N sets with the same number of angles, the QR sets often provide superior solutions for models which have material interfaces along any of the coordinate axes and/or particle streaming through gaps that are parallel to a coordinate axis [27], such as the cavity gap between an LWR's RPV and bioshield.

Figure 2-3 shows the quadrature ordinates and weights for the level symmetric S8 and S16 quadratures, which are widely used in RPV fluence calculations. Figure 2-4 shows the ordinates

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[3] The notation S_N refers to a quadrature set with N/2 direction cosines with respect to each of the coordinate axes. Thus, for example, the S8 quadrature shown in Figure 2-3 in has four direction cosines with respect to the X, Y, and Z axes. While the value of *N* is often written as a subscript, the convention in this report is to avoid subscripts, which could become difficult to read for some of the QR quadrature sets considered in Section 6.

and weights for S16 and QR8T[4] quadratures, each with 36 angles per octant in a triangular arrangement. Note that the QR8T quadrature has ordinates closer to each of the coordinate system axes. Consequently, QR8T quadrature is likely to be more appropriate than S16 quadrature for problems in which particle streaming near the coordinate axes is a significant transport path. This behavior is illustrated in Section 6.

Numerous other types of quadrature sets have been developed in attempts to improve the accuracy and/or efficiency of discrete ordinates solutions. Carew et al. [28], [29] developed uniformly distributed equal weight quadratures and uniform Gauss weight quadratures to provide the ability to systematically increase quadrature order while maintaining positive weights. This work was motivated by the inability of the standard level symmetric SN sets to be refined beyond order 20. Longoni and Haghighat [30] developed an ordinate splitting technique for problems in which the particle flux is peaked along certain directions of the unit sphere. Ahrens [31] developed new quadratures that are invariant under the icosahedral rotation group, but not under 90°and 180°rotations. Fromowitz and Zeigler [32] developed large quadrature sets with more than approximately 1,000 angles per octant. These were developed to reduce ray effects[5] in problems that have significant regions with low-scattering or nonscattering media.

A recent paper by Manalo, Ahrens, and Sjoden [33] provides an overview of their work in quadrature development. While many of the quadrature sets discussed in the references can be used directly in standard discrete ordinates radiation transport codes such as TORT, PARTISN, and Denovo, methods such as the ordinate splitting technique [30] require modifications to the transport sweep routines.

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[4] For QR quadratures the notation QR*N*T refers to a QR set with N direction cosines with respect to the Z-axis and a triangular arrangement of azimuthal angles on the polar levels. The notation QR*M*x*N* refers to a QR set with M polar levels and N azimuthal angles on each polar level.

[5] Ray effects can occur in multidimensional discrete ordinates calculations, particularly those with highly localized sources and regions with minimal or no scattering. Ray effects are characterized by nonphysical oscillations in the angular flux solution and even in the scalar flux. Examples of ray effects are presented in Section 6.

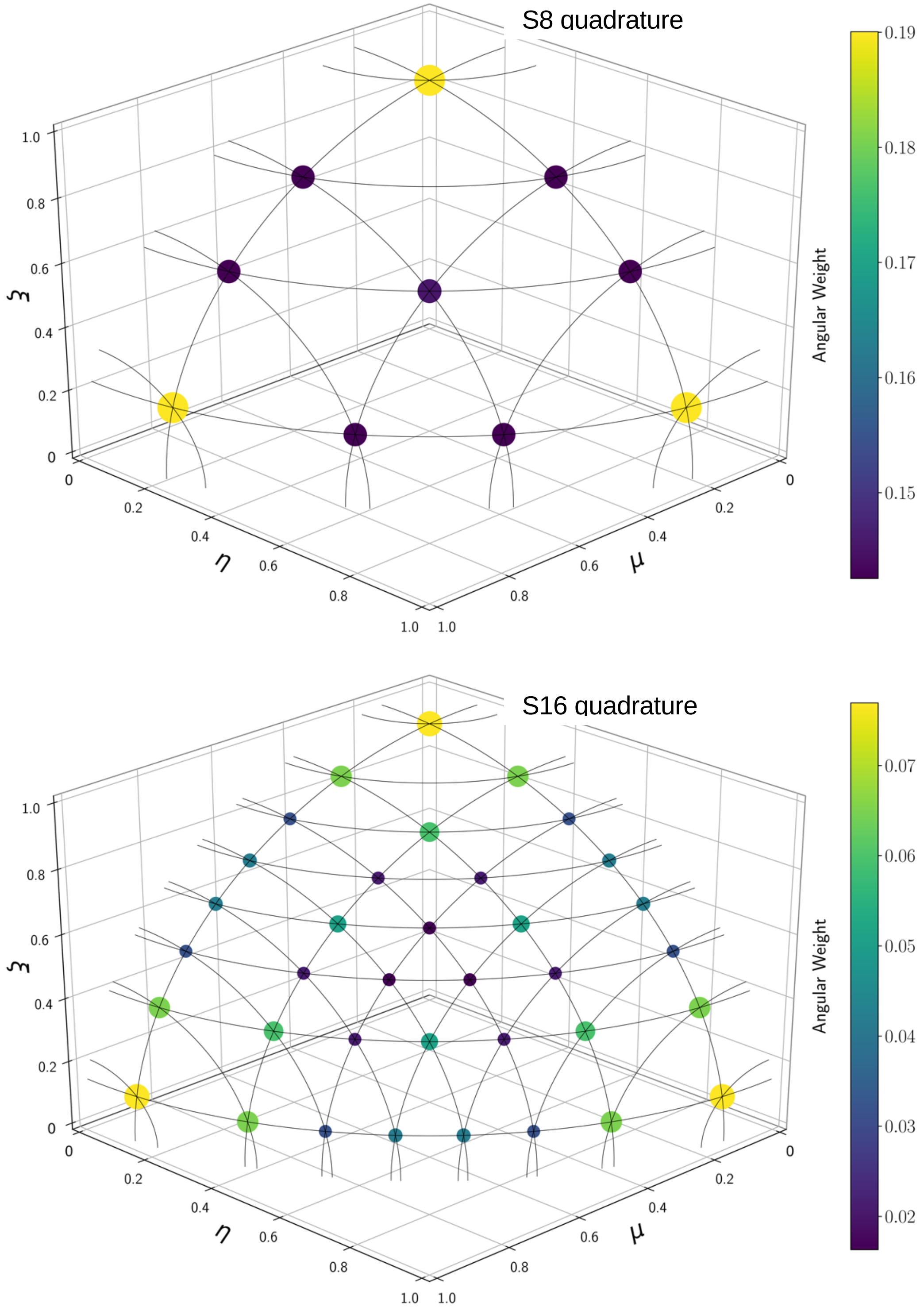

Figure 2-3 Level symmetric S8 and S16 quadrature ordinates and weights in one octant of the unit sphere. The circles represent the direction cosines in X, Y, and Z on the unit sphere. Associated weights are indicated by circle color and size. These sets are rotationally invariant

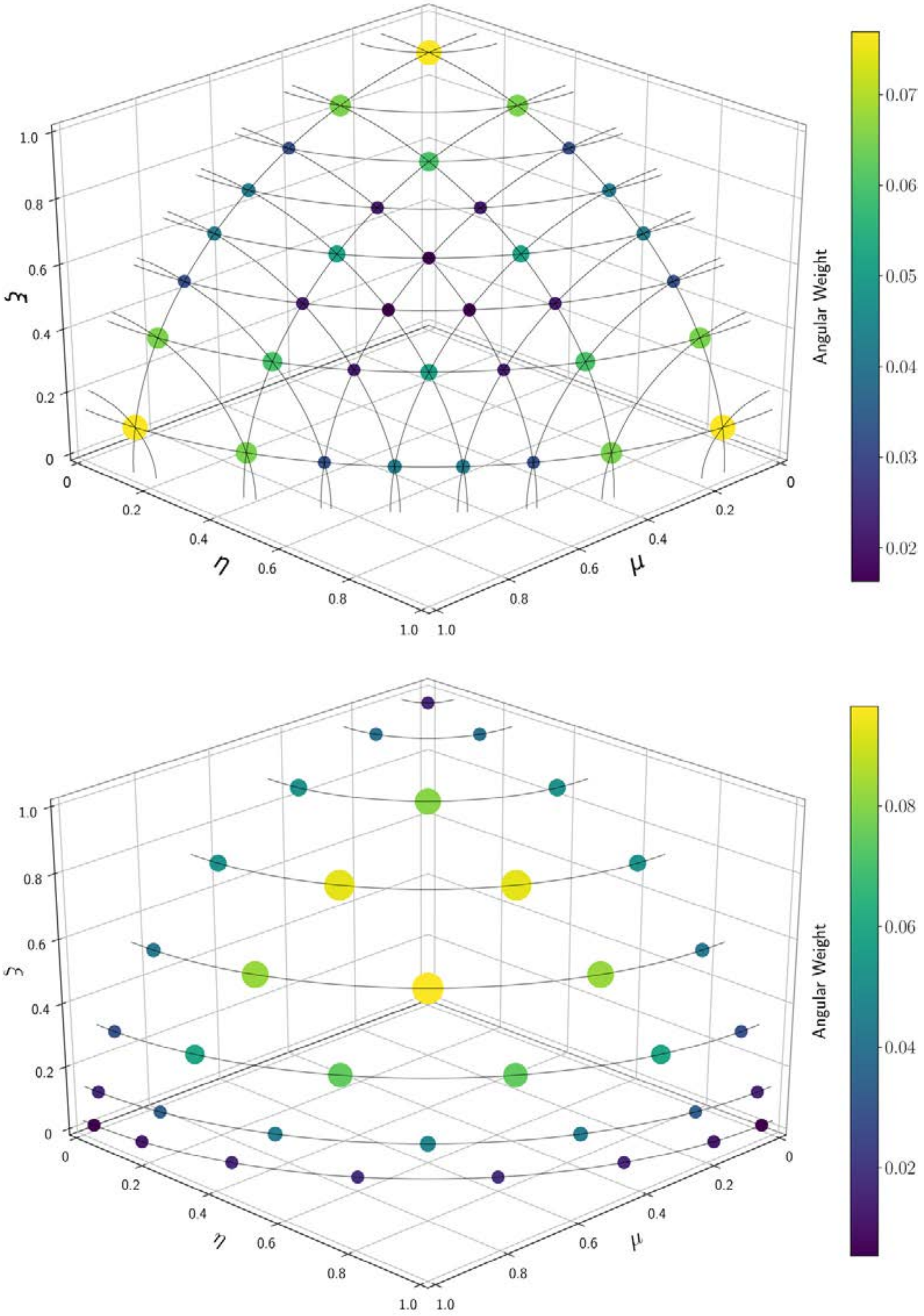

Figure 2-4 S16 and QR8T quadrature ordinates and weights in one octant of the unit sphere. The circles represent the direction cosines in X, Y, and Z on the unit sphere. The associated weights are indicated by circle color and size. The S16 set is rotationally invariant, while the QR8T set is not. Both sets have eight polar levels and 36 angles per octant

2.1.4 Solution of the discrete ordinates transport equations

Discretization of a transport model in energy, space, and angle in a discrete ordinates calculation produces a set of linear equations which are solved iteratively until a specified convergence criterion is met. Because the number of unknowns for discrete ordinates calculations is generally very large, substantial computing resources are required for many applications of this method. For example, the Denovo calculations for this study typically included more than 10^{10} unknowns based on mesh spacing, MG library, and quadrature selection.

Because these calculations are computationally expensive, a variety of acceleration techniques have been developed to reduce the time required to obtain a converged solution. The most commonly used acceleration techniques in modern discrete ordinates codes include partial-current rebalance (PCR), diffusion synthetic acceleration (DSA), and transport synthetic acceleration (TSA). The primary acceleration technique in Denovo is based on Krylov methods, which can be substantially more efficient than other acceleration schemes.

The usual output of a discrete ordinates calculation is the scalar flux in every mesh cell in each energy. The scalar fluxes can be combined with MG response functions to obtain other quantities of interest, such as reaction rates, or dose rates.

2.2 Stochastic calculations

Radiation transport computer programs based on stochastic methods are generally referred to as *Monte Carlo* codes. Monte Carlo radiation transport calculations are based on the simulation of particle histories. Each history is based on sampling probability distribution functions (PDFs) that govern each event as a particle is born (e.g., a neutron is created by fission), undergoes various interactions as it traverses through the model phase space, and is finally absorbed or escapes the boundary of the model.

As particle histories are accumulated, the model phase space is populated with a distribution of particle positions, energies, and directions. The population in one or more region(s) of interest can be obtained through the use of particle tallies. These tallies can be very localized, or they can encompass large portions of the model (Section 2.2.3).

Monte Carlo calculations inherently provide higher-fidelity solutions than discrete ordinates calculations, as they do not require the discretization in energy, space, and angle imposed by all discrete ordinates codes. Because of this, Monte Carlo simulations are generally considered to be the most accurate method for high-fidelity radiation transport calculations. Until the advent of hybrid radiation transport (Section 2.3), Monte Carlo calculations were generally used on a very limited basis due to the amount of central processing unit (CPU) time required to achieve a well-converged solution.

The Monte Carlo calculations in this study used the MCNP [34] and Shift [35] computer codes.

2.2.1 Continuous-energy cross sections

Monte Carlo calculations can use either CE or MG cross-section data. CE cross-section libraries should be used whenever possible, as they provide a higher fidelity modeling of the physics involved in particle transport.

CE libraries used in Monte Carlo calculations include data for all reaction types that are present in the Evaluated Nuclear Data Files (ENDF) data [36], [37]. The cross-section values are given on an energy grid that is sufficiently dense that linear-linear interpolation between the energy grid points reproduces the evaluated cross sections within a specified tolerance. The data libraries used by MCNP have cross sections that are reproduced to a tolerance of 1% or less, with many of the more recent evaluations having tolerances of 0.1%. The data libraries used by Shift have tolerances of 0.1%. In addition to the cross-section values, CE libraries include kinematic data that provide PDFs for the energy and angular distributions of secondary particles.

2.2.2 Spatial modeling capabilities

Unlike discrete ordinates calculations, in which the problem geometry is defined based on a mesh grid, Monte Carlo calculations provide the ability to exactly model the majority of the geometric features in most radiation transport problems. Both MCNP and Shift allow modeling of linear and quadratic surfaces (planes, spheres, cylinders, cones, ellipsoids, hyperboloids, paraboloids), as well as elliptical or circular torii with axes parallel to the X-, Y-, or Z-axis.

This aspect of Monte Carlo modeling is particularly beneficial in vessel fluence analyses, as the vessel, closure head, nozzles, and reactor vessel internals (RVIs) can all be represented without the meshing artifacts that occur with discrete ordinates codes. Because there are no approximations made when modeling the surfaces listed in the previous paragraph, Monte Carlo models are sometimes referred to as *exact geometric models*. It must be kept in mind, though, that the *exactness* refers to the representation of the model as defined by the analyst. It is incumbent on the analyst to construct a model that is a faithful representation of the geometry of the system being modeled and to consider modeling issues (e.g., tolerances, as-built dimensions) that may cause the geometry model to deviate from the actual system being modeled.

2.2.3 Monte Carlo tallies

Radiation transport calculations that are performed using the discrete ordinates method provide a solution that contains the particle flux as a function of position and energy throughout the model phase space. In contrast, the output of a Monte Carlo calculation provides the flux—or response(s) based on the flux—only for locations and energy intervals that are specified in the problem input. These user defined regions of interest in the model are referred to as *tallies*. The most commonly used types of Monte Carlo tallies are briefly defined below, and further details can be found in the literature [34], [38], [10].

Cell tallies are used to obtain the particle flux in one or more cells that are part of the model geometry definition. For example, if the core barrel is modeled as a single cell, then a cell tally for the core barrel will provide the average flux in the core barrel over specified energy intervals.

Surface tallies are used to obtain the particle flux crossing a given surface that is a boundary between two adjacent cells.

Point detector tallies are deterministic estimates of the flux at a point in space rather than the flux averaged over a cell or surface. Point detectors are also referred to as *next event estimators*, as they involve the computation of the contribution to a point detector tally at source and collision events throughout a particle history, as if the next event were a particle trajectory directly to the detector point without further collision.

Mesh tallies provide estimates of the flux in every voxel of a Cartesian or cylindrical mesh that is superimposed over the problem geometry. In the limiting case, a mesh tally can provide a global solution with spatial resolution that can be comparable to, or even finer than, the spatial mesh of a discrete ordinates calculation of the same model. Until fairly recently, the use of mesh tallies in many Monte Carlo simulations was impractical because the problem run times that would be necessary to achieve acceptable convergence were unacceptably long. With the advent of hybrid radiation transport methods (Section 2.3), highly detailed mesh tallies are now feasible for many shielding analyses, including vessel fluence calculations.

Monte Carlo tallies provide estimates of the mean and variance for the tally quantities of interest. The standard approach for reporting tally results is to provide the mean and the relative error. Given a mean value μ and a variance σ^2, the tally relative error *RE* is given by

$$RE = \frac{\sigma}{\mu}. \tag{7}$$

Monte Carlo tallies are generally considered reliable if the relative error is less than 10%, except in the case of point detectors. Because point detectors are more prone to false convergence than other tally types, it is recommended that they be converged to less than 5% relative error for reliability.

2.2.4 Analog Monte Carlo and variance reduction methods

The simplest type of Monte Carlo radiation transport calculation is an analog calculation. In analog calculations, the natural probabilities for all the events that occur during a particle history are used. This approach is referred to as *analog* because it is directly analogous to the events that occur naturally during each particle's history.

Analog Monte Carlo simulations can work well when a significant fraction of the source particles contribute to the tally (or tallies) of interest. For deep penetration shielding calculations, though, the fraction of source particles that reach a tally region can be very small. For these problems, an analog simulation is not feasible because few of the source particles reach the tally region, and the statistical error associated with the tally mean is unacceptably high.

An example is a simplified one-dimensional radial geometry that is representative of the PWR reference model at the core midplane. For an analog calculation with 10^7 source neutrons, only 188 neutrons "survive" to reach the RPV. Obtaining a well-converged solution to this type of problem requires either extremely long computational times or the use of variance reduction methods, which are used to reduce the statistical uncertainties associated with each tally value.

Before discussing variance reduction methods, it is necessary to introduce the concept of a particle's *weight*. *Weight* can be defined as "an adjustment for deviating from a direct physical simulation of the transport process" [34]. In an analog Monte Carlo simulation, every particle has a unit weight, as no adjustments are made to the natural probability distributions that govern each event in the particle histories.

Most variance reduction techniques alter or bias the natural probability distributions in an effort to improve the statistical convergence of the problem tallies. The purpose of these biasing techniques is to increase the number of particles that contribute to the tallies of interest without erroneously affecting the mean tally results. At every instance in which a biased distribution is sampled, the particle weight must be adjusted so that the biased weight is given by

$$w_{biased} = w_{unbiased} \times \frac{pdf_{unbiased}}{pdf_{biased}}. \tag{8}$$

Some of the more commonly used variance reduction methods are briefly discussed in this section. Further details on these and other variance reduction methods can be found in the references.

1. **Source biasing**

   Source biasing can be illustrated using the example of Monte Carlo calculations of RPV fluence. It is well known that within the beltline region of a typical LWR, the outermost fuel assemblies dominate the neutron flux levels in the RPV. Furthermore, for the calculation of the flux of neutrons with energies above 1 MeV, it is clear that only neutrons born with an energy above 1 MeV have any chance of reaching the RPV with an energy of at least 1 MeV. It is also known that the probability of a source neutron reaching the RPV increases as the source energy increases.

   a. The fraction of source neutrons that reach the RPV can thus be increased by biasing the probabilities that govern the spatial and energy sampling of the source. The simplest example would be a case in which the spatial distribution of source neutrons in the core is uniform. In this case, any location within the core has an equal probability of being sampled as source particles are generated. Because neutrons born near the outer edge of the core have a significantly higher probability of reaching the vessel compared to neutrons born in the interior modules, the distribution used to sample the starting location can be biased so that more neutrons are born near the outer edge of the core. In doing so, the weight assigned to each source particle must be modified by the ratio of the unbiased and biased probability distributions, as shown in Eq. (8).
   b. In a similar manner, the energy distribution of the source neutrons can be biased by sampling from a probability distribution that is more heavily weighted toward higher neutron energies. An example of the effects of spatial and energy biasing for an RPV fluence calculation is provided in Section 2.3.4.

2. **Implicit capture**

   Implicit capture is a variance reduction method that is particularly useful for deep penetration shielding problems in which a particle's history may be terminated very close to a tally region. When implicit capture is used, a particle is not terminated if it undergoes an absorption reaction. Instead, the particle undergoes a scattering interaction and has its weight reduced. Given an absorption cross section Σ_a and a total cross section Σ_t, a particle whose weight is w_i is scattered and assigned a weight w_o of

$$w_o = w_i \times \left(1 - \frac{\Sigma_a}{\Sigma_t}\right) \tag{9}$$

3. Particle splitting and Russian roulette

Particle splitting and Russian roulette are variance reduction methods used to control the population of particles in various regions of the problem space. Using these methods, many particles of low weight are tracked in important regions, whereas in unimportant regions, only a few particles of high weight are tracked. The weight of each particle is adjusted at each splitting or rouletting event to ensure that the simulation remains unbiased.

An example of splitting would be a problem in which an optically thick shield—that is, a shield with a thickness of many mean free paths—is placed between a source and a detector where the flux is to be tallied in the simulation. The shield can be split into multiple layers in the Monte Carlo model, with the particles being split each time they exit one layer and enter a layer nearer the detector. With each split of a single particle into *N* particles, the weight of the *N* particles is reduced by a factor of 1/*N*.

The Russian roulette technique is essentially the opposite of splitting. Particles that are moving into unimportant regions of the model or whose weight has fallen below a specified value can be rouletted. In this process, a random number is generated and compared to a parameter such as 1/*d*, where *d* is a parameter in the range [2,10] [38]. If the random number is greater than 1/*d*, then the history is terminated. If it is not, then the history is continued, and the particle weight is increased by a factor of *d*.

Splitting and rouletting can also be performed based on energy for cases in which certain energy ranges are more important than others.

4. Weight windows

The weight-window variance reduction technique provides splitting and rouletting of particles as a function of space or of both space and energy. The technique is illustrated in Figure 2-5.

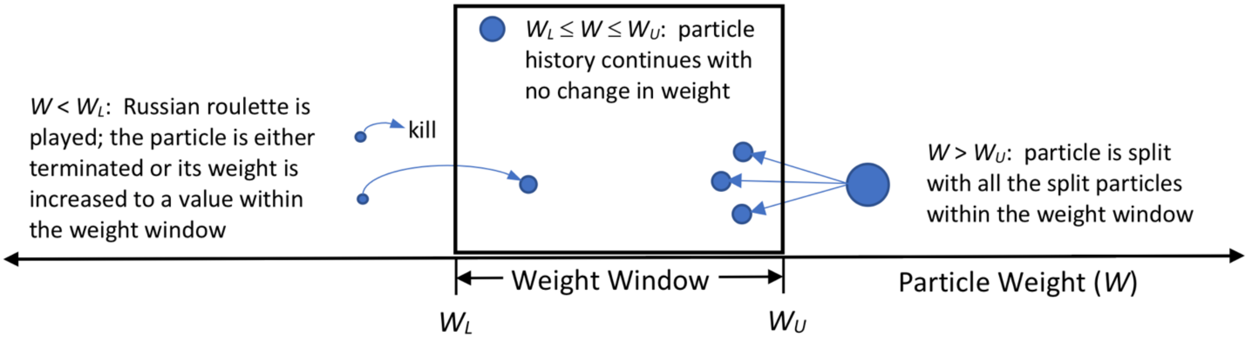

Figure 2-5 Illustration of weight windows as a variance reduction method for Monte Carlo calculations. Particles of weight *W* enter the weight window, which has lower and upper bounds of W_L and W_U. The size of each particle in the figure is proportional to its weight

For each phase space (space or space-energy) cell in the weight-window map, the user supplies a lower weight bound W_L. The upper weight bound W_U is calculated as a user specified multiple of the lower weight bound. These bounds define a window of acceptable particle weights. If a particle's weight is above the upper bound, then it is split so that all of the split particles are within the weight window. In the illustration, the particle that enters with a weight W above the weight window's upper boundary W_U is split into three particles, each of which has weight $W/3$. If a particle's weight is below the lower bound, then Russian roulette is played. The particle history is either terminated, or the particle continues with its weight increased to a value within the window. In the illustration, the particle is killed with a 50% probability or continued with a weight of 2W. Particles with weights within the lower and upper bounds are continued with no change in weight.

Judicious use of variance reduction techniques can make it possible to obtain well-converged Monte Carlo simulations in substantially less time than in analog simulations. The hybrid radiation transport method provides an efficient, effective means of generating space-energy weight windows and source biasing parameters that can reduce Monte Carlo run times by orders of magnitude.

2.3 Hybrid methods

Hybrid methods are a class of techniques used to obtain a solution to the Boltzmann transport equation using a combination of deterministic and stochastic calculations. The deterministic calculations in hybrid calculation sequences are used to generate variance reduction (VR) parameters (space- and energy-dependent weight windows and source biasing parameters) that are then used in Monte Carlo transport calculations to obtain the desired quantities of interest. Hybrid calculations can be run using ADVANTG [39] and MCNP, and they can also be run with the Denovo and Shift codes in Exnihilo, the massively parallel radiation transport code suite developed at Oak Ridge National Laboratory (ORNL).

2.3.1 Particle importance and adjoint flux calculations

Hybrid radiation transport methods are based on the concept of *particle importance*. A particle's importance is a measure of how likely it is to contribute to a response of interest, such as the flux or a reaction rate at a particular location within a particular energy range. For example, a neutron with an energy of 2 MeV in the downcomer region of a PWR has a much greater probability of contributing to the fast fluence in the RPV than a neutron of the same energy in an inner assembly of the core. Therefore, the neutron in the downcomer has a greater importance than the equivalent-energy neutron within the core.

The particle importance as a function of space and energy can be obtained by solving the adjoint form of the Boltzmann transport equation [9]. In an adjoint calculation, the adjoint source is the response of interest at a specified location or locations. For example, if the response of interest is the fast fluence in the beltline region of the RPV, then the adjoint source is taken to be the flux of neutrons with energy greater than 1 MeV within the RPV over the axial extent of the beltline region. Figure 8 shows the resulting adjoint flux, and hence the importance, for neutrons that will reach the RPV with energies greater than 1 MeV. As expected, the outermost fuel pins are the most important core regions with respect to RPV fluence.

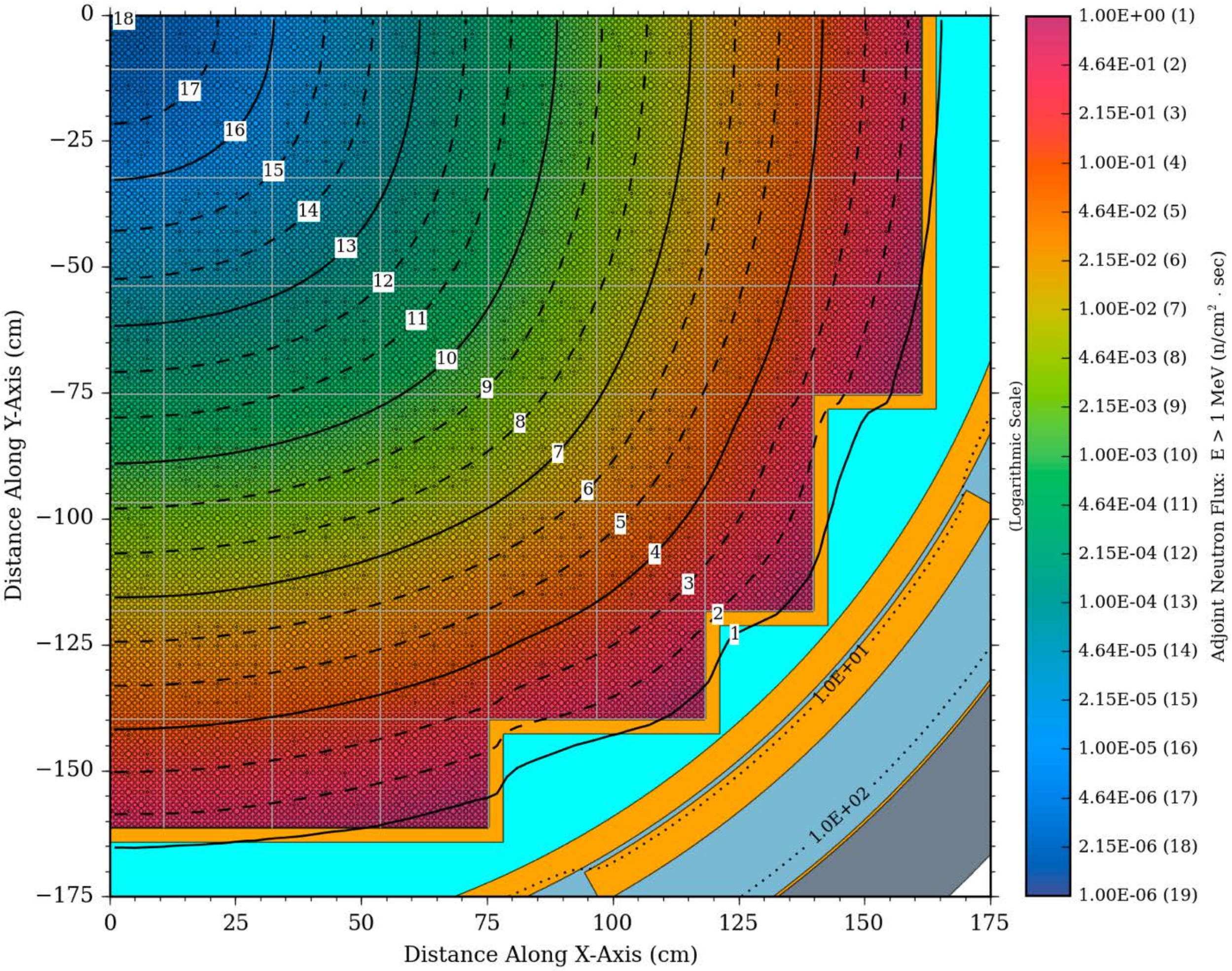

Figure 2-6 Adjoint fast (E > 1 MeV) neutron flux for the PWR reference model with homogenized fuel assemblies. Plan view at Z= 200 cm. The adjoint source region is in the reactor from an elevation of 0–400 cm

2.3.2 Hybrid calculations using the CADIS methodology

The Consistent Adjoint Driven Importance Sampling (CADIS) methodology [40], [41] was developed to generate space-energy weight windows and consistently biased sources to accelerate the convergence of source-detector type problems in which a single localized response (tally) is the quantity of interest in a Monte Carlo simulation. The basic steps of a hybrid calculation sequence using the CADIS methodology are listed here. A detailed derivation can be found in the references.

1. The adjoint flux for the problem of interest is solved using a discrete ordinates calculation. The adjoint source is modeled as the response of interest (e.g., the fast flux) at the location where the Monte Carlo tally will be computed. As noted in Section 2.3.1, the adjoint scalar flux $\varphi^{+}_{g}(r)$ represents the importance of a particle such as a neutron at a given location and energy in contributing to the response of interest at the adjoint source location (i.e., the Monte Carlo tally location).

2. Weight-window target values $w_g(\mathbf{r})$ for the Monte Carlo calculations are constructed as

$$w_g(r) = \frac{R}{\phi_g^+(\boldsymbol{r})}, \tag{10}$$

   where *R* is an estimate of the response of interest based on the adjoint calculation.

3. A source distribution *q* that is biased in space and energy in a manner that is consistent with the weight windows is constructed as

$$\hat{q}_g(\boldsymbol{r}) = \frac{q_g(\boldsymbol{r})\phi_g^+(\boldsymbol{r})}{R}, \tag{11}$$

   where $q_g(\mathbf{r})$ is the source for the forward transport problem.

4. The Monte Carlo calculation is run using the space-energy weight windows from Eq. (10) and the biased source definition from Eq. (11).

The CADIS methodology has been applied to a wide range of shielding calculations. It can provide speedups of orders of magnitude in obtaining a well-converged Monte Carlo solution.

2.3.3 Hybrid calculations using the FW-CADIS methodology

While the CADIS method is very effective for providing substantial speedups in the convergence of Monte Carlo simulations for a single tally, it is not well suited to Monte Carlo simulations in which tallies at multiple locations or mesh tallies are the quantities of interest. To converge multiple tallies to the same relative uncertainty in a single Monte Carlo simulation using CADIS, the adjoint source corresponding to each tally must be weighted inversely with the expected tally value [42].

In the FW-CADIS (Forward-Weighted CADIS) method, a forward discrete ordinates calculation, is performed to estimate the response of interest at each location to be tallied in the Monte Carlo simulation. The inverse of those response values is then used to weight the adjoint source strength at each location. Using this adjoint source, the CADIS methodology outlined in Steps 1–4 of Section 2.3.2 is used to construct weight windows and a biased source for use in the Monte Carlo simulation.

2.3.4 An FW-CADIS example

The FW-CADIS methodology can be illustrated with the following example. Consider a Monte Carlo simulation that is performed to obtain the fast (E > 1 MeV) neutron flux in the RPV of the PWR reference model over an axial range that extends from the top of the lower head to the bottom of the closure head. The flux will be obtained using a cylindrical mesh tally in a Monte Carlo simulation. The first step of the FW-CADIS sequence is a discrete ordinates calculation that provides an estimate of the fast flux for the entire solution space. The inverse of the discrete ordinates forward flux is then used to weight an adjoint source that corresponds to the cylindrical mesh tally for the Monte Carlo simulation.

Using the adjoint source derived from the forward discrete ordinates solution, an adjoint discrete ordinates calculation is then run to construct the space-energy weight windows and the biased source for the Monte Carlo simulation.

Figure 2-7 illustrates the weight window lower bounds for two energy groups from the SCALE [43] 27N19G cross-section library that is often used for generating variance reduction parameters. (Recall from Eq. 10 that the weight window bounds are inversely proportional to the adjoint flux, so regions of high importance have correspondingly low weight window bounds.) The left side of the figure shows the weight window's lower bounds for neutrons with energies between 6.3763 and 20.0 MeV (group 1 of the 27-group structure). The right side shows the lower bounds for neutrons with energies between 0.90718 and 1.4227 MeV (group 5). It is clear from this figure that the higher energy neutrons are much more important in all regions of the model space and that they are attenuated much less rapidly. (Note that since weight window bounds are inversely proportional to the importance estimate provided by the adjoint flux, locations with higher weight window bounds are less likely to contribute to the tallies of interest.)

Because the outer regions of the core have higher importance with regard to contributions to the flux in the RPV, the spatial source distribution is biased so that more particles are sampled from the outer assemblies, with their source weights reduced to maintain an unbiased[6] simulation. Figure 2-8 shows the spatial distribution and particle weights for 10^6 source neutrons in an MCNP simulation. The left side of the figure shows the distribution for a case with no VR applied. As expected, all the source neutrons have a weight of 1.0, and they are distributed uniformly throughout the core for this example calculation, which has a spatially uniform source. The right side illustrates the spatial distribution for an MCNP simulation using variance reduction parameters from ADVANTG. The spatial distribution that will be sampled in the Monte Carlo simulation is strongly biased toward the outer assemblies, with the source weights being reduced by several orders of magnitude because of the higher sampling probability from the biased distribution. Only a few source points are sampled from the interior portion of the core, and their weights are increased by several orders of magnitude to offset their low sampling probability.

In addition to the spatial biasing of the source in the reactor core, the energy distribution is biased in a manner that is consistent with the weight windows. Figure 2-9 shows the adjoint flux spectra at five locations within the core. In addition to the decrease in the magnitude of the adjoint flux at locations further from the edge of the core, it is also clear that the spectrum of the adjoint flux changes significantly as a function of location. In particular, for the two locations furthest from the edge of the core (Locations 1 and 2), the variation in the spectrum is 10 orders of magnitude or more. This is consistent with the fact that neutrons born with energies in the lower part of the energy range of interest are much less likely to be transported from the inner assemblies to the RPV than neutrons born near the periphery of the core.

The effect of biasing the source in energy is illustrated in Figure 2-10. The left side of this figure shows the energy distribution for neutrons born with energies greater than 1 MeV when sampled from a Watt fission spectrum. The right side shows the energy distribution for the biased source. It can be seen that the source is biased toward higher energies, which is

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[6] In Monte Carlo terminology, an unbiased simulation is sometimes referred to as a *fair game*.

consistent with the greater importance of high-energy neutrons contributing to the fast neutron flux in the RPV.

The result of applying the weight windows and source biasing to a Monte Carlo simulation is illustrated in Figure 2-11. The left side of this figure shows the collision locations and particle weights for the small number of the 10^6 source neutrons that are transported to the RPV in a simulation with no variance reduction applied. Note that some of the particle weights are below 1.0, even though weight windows have not been applied. These variations are due to the use of the default implicit capture method in the MCNP simulation. The right side of the figure shows the collision locations and particle weights for the simulation using ADVANTG-generated weight windows and source biasing. Here it can be seen that there is an increase of several orders of magnitude in the number of particles experiencing collisions in the RPV, with all of those collisions occurring with particles whose weight has been reduced to provide an unbiased simulation.

2.3.5 Accuracy considerations for the discrete ordinates forward and adjoint calculations in hybrid calculations

One of the key factors of the hybrid radiation transport method is that the deterministic calculations performed to generate space-energy weight windows and source biasing parameters do not need to be highly accurate. Moderate-fidelity discrete ordinates calculations are capable of producing variance reduction parameters that are highly effective in reducing the computational time required to achieve well-converged Monte Carlo tallies. Consequently, the Denovo calculations that are run to generate variance reduction parameters with ADVANTG and Shift are typically performed using broad-group cross-section libraries and relatively coarse angular and spatial discretizations. This approach reduces the discrete ordinates run time and computational resource requirements (e.g., number of processors and amount of memory) while still providing highly effective variance reduction.

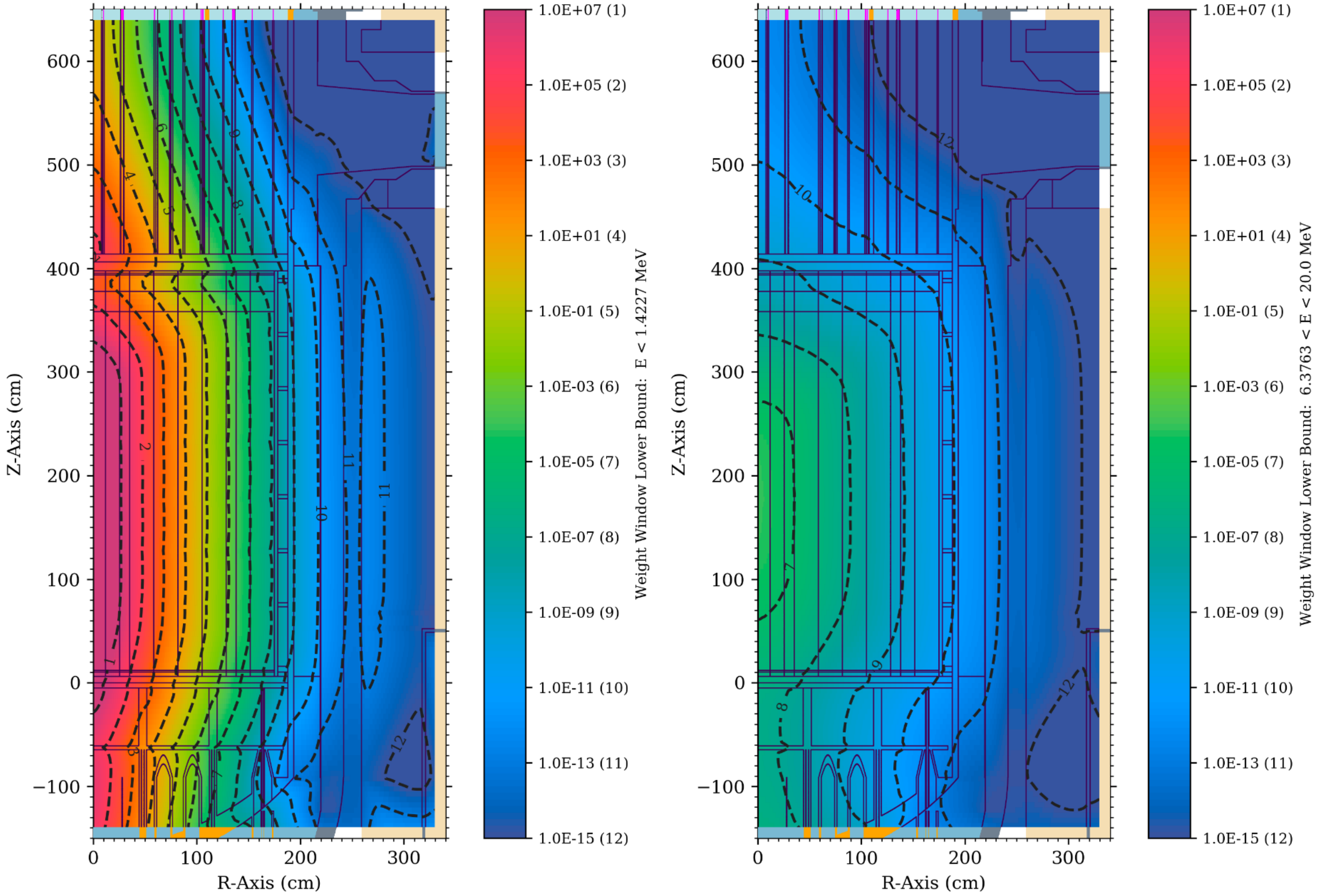


Figure 2-7 **ADVANTG-generated weight-window lower bounds for neutrons with energies from 6.3763–20.0 MeV and for those with energies from 0.90718–1.4227 MeV. The adjoint source region for the ADVANTG calculation is the RPV, including the inlet and outlet nozzles, from Z=-100 cm to 648 cm**

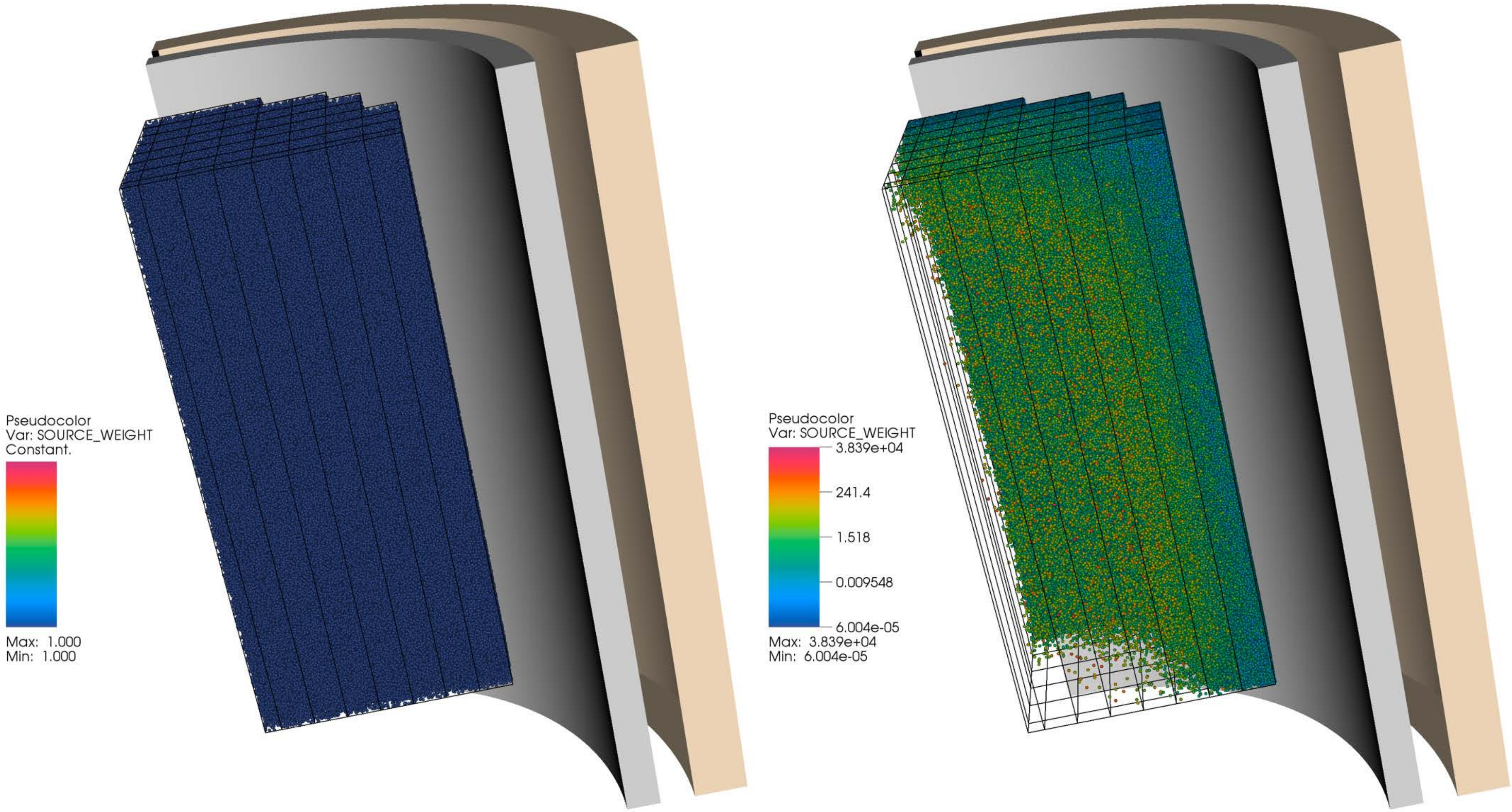
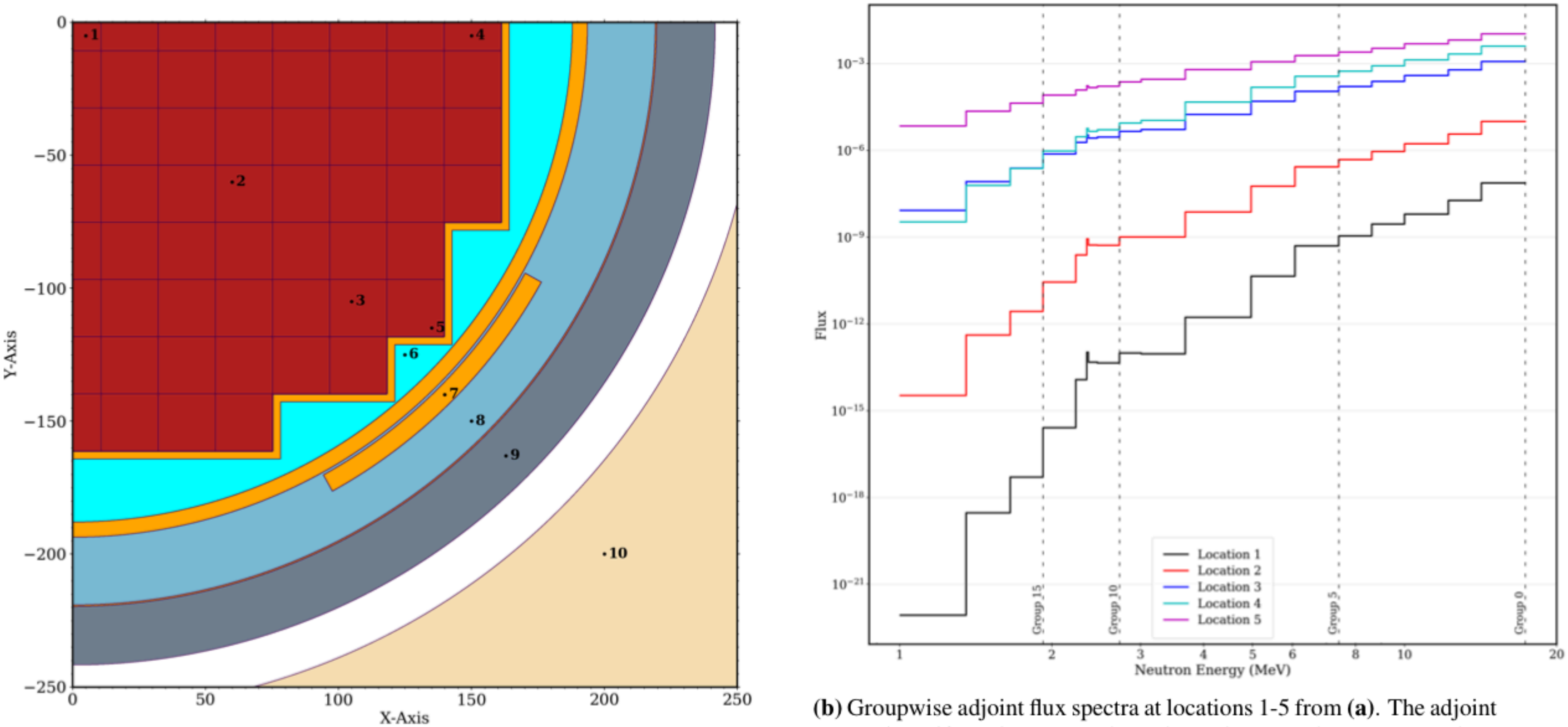


Figure 2-8 **Spatial distribution and weights of 10^6 source neutrons for an analog MCNP simulation and an MCNP simulation using weight windows and source biasing generated by ADVANTG. The adjoint source region for the ADVANTG calculation is the RPV, including the inlet and outlet nozzles, from Z = -100 cm to 648 cm. Note that the biased source samples are primarily from the outermost assemblies, which is consistent with the adjoint fluxes and weight windows shown in Figure 2-6 and Figure 2-7**



(a) Geometry plan view at z = 200 cm for the PWR reference model with homogenized assemblies. The location numbers correspond to the flux spectra shown in **(b)**.

(b) Groupwise adjoint flux spectra at locations 1-5 from **(a)**. The adjoint source is uniform in energy throughout the RV. The group numbers correspond to the BUGLE-B7 group structure. The local peaks in each spectrum for the energy group from 2.3457 to 2.3653 MeV are caused by a resonance minimum in the O-16 cross section in that energy range.

Figure 2-9 Plan view of the PWR reference model with homogenized assemblies at Z = 200 cm, and the adjoint flux spectra at five locations in the core. The slight uptick in the adjoint flux spectrum in group 13 is due to an antiresonance (resonance minimum) in the ^{16}O scattering cross section

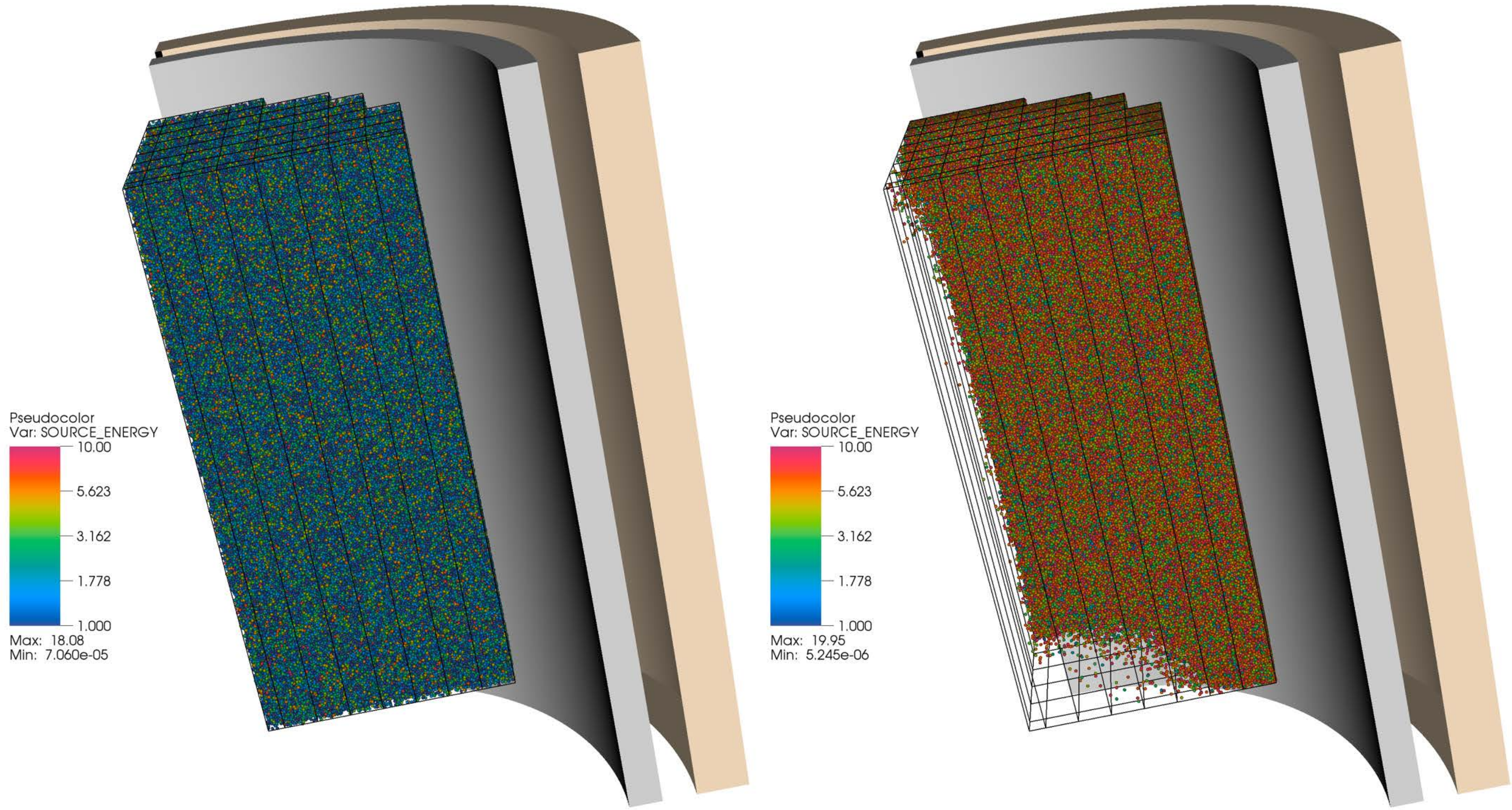
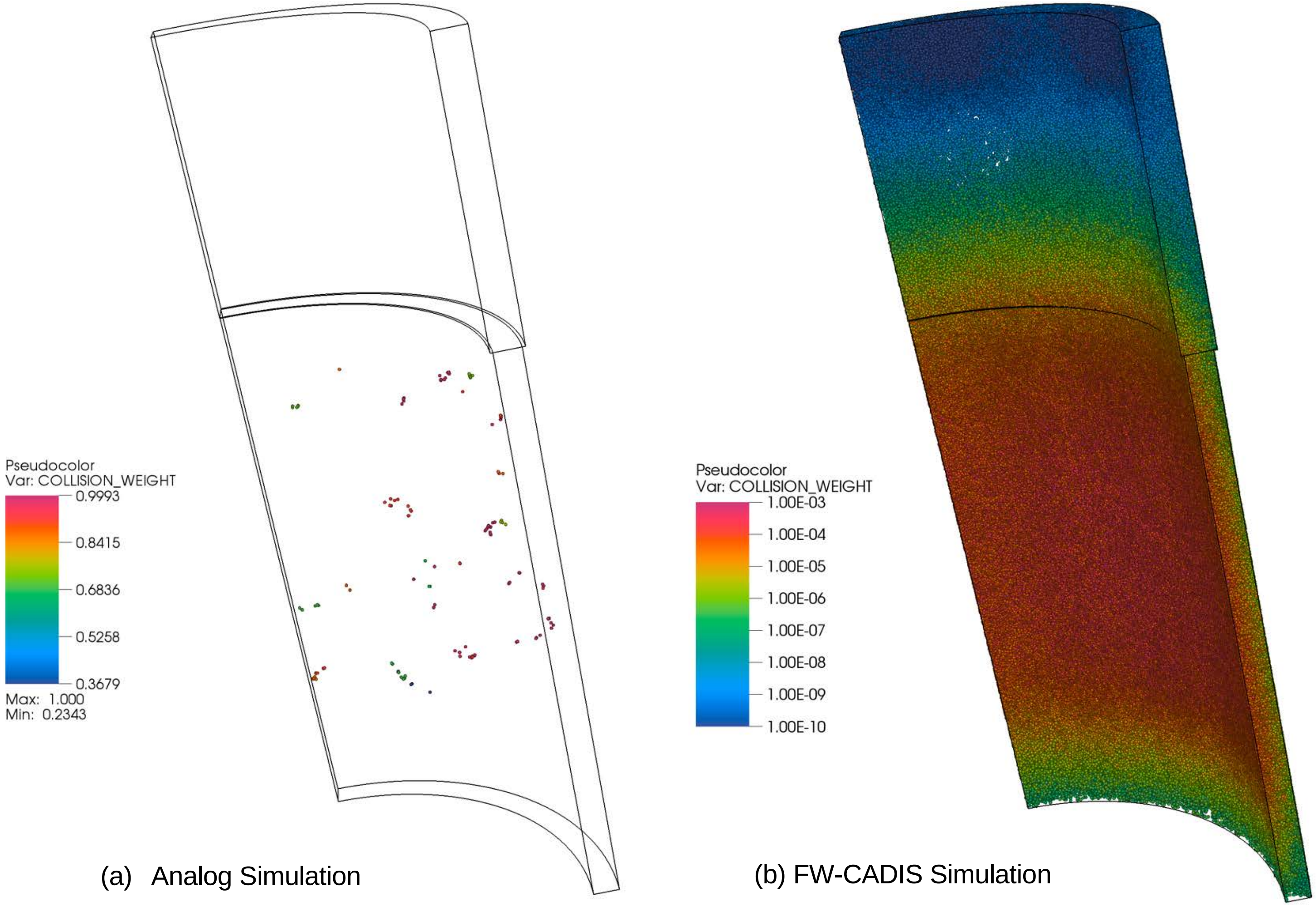


Figure 2-10 Spatial distribution and energies of 10^6 source neutrons for an analog MCNP simulation and an MCNP simulation using weight windows and source biasing generated by ADVANTG. The adjoint source region for the ADVANTG calculation is the RPV, including the inlet and outlet nozzles, from Z = -100 cm to 648 cm. Note that the biased source samples are primarily from the outermost assemblies, consistent with the adjoint fluxes and weight windows shown in Figure 2-6 and Figure 2-7



(a) Analog Simulation

(b) FW-CADIS Simulation

Figure 2-11 Spatial distribution and weights of fast (E > 1 MeV) neutrons that enter the RPV from the sampling of 10^6 source neutrons. View (a) is for an analog simulation, and view (b) is for a simulation using weight windows and source biasing parameters from ADVANTG

2.4 Computer codes used in this study

The radiation transport calculations used for the analyses in this report were performed using the deterministic and hybrid methods. The codes used for these calculations are briefly described in this section.

2.4.1 Deterministic calculations

The deterministic calculations used for the work presented in this report, including the deterministic portion of all hybrid calculations, were performed using the Denovo code, which has superseded the 2D DORT and 3D TORT codes in the DOORS package [15]. Codes in the DOORS package were developed at ORNL but are no longer maintained. The final version of DOORS (DOORS3.2a) was released in May 2007. Denovo has been used for all subsequent discrete ordinates code development at ORNL.

While DORT and TORT support cylindrical and Cartesian geometries, Denovo is limited to Cartesian geometry. However, the use of mixed materials (Section 2.1.2) in Denovo reduces the solution artifacts that can occur with Cartesian representation of cylindrical surfaces, such as the inner and outer radius of the RPV, when no material mixing is performed. Comparison of Denovo and hybrid calculations (which provide exact modeling of both cylindrical and Cartesian geometry in the Monte Carlo calculation) in later sections of this report demonstrates that with adequate space, energy, and angular discretization, Denovo is capable of producing solutions that capture the peak fast fluence values at the inner surfaces of RPVs.

2.4.2 Hybrid calculations

The hybrid calculations presented in this report were run using two hybrid sequences: MCNP/ADVANTG and Shift. In the first approach, the ADVANTG code is used to generate space-energy weight windows and a consistently biased source for use in MCNP Monte Carlo calculations. The deterministic portion of the hybrid sequence, which is driven by ADVANTG, uses the Denovo discrete ordinates code. After the variance reduction parameters are generated by ADVANTG, they are used in an MCNP calculation to obtain the final Monte Carlo solution.

When hybrid calculations are run using Shift, Denovo is first executed to generate the variance reduction parameters (weight windows and source biasing), as with the ADVANTG/MCNP sequence. Shift then runs a Monte Carlo simulation using those variance reduction parameters in the same manner as the ADVANTG/ MCNP sequence.

Because the deterministic calculations in a hybrid sequence only require moderate fidelity to generate effective variance reduction parameters, the Denovo calculations in the ADVANTG/MCNP hybrid sequence are typically performed with space, energy, and angular discretizations that are relatively coarse compared to what would be appropriate for a deterministic-only calculation.

3 LITERATURE REVIEW OF CALCULATIONAL METHODS USED FOR REACTOR PRESSURE VESSEL FLUENCE CALCULATIONS

The RPV fluence calculational methodologies developed by nuclear power industry companies that follow Regulatory Guide 1.190 and are approved by the NRC are predominantly based on the 2D/1D FLUX synthesis technique, which synthesizes 3D solutions from 2D and 1D calculations [44], [45], [46]. The 2D/1D flux synthesis technique has been shown to provide acceptable results when applied to RPV dosimetry at axial regions that directly surround the active core height. However, this methodology has limitations when used in the extended beltline region of the RPV. Recently, the 3D RAPTOR-M3G code [47] has been approved by the NRC as a fluence methodology for the RPV traditional beltline region.

There is no currently approved methodology for fluence analysis in the RPV extended beltline region. However, numerous 60-calendar-year NPP heat-up and cooldown limit curve reports have been submitted to the NRC that have used the 2D/1D fluence rate methodology in the RPV extended beltline region [48], [49], [50], [51], [52].

Recent NPP subsequent license renewal (SLR) applications for 80-calendar-year operation ([53], [54], [55]) have also used the 2D/1D fluence rate synthesis methodology in the RPV extended beltline region. Volume 2 of NUREG-2191 [56] states the following in Section X.M2:

> *… The methods developed and approved using the guidance contained in RG 1.190 are specifically intended for determining neutron fluence in the region of the RPV close to the active fuel region of the core and are not intended to apply to vessel regions significantly above and below the active fuel region of the core, nor to RVI components. Therefore, the use of RG 1.190-adherent methods to estimate neutron fluence for the RPV regions significantly above and below the active fuel region of the core and RVI components may require additional justification, even if those methods were approved by the NRC for RPV neutron fluence calculations …*

The following excerpts from the SLR applications provide a representative sample of the statements that were made to justify the use of the 2D/1D fluence rate synthesis methodology in the RPV extended beltline region:

> *… End-of-license (life) 80-year fluence in PTN [Turkey Point] RPV regions above the active fuel region (e.g., in nozzle locations) are currently projected to exceed the* 1×10^{17} *n/cm*^{2} *threshold prior to the end of the [subsequent period of extended operation] SPEO, whereas RPV locations below the active fuel region do not, as described in Section 4.2.1. FPL follows related industry efforts, such as those from the Pressurized Water Reactor Owners Group (PWROG), and will use the information from those efforts to provide additional justification for fluence determinations in those areas prior to entering the SPEO* [53].

> *… The methods and assumptions used for the original beltline region are considered appropriate for the beltline region that has been extended to encompass materials projected to experience fluence in excess of* 1×10^{17} *n/cm*^{2} *(E > 1 MeV) at 70 EFPY, since the extended region does not extend significantly above or below the active fuel region and no additional reactor vessel plate materials (heat numbers) or welds are projected to experience fluence in excess of* $1 \text{ x } 10^{17}$ *n/cm*^{2} *(E > 1 MeV) … [55].*

> *… Some of the inlet and outlet nozzles are projected to experience neutron fluence in excess of 1 × 10^{17} n/cm^2. These inlet and outlet nozzles are treated as extended beltline material for subsequent license renewal … Studies to date have shown that the DORT model calculates fluence in the Z direction above the core more conservatively than three-dimensional models such as RAPTOR-M3G … The fluence projections used in the SLR application conservatively utilized a constant material mixture of 90% water and 10% steel above and below the core. A sensitivity study was performed to show that this assumption was conservative compared to an analysis based upon more representative plant specific material mixture data above and below the core" [54].*

Publications focused on fluence calculations in the RPV extended beltline region or on RVI components above and below the core height are limited [57], [58], [59], [60], [61]. Hopkins et al. [57] present results from analysis of ex-vessel capsules in a PWR at nozzle support elevations using the 2D/1D fluence rate synthesis technique. Comparison of calculations to measurements demonstrates the limitations of this analysis methodology in the region of the nozzles and supports. In a paper by Chen et al. [58], 3D transport calculations and 2D/1D fluence rate synthesis calculations are compared at the core barrel inner and outer radii in a PWR. Their work concludes that the 2D/1D fluence rate synthesis technique cannot accurately capture some important details of 3D geometry. Even within the active core height, the 2D/1D fluence rate synthesis technique can provide inadequate results with complex geometries, such as RPV internal structures with irregular shapes. Lippincott and Manahan [59] demonstrate that in regions above and below the core of a BWR, the separability of the azimuthal and axial fluence rate shape in the 2D/1D fluence rate synthesis technique does not hold. Amiri et al. [60] present the results of a dosimetry evaluation that was performed for above-core zirconium alloy samples in a PWR. The results indicate that the use of core-averaged, cycle-specific axial power distributions can cause significant underestimation of fluence with the 2D/1D fluence rate synthesis technique. Fischer and Kim [61] performed retrospective dosimetry analysis for PWR top support plug samples from scrap surveillance capsule material. Their comparisons of measurements with calculations using the 3D RAPTOR-M3G code generally demonstrate good agreement.

An objective of the ongoing PWROG program is to qualify the fluence determination in the RPV extended beltline locations. This program involves collecting measurements in the RPV extended beltline regions of operating PWRs [62]. The measurement data collected from this PWROG program will provide valuable information for validating fluence methodologies in the RPV extended beltline region.

4 ANALYSIS MODELS

4.1 PWR model

The PWR reference model is based on Watts Bar Nuclear Plant Unit 1 (WBN1). WBN1 is a Westinghouse four-loop design with a licensed power of 3,456 MWt. The fuel assemblies are a Westinghouse 17 × 17 design with three ^{235}U enrichments: 2.11, 2.619, and 3.1 wt%. Geometry and material specifications for the core were obtained from Godfrey [63]. Materials and dimensions for internal structures, the RPV, and the inlet and outlet nozzles were adopted from several sources [64], [65], [66], [67], [68], [69]. This four-loop model has quarter-core symmetry, and calculations were performed using a quarter-core model.

The model was developed using MCNP geometry. The MCNP model can be used to run MCNP with ADVANTG for hybrid radiation transport, Shift through Omnibus for hybrid radiation transport, and Denovo through Omnibus for deterministic calculations. Selected parameters in the PWR model are provided in Table 4-1. The material definitions are provided in Table 4-2.

Elevation and plan views of the PWR model are shown in Figure 4-1 through Figure 4-5.

Table 4-1 Selected model parameters for the PWR reference model

Parameter	Measurement
Thermal power	3,456 MW(t)
Core operating pressure	2,250 PSIA
Coolant temp:	
Inlet	559 °F
Outlet	622.5 °F
Core	592.5 °F
Baffle plate thickness	2.85 cm
Core barrel:	
Inner radius	187.96 cm
Outer radius	193.68 cm
Neutron pad:	
Inner radius	194.64 cm
Outer radius	201.63 cm
RPV liner inner radius:	
Below Z = 402.59 cm	219.15 cm
Above Z = 402.59 cm	216.45 cm
RPV inner radius:	
Below Z = 402.59 cm	219.71 cm
Above Z = 402.59 cm	217.01 cm
RPV outer radius:	
Below Z = 402.59 cm	241.7 cm
Above Z = 402.59 cm	244.32 cm
Bioshield inner radius:	
Below Z = 630.48 cm	259.08 cm
Above Z = 630.48 cm	277.73 cm

Table 4-2 Material definitions in the PWR reference model. The isotope identifiers are of the form ZZAAA, where ZZ is the atomic number (e.g., 8 for oxygen) and AAA is the atomic mass (e.g., 16). The units are atoms/b·cm, where 1 b = 10^{-24} cm^2

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
UO$_2$ fuel; 2.11% enrichment	10.257	8016	4.5758E-02
		92234	4.0480E-06
		92235	4.8879E-04
		92236	2.2375E-06
		92238	2.2384E-02
UO$_2$ fuel; 2.619% enrichment	10.257	8016	4.5760E-02
		92234	5.0949E-06
		92235	6.0671E-04
		92236	2.7680E-06
		92238	2.2266E-02
UO$_2$ fuel; 3.1% enrichment	10.257	8016	4.5763E-02
		92234	6.1184E-06
		92235	7.1811E-04
		92236	3.2985E-06
		92238	2.2154E-02
Zircaloy 4	6.56	24050	3.3011E-06
		24052	6.3658E-05
		24053	7.2184E-06
		24054	1.7968E-06
		26054	8.6828E-06
		26056	1.3630E-04
		26057	3.1478E-06
		26058	4.1891E-07
		40090	2.1886E-02
		40091	4.7728E-03
		40092	7.2953E-03
		40094	7.3931E-03
		40096	1.1911E-03
		50112	4.6805E-06
		50114	3.1847E-06
		50115	1.6406E-06
		50116	7.0159E-05
		50117	3.7058E-05
		50118	1.1687E-04
		50119	4.1449E-05
		50120	1.5721E-04
		50122	2.2341E-05
		50124	2.7938E-05
		72174	3.5413E-09
		72176	1.1642E-07
		72177	4.1167E-07
		72178	6.0379E-07
		72179	3.0145E-07
		72180	7.7642E-07

Table 4-2. Material definitions in the PWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Inconel	8.19	14028	4.0487E-03
		14029	2.0568E-04
		14030	1.3574E-04
		22046	2.1251E-04
		22047	1.9165E-04
		22048	1.8989E-03
		22049	1.3936E-04
		22050	1.3343E-04
		24050	6.1820E-04
		24052	1.1921E-02
		24053	1.3518E-03
		24054	3.3649E-04
		26054	3.6134E-04
		26056	5.6723E-03
		26057	1.3100E-04
		26058	1.7433E-05
		28058	4.1759E-02
		28060	1.6086E-02
		28061	6.9923E-04
		28062	2.2295E-03
		28064	5.6778E-04
Stainless steel 304	8.0	6012	3.1745E-04
		6013	3.4334E-06
		14028	1.5819E-03
		14029	8.0363E-05
		14030	5.3038E-05
		15031	6.9991E-05
		24050	7.6489E-04
		24052	1.4750E-02
		24053	1.6725E-03
		24054	4.1633E-04
		25055	1.7538E-03
		26054	3.4476E-03
		26056	5.4121E-02
		26057	1.2499E-03
		26058	1.6634E-04
		28058	5.3084E-03
		28060	2.0448E-03
		28061	8.8885E-05
		28062	2.8340E-04
		28064	7.2175E-05

Table 4-2. Material definitions in the PWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Low-Alloy carbon steel (RPV)	7.7879	6012	9.7048E-04
		6013	1.0496E-05
		14028	3.4216E-04
		14029	1.7326E-05
		14030	1.1501E-05
		24050	5.5180E-06
		24052	1.0641E-04
		24053	1.2065E-05
		24054	3.0035E-06
		25055	1.1200E-03
		26054	4.8320E-03
		26056	7.5117E-02
		26057	1.7199E-03
		26058	2.2931E-04
		28058	3.0311E-04
		28060	1.1588E-04
		28061	5.0171E-06
		28062	1.5940E-05
		28064	4.0403E-06
Pyrex	2.2458	5010	9.6145E-04
		5011	3.8944E-03
		8016	4.6688E-02
		14028	1.8164E-02
		14029	9.2273E-04
		14030	6.0898E-04
B_4C	1.76	5010	1.5269E-02
		5011	6.1458E-02
		6012	1.8976E-02
		6013	2.0524E-04
AgInCd	10.2	47107	2.3616E-02
		47109	2.1940E-02
		48106	3.4152E-05
		48108	2.4316E-05
		48110	3.4124E-04
		48111	3.4971E-04
		48112	6.5927E-04
		48113	3.3387E-04
		48114	7.8494E-04
		48116	2.0464E-04
		49113	3.4426E-04
		49115	7.6804E-03

Table 4-2. Material definitions in the PWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Upper core plate support column and control rod guide tube	5.581	1001	1.6540E-02
		5010	3.5277E-06
		5011	1.4199E-05
		6012	2.1163E-04
		6013	2.2890E-06
		8016	8.2701E-03
		14028	1.0546E-03
		14029	5.3575E-05
		14030	3.5358E-05
		15031	4.6661E-05
		24050	5.0993E-04
		24052	9.8334E-03
		24053	1.1150E-03
		24054	2.7756E-04
		25055	1.1692E-03
		26054	2.2984E-03
		26056	3.6080E-02
		26057	8.3326E-04
		26058	1.1089E-04
		28058	3.5389E-03
		28060	1.3632E-03
		28061	5.9257E-05
		28062	1.8894E-04
		28064	4.8116E-05
Type 04 concrete	2.35	1001	7.7679E-03
		8016	4.4081E-02
		11023	1.0479E-03
		12024	1.1744E-04
		12025	1.4868E-05
		12026	1.6370E-05
		13027	2.3884E-03
		14028	1.4675E-02
		14029	7.4547E-04
		14030	4.9199E-04
		16032	5.3526E-05
		16033	4.2261E-07
		16034	2.3948E-06
		16036	5.6349E-09
		19039	6.4646E-04
		19040	8.1103E-08
		19041	4.6653E-05
		20040	2.8262E-03
		20042	1.8862E-05
		20043	3.9357E-06
		20044	6.0814E-05
		20046	1.1661E-07

Table 4-2. Material definitions in the PWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Type 04 concrete (continued)		20048	5.4518E-06
		26054	1.8281E-05
		26056	2.8697E-04
		26057	6.6274E-06
		26058	8.8198E-07
Homogenized top nozzle	3.9041	1001	2.6768E-02
		2004	3.6442E-09
		5010	2.1140E-05
		5011	7.5484E-06
		6012	1.1046E-04
		6013	1.1947E-06
		8016	1.3384E-02
		14028	5.5045E-04
		14029	2.7963E-05
		14030	1.8455E-05
		15031	2.4355E-05
		24050	2.6615E-04
		24052	5.1325E-03
		24053	5.8199E-04
		24054	1.4487E-04
		25055	6.1027E-04
		26054	1.1997E-03
		26056	1.8832E-02
		26057	4.3492E-04
		26058	5.7879E-05
		28058	1.8471E-03
		28060	7.1151E-04
		28061	3.0929E-05
		28062	9.8615E-05
		28064	2.5114E-05
		47107	1.6658E-03
		47109	1.5476E-03
		48106	2.4090E-06
		48108	1.7152E-06
		48110	2.4071E-05
		48111	2.4668E-05
		48112	4.6503E-05
		48113	2.3550E-05
		48114	5.5369E-05
		48116	1.4435E-05
		49113	2.4283E-05
		49115	5.4176E-04

Table 4-2. Material definitions in the PWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Homogenized bottom nozzle	3.6885	1001	2.7718E-02
		5010	5.9117E-06
		5011	2.3795E-05
		6012	1.2989E-04
		6013	1.4049E-06
		8016	1.3859E-02
		14028	6.4729E-04
		14029	3.2883E-05
		14030	2.1702E-05
		15031	2.8639E-05
		24050	3.1298E-04
		24052	6.0355E-03
		24053	6.8438E-04
		24054	1.7036E-04
		25055	7.1763E-04
		26054	1.4107E-03
		26056	2.2145E-02
		26057	5.1143E-04
		26058	6.8062E-05
		28058	2.1721E-03
		28060	8.3669E-04
		28061	3.6370E-05
		28062	1.1596E-04
		28064	2.9533E-05
Inlet coolant	0.7419	1001	4.9548E-02
		5010	1.0568E-05
		5011	4.2535E-05
		8016	2.4774E-02
Core average coolant	0.7025	1001	4.6917E-02
		5010	1.0006E-05
		5011	4.0276E-05
		8016	2.3458E-02
Outlet coolant	0.6584	1001	4.3971E-02
		5010	9.3782E-06
		5011	3.7748E-05
		8016	2.1986E-02

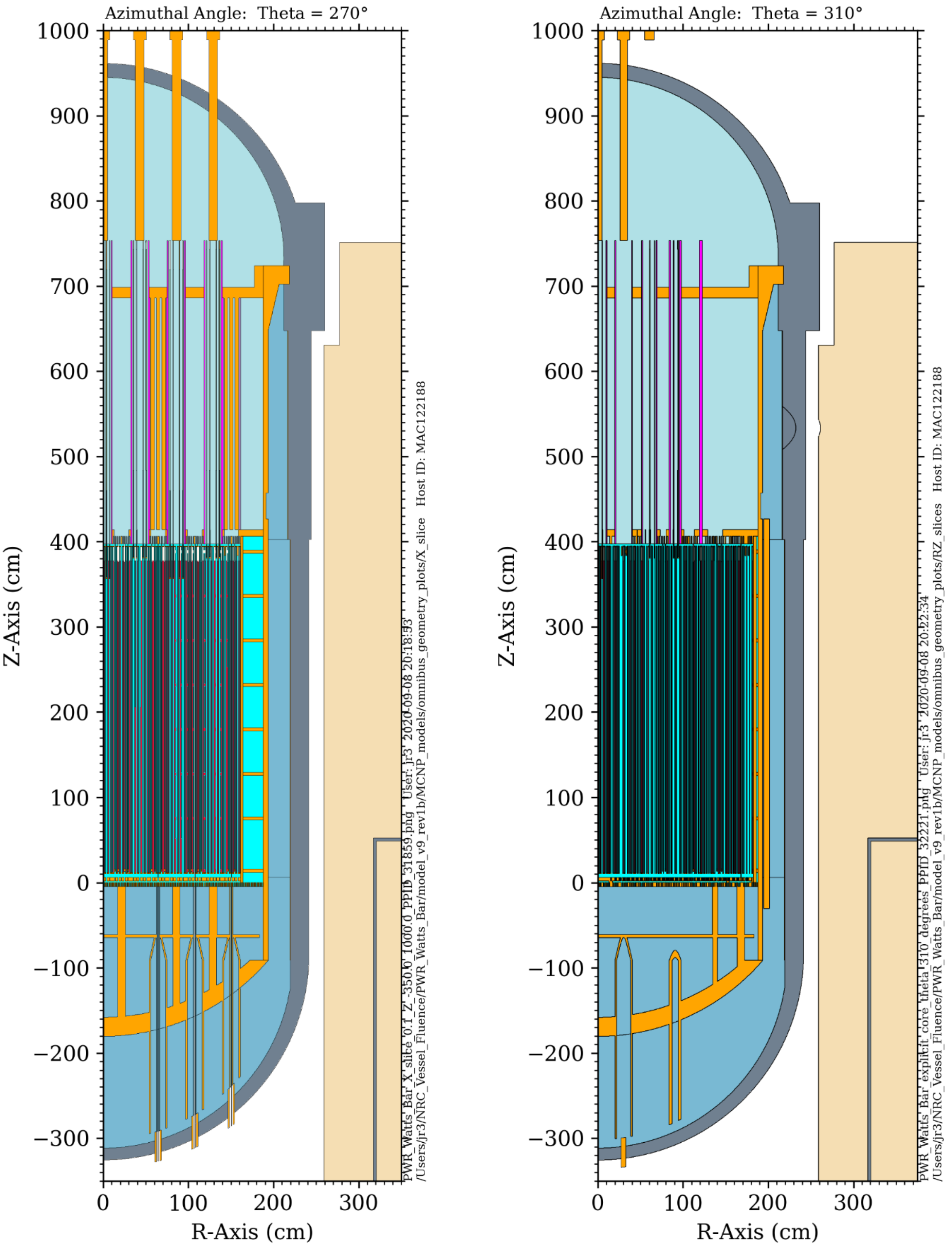


Figure 4-1 **Elevation views of the PWR model at azimuthal angles of 270° and 310°. These are the azimuthal locations with the maximum and minimum amounts of water, respectively, between the core and the RPV**

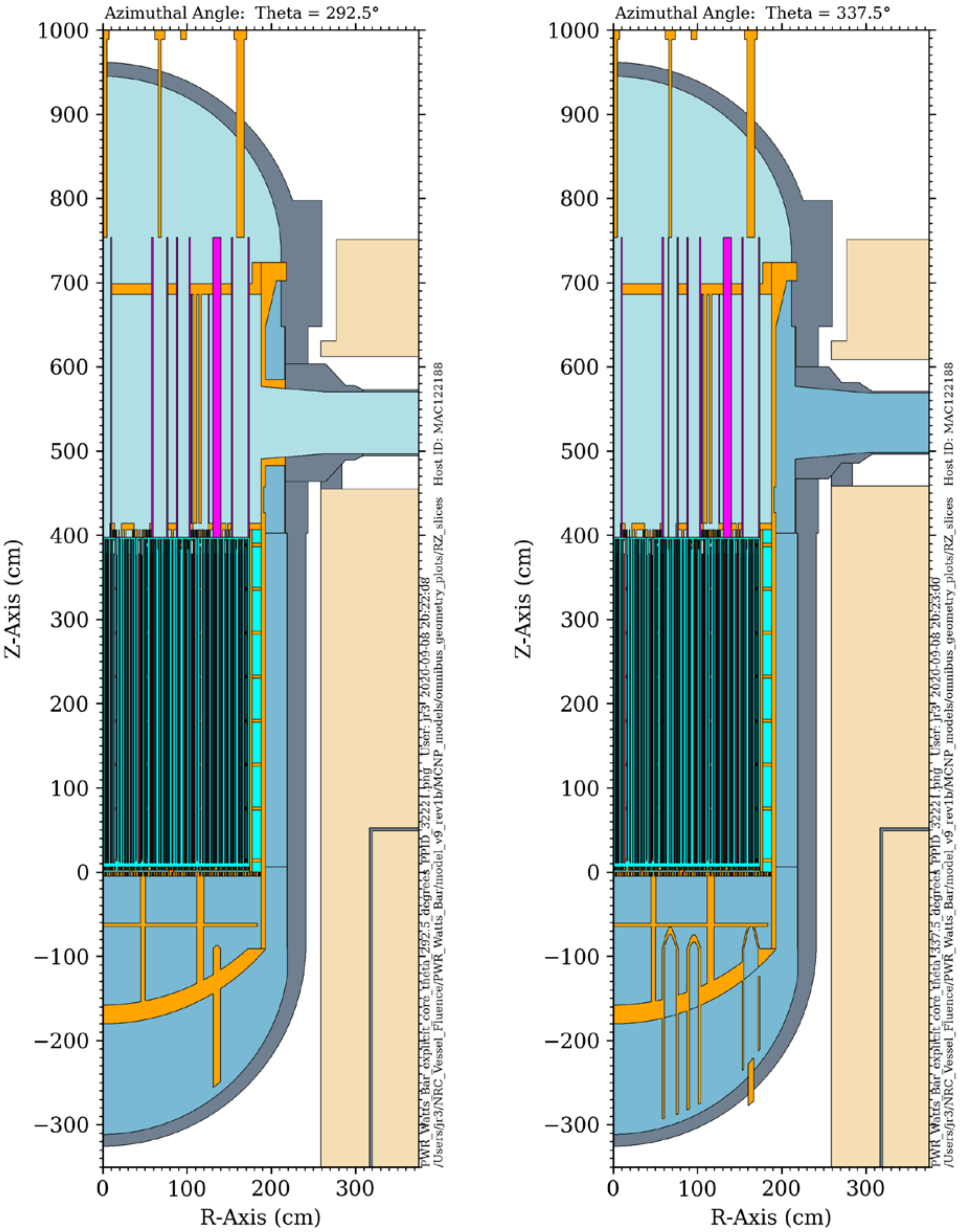


Figure 4-2 Elevation views of the PWR model at azimuthal angles of 292.5° and 337.5°. These are the azimuthal locations of the outlet and inlet nozzles, respectively

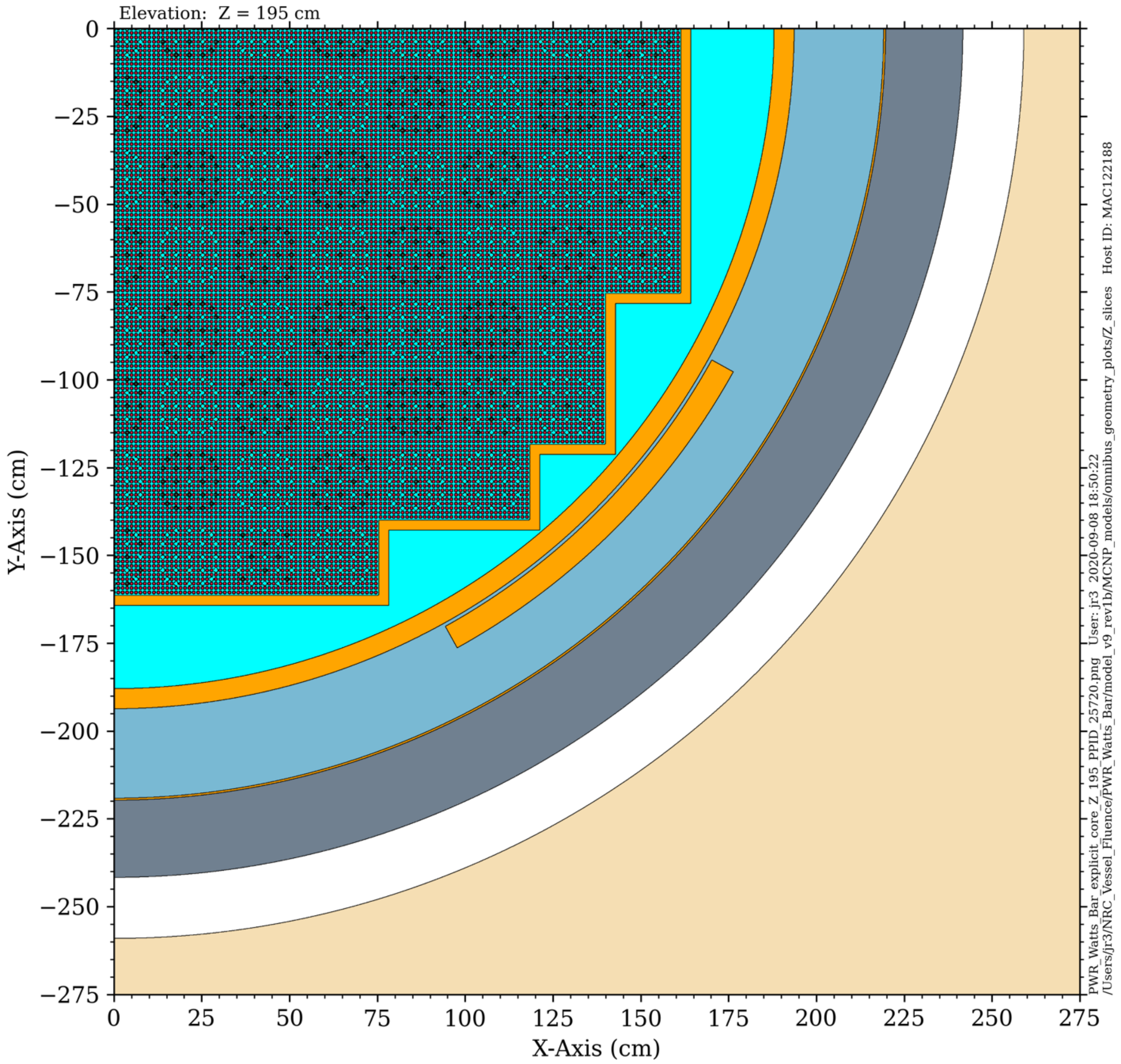


Figure 4-3 Plan view of the PWR model at the core midplane (Z = 195 cm)

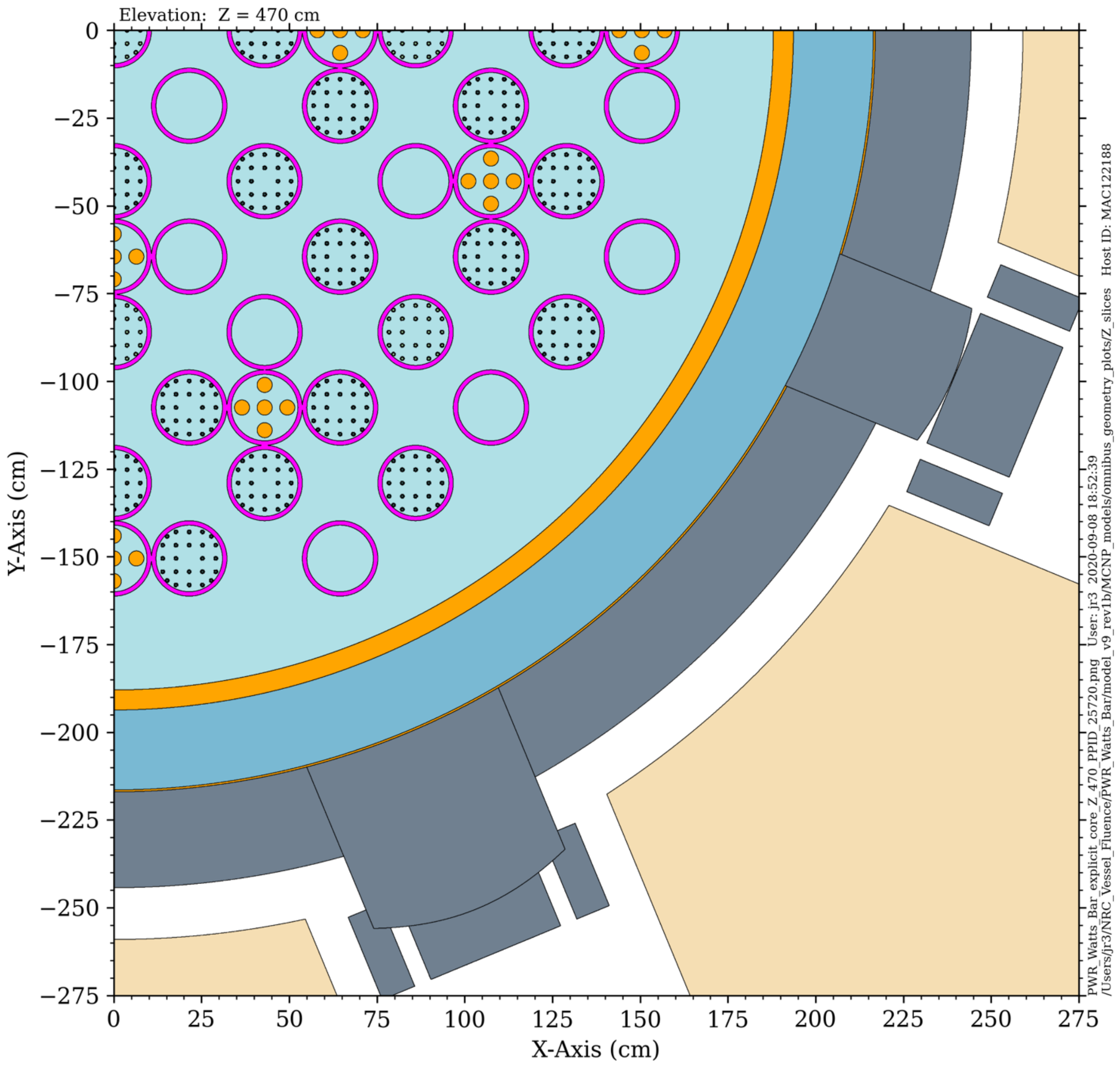


Figure 4-4 Plan view of the PWR model at an elevation of Z = 470 cm. This elevation intersects the vessel supports and the bottom portion of the inlet and outlet nozzles

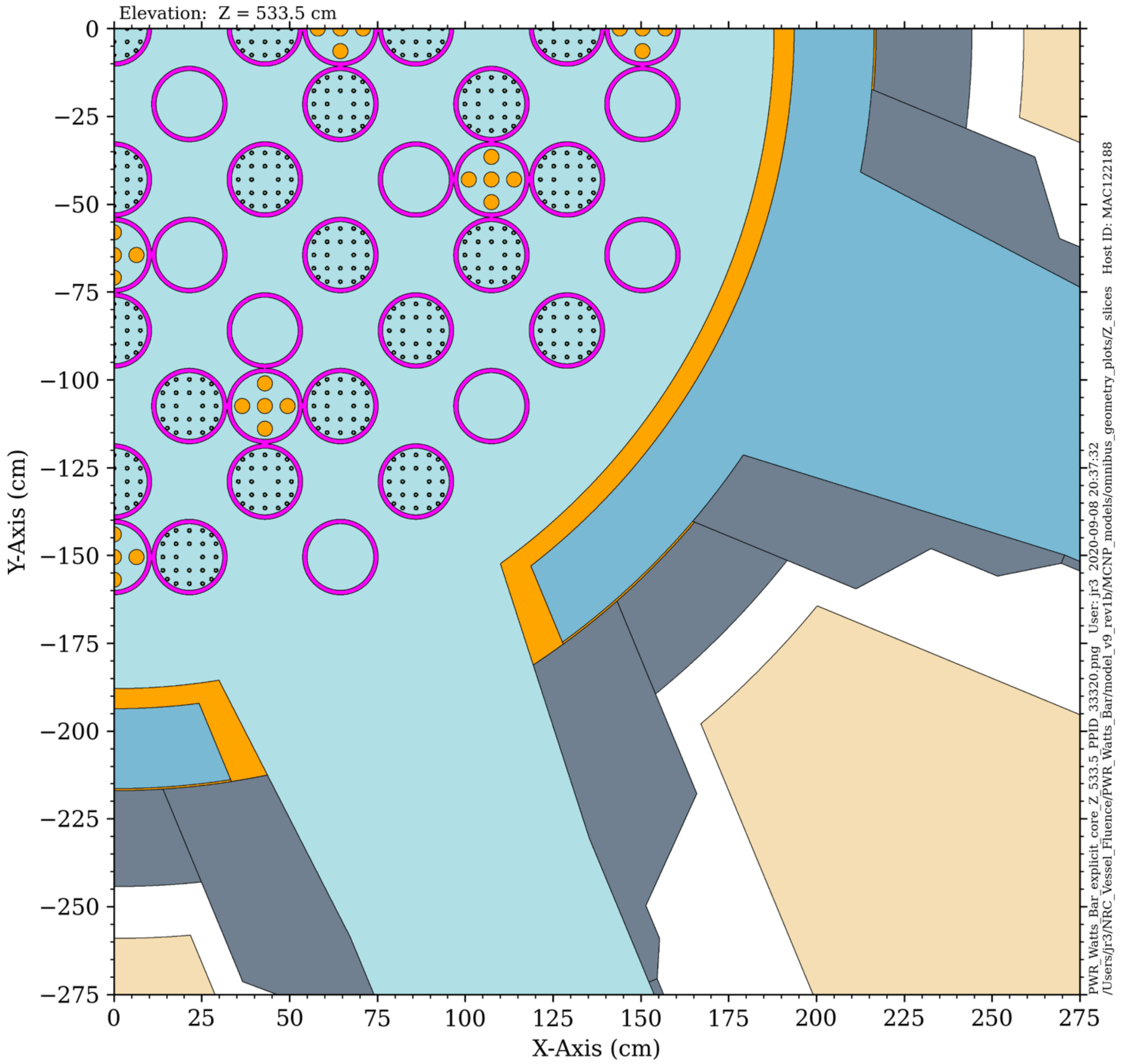


Figure 4-5 Plan view of the PWR model at an elevation of Z = 533.5 cm. This elevation is through the centerline of the inlet and outlet nozzles

4.2 BWR model

The BWR reference model is based on Hatch Unit 2, which is a GE-4 design with a licensed power of 2,804 MWt. Modeling dimensions and materials were taken from NUREG/CR-6115 [70] and the Hatch Nuclear Plant Unit No. 2 Updated Final Safety Analysis Report [71]. The initial model used a core design with fuel assemblies based on a GE 7 × 7 design with four different ^{235}U enrichments. An update to the model during the course of the project replaced the 7 × 7 assembly design with the GE14 design, which has a 10 × 10 lattice. Figure 4-6 shows a typical fuel lattice arrangement in the GE14 assembly design [72]. This design has seven ^{235}U enrichment levels, as well as natural uranium. There are seven axial zones: natural uranium at the bottom of each rod (NAT), the power shaping zone (PSZ), the dominant zone (DOM), the plenum zone (PLE), the vanished rod zone (VAN), the natural uranium vanished rod zone (N-V), and the natural uranium top zone (N-T). Some fuel pins in the PSZ, DOM, PLE, and VAN zones contain Gd as a burnable poison. Void fractions (VFs) in the fuel assembly axial zones are discussed in the parameter study of Section 5.4. The baseline model uses average VFs based on data from NUREG/CR-7224 [73].

The model was developed using MCNP geometry. The MCNP model can be used to run MCNP with ADVANTG for hybrid radiation transport, Shift through Omnibus for hybrid radiation transport, and Denovo through Omnibus for deterministic calculations. Key dimensions in the BWR model are provided in Table 4-3. The material definitions are provided in Table 4-4.

Elevation and plan views of the BWR model are shown in Figure 4-7 through Figure 4-13. Full- and quarter-core models were constructed. Because the model has quarter-core symmetry at elevations below the feedwater inlet nozzle, quarter-core calculations were performed to reduce memory and CPU requirements for the MCNP, Shift, and Denovo calculations.

zone height (inches)	axial zone	zone length (inches)
145.24	N-T	6.00
139.24	N-V	6.00
133.24	VAN	37.24
96.00	PLE	12.00
84.00	DOM	30.00
54.00	PSZ	48.00
6.00	NAT	6.00
0.00		

N-T

	A	B	C	D	E	F	G	H	I	J
1	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
2	0.71	V	0.71	V	0.71	E	V	E	V	0.71
3	0.71	0.71	E	0.71	E	0.71	E	0.71	E	0.71
4	0.71	V	0.71	E	0.71	WR		0.71	V	0.71
5	0.71	0.71	E	0.71	V			0.71	0.71	0.71
6	0.71	E	0.71	WR		V	E	0.71	E	0.71
7	0.71	V	E			E	0.71	E	V	0.71
8	0.71	E	0.71	0.71	0.71	0.71	E	0.71	E	0.71
9	0.71	V	E	V	0.71	E	V	E	V	0.71
10	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71

X = Enrichment (wt% ^{235}U)
V = Vanished Rod
E = Empty Rod
WR = Water Rod

X
Y

X = Enrichment (wt% ^{235}U)
Y = Gd_2O_3 content (wt %)

Reproduced similar to and using data from Ref. 19.

M. L. Fensin, "Optimum Boiling Water Reactor Fuel Design Stragegies to Enhance Reactor Shutdown by Standby Liquid Liquid Control System," M.S. Thesis, University of Florida, 2004.

N-V

	A	B	C	D	E	F	G	H	I	J
1	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
2	0.71	V	0.71	V	0.71	0.71	V	0.71	V	0.71
3	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
4	0.71	V	0.71	0.71	0.71	WR		0.71	V	0.71
5	0.71	0.71	0.71	0.71	V			0.71	0.71	0.71
6	0.71	0.71	0.71	WR		V	0.71	0.71	0.71	0.71
7	0.71	V	0.71			0.71	0.71	0.71	V	0.71
8	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
9	0.71	V	0.71	V	0.71	0.71	V	0.71	V	0.71
10	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71

VAN

	A	B	C	D	E	F	G	H	I	J
1	1.60	2.80	3.20	3.95	3.95	3.95	3.95	3.95	3.95	2.80
2	2.80	V	3.20	V	3.60	3.95	V	4.40 6.00	V	3.95
3	3.20	3.20	4.40	4.40	4.90 6.00	4.40	4.40 6.00	4.40	4.40 6.00	4.90
4	3.95	V	4.40	4.90 8.00	3.95	WR		4.90	V	4.90
5	3.95	3.60	4.90 6.00	3.95	V			4.90	4.90	4.90
6	3.95	3.95	4.40	WR		V	4.90 8.00	4.90	4.90 8.00	4.90
7	3.95	V	4.40 6.00			4.90 8.00	4.90	4.90	V	4.90
8	3.95	4.40 6.00	4.40	4.90	4.90	4.90	4.90	4.90	4.90 8.00	4.90
9	3.95	V	4.40 6.00	V	4.90	4.90 8.00	V	4.90 8.00	V	4.90
10	2.80	3.95	4.90	4.90	4.90	4.90	4.90	4.90	4.90	3.20

PLE

	A	B	C	D	E	F	G	H	I	J
1	1.60	2.80	3.20	3.95	3.95	3.95	3.95	3.95	3.95	2.80
2	2.80	E	3.20	E	3.60	3.95	E	4.40 6.00	E	3.95
3	3.20	3.20	4.40	4.40	4.90 6.00	4.40	4.40 6.00	4.40	4.40 6.00	4.90
4	3.95	E	4.40	4.90 8.00	3.95	WR		4.90	E	4.90
5	3.95	3.60	4.90 6.00	3.95	E			4.90	4.90	4.90
6	3.95	3.95	4.40	WR		E	4.90	4.90	4.90 8.00	4.90
7	3.95	E	4.40 6.00			4.90	4.90	4.90 8.00	E	4.90
8	3.95	4.40 6.00	4.40	4.90	4.90	4.90	4.90 8.00	4.90	4.90 8.00	4.90
9	3.95	E	4.40 6.00	E	4.90	4.90 8.00	E	4.90 8.00	E	4.90
10	2.80	3.95	4.90	4.90	4.90	4.90	4.90	4.90	4.90	3.20

DOM

	A	B	C	D	E	F	G	H	I	J
1	1.60	2.80	3.20	3.95	3.95	3.95	3.95	3.95	3.95	2.80
2	2.80	2.80	3.20	3.95	3.60	3.95 8.00	3.95	4.40 6.00	3.95	3.95
3	3.20	3.20	4.40 3.00	4.40	4.90 6.00	4.40	4.40 6.00	4.40	4.40 6.00	4.90
4	3.95	3.95	4.40	4.90 8.00	3.95	WR		4.90	4.90	4.90
5	3.95	3.60	4.90 6.00	3.95	4.90			4.90	4.90	4.90
6	3.95	3.95 8.00	4.40	WR		4.90	4.90	4.90	4.90 8.00	4.90
7	3.95	3.95	4.40 6.00			4.90	4.90	4.90 8.00	4.90	4.90
8	3.95	4.40 6.00	4.40	4.90	4.90	4.90	4.90 8.00	4.90	4.90 8.00	4.90
9	3.95	3.95	4.40 6.00	4.90	4.90	4.90 8.00	4.90	4.90 8.00	4.90	4.90
10	2.80	3.95	4.90	4.90	4.90	4.90	4.90	4.90	4.90	3.20

PSZ

	A	B	C	D	E	F	G	H	I	J
1	1.60	2.80	3.20	3.95	3.95	3.95	3.95	3.95	3.95	2.80
2	2.80	2.80	3.20	3.95	3.60	3.95 8.00	3.95	4.40 8.00	3.95	3.95
3	3.20	3.20	4.40 3.00	4.40	4.90 8.00	4.40	4.40 8.00	4.40	4.40 8.00	4.90
4	3.95	3.95	4.40	4.90 8.00	3.95	WR		4.90	4.90	4.90
5	3.95	3.60	4.90 8.00	3.95	4.90			4.90	4.90	4.90
6	3.95	3.95 8.00	4.40	WR		4.90	4.90	4.90	4.90 8.00	4.90
7	3.95	3.95	4.40 8.00			4.90	4.90	4.90 8.00	4.90	4.90
8	3.95	4.40 8.00	4.40	4.90	4.90	4.90	4.90 8.00	4.90	4.90 8.00	4.90
9	3.95	3.95	4.40 8.00	4.90	4.90	4.90 8.00	4.90	4.90 8.00	4.90	4.90
10	2.80	3.95	4.90	4.90	4.90	4.90	4.90	4.90	4.90	3.20

NAT

	A	B	C	D	E	F	G	H	I	J
1	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
2	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
3	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
4	0.71	0.71	0.71	0.71	0.71	WR		0.71	0.71	0.71
5	0.71	0.71	0.71	0.71	0.71			0.71	0.71	0.71
6	0.71	0.71	0.71	WR		0.71	0.71	0.71	0.71	0.71
7	0.71	0.71	0.71			0.71	0.71	0.71	0.71	0.71
8	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
9	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
10	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71

Figure 4-6 Typical GE14 10 × 10 fuel lattice

Table 4-3 Selected model parameters for the BWR reference model

Parameter	Measurement
Thermal power	2,804 MW(t)
Core operating pressure	1,246 PSIA
Coolant temp	
Feedwater inlet	425.8 °F
Recirc pump suction	534 °F
Recirc pump discharge	535 °F
Core inlet	534.6 °F
Core outlet	553.3 °F
Steam dome	551.8 °F
Shroud	
Inner radius	222.32 cm
Outer radius	225.50 cm
RPV	
Liner inner radius	277.34 cm
Inner radius	278.13 cm
Outer radius	293.05 cm
Concrete bioshield	
Inner radius	309.05 cm
Thickness*	30.48 cm

* The thickness used in the calculations.

Table 4-4 Material definitions in the BWR reference model

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
UO_2 fuel; 0.71% enrichment	10.5	8016	4.6848E-02
		92234	1.5054E-06
		92235	1.6842E-04
		92236	7.7146E-07
		92238	2.3253E-02
UO_2 fuel; 1.6% enrichment	10.4	8016	4.6394E-02
		92234	3.3591E-06
		92235	3.7582E-04
		92236	1.7214E-06
		92238	2.2816E-02
UO_2 fuel; 2.8% enrichment	10.4	8016	4.6400E-02
		92234	5.8783E-06
		92235	6.5767E-04
		92236	3.0124E-06
		92238	2.2533E-02
UO_2 fuel; 3.2% enrichment	10.4	8016	4.6402E-02
		92234	6.7181E-06
		92235	7.5162E-04
		92236	3.4428E-06
		92238	2.2439E-02
UO_2 fuel; 3.6% enrichment	10.4	8016	4.6404E-02
		92234	7.5578E-06
		92235	8.4556E-04
		92236	3.8731E-06
		92238	2.2345E-02
UO_2 fuel; 3.95% enrichment	10.4	8016	4.6406E-02
		92234	8.2925E-06
		92235	9.2777E-04
		92236	4.2496E-06
		92238	2.2263E-02
UO_2 fuel; 4.4% enrichment	10.4	8016	4.6408E-02
		92234	9.2372E-06
		92235	1.0335E-03
		92236	4.7337E-06
		92238	2.2157E-02
UO_2 fuel; 4.9% enrichment	10.4	8016	4.6411E-02
		92234	1.0287E-05
		92235	1.1509E-03
		92236	5.2716E-06
		92238	2.2039E-02

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
UO_2 fuel; 3.95% enrichment 8.0 wt% Gd_2O_3	10.26	8016	4.6210E-02
		64152	5.4544E-06
		64154	5.9453E-05
		64155	4.0363E-04
		64156	5.5826E-04
		64157	4.2681E-04
		64158	6.7744E-04
		64160	5.9616E-04
		92234	7.5264E-06
		92235	8.4206E-04
		92236	3.8570E-06
		92238	2.0206E-02
UO_2 fuel; 4.4% enrichment 3.0 wt% Gd_2O_3	10.26	8016	4.5944E-02
		64152	2.0454E-06
		64154	2.2295E-05
		64155	1.5136E-04
		64156	2.0935E-04
		64157	1.6005E-04
		64158	2.5404E-04
		64160	2.2356E-04
		92234	8.8394E-06
		92235	9.8896E-04
		92236	4.5299E-06
		92238	2.1203E-02
UO_2 fuel; 4.4% enrichment 6.0 wt% Gd_2O_3	10.26	8016	4.6105E-02
		64152	4.0908E-06
		64154	4.4590E-05
		64155	3.0272E-04
		64156	4.1869E-04
		64157	3.2010E-04
		64158	5.0808E-04
		64160	4.4712E-04
		92234	8.5661E-06
		92235	9.5837E-04
		92236	4.3898E-06
		92238	2.0547E-02

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
UO_2 fuel; 4.4% enrichment 8.0 wt% Gd_2O_3	10.26	8016	4.6212E-02
		64152	5.4544E-06
		64154	5.9453E-05
		64155	4.0362E-04
		64156	5.5826E-04
		64157	4.2681E-04
		64158	6.7744E-04
		64160	5.9616E-04
		92234	8.2291E-06
		92235	9.3798E-04
		92236	4.2964E-06
		92238	2.0110E-02
UO_2 fuel; 4.9% enrichment 6.0 wt% Gd_2O_3	10.26	8016	4.6107E-02
		64152	4.0908E-06
		64154	4.4590E-05
		64155	3.0272E-04
		64156	4.1869E-04
		64157	3.2010E-04
		64158	5.0808E-04
		64160	4.4712E-04
		92234	9.4481E-06
		92235	1.0673E-03
		92236	4.8886E-06
		92238	2.0438E-02
UO_2 fuel; 4.9% enrichment 8.0 wt% Gd_2O_3	10.26	8016	4.6214E-02
		64152	5.4544E-06
		64154	5.9453E-05
		64155	4.0363E-04
		64156	5.5826E-04
		64157	4.2681E-04
		64158	6.7744E-04
		64160	5.9616E-04
		92234	9.3364E-06
		92235	1.0446E-03
		92236	4.7846E-06
		92238	2.0003E-02
B_4C	1.7643	5010	1.5306E-02
		5011	6.1610E-02
		6012	1.9023E-02
		6013	2.0575E-04
Al_2O_3	3.97	8016	7.0363E-02
		13027	4.6895E-02

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Zircaloy-2	6.56	24050	3.3012E-06
		24052	6.3660E-05
		24053	7.2185E-06
		24054	1.7968E-06
		26054	5.5819E-06
		26056	8.7624E-05
		26057	2.0236E-06
		26058	2.6931E-07
		28058	2.5201E-05
		28060	9.7075E-06
		28061	4.2198E-07
		28062	1.3454E-06
		28064	3.4265E-07
		40090	2.1891E-02
		40091	4.7738E-03
		40092	7.2969E-03
		40094	7.3947E-03
		40096	1.1913E-03
		50112	4.6806E-06
		50114	3.1847E-06
		50115	1.6406E-06
		50116	7.0161E-05
		50117	3.7059E-05
		50118	1.1687E-04
		50119	4.1450E-05
		50120	1.5721E-04
		50122	2.2341E-05
		50124	2.7939E-05
		72174	3.5412E-09
		72176	1.1642E-07
		72177	4.1167E-07
		72178	6.0378E-07
		72179	3.0145E-07
		72180	7.7642E-07
Zircaloy-4	6.56	24050	3.3012E-06
		24052	6.3660E-05
		24053	7.2185E-06
		24054	1.7968E-06
		26054	8.6829E-06
		26056	1.3630E-04
		26057	3.1478E-06
		26058	4.1892E-07
		40090	2.1888E-02
		40091	4.7733E-03
		40092	7.2961E-03

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Zircaloy-4 (continued)		40094	7.3940E-03
		40096	1.1912E-03
		50112	4.6806E-06
		50114	3.1847E-06
		50115	1.6406E-06
		50116	7.0160E-05
		50117	3.7059E-05
		50118	1.1687E-04
		50119	4.1450E-05
		50120	1.5721E-04
		50122	2.2341E-05
		50124	2.7939E-05
Stainless steel 304	8.0	6012	3.1774E-04
		6013	3.4366E-06
		14028	3.1641E-03
		14029	1.6067E-04
		14030	1.0591E-04
		15031	6.2216E-05
		16032	5.7049E-05
		16033	4.5673E-07
		16034	2.5781E-06
		16036	1.2019E-08
		24050	7.8503E-04
		24052	1.5139E-02
		24053	1.7166E-03
		24054	4.2730E-04
		25055	1.3154E-03
		26054	3.3955E-03
		26056	5.3302E-02
		26057	1.2310E-03
		26058	1.6382E-04
		28058	5.3085E-03
		28060	2.0448E-03
		28061	8.8886E-05
		28062	2.8341E-04
		28064	7.2176E-05

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Low-alloy carbon steel (RPV)	7.7879	6012	9.7048E-04
		6013	1.0496E-05
		14028	3.4216E-04
		14029	1.7326E-05
		14030	1.1501E-05
		24050	5.5181E-06
		24052	1.0641E-04
		24053	1.2065E-05
		24054	3.0035E-06
		25055	1.1200E-03
		26054	4.8320E-03
		26056	7.5117E-02
		26057	1.7199E-03
		26058	2.2932E-04
		28058	3.0311E-04
		28060	1.1588E-04
		28061	5.0171E-06
		28062	1.5940E-05
		28064	4.0403E-06
Jet pumps, risers	7.9273	24050	6.7422E-04
		24052	1.3002E-02
		24053	1.4742E-03
		24054	3.6698E-04
		26054	3.7636E-03
		26056	5.8508E-02
		26057	1.3396E-03
		26058	1.7862E-04
		28058	4.7081E-03
		28060	1.8000E-03
		28061	7.7929E-05
		28062	2.4758E-04
		28064	6.2756E-05
Upper axial reflector	2.5675	1001	9.8222E-03
		6012	3.7936E-05
		6013	4.1031E-07
		8016	4.9112E-03
		14028	1.3326E-04
		14029	6.7477E-06
		14030	4.4792E-06
		24050	1.2247E-04
		24052	2.3590E-03
		24053	2.6746E-04
		24054	6.6582E-05
		25055	2.4594E-04
		26054	5.5654E-04
		26056	8.6518E-03

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Upper axial reflector (cont.)		26057	1.9809E-04
		26058	2.6412E-05
		28058	9.4444E-04
		28060	3.6106E-04
		28061	1.5632E-05
		28062	4.9644E-05
		28064	1.2589E-05
		40090	3.8652E-03
		40091	8.4289E-04
		40092	1.2884E-03
		40094	1.3057E-03
		40096	2.1035E-04
Top guide	1.3466	1001	1.2153E-02
		8016	6.0767E-03
		40090	3.9563E-03
		40091	8.6278E-04
		40092	1.3188E-03
		40094	1.3365E-03
		40096	2.1531E-04
Type 04 concrete	2.35	1001	7.7679E-03
		8016	4.4081E-02
		11023	1.0479E-03
		12024	1.1744E-04
		12025	1.4868E-05
		12026	1.6370E-05
		13027	2.3884E-03
		14028	1.4675E-02
		14029	7.4547E-04
		14030	4.9199E-04
		16032	5.3526E-05
		16033	4.2261E-07
		16034	2.3948E-06
		16036	5.6349E-09
		19039	6.4646E-04
		19040	8.1103E-08
		19041	4.6653E-05
		20040	2.8262E-03
		20042	1.8862E-05
		20043	3.9357E-06
		20044	6.0814E-05
		20046	1.1661E-07
		20048	5.4518E-06
		26054	1.8281E-05
		26056	2.8697E-04
		26057	6.6274E-06
		26058	8.8198E-07

Table 4-4 Material definitions in the BWR reference model (continued)

Material	Density (g/cm^3)	Isotope	Atom density (atoms/b·cm)
Core inlet coolant	0.7537	1001	5.0399E-02
		8016	2.5200E-02
Core outlet coolant (within the shroud dome)	0.0389	1001	2.5991E-03
		8016	1.2995E-03
Recirc pump suction	0.7540	1001	5.0421E-02
		8016	2.5210E-02
Recirc pump discharge	0.7549	1001	5.0480E-02
		8016	2.5240E-02
Feedwater inlet	0.8463	1001	5.6591E-02
		8016	2.8296E-02
Coolant – Zone Nat*	0.7386	1001	4.9392E-02
		8016	2.4696E-02
Coolant – Zone PSZ*	0.7160	1001	4.7880E-02
		8016	2.3940E-02
Coolant – Zone DOM*	0.6142	1001	4.1076E-02
		8016	2.0538E-02
Coolant – Zone PLE*	0.4710	1001	3.1500E-02
		8016	1.5750E-02
Coolant – Zone VAN*	0.3957	1001	2.6460E-02
		8016	1.3230E-02
Coolant – Zone N-V*	0.3241	1001	2.1672E-02
		8016	1.0836E-02
Coolant – Zone N-T*	0.3241	1001	2.1672E-02
		8016	1.0836E-02

* Densities within the fuel assemblies correspond to the average VF (VF) condition. See Section 5.4 for a discussion of the variation in VFs.

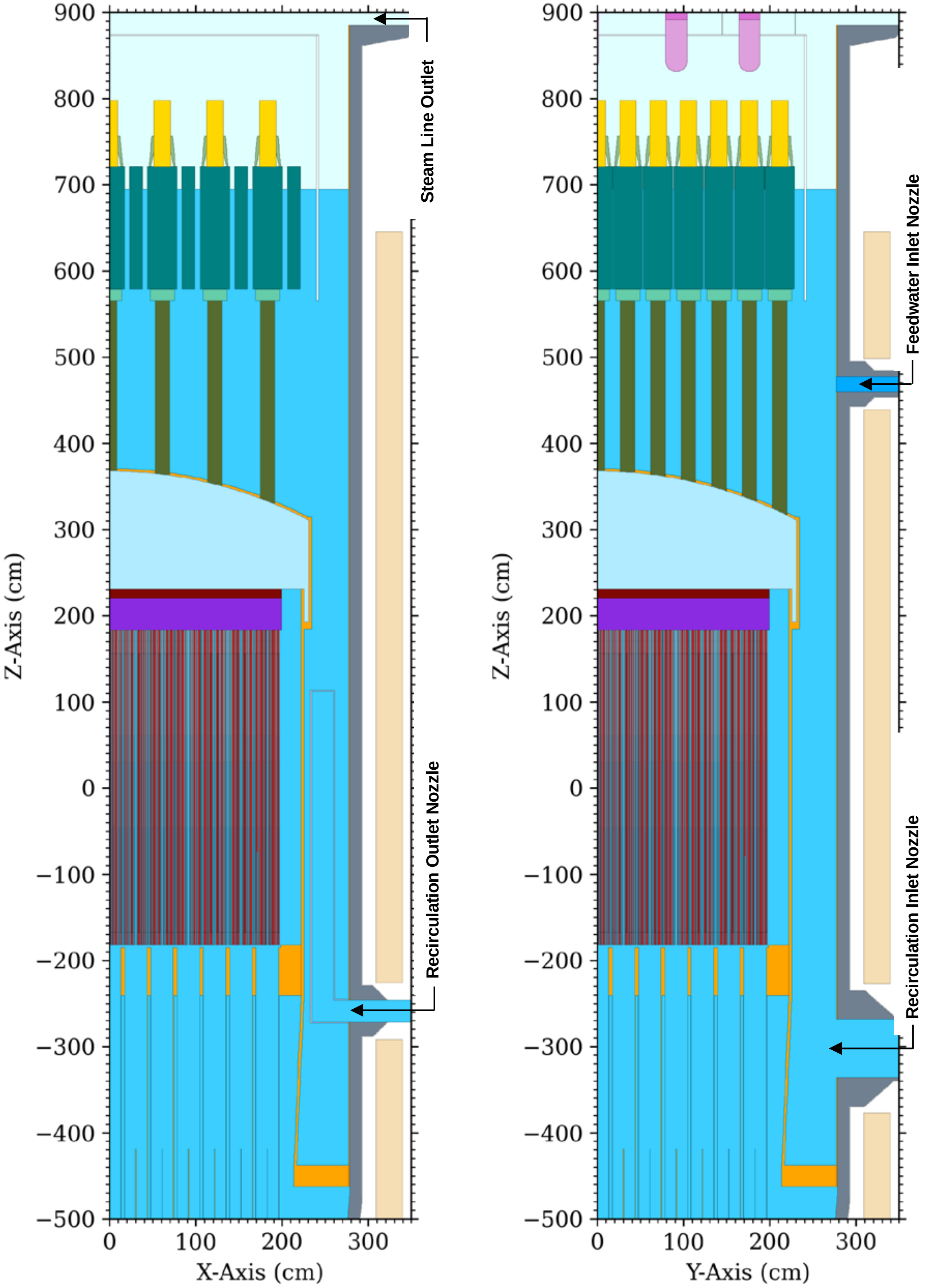


Figure 4-7 Elevation views of the BWR model at locations through the first row of fuel pins nearest the X- and Y-reflecting boundaries

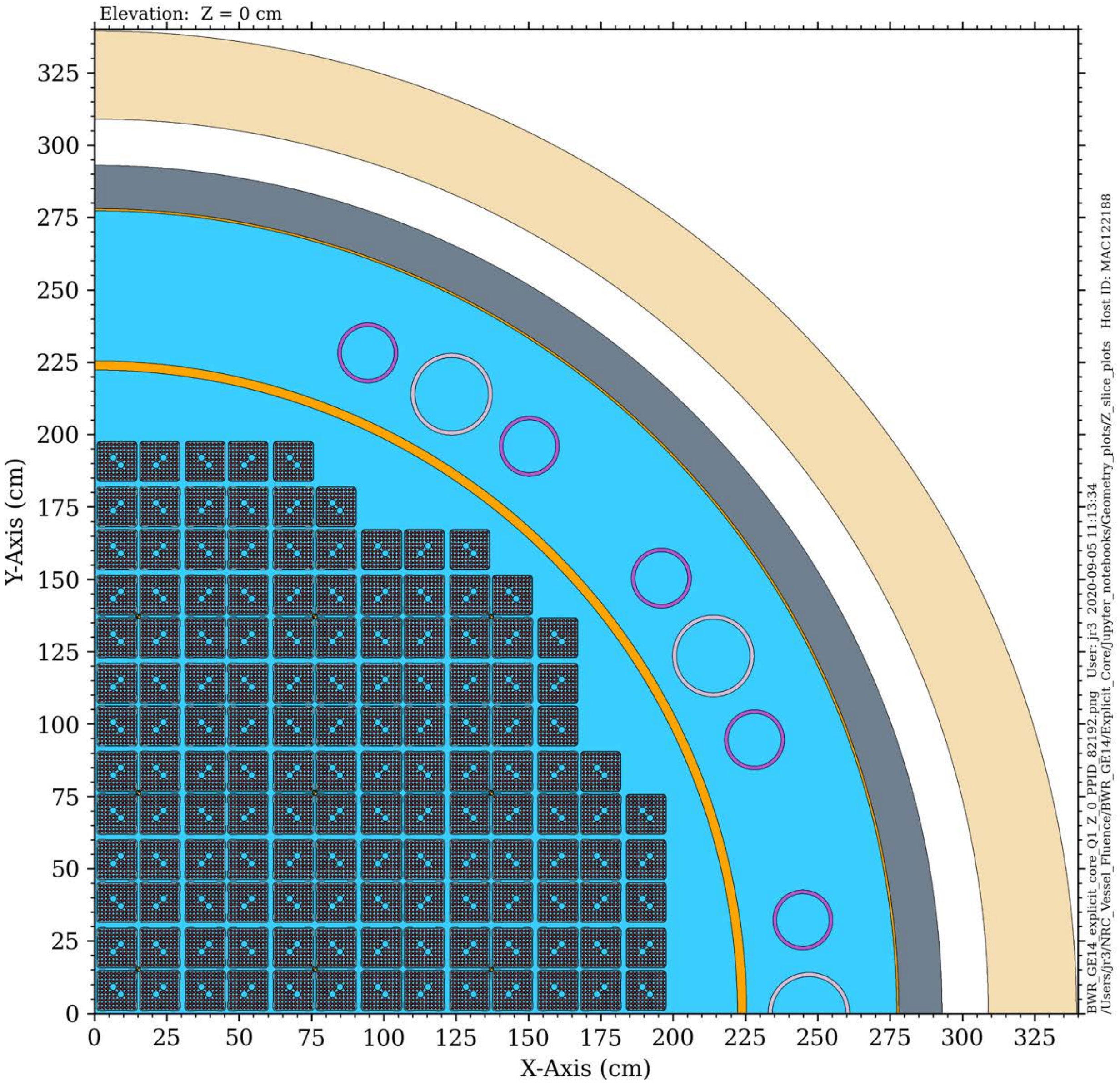


Figure 4-8 Plan view of the BWR model at the core midplane (Z = 0)

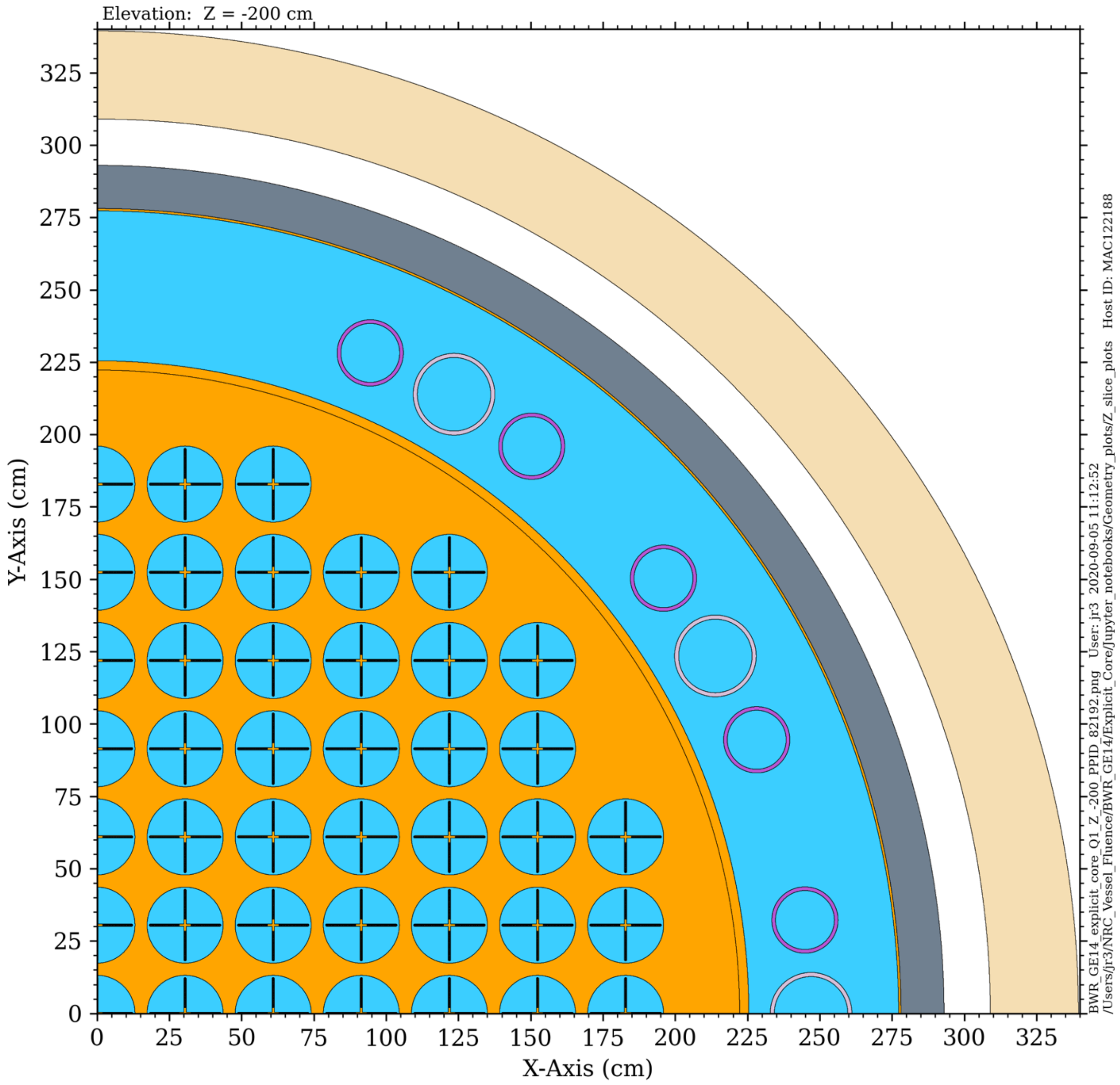


Figure 4-9 Plan view of the BWR model through the bottom core pate (Z = -200 cm)

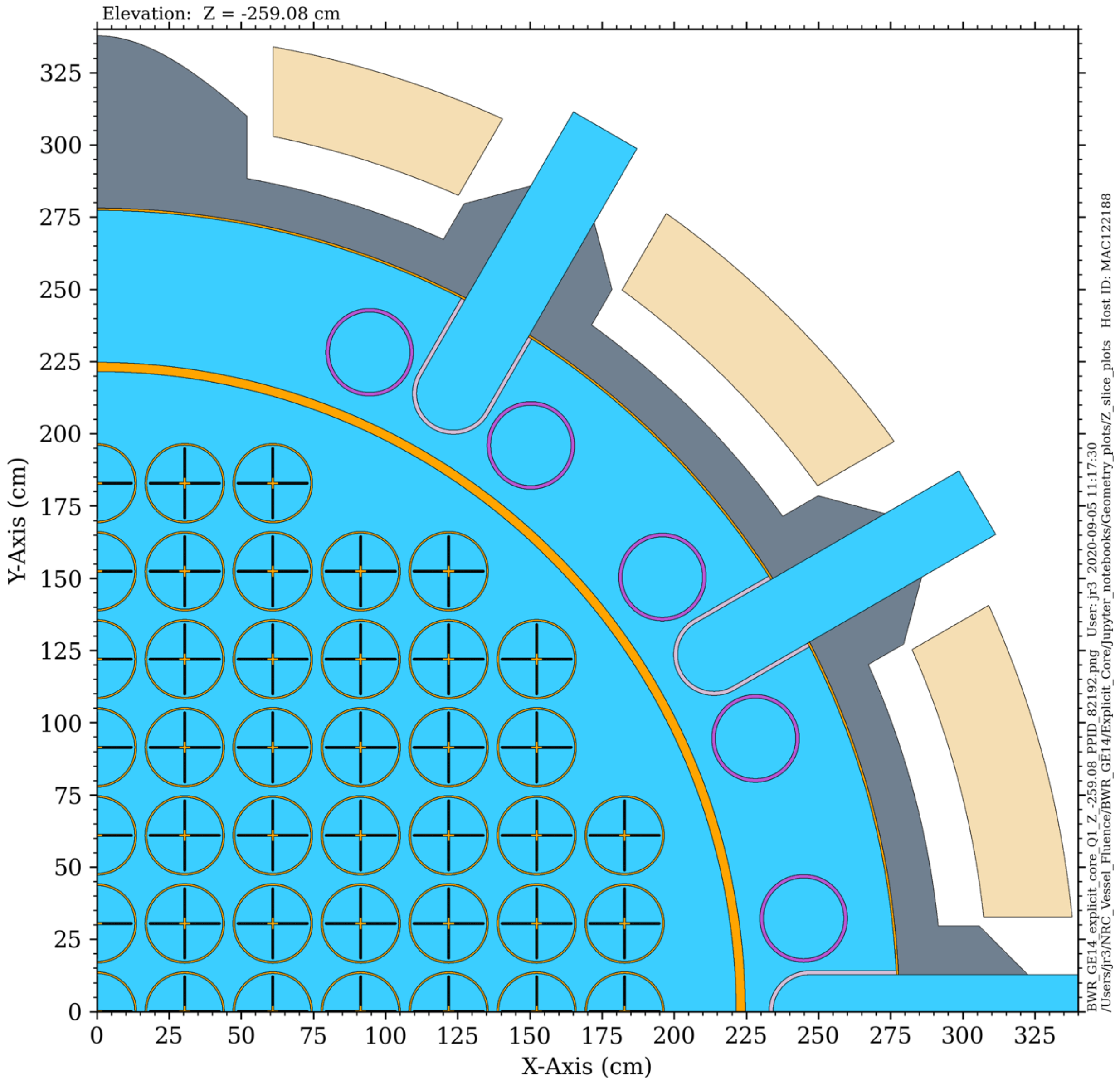


Figure 4-10 Plan view of the BWR model at the midplane of the recirculation outlet nozzles (Z = -259.08 cm)

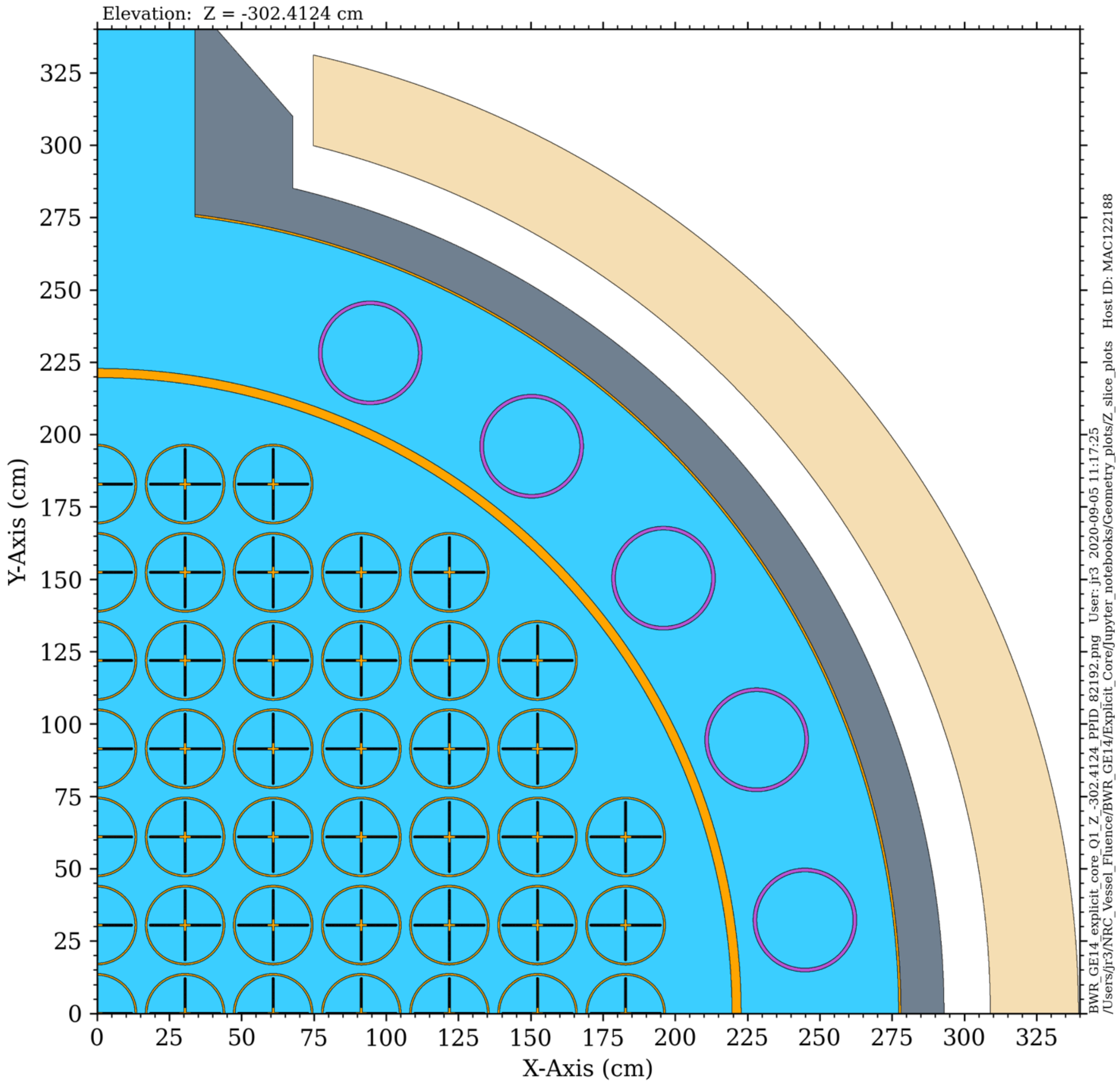


Figure 4-11 Plan view of the BWR model at the midplane of the recirculation inlet nozzles (Z = -302.41 cm)

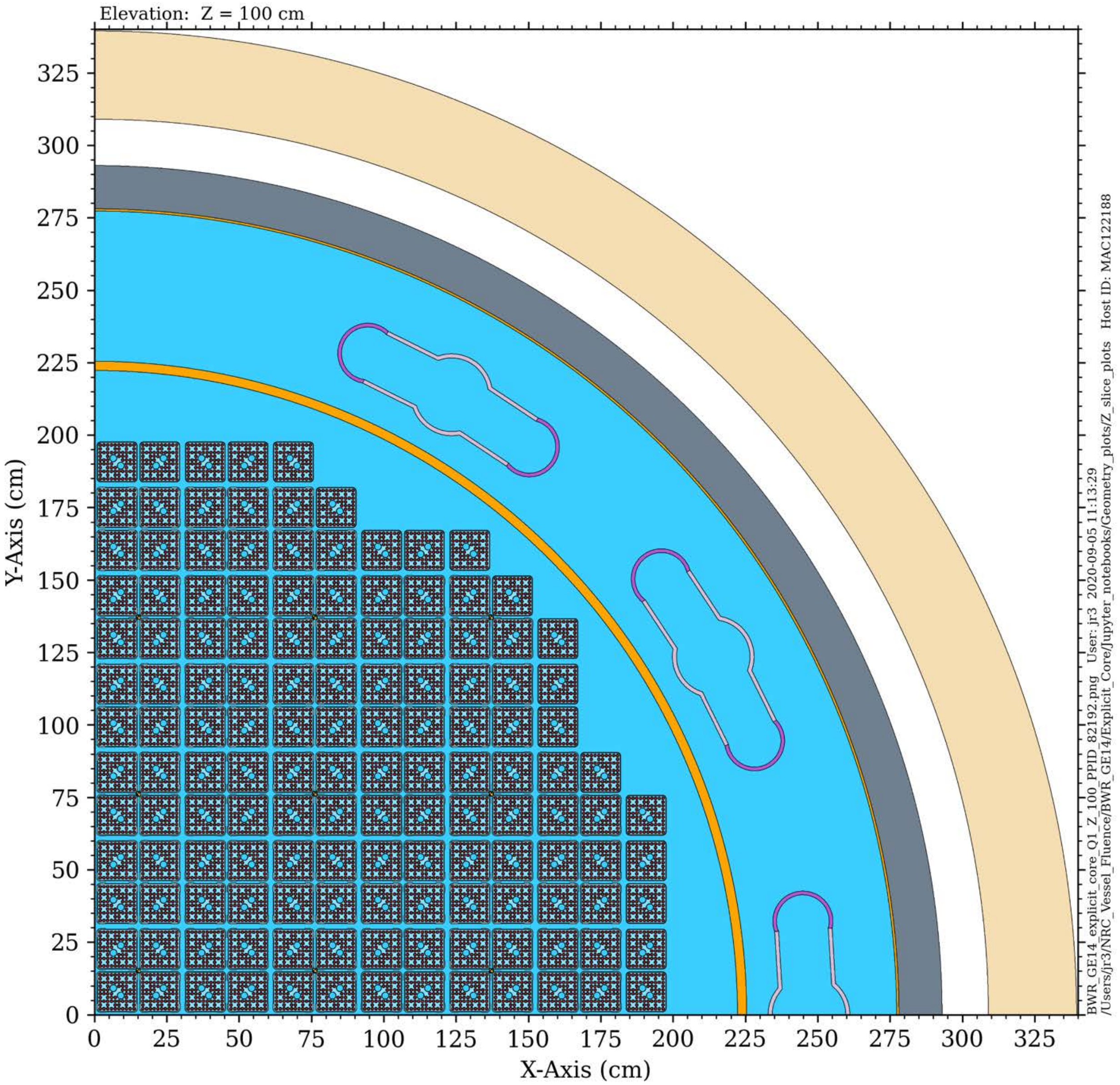


Figure 4-12 Plan view of the BWR model through the jet pump to riser connection (Z = 100 cm)

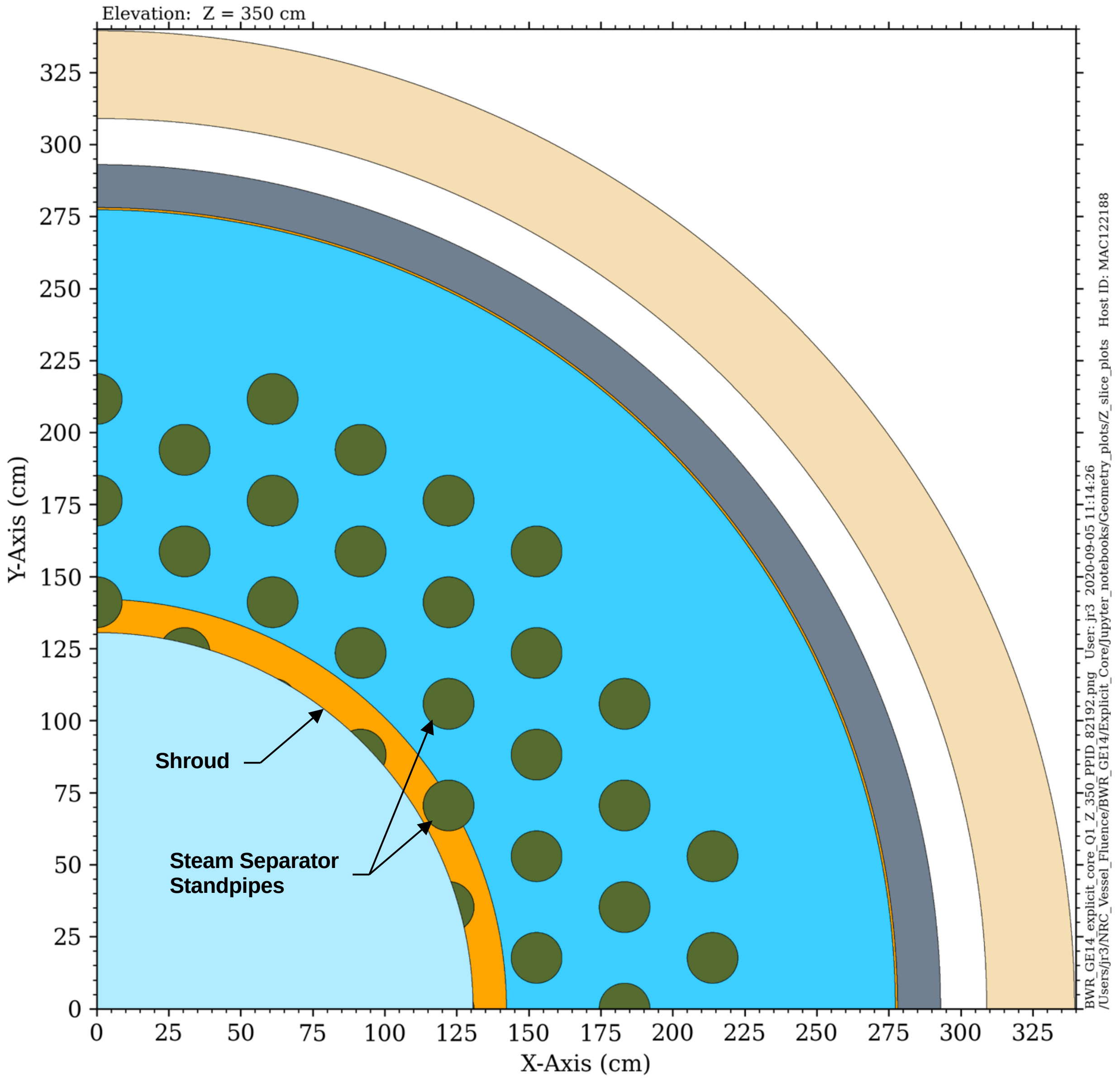


Figure 4-13 Plan view of the BWR model through the shroud dome (Z = 350 cm)

5 SENSITIVITY ANALYSES OF SELECTED PHYSICAL PARAMETERS FOR EXTENDED BELTLINE FLUENCE CALCULATIONS

5.1 Baseline calculations

Baseline calculations with the PWR and BWR models were performed using hybrid FW-CADIS radiation transport with the Shift Monte Carlo code. CE cross sections based on ENDF/B-VII.1 were used. These calculations provide high-fidelity solutions that are typically converged to within 1% in mesh tally voxels in the regions of interest. Parameter studies with the PWR and BWR models employ pinwise spatially uniform core sources. While not representative of any actual operating condition, this distribution provides a convenient means of isolating the effects of individual parameters—whether modifications in the physical construction of the models or variations in analysis parameters. The fission nuclides included in the source definitions are discussed in Section 5.1 and Section 5.2. All radiation transport calculations in this report are normalized to the full-power values noted in Sections 4.1 and 4.2.

The baseline calculations in Sections 5.1.1 and 5.1.2 form the foundation for many of the parameter studies addressed in Section 5. In some cases, particularly for those presented in Section 6, calculations are run using a neutron source representing a single fissile isotope—typically ^{235}U—for parameter studies. These cases are noted in the relevant sections.

The purpose of the parameter studies described in this section was to identify sensitivities to physical parameters and modeling techniques that may be more pronounced in the extended beltline region than in the traditional beltline region. This information provides insights into optimal calculation strategies.

Much of the data analysis in this report is based on interpretation of mesh tally plots that provide information on the model geometry and the solution (or ratio of solutions in parameter studies) being plotted. Examples of mesh tally plots are provided in APPENDIX A. Examination of those plots will aid in understanding the features of the 2D data plots throughout Sections 5 and 6.

5.1.1 PWR model

The baseline calculation for the PWR model employs a source with fission fractions based on a pseudo beginning-of-life (BOL) source. Fission fractions for six fissile isotopes (^{235}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu) as a function of PWR burnup were obtained from NUREG/CR-6115, Table 2.1.1.1 [70]. The fractions for ^{240}Pu and ^{242}Pu were excluded since they are below 0.05%, even at the highest reported burnup values. The remaining data are provided below in Table 5-1.

Table 5-1. Fraction of fissions by isotope as a function of burnup for a PWR

Exposure (MWD)	^{235}U	^{238}U	^{239}Pu	^{241}Pu
150	0.9281	0.06172	0.01018	4.258E-7
500	0.9051	0.06214	0.03270	1.499E-5
10,000	0.5570	0.07074	0.3395	0.03342
20,000	0.3564	0.07819	0.4647	0.09988
40,000	0.1231	0.09120	0.5828	0.2035

For fresh fuel, there is no fission from the Pu isotopes, so only the ^{235}U and ^{238}U spectra are required. The pseudo-BOL source was obtained by using the ^{238}U fission fraction at the lowest recorded burnup (6.172% at 150 MWD) and assigning the remainder (93.828%) to ^{235}U. Note that of all the fissile isotopes considered, the ^{238}U fission fraction has the smallest variation with burnup. With increasing burnup, the relative contributions of ^{235}U, ^{239}Pu, and ^{241}Pu change significantly, but their sum varies by only a few percent throughout the lifetime of the fuel.

Plan views of the fast ($E > 1$ MeV) neutron flux for the baseline PWR calculation are shown in Figure 5-1, Figure 5-2, and Figure 5-3. Figure 5-1 shows the fast flux at the core midplane ($Z = 195$ cm), where the highest flux levels occur. Figure 5-2 and Figure 5-3 show the fast neutron flux at $Z = -70$ cm and at $Z = 470$ cm, respectively. These elevations are approximately 80 cm below and 90 cm above the axial extents of the fuel (Figure 4-2).

The peak fast neutron flux at the core midplane for the baseline solution is 4.31×10^{10} n/cm^2•sec. (Table 1 of IAEA's report [6] lists a fast flux of 4×10^{10} n/cm^2•sec for a Westinghouse PWR.) The peak fast flux levels in the RPV at the elevations of $Z = -70$ cm and $Z = 470$ cm are approximately three orders of magnitude lower than the peak flux in the RPV at the core midplane. The elevation at $Z = 470$ cm is noteworthy, as it includes critical weld locations for the nozzles, as well as the vessel supports.

If the spatially uniform source of this baseline calculation were realistic, then the peak EOL fast fluence levels at $Z = -70$ cm and $Z = 470$ cm would be less than 1×10^{17} n/cm^2, which is the value above which 10 CFR Part 50 [3], Section III, Appendix H, requires monitoring of beltline materials using a surveillance program complying with ASTM E185-82 [4]. With typical axial power distributions, the neutron flux levels in the upper and lower regions of the fuel are lower than the core average, so the fast flux values at these elevations with the baseline model are likely to be conservative with respect to lifetime exposure.

Elevation views of the fast neutron flux for the baseline PWR calculation are shown in Figure 5-4, Figure 5-5, and Figure 5-6. Figure 5-4 is an elevation view at 270.5°, which is the azimuthal location with the maximum amount of water between the core and the RPV. Figure 5-5 is an elevation view at 315.5°, which is at or near the location of the peak fast flux in the RPV (Figure 5-1). Figure 5-6 shows the fast flux at an azimuthal angle of 292.5°, which is through the centerline of the outlet nozzle.

The elevation views show clear evidence of a condition known as *cavity streaming*, in which fast neutrons that enter the cavity gap between the RPV and the concrete bioshield scatter into directions that transport them vertically upward and downward in the gap. At elevations of ~50 to ~60 cm above and below the active fuel region, the fast flux radial profile through the RPV is no longer monotonically decreasing from the RPV inner surface to the RPV outer surface.

The effect of cavity streaming neutrons is illustrated further in Figure 5-7 and Figure 5-8. These plots provide 1D normalized radial fast flux profiles through the RPV at 270.5° and 315.5°, respectively. At each of these azimuthal locations, 1D radial profiles are shown at elevations ranging from ~100 cm below the core to ~110 cm above the core. Each plot also includes a solid dashed line that represents the attenuation formula in Eq. (3) of Regulatory Guide (RG) 1.99 [74]. A number of observations can be made from these two figures.

1. At elevations within or nearly within the height of the fuel ($Z = 0$ cm, $Z = 195$ cm, and $Z = 400$ cm), the fast neutron flux is attenuated with a nearly exponential behavior that decreases more rapidly than the RG 1.99 formula would suggest. The RG 1.99 equation was developed based on the attenuation behavior of dpa rates rather than fast flux, and at

elevations within the core height the equation is conservative for predictions of fast flux attenuation.

2. At elevations outside of an axial range extending from ~50 cm below the fuel (Z = -40 cm) to ~50 cm above the fuel (Z = 430 cm), the slope of the fast flux profile changes, and there is no longer a monotonic fast flux decrease through the entire RPV thickness.
3. For elevations lower than ~-75 cm and higher than ~480 cm, the peak fast flux is no longer at the inner surface of the RPV, but rather at the outer surface. In these regions, the radial flux profile is dominated by cavity streaming neutrons. Note that based on the discussion above regarding peak EOL fluences at Z = -70 cm and Z = 470 cm, these locations would be expected to have EOL fluence levels below the monitoring threshold of 1×10^{17} n/cm^2.

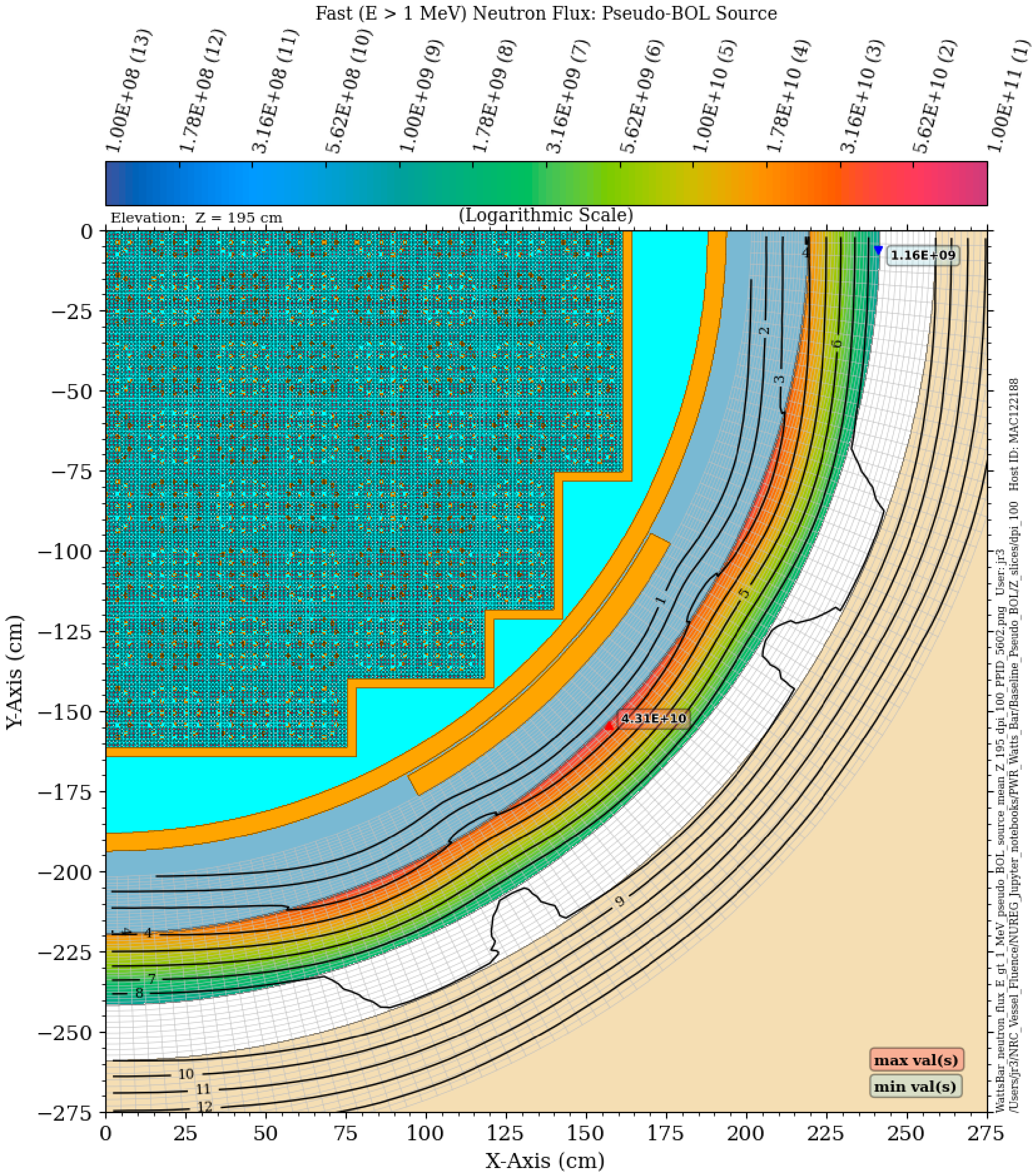


Figure 5-1 **Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Plan view at the core midplane elevation**

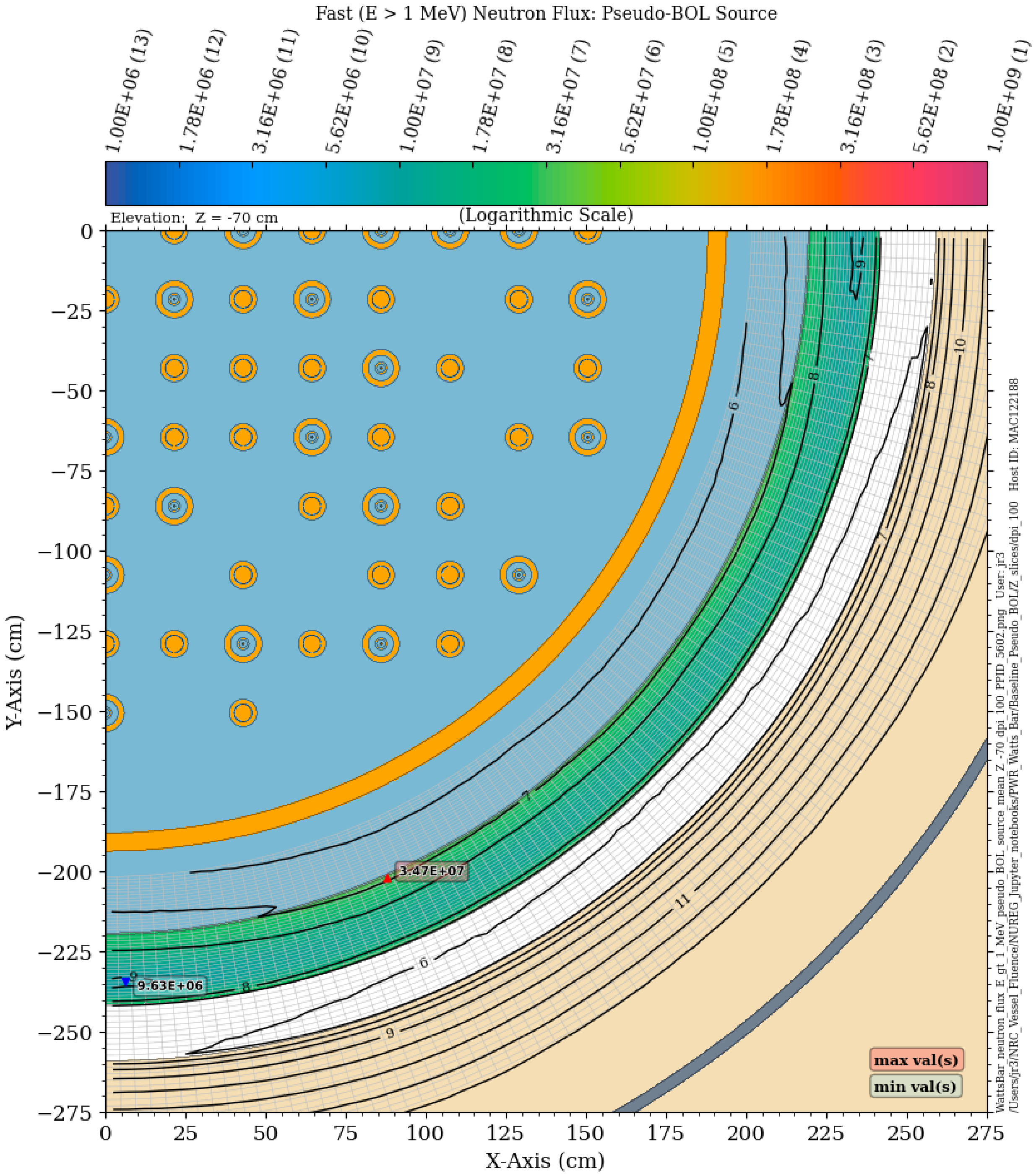


Figure 5-2 Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Plan view at an elevation of Z = -70 cm

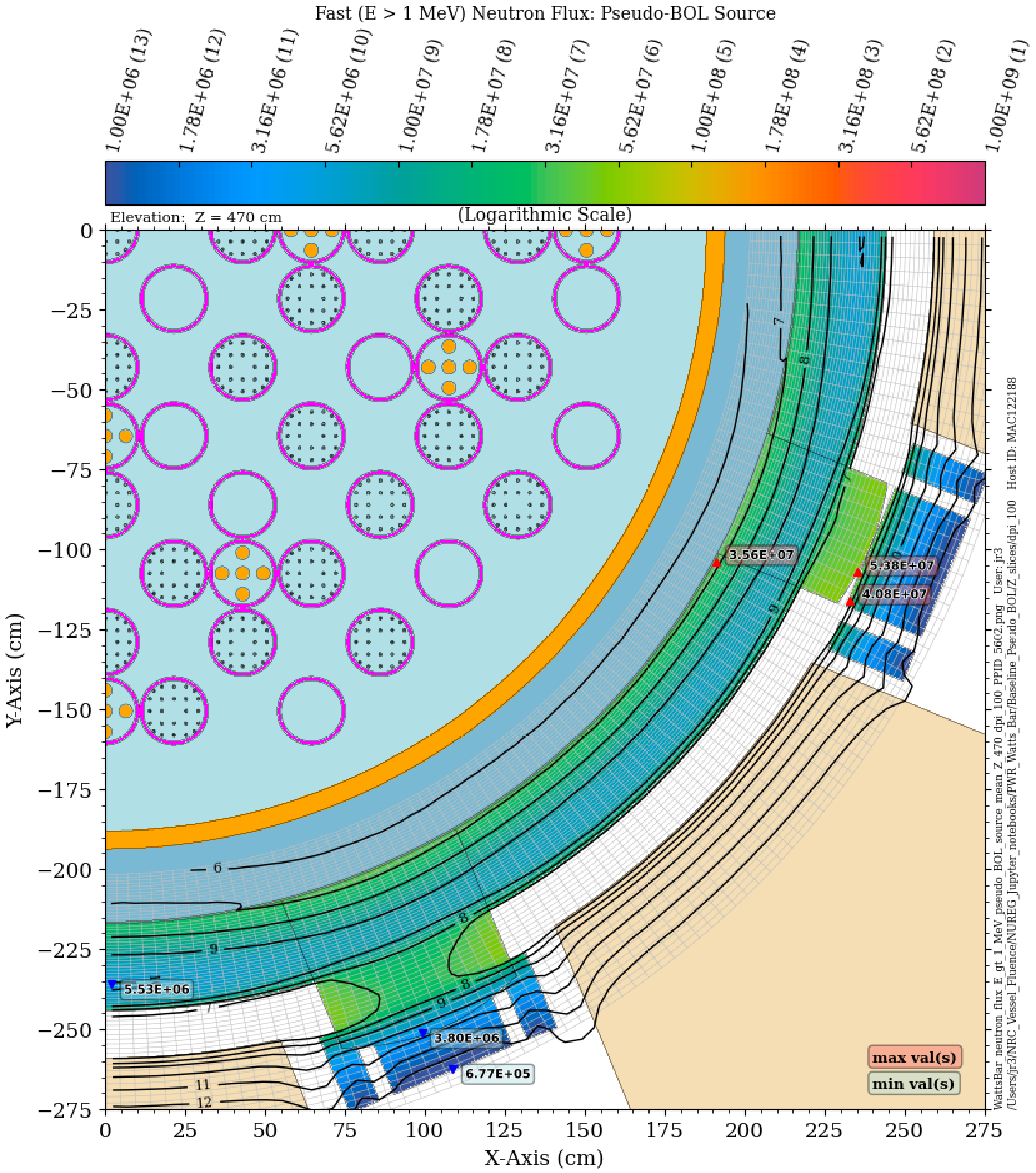


Figure 5-3 Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Plan view through the vessel supports at an elevation of Z = 470 cm

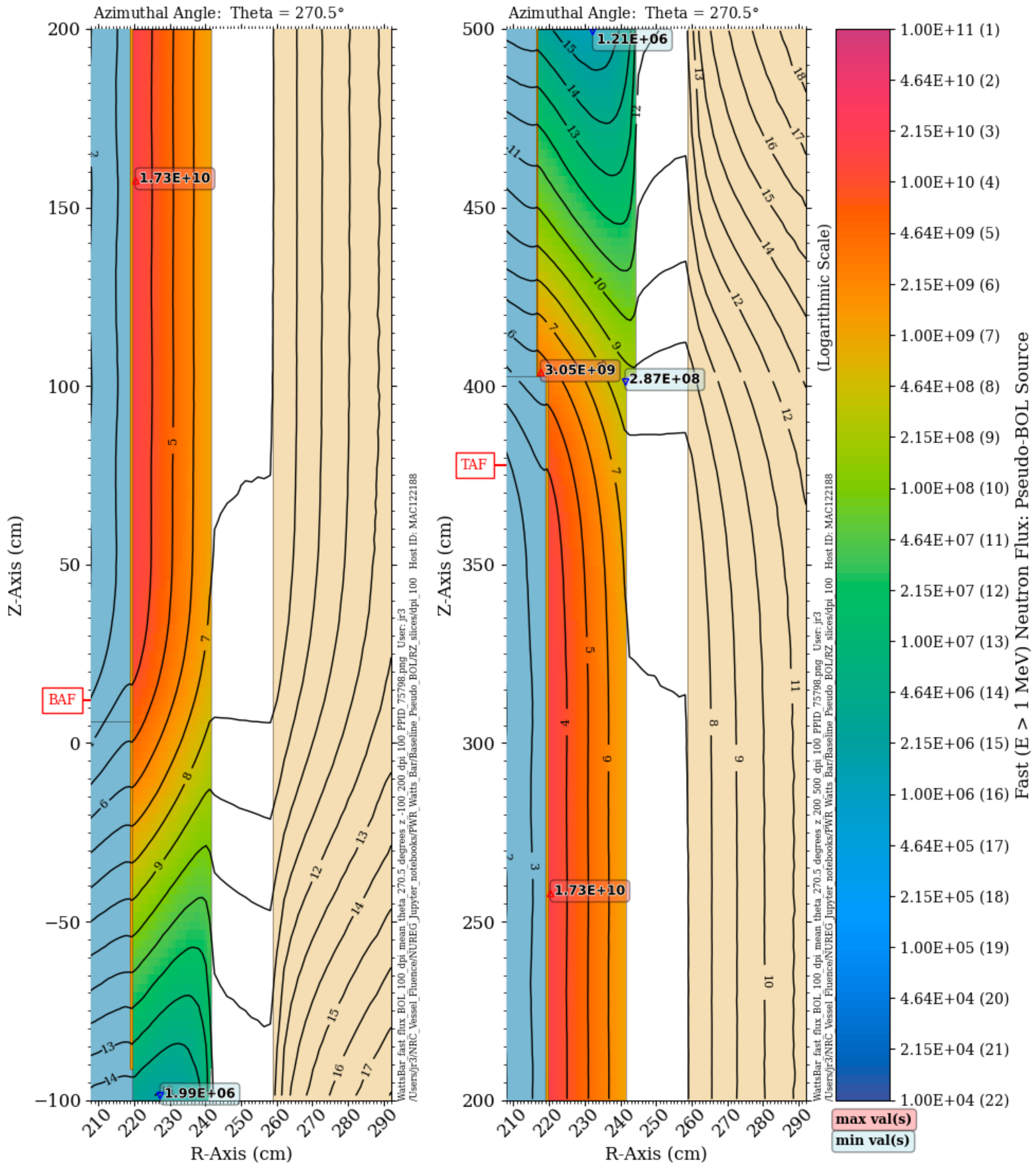


Figure 5-4 Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Elevation view at an azimuthal angle of 270.5°. This is the azimuthal location with the maximum amount of water between the core and the RPV

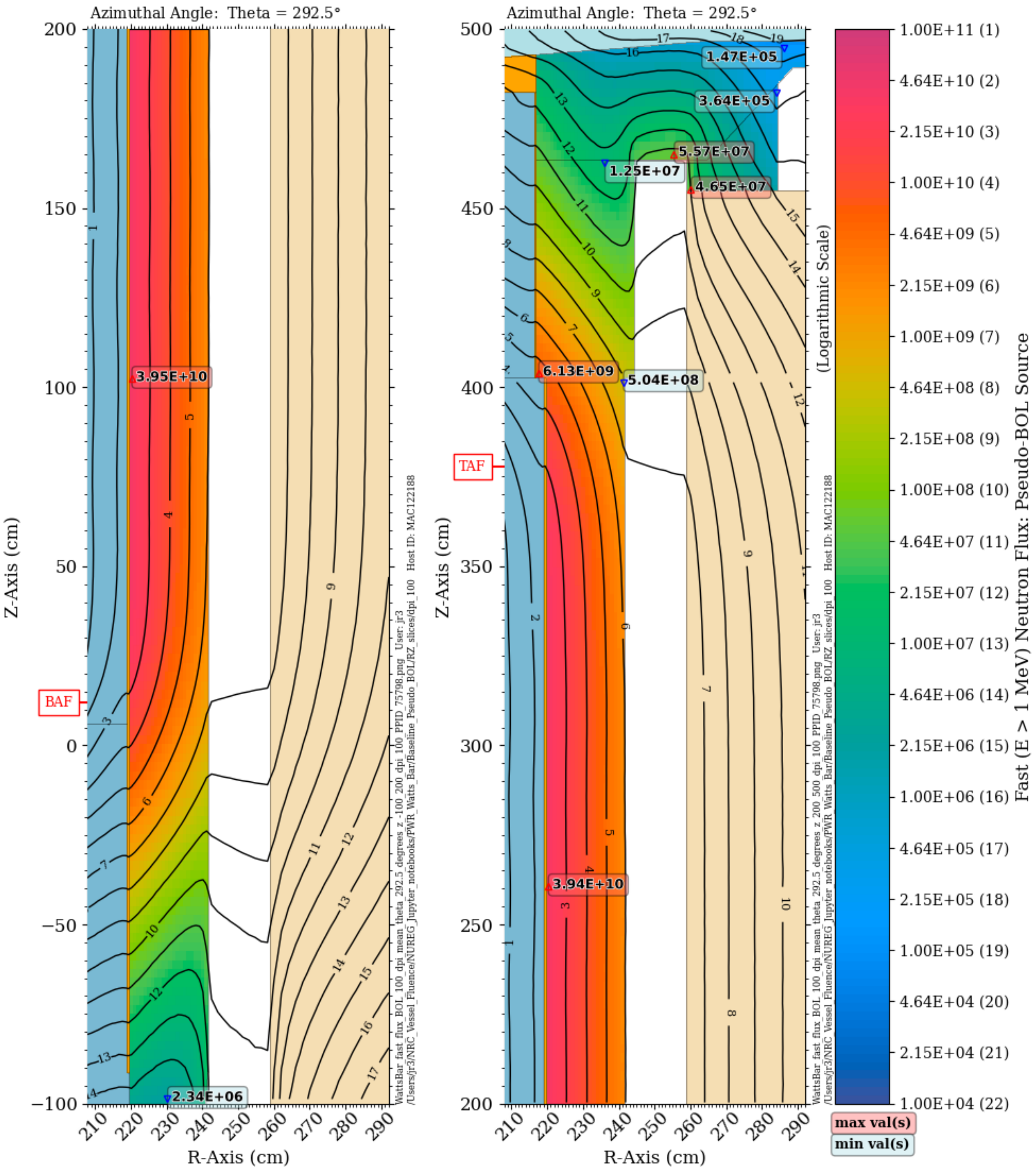


Figure 5-5 **Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Elevation view at an azimuthal angle of 292.5°**

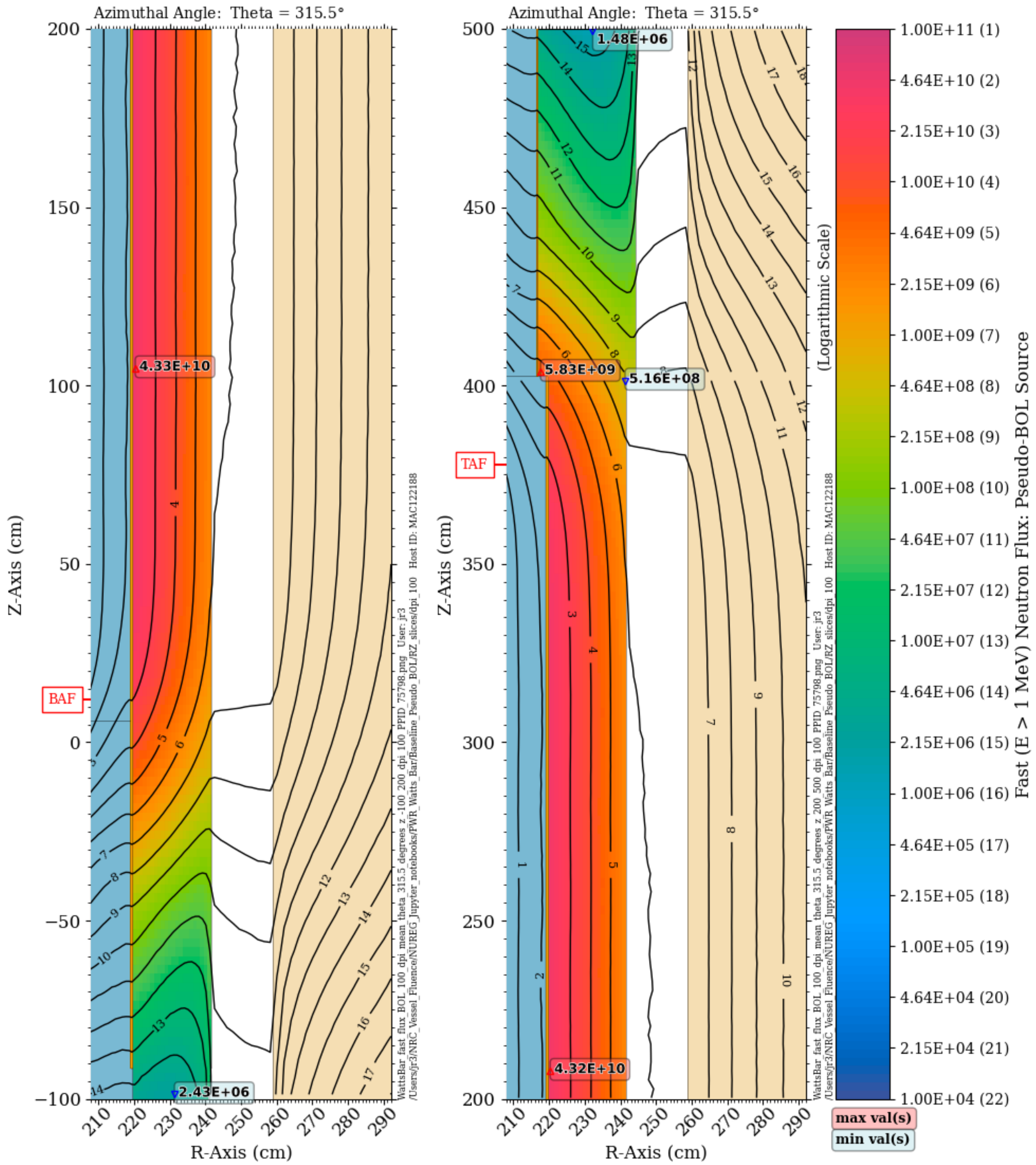


Figure 5-6 **Fast neutron flux in the baseline PWR model with a pseudo-BOL fission source. Elevation view at an azimuthal angle of 315.5°**

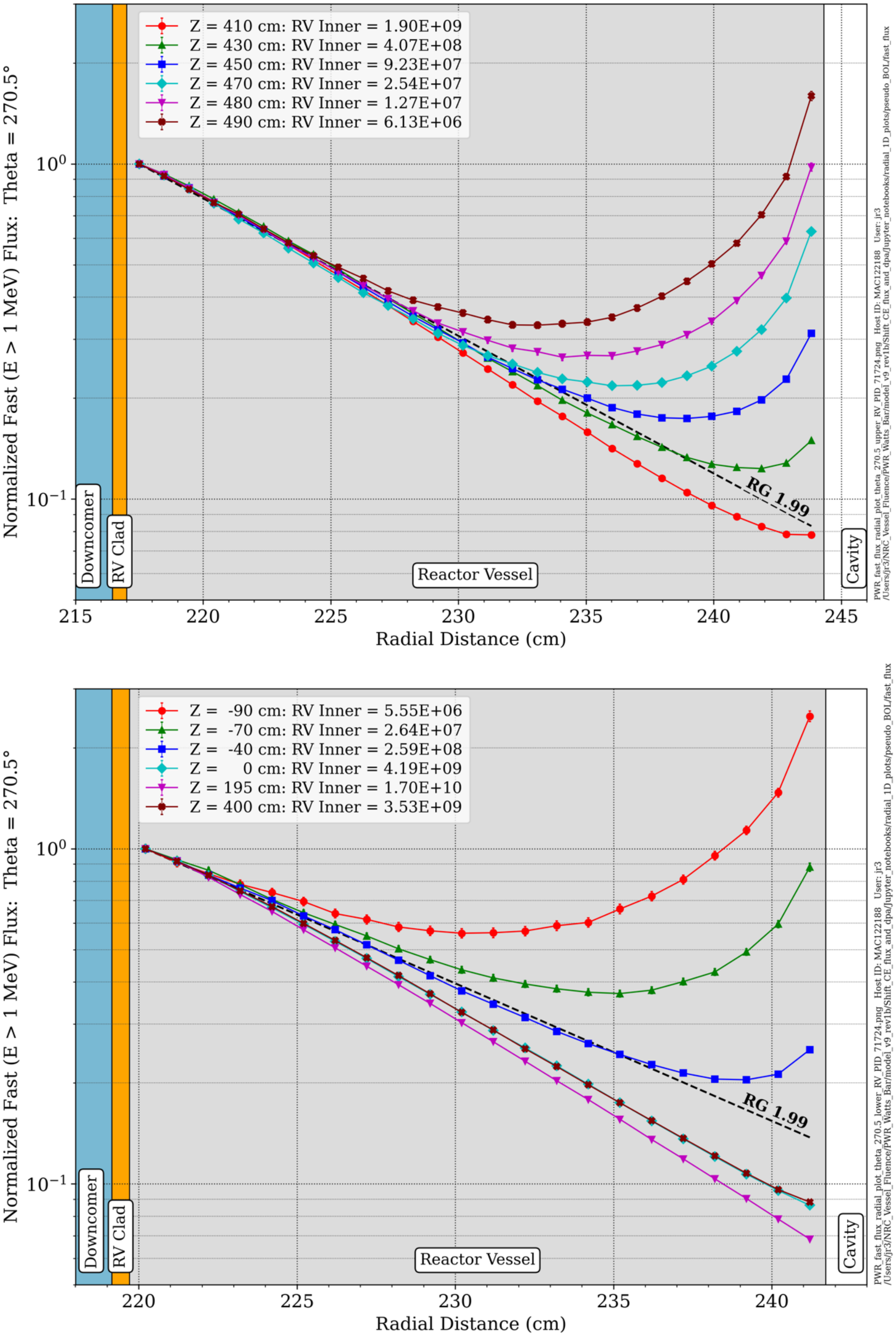


Figure 5-7 **Normalized radial fast neutron flux profiles in the PWR model with a pseudo-BOL fission source. Azimuthal angle of 270.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation**

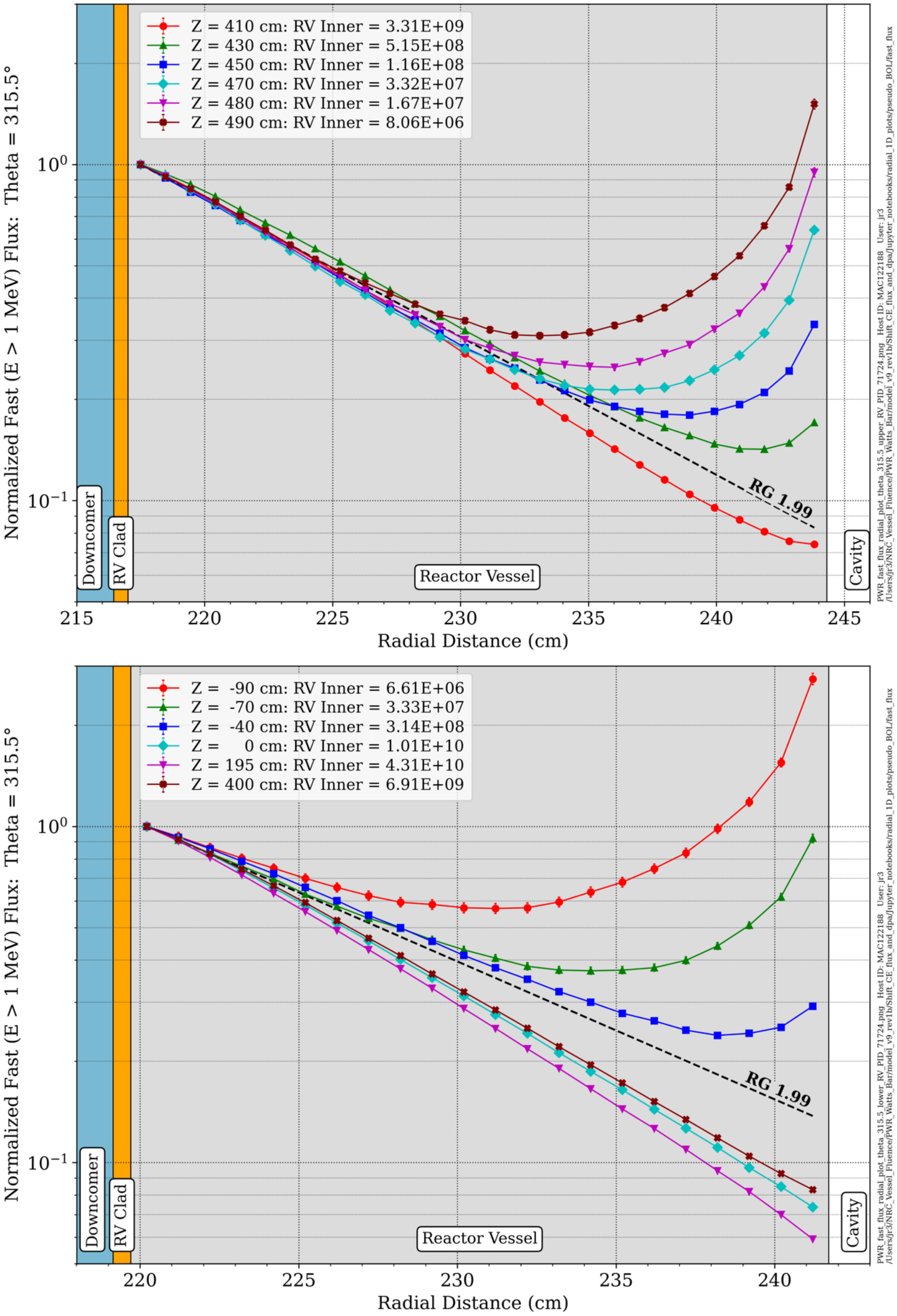


Figure 5-8 **Normalized radial fast neutron flux profiles in the PWR model with a pseudo-BOL fission source. Azimuthal angle of 315.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation**

5.1.2 BWR model

The baseline calculation for the BWR model employs a source with fission fractions based on a pseudo-BOL source. Fission fractions for four fissile isotopes (^{235}U, ^{238}U, ^{239}Pu, and ^{241}Pu) as a function of BWR burnup were obtained from NUREG/CR-6115, Table 2.2.1.1 [70]. The data are provided below in Table 5-2.

Table 5-2. Fraction of fissions by isotope as a function of burnup for a BWR

Exposure (MWD/T)	^{235}U	^{238}U	^{239}Pu	^{241}Pu
4.33	0.7651	0.0733	0.1566	0.0
5.92	0.7190	0.0733	0.1936	0.0059
6.62	0.7085	0.0733	0.2096	0.0074
11.13	0.6153	0.0766	0.2814	0.0220
14.12	0.5630	0.0766	0.3220	0.0373
15.66	0.5370	0.0766	0.3400	0.0440
17.88	0.5070	0.0766	0.3580	0.0533

For fresh fuel, there is no fission from the Pu isotopes, so only the ^{235}U and ^{238}U spectra are required. The pseudo-BOL source was obtained by using the ^{238}U fission fraction at the lowest tabulated burnup (7.33% at 4.33 MWD/T) and assigning the remainder (92.67%) to ^{235}U. Note that, as with the PWR model, there is very little change in the ^{238}U fission fraction with burnup. With increasing burnup, the relative contributions of the ^{235}U, ^{239}Pu, and ^{241}Pu change significantly, but their sum remains nearly constant throughout the lifetime of the fuel.

Plan views of the fast neutron flux in the BWR model are shown in Figure 5-9, Figure 5-10, and Figure 5-11. Figure 5-9 shows the fast flux at the core midplane, where the highest flux levels occur. Note that the maximum fast flux level at the core midplane in the BWR RPV is nearly an order of magnitude lower than that at the core midplane in the PWR RPV (Figure 5-1). This difference is consistent with PWR and BWR fluence estimates in [6] and [7]. Figure 5-10 shows the fast flux at an elevation through the recirculation outlet nozzles at Z = -250 cm. The peak RPV fast flux at this elevation is approximately two orders of magnitude lower than at the core midplane. Figure 5-11 shows the fast flux above the shroud dome at an elevation of Z = 375 cm. At this elevation, the peak RPV fast flux level is also approximately two orders of magnitude lower than at the core midplane. If the spatially uniform source of this baseline calculation were realistic, then the peak EOL fast fluence at Z = -250 cm and Z = 375 cm would be less than 1×10^{17} n/cm^2, which is the threshold for requirement of a surveillance program, as noted in Section 5.1.1.

Elevation views of the fast neutron flux for the baseline BWR calculation are shown in Figure 5-12 and Figure 5-13. Figure 5-12 is an elevation view at 0.5°, which is the location with the minimum fast flux incident to the RPV. Figure 5-13 is an elevation view at 44.5°, which has the maximum fast flux incident to the RPV. By comparison with the PWR figures in Section 5.1.1, it is clear that the effects of cavity streaming neutrons are less pronounced in the BWR model than the PWR model, particularly at elevations above the top of the core. This is also illustrated in Figure 5-14 and Figure 5-15. At all elevations above the core midplane up to a distance of almost 200 cm above the top of the fuel, the normalized fast flux decreases monotonically (or nearly so) in the RPV, and it is bounded by (i.e., below) RG 1.99 Eq. (3). The fast neutron flux profile through the BWR RPV only ceases to display monotonic attenuation at the 44.5° azimuthal angle for elevations more than approximately 50 cm below the active core. At increasing distances below the active core, the fast neutron flux profile in the RPV is dominated by cavity streaming neutrons.

The reason for this behavior in the BWR model is the significant difference in the neutron flux attenuation at elevations below and above the fuel. For the lower elevations, the neutron flux is significantly attenuated by the water, and to a lesser extent, by the structures between the core and the RPV. This is clearly demonstrated in the lower axial portions of Figure 5-12 and Figure 5-13. At locations above the core midplane, there is less neutron attenuation from the core to the RPV due to (1) the coolant density within the fuel assemblies decreasing with increasing height as the VFs increase, and (2) neutrons that have transport paths through the shroud dome (with coolant in the steam phase) have significantly less attenuation than those transported through coolant in the liquid phase. Thus, for the BWR model, cavity, streaming is significant only for elevations in the lower extended beltline region, where EOL fast fluence values are likely to be less than 1×10^{17} n/cm^2.

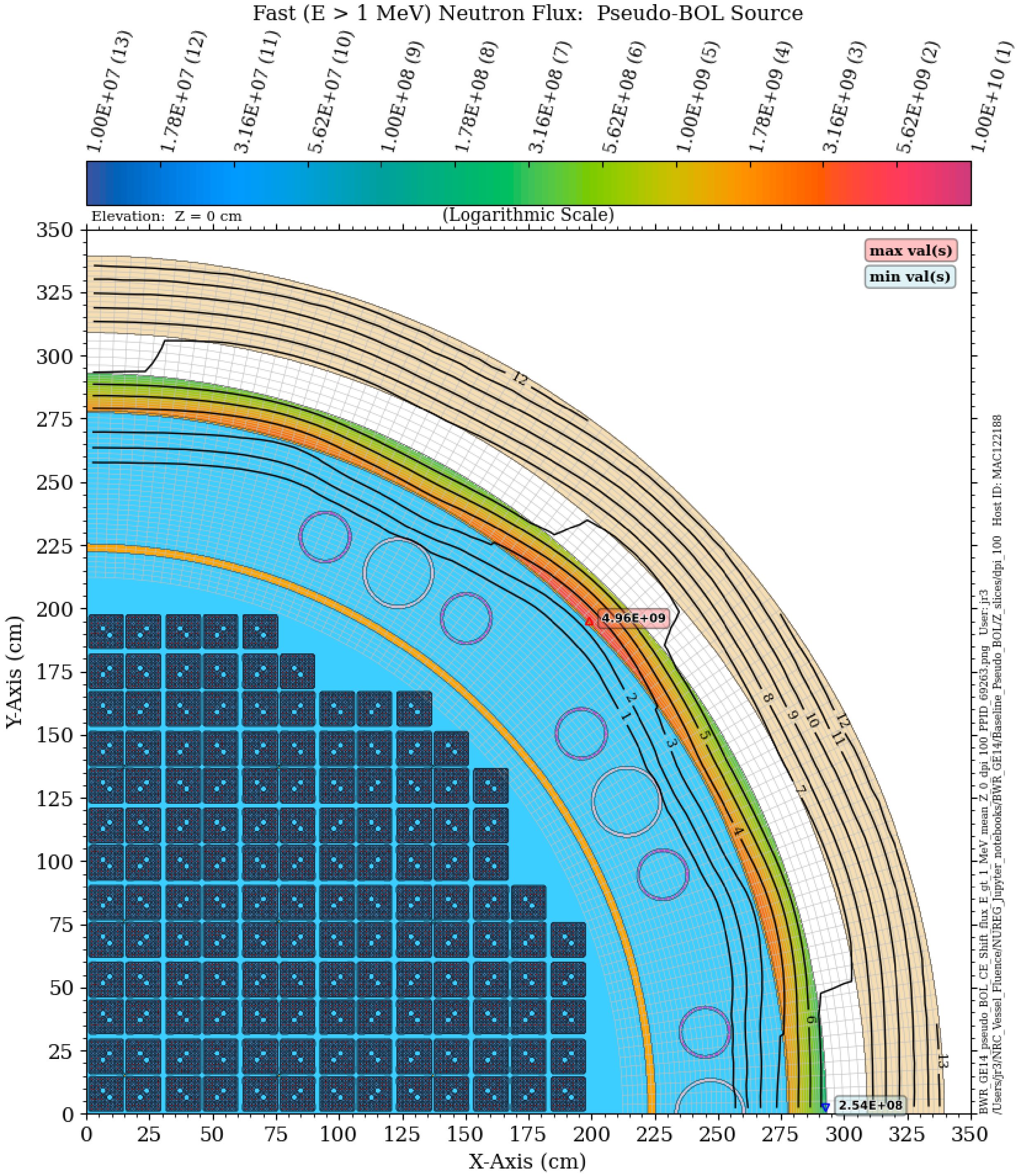


Figure 5-9 Fast neutron flux in the baseline BWR model with a pseudo-BOL fission source. Plan view at the core midplane

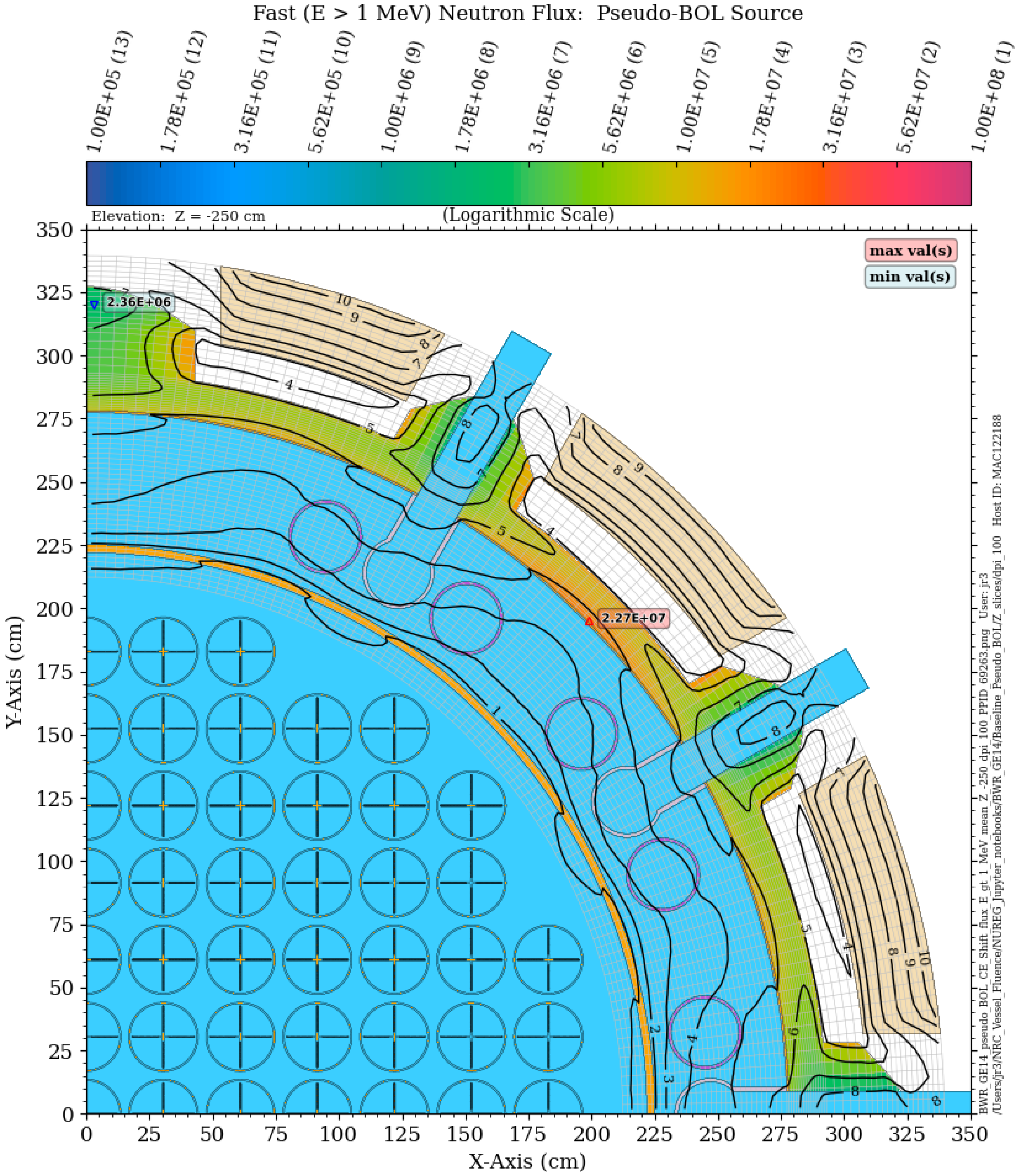


Figure 5-10 Fast neutron flux in the baseline BWR model with a pseudo-BOL fission source. Plan view through the recirculation outlet nozzles at an elevation of Z = -250 cm

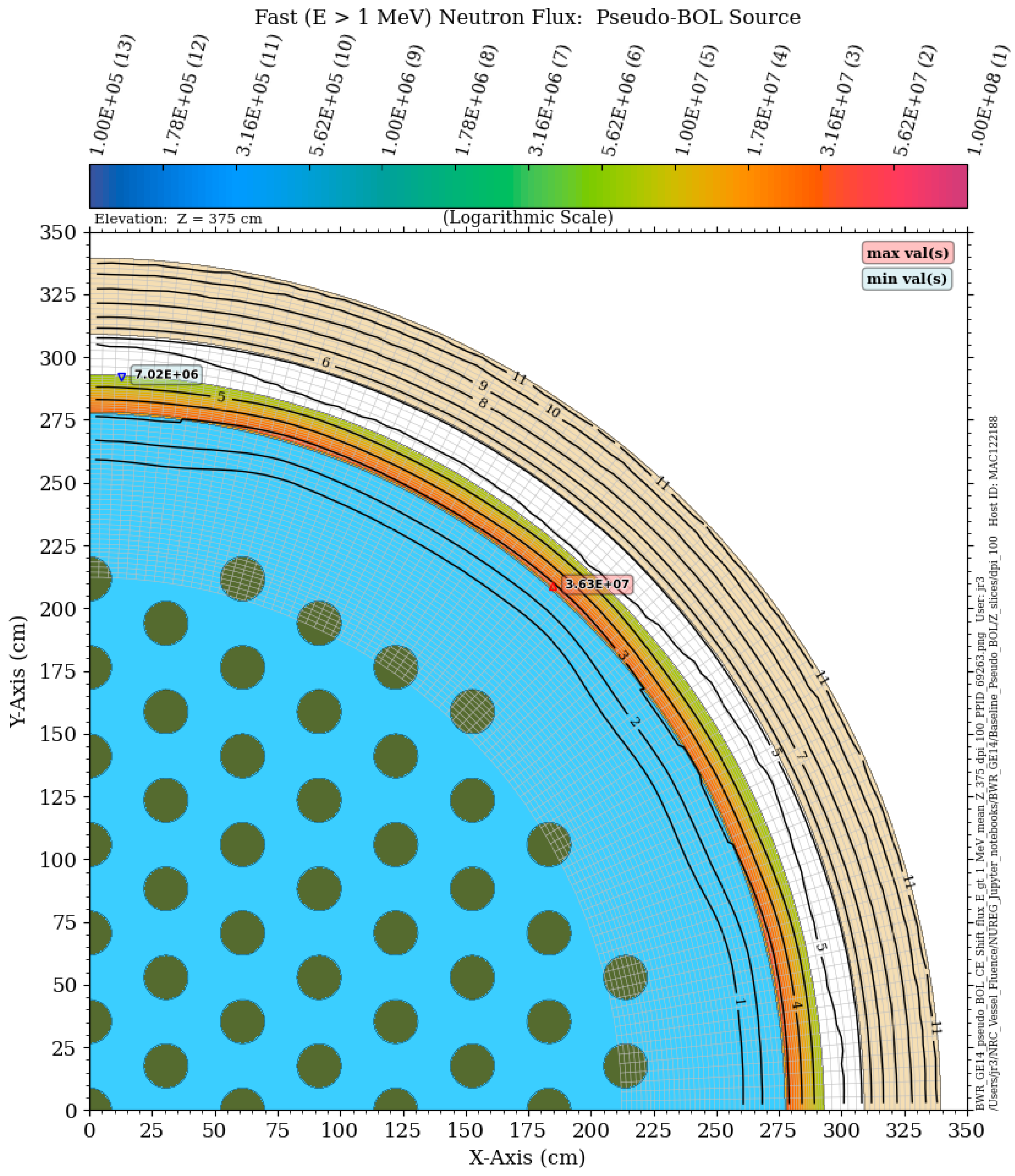


Figure 5-11 Fast neutron flux in the baseline BWR model with a pseudo-BOL fission source. Plan view above the core shroud dome at an elevation of Z = 375 cm

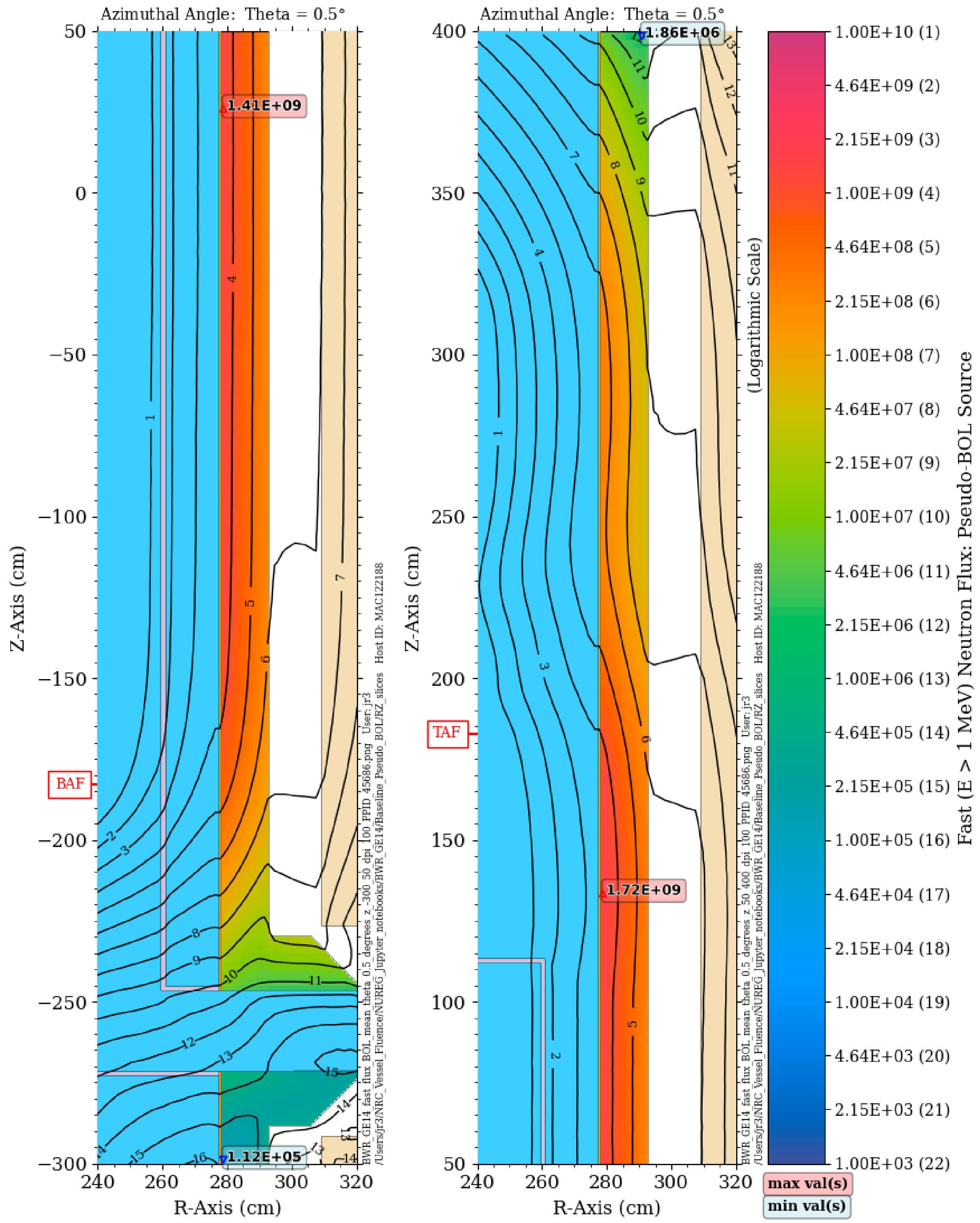


Figure 5-12 Fast neutron flux in the baseline BWR model with a pseudo-BOL fission source. Elevation view through the recirculation outlet nozzle at an azimuthal angle of 0.5°

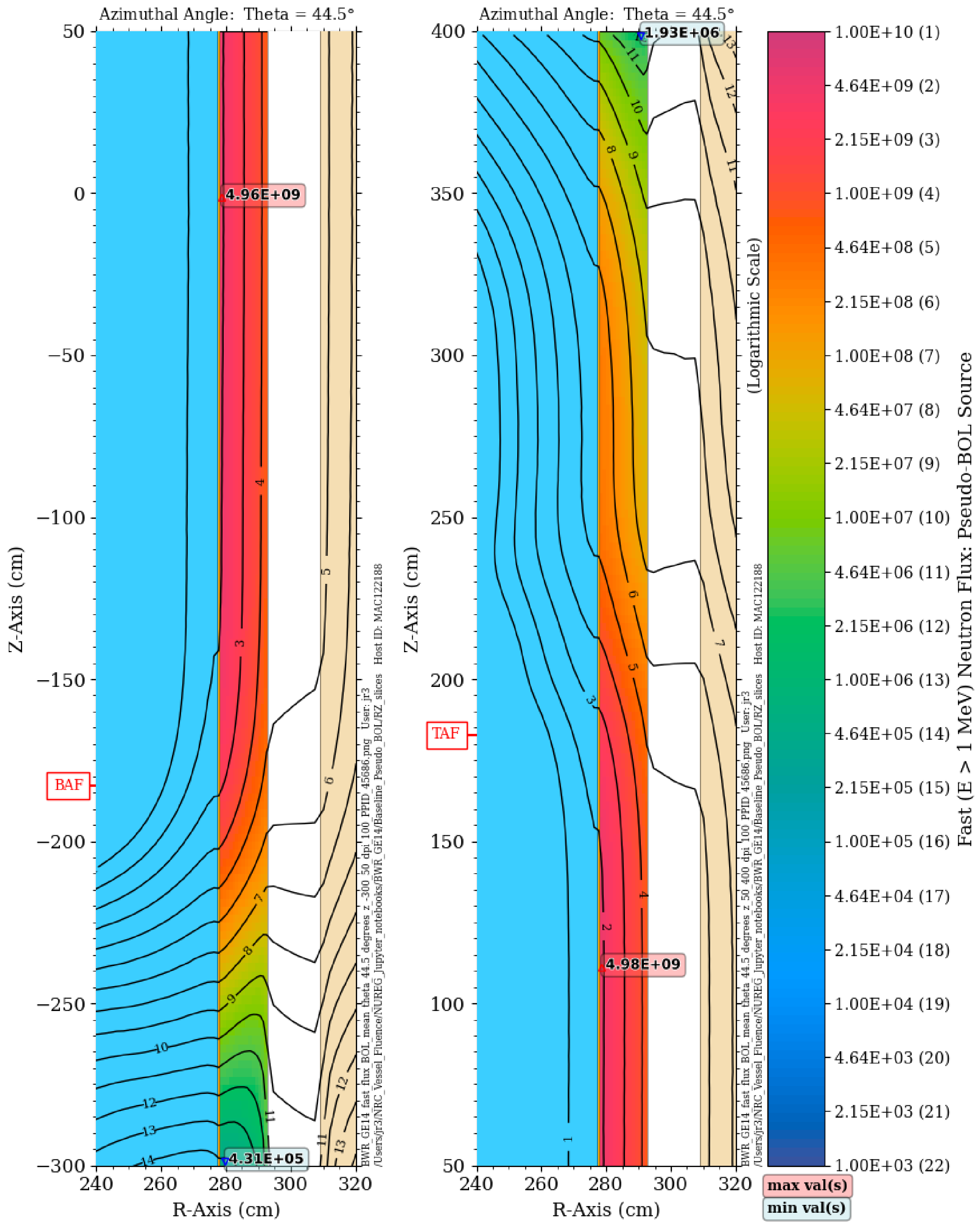


Figure 5-13 Fast neutron flux in the baseline BWR model with a pseudo-BOL fission source. Elevation view at an azimuthal angle of 44.5°

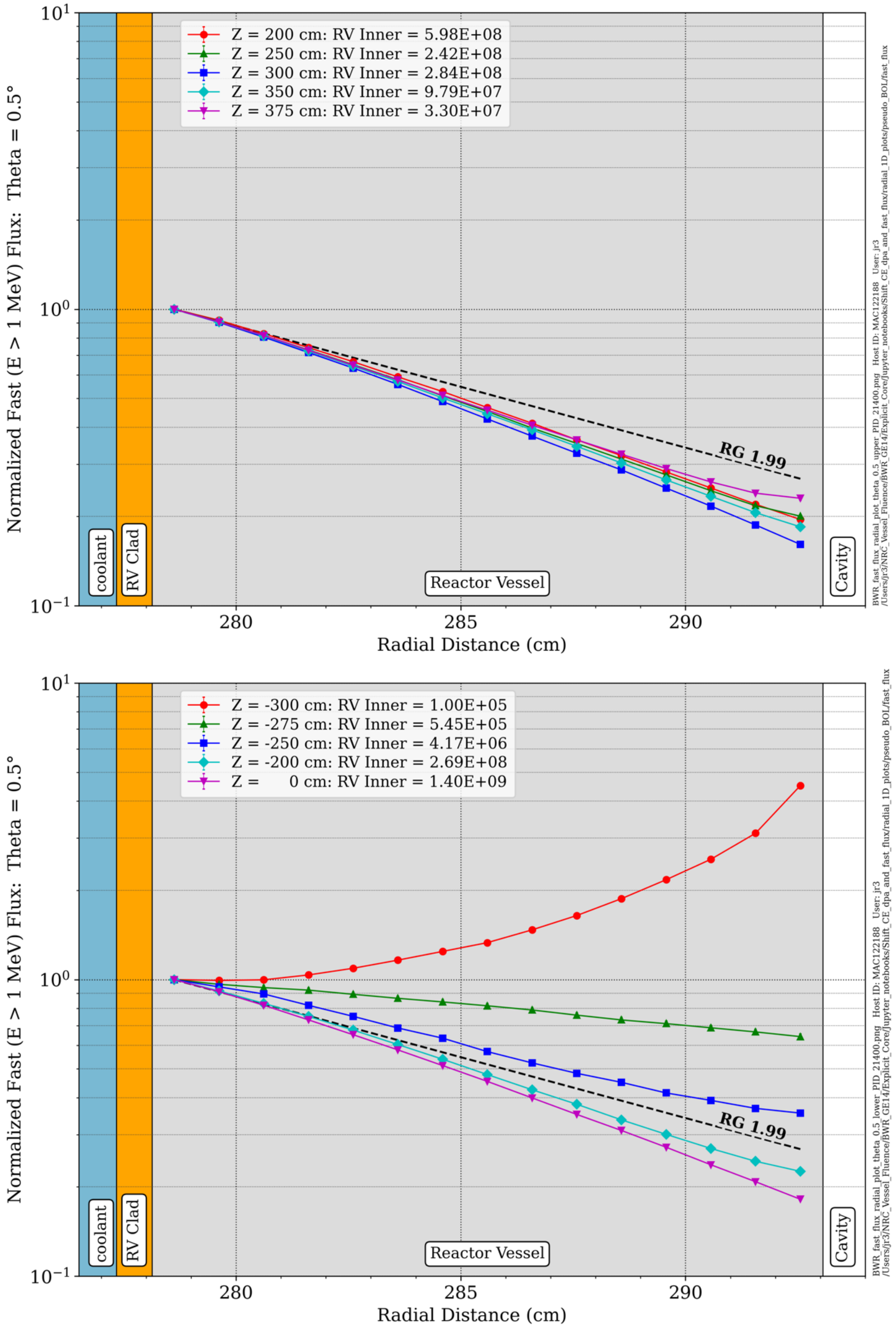


Figure 5-14 Normalized radial fast neutron flux profiles in the BWR model with a pseudo-BOL fission source. Azimuthal angle of 0.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation

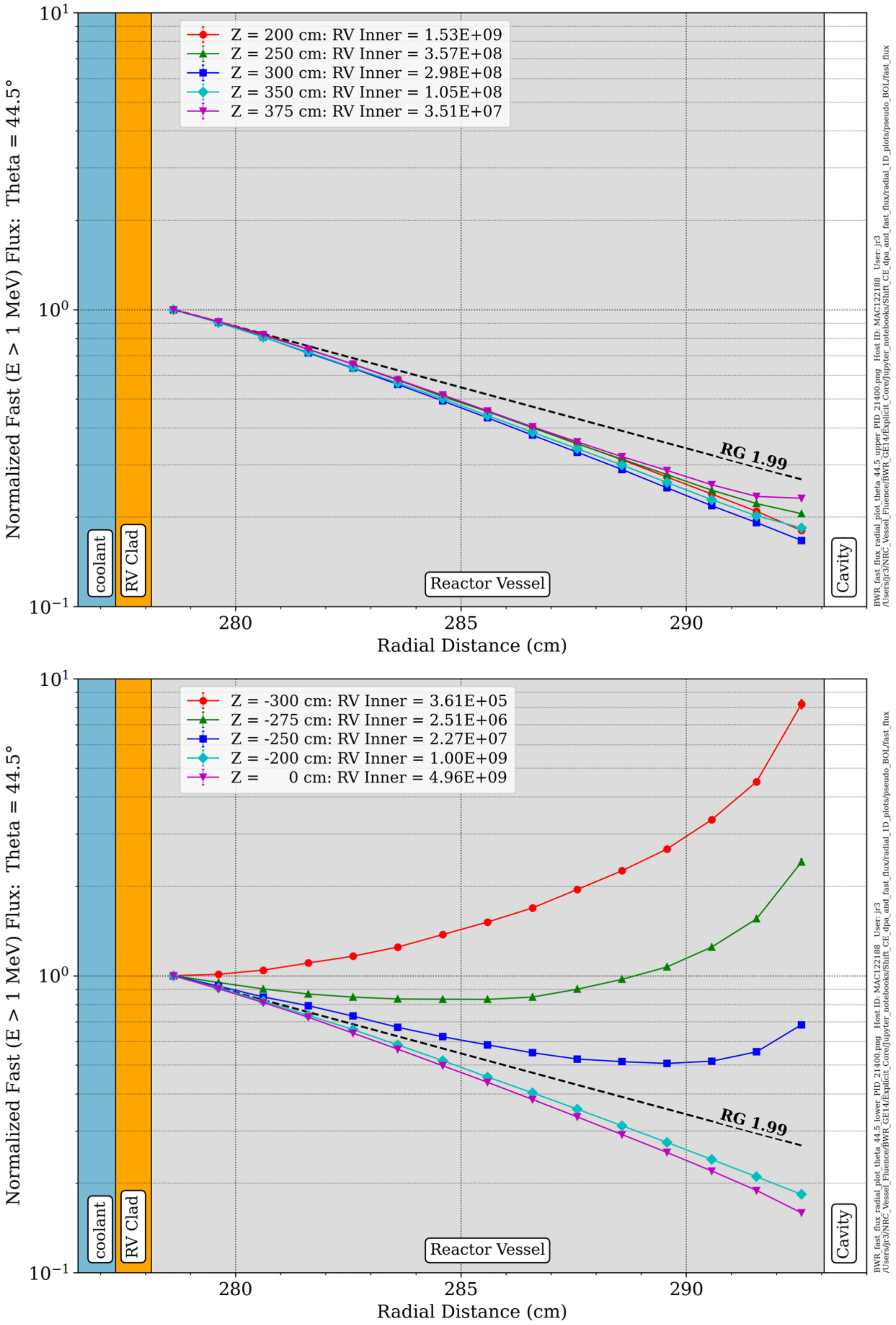


Figure 5-15 Normalized radial fast neutron flux profiles in the BWR model with a pseudo-BOL fission source. Azimuthal angle of 44.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation

5.2 Fission spectrum changes with burnup

As LWR fuel is burned up, the fraction of fissions by isotope changes. In general, the fraction of fissions from ^{235}U decreases monotonically, while the fission fractions from ^{239}Pu, ^{241}Pu, and, to a lesser degree ^{238}U, increase monotonically. While some fission can occur in ^{240}Pu and ^{242}Pu, the fraction of fissions in those isotopes is very low, even in high-burnup assemblies.

Changes in the fission fraction contributions with burnup affect RPV fluence in three ways:

1. They result in changes in the energy spectrum of the fission neutrons. Relative to ^{235}U, the spectrum of prompt fission neutrons from ^{238}U is “softer,” i.e., shifted toward lower energies, while the spectra for ^{239}Pu and ^{241}Pu are “harder,” i.e., shifted toward higher energies. This is shown in Figure 5-16.
2. They cause changes in the value of $\bar{\nu}$, the average number of neutrons emitted per fission. The values of $\bar{\nu}$ for ^{235}U, ^{238}U, ^{239}Pu, and ^{241}Pu are provided in Table 5-3 based on cross-section data in [43]. With increasing fuel burnup, the average number of neutrons released per fission increases due to the increasing contributions of ^{239}Pu and ^{241}Pu.
3. They lead to changes in K, the energy released in fission. Values of K for ^{235}U, ^{238}U, ^{239}Pu, and ^{241}Pu were obtained from work by James [75] and are provided in Table 5-3. The number of fissions required per second for a given power level is inversely proportional to K. Consequently, the fission rate per unit power is slightly lower with the Pu isotopes compared to with the U isotopes. However, this effect is small (less than 4%) compared to the increasing values of $\bar{\nu}$.

Table 5-3 Values of nu-bar ($\bar{\nu}$) and kappa (K) used in the fissile isotope parameter study

Isotope	$\bar{\nu}$ (neutrons/fission)	K (MeV/fission)
^{235}U	2.44	202.7
^{238}U	2.56	205.9
^{239}Pu	2.88	207.2
^{241}Pu	2.95	210.6

The baseline calculations in Sections 5.1.1 and 5.1.2 employed pseudo-BOL sources based on data from Table 5-1 and Table 5-2. In the same manner, pseudo end-of-life (pseudo EOL) sources were used to evaluate fast flux levels based on the maximum-depletion data in Table 5-1 and Table 5-2. A comparison of the pseudo BOL and pseudo EOL sources is shown in Table 5-4. The BWR EOL ^{241}Pu fraction includes 5.33% from Table 5-2 and the residual fraction of 0.51%.

Table 5-4 Isotopic fission fractions for the pseudo BOL and pseudo EOL sources used in the PWR and BWR models

Isotope	PWR		BWR	
	BOL	EOL	BOL	EOL
^{235}U	93.828%	12.31%	92.67%	50.70%
^{238}U	6.172%	9.12%	7.33%	7.66%
^{239}Pu	0.0%	58.28%	0.0%	35.80%
^{241}Pu	0.0%	20.35%	0.0%	5.84%

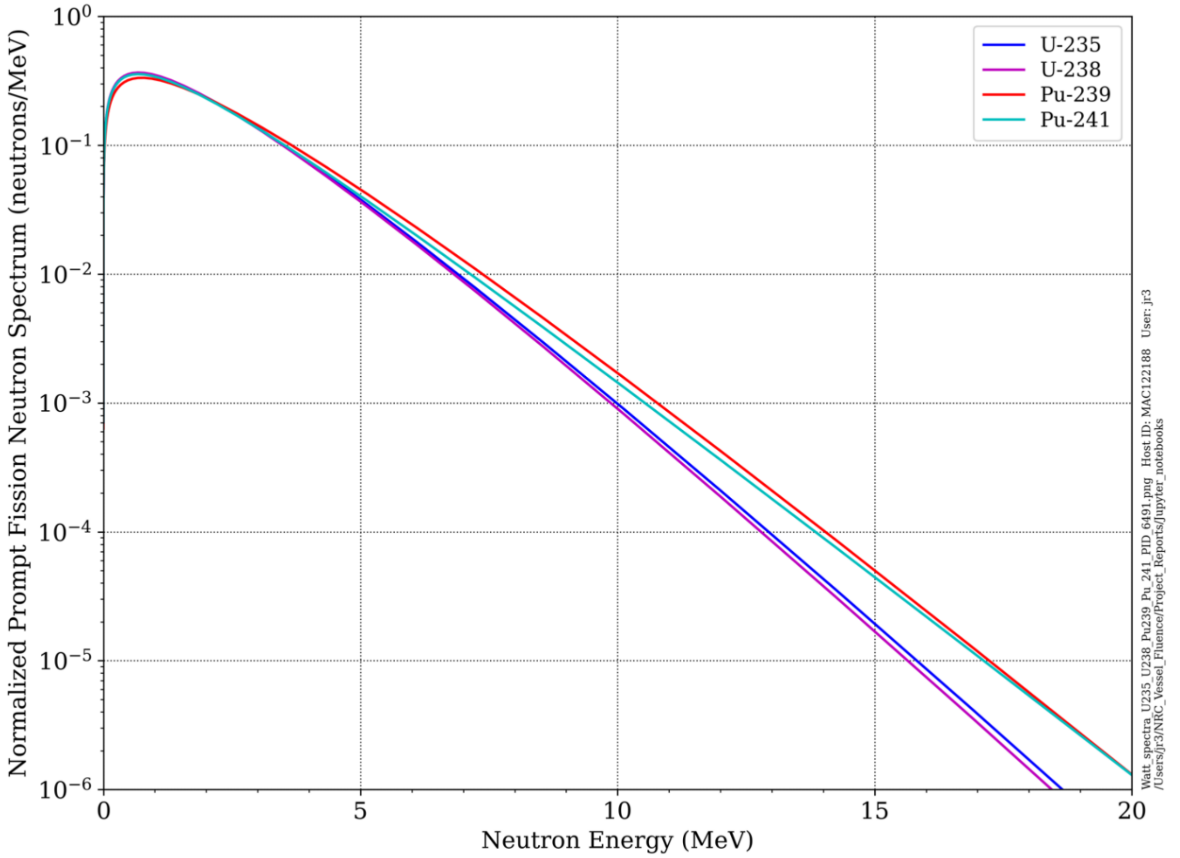


Figure 5-16 Prompt fission neutron spectra of ^{235}U, ^{238}U, ^{239}Pu, and ^{241}Pu. All spectra are normalized to an integrated value of 1.0

If there were no differences in the prompt fission neutron spectra for the four isotopes of interest, then the fast flux in the RPV (and at all locations) would be expected to increase with burnup based on changes in $\bar{\nu}$ and K. Using the data from Table 5-3 and Table 5-4, the expected flux increase from the pseudo BOL to the pseudo EOL sources would be ~12.5% for the PWR model and ~6.5% for the BWR model. However, as discussed in the following paragraph, the shift in the prompt neutron fission spectrum with increasing burnup plays an important role in the overall effect of changes in the fission fractions.

A significant amount of neutron attenuation between the outer fuel assemblies and the RPV occurs in water, especially for the BWR model. Hydrogen is particularly effective in moderating (reducing the energy of) neutrons, as a neutron can lose more than 99.9999% of its energy in a single scatter with ^{1}H. In contrast, a neutron can lose at most ~22.1% of its energy in a single scatter from ^{16}O, and at most, it will only lose ~6.9% in a single scatter from ^{56}Fe. Because the neutron scattering cross section in hydrogen decreases monotonically with neutron energy for energies greater than about 10 keV, a shift in the neutron energy spectrum toward higher energies (as is the case with ^{239}Pu and ^{241}Pu relative to ^{235}U and ^{238}U) will lead to higher fast neutron flux levels in the RPV, even with no consideration of changes in $\bar{\nu}$ and K.

Therefore, it is clear that the fast neutron flux in the RPV of both the PWR and BWR models will increase with increasing fuel burnup. This effect is quantitively addressed in Section 5.2.1 and Section 5.2.2.

5.2.1 Fission spectrum effects in the PWR model

The fast flux levels in the PWR with a pseudo EOL fission source were calculated using the EOL fission fractions from Table 5-4. Ratio plots showing the increase in the fast neutron flux in the RPV from the pseudo EOL relative to the pseudo BOL source are shown in Figure 5-17 through Figure 5-22. At the core midplane, the increase in the fast flux in the RPV ranges from ~23 to 34%. As noted above, ~12.5% of this increase can be attributed to the changes in $\bar{\nu}$ and K. The minimum increases occur in the azimuthal portion of the RPV that is aligned with the neutron pad. This portion of the RPV has the least amount of water between the core and the RPV due to the proximity of the corner fuel assemblies to the core barrel, as well as the displacement of water by steel due to the neutron pad. In contrast, the maximum increase occurs at an azimuthal location with substantially more water between the core and the RPV. This results in a lower fast flux magnitude, but it causes an increase in the EOL/BOL ratio because the spectrum changes are magnified by transmission through a greater amount of water.

At the elevations of Z = -70 cm and Z = 470 cm, the EOL/BOL ratios are greater than at the core midplane, ranging from ~30 to 50%, with the maximum ratios occurring near the inner surface of the RPV. These higher ratios are consistent with the increase in the amount of water through which neutrons are transported to reach the inner portion of the RPV at elevations above and below the active fuel height.

5.2.2 Fission spectrum effects in the BWR model

The fast flux levels in the BWR with a pseudo EOL fission source were calculated using the EOL fission fractions from Table 5-4. Ratio plots of the increase in the fast neutron flux in the RPV from the pseudo EOL to the pseudo BOL sources are shown in Figure 5-23 through Figure 5-27. At the core midplane elevation, the fast flux in the RPV increases by ~20–24%. As noted above, ~6.5% of this increase is due to changes in $\bar{\nu}$ and K, and the remainder is due to the effect of the harder neutron spectrum at EOL. At an elevation of Z = -250 cm, the fast flux increases by ~22–30%. This slightly greater increase is consistent with neutron transport through a greater amount of water than at the core midplane. In a similar manner, the fast flux at Z = 375 cm increases by 24–28%.

BWR cores with higher burnup than that specified in Table 5-3 will experience even greater increases in the RPV fast flux as the contributions from ^{239}Pu and ^{241}Pu increase. A conservative upper limit for this increase can be obtained by comparing the flux with a ^{239}Pu source to that with a ^{235}U source. The ^{239}Pu-to-^{235}U fast flux ratios at the three selected elevations are shown in Figure 5-28, Figure 5-29, and Figure 5-30. While these are a conservative upper limit, as noted, they do indicate that higher burnup BWR fuel would be likely to result in greater EOL/BOL fast flux ratios in the RPV.

5.2.3 Summary

The fast flux levels in the RPV of a PWR and BWR are likely to increase with burnup levels as the contribution of neutrons from the fission of ^{239}Pu and ^{241}Pu increases. These Pu isotopes have a greater average number of neutrons per fission and harder neutron spectra compared to ^{235}U and ^{238}U. The use of core designs with higher fuel burnup could lead to even greater increases in the fast flux as increasing levels of fission occur in the Pu isotopes. The actual variation in the fast neutron flux levels in the RPV as a function of core lifetime will depend on core loading patterns and operating parameters.

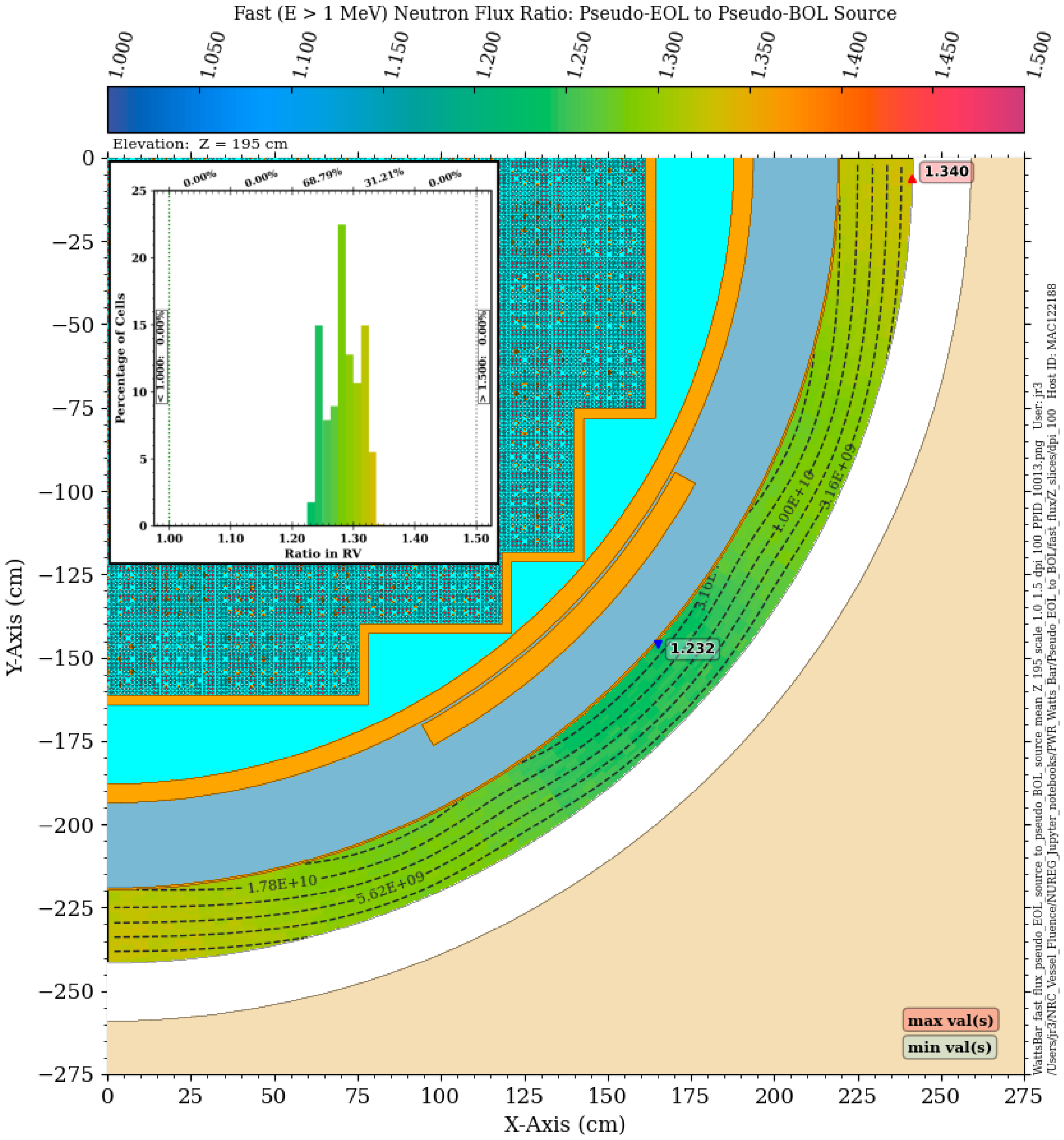


Figure 5-17 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Plan view at an elevation of Z = 195 cm. The dashed contour lines represent the fast flux with the pseudo-BOL source

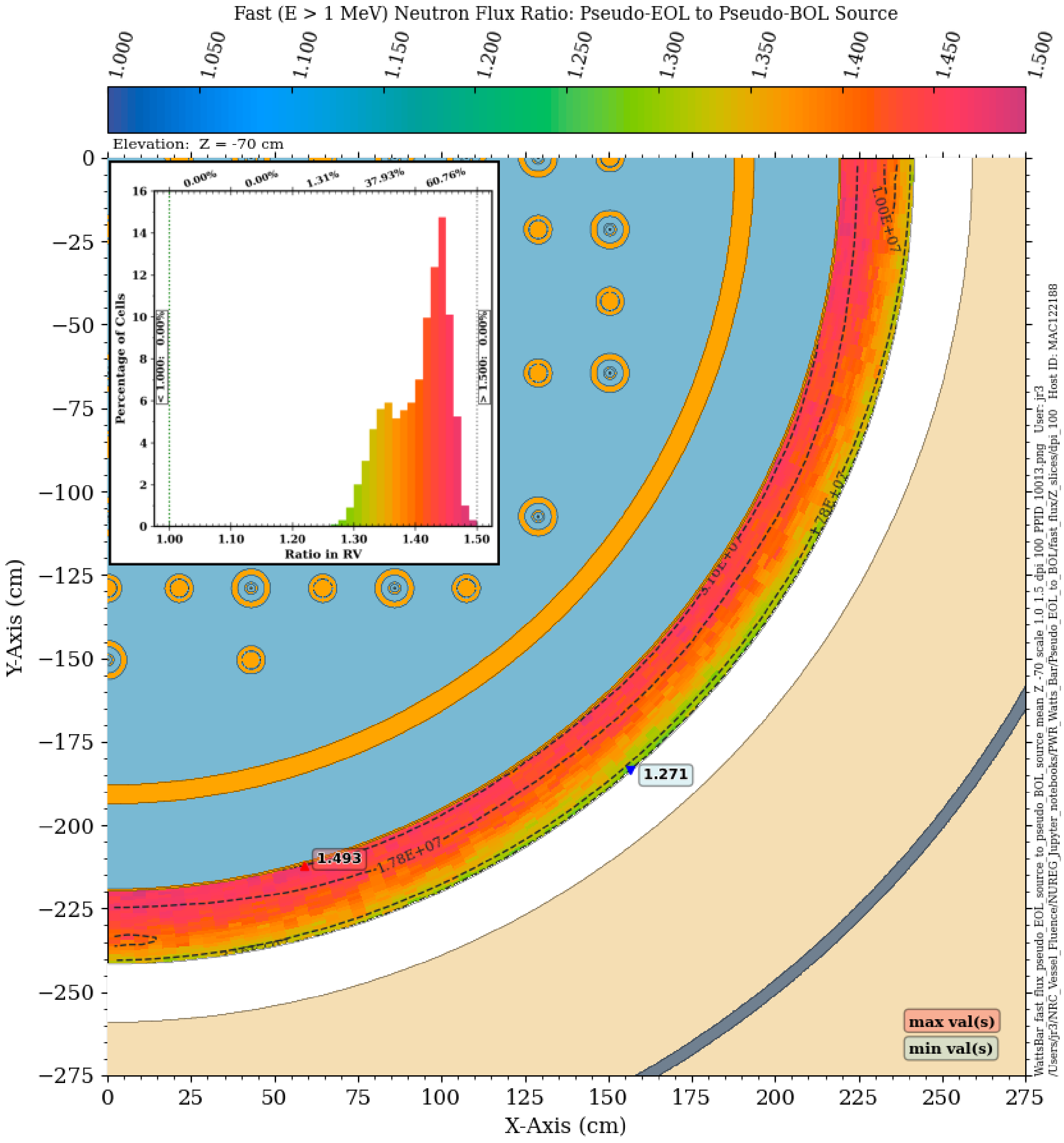


Figure 5-18 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Plan view at an elevation of Z = -70 cm. The dashed contour lines represent the fast flux with the pseudo-BOL source

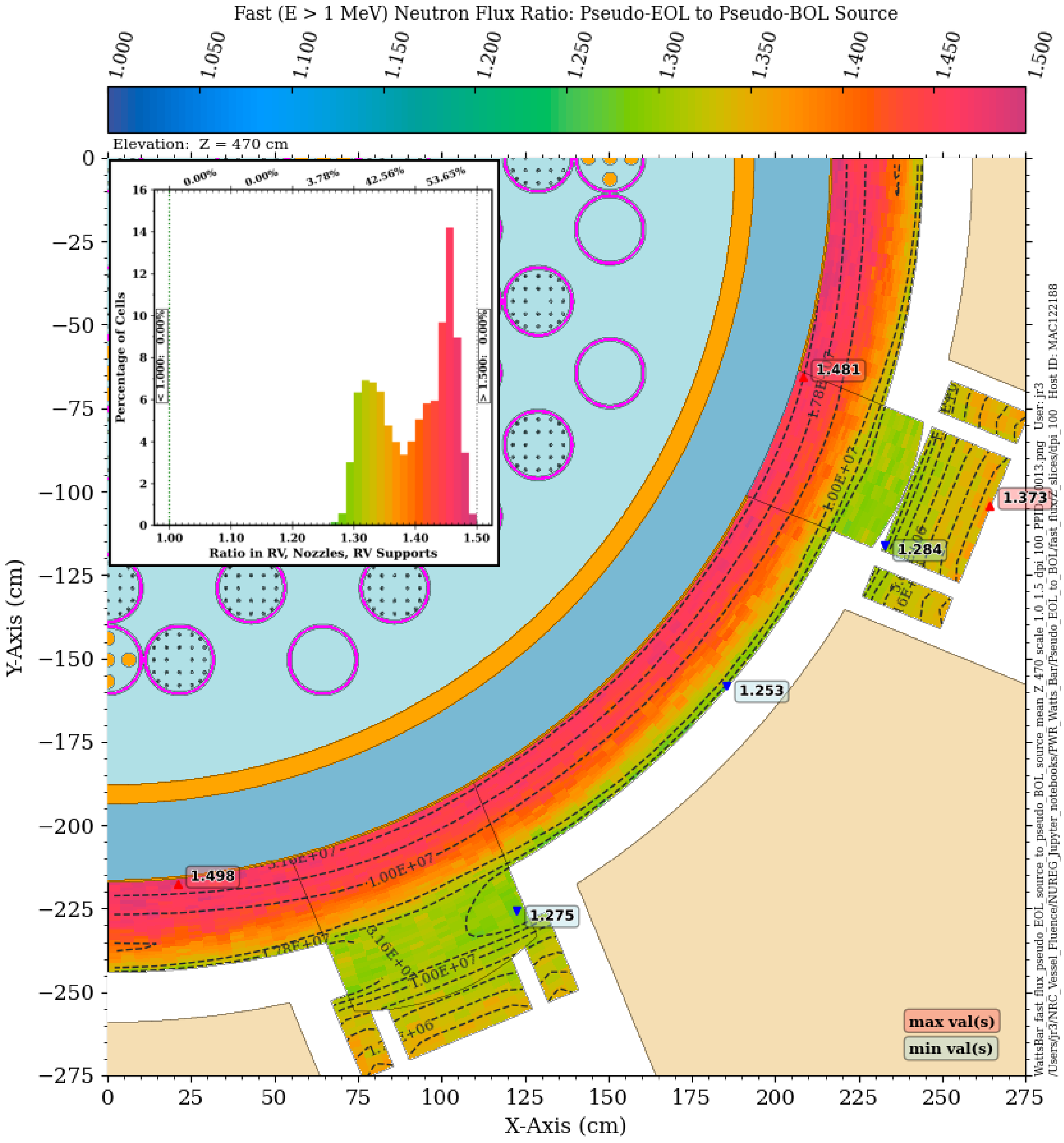


Figure 5-19 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Plan view at an elevation or Z = 470 cm. The dashed contour lines represent the fast flux with the pseudo-BOL source

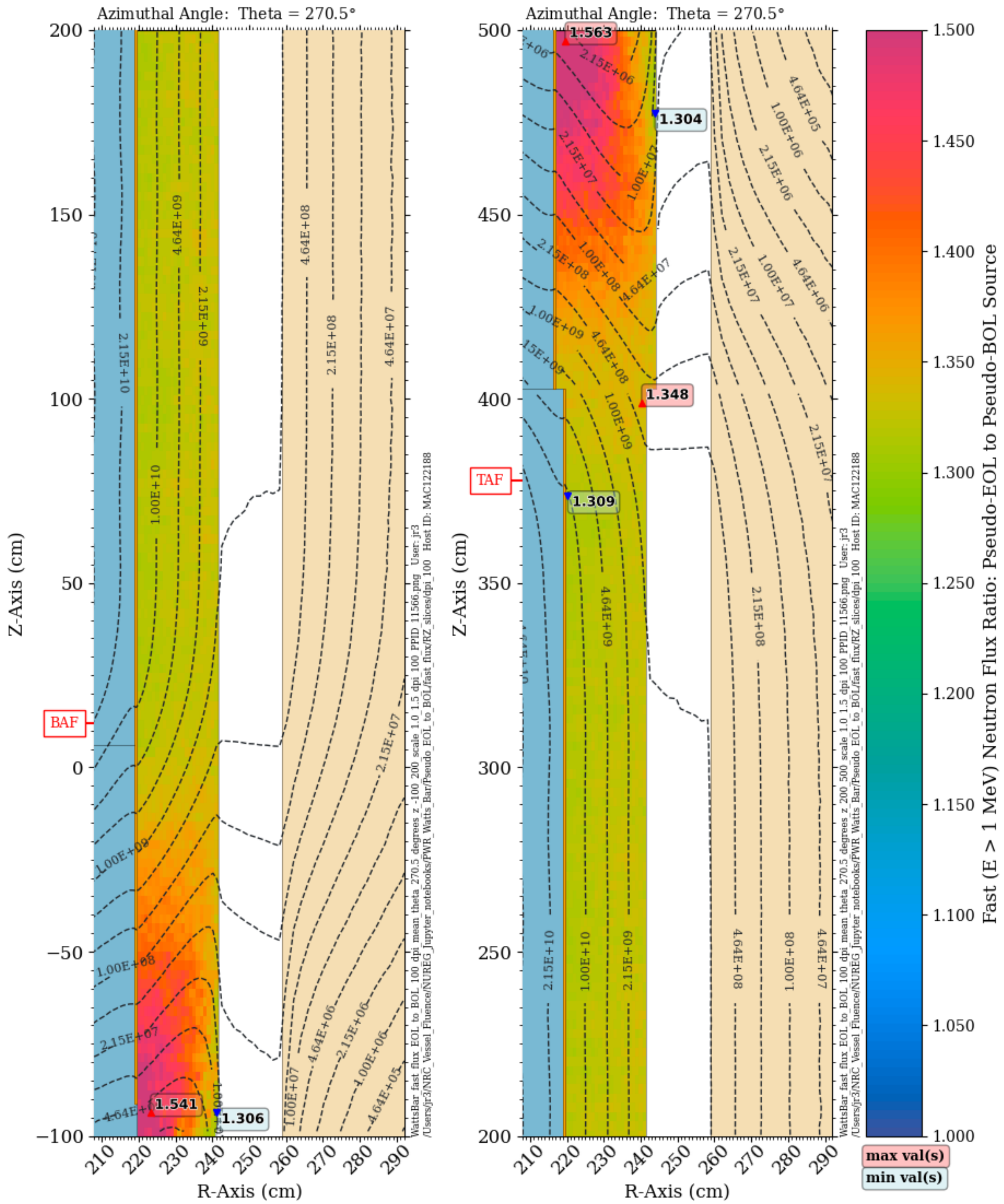


Figure 5-20 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Elevation view at an azimuthal angle of 270.5°. The dashed contour lines represent the fast flux with the pseudo BOL source

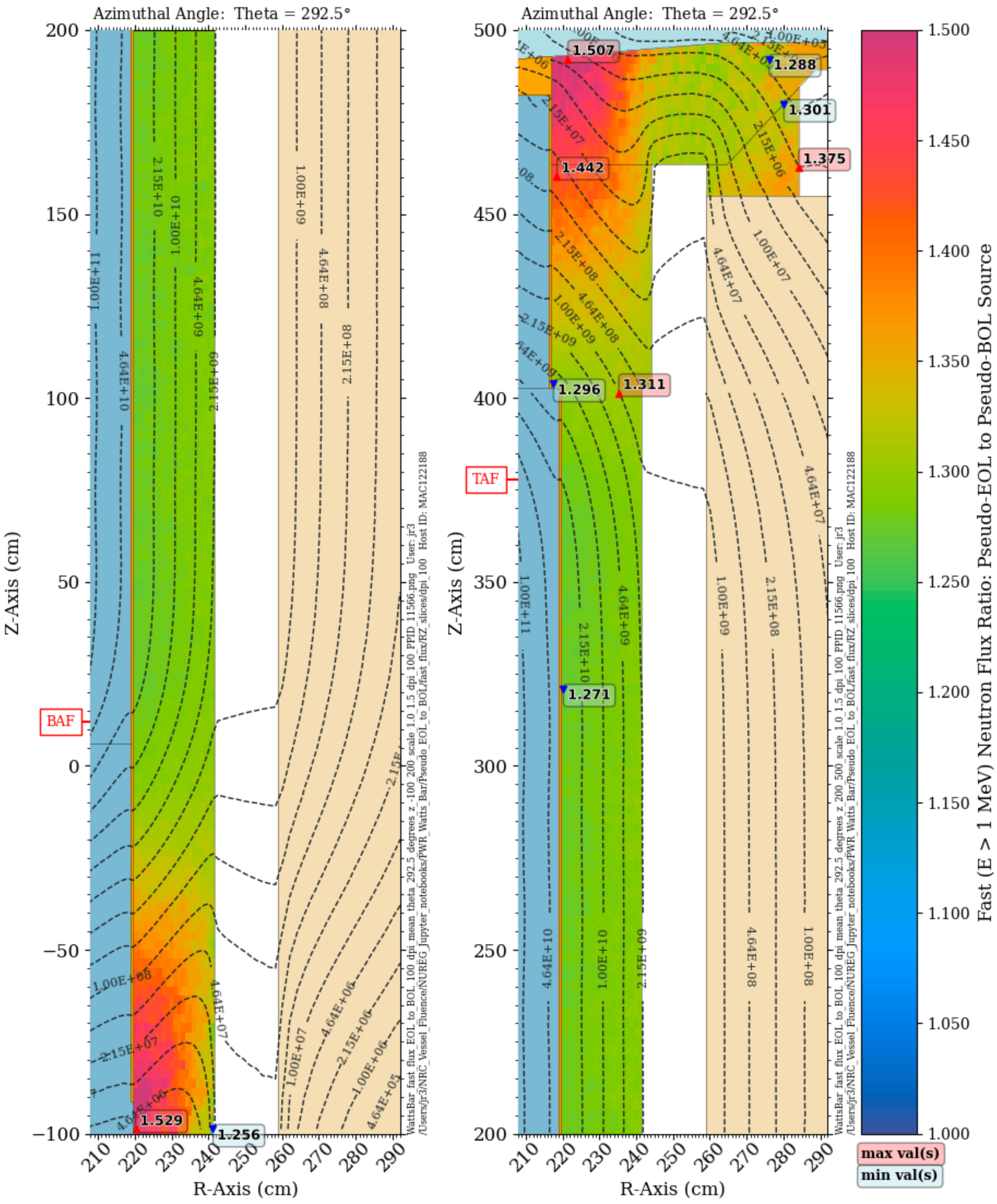


Figure 5-21 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Elevation view at an azimuthal angle of 292.5°. The dashed contour lines represent the fast flux with the pseudo-BOL source

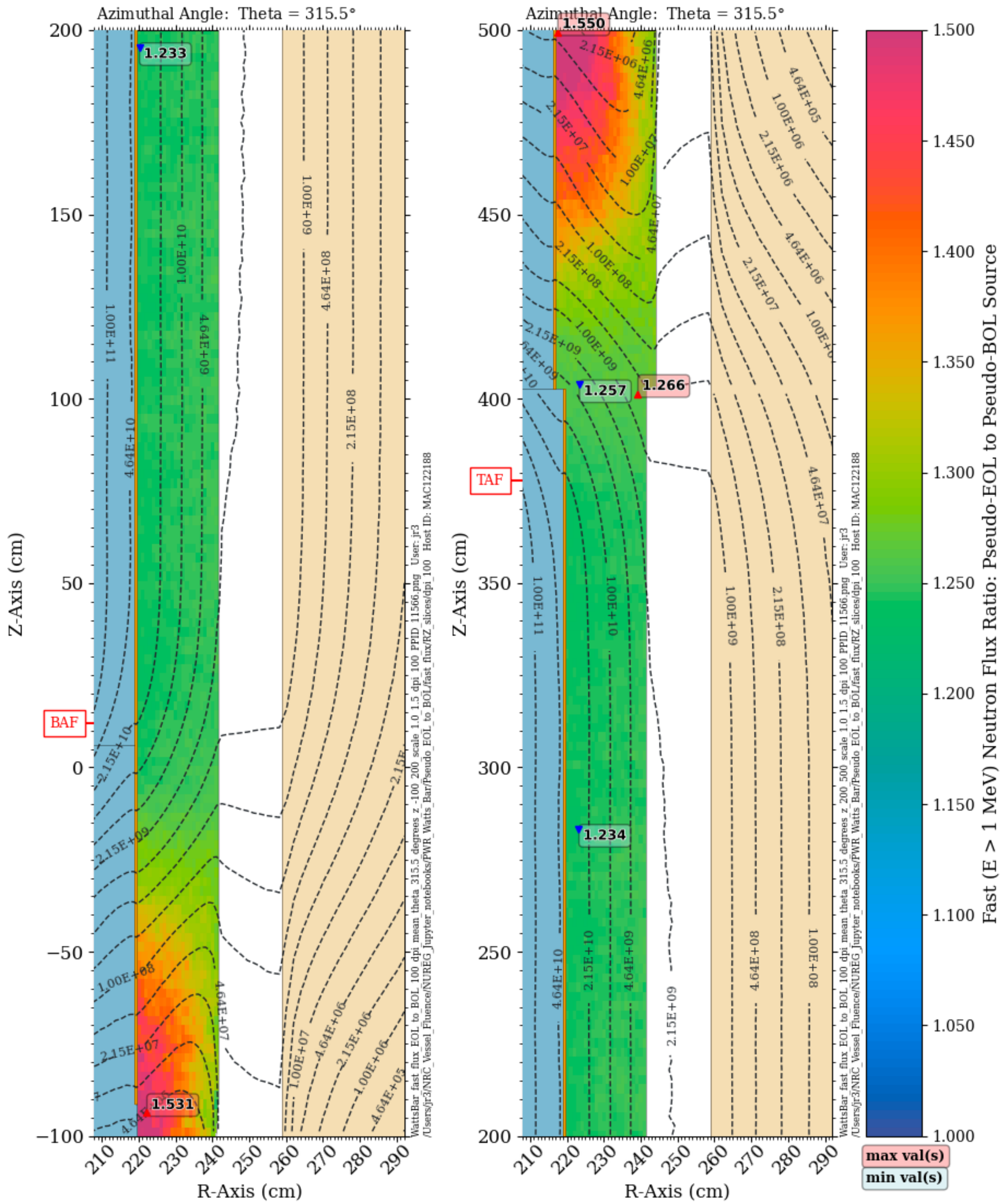


Figure 5-22 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the PWR model. Elevation view at an azimuthal angle of 315.5°. The dashed contour lines represent the fast flux with the pseudo BOL source

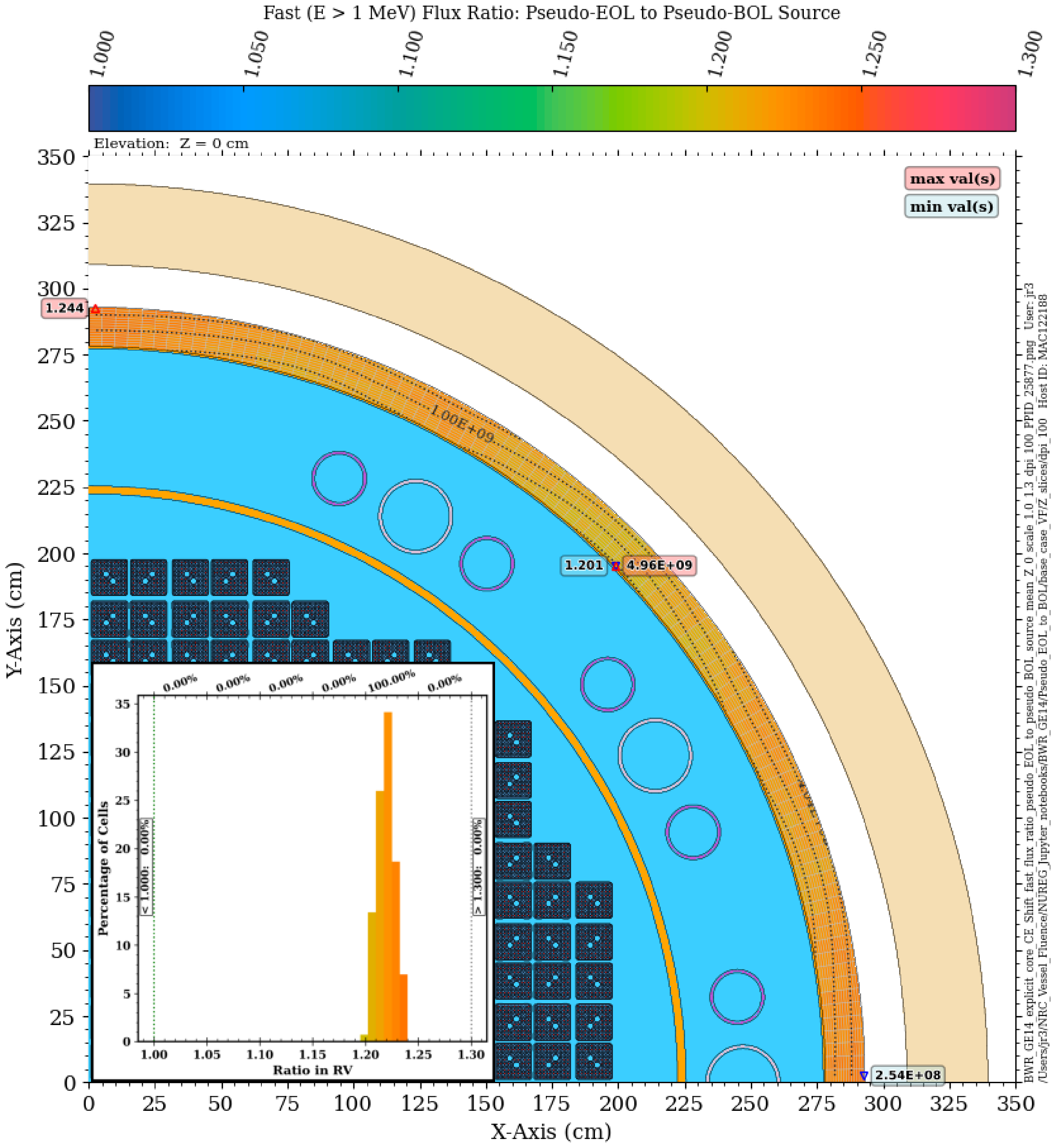


Figure 5-23 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the BWR model. Plan view at the core midplane. The dashed contour lines represent the fast flux with the pseudo BOL source

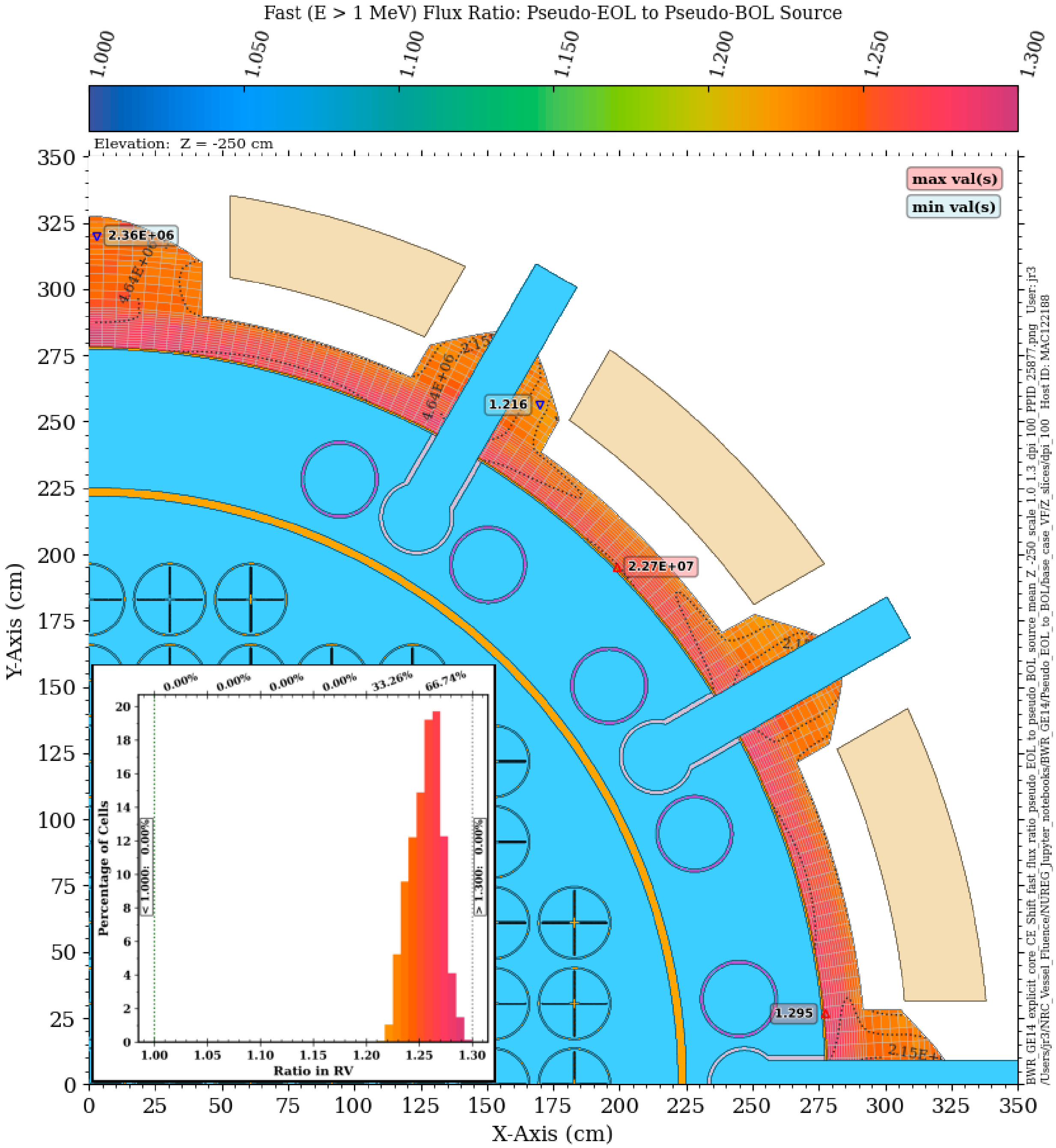


Figure 5-24 **Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the BWR model. Plan view through the recirculation outlet nozzles at Z = -250 cm. The dashed contour lines represent the fast flux with the pseudo-BOL source**

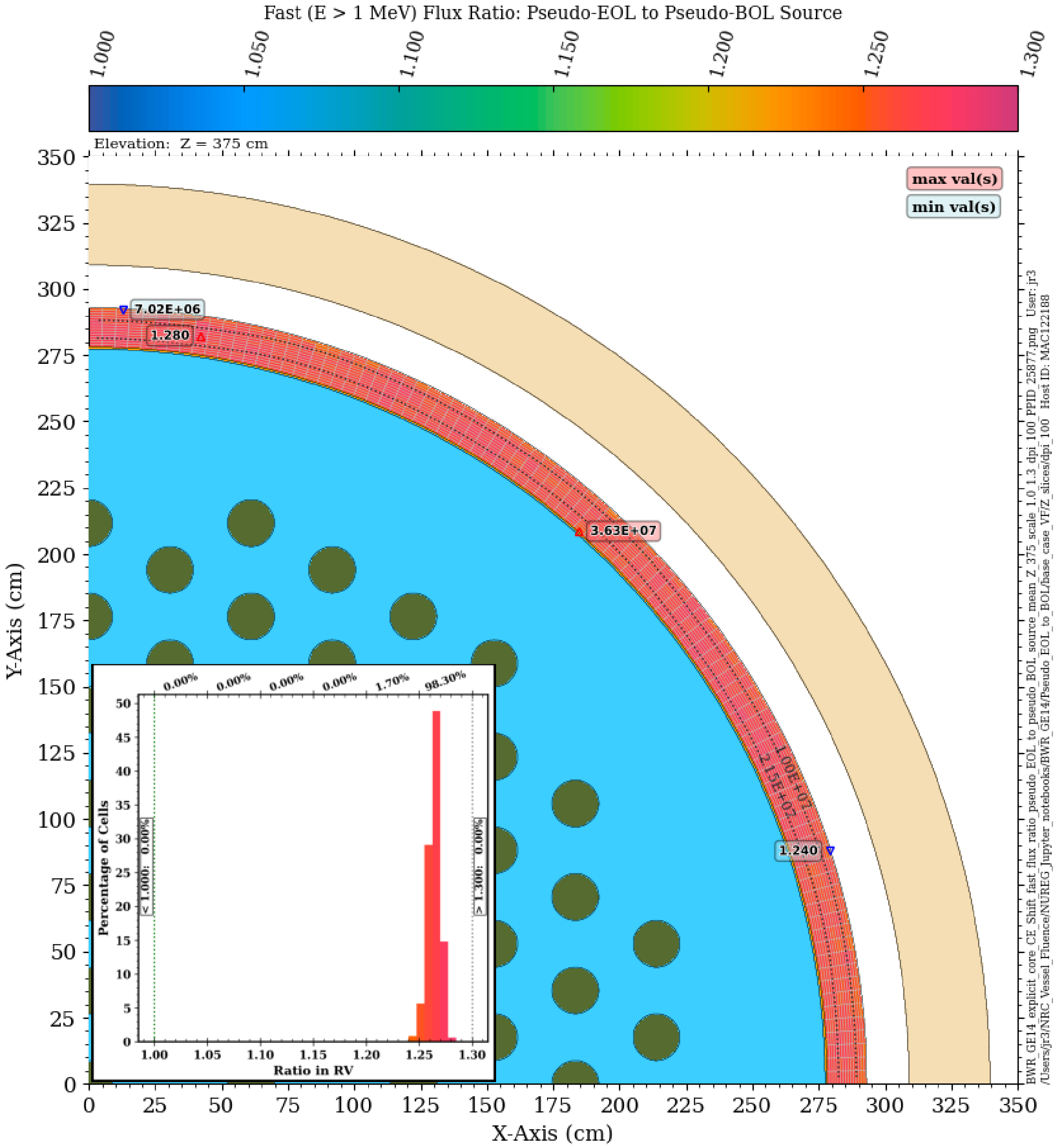


Figure 5-25 **Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the BWR model. Plan view above the core shroud at Z = 375 cm. The dashed contour lines represent the fast flux with the pseudo BOL source**

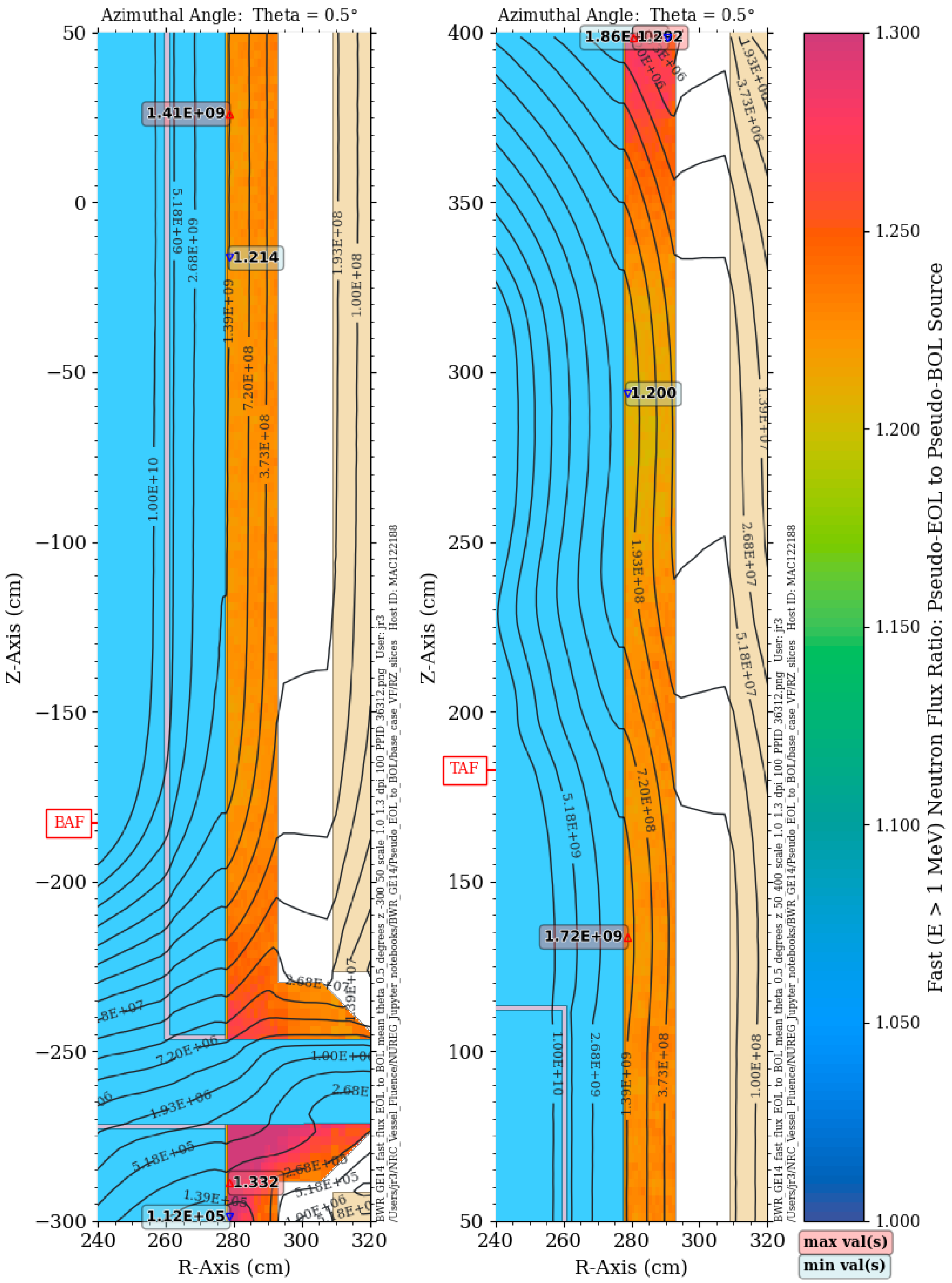


Figure 5-26 **Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the BWR model. Elevation view at an azimuthal angle of 0.5°. The contour lines represent the fast flux with the pseudo BOL source**

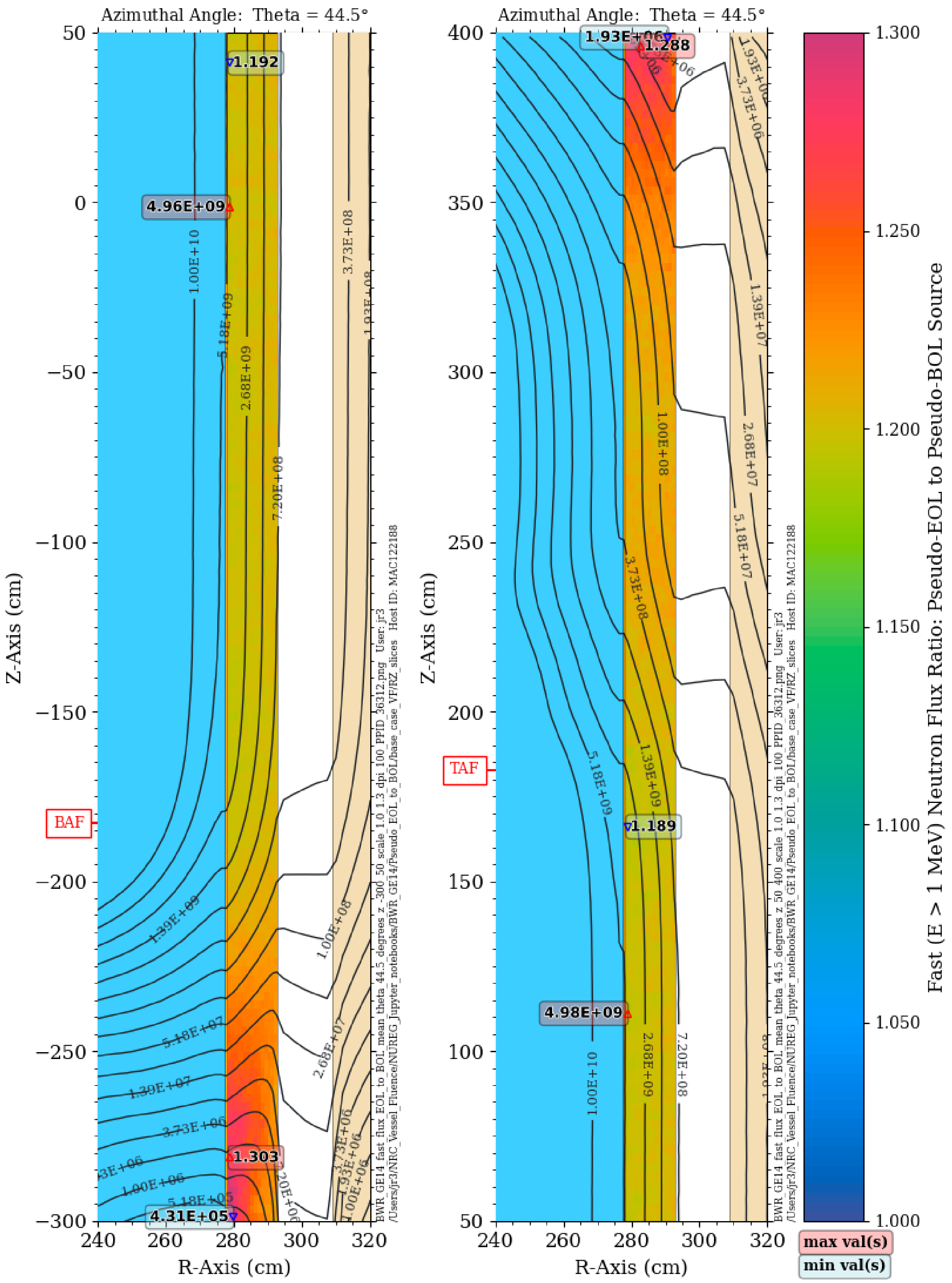


Figure 5-27 Fast neutron flux ratio for a pseudo EOL source relative to a pseudo BOL source in the BWR model. Elevation view at an azimuthal angle of 0.5°. The contour lines represent the fast flux with the pseudo BOL source

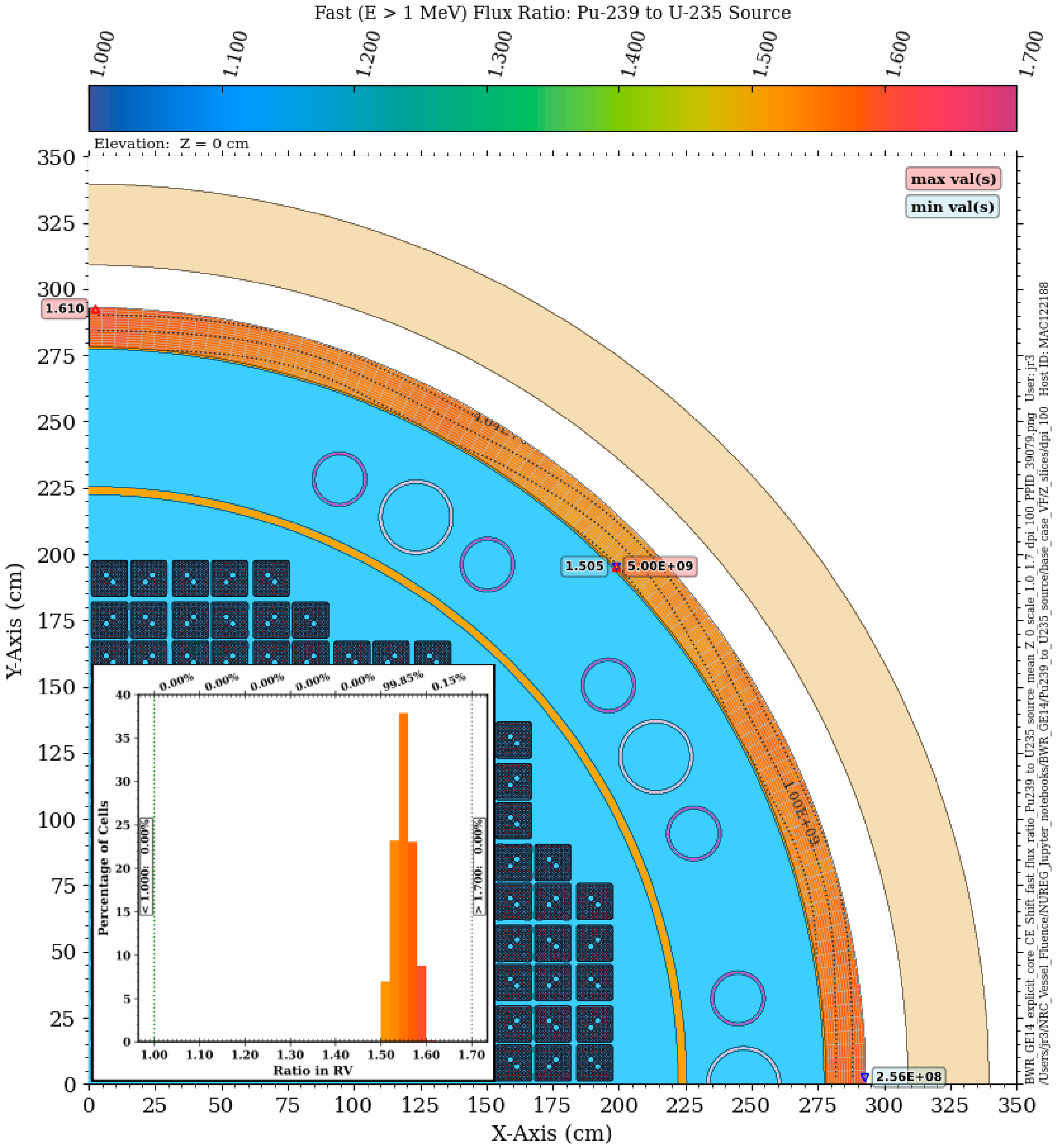


Figure 5-28 **Fast neutron flux ratio for a ^{239}Pu source relative to a ^{235}U source in the BWR model. Plan view at the core midplane. The dashed contour lines represent the fast flux with a ^{235}U source**

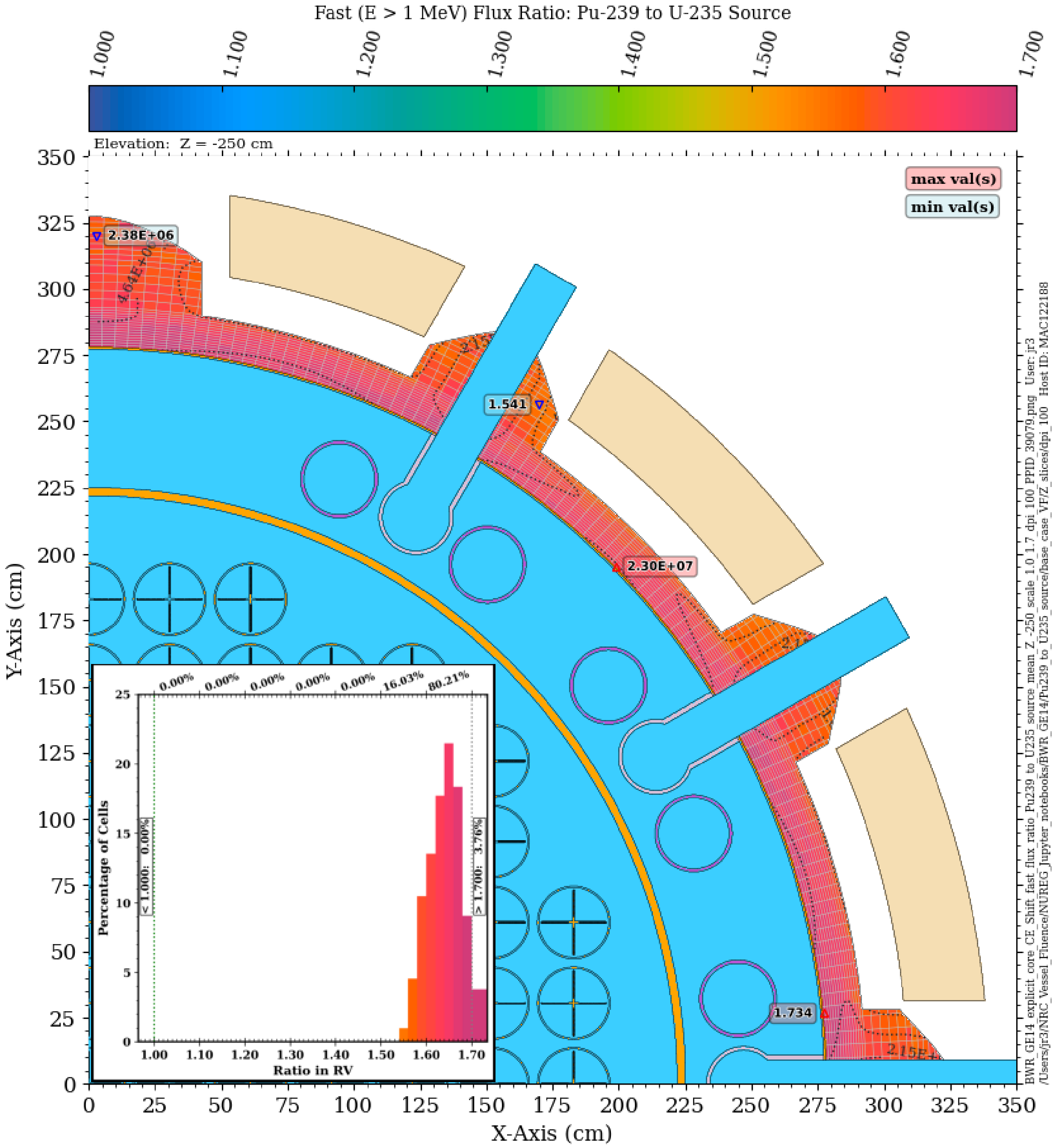


Figure 5-29 **Fast neutron flux ratio for a ^{239}Pu source relative to a ^{235}U source in the BWR model. Plan view through the recirculation outlet nozzles at Z = -250 cm. The dashed contour lines represent the fast flux with a ^{235}U source**

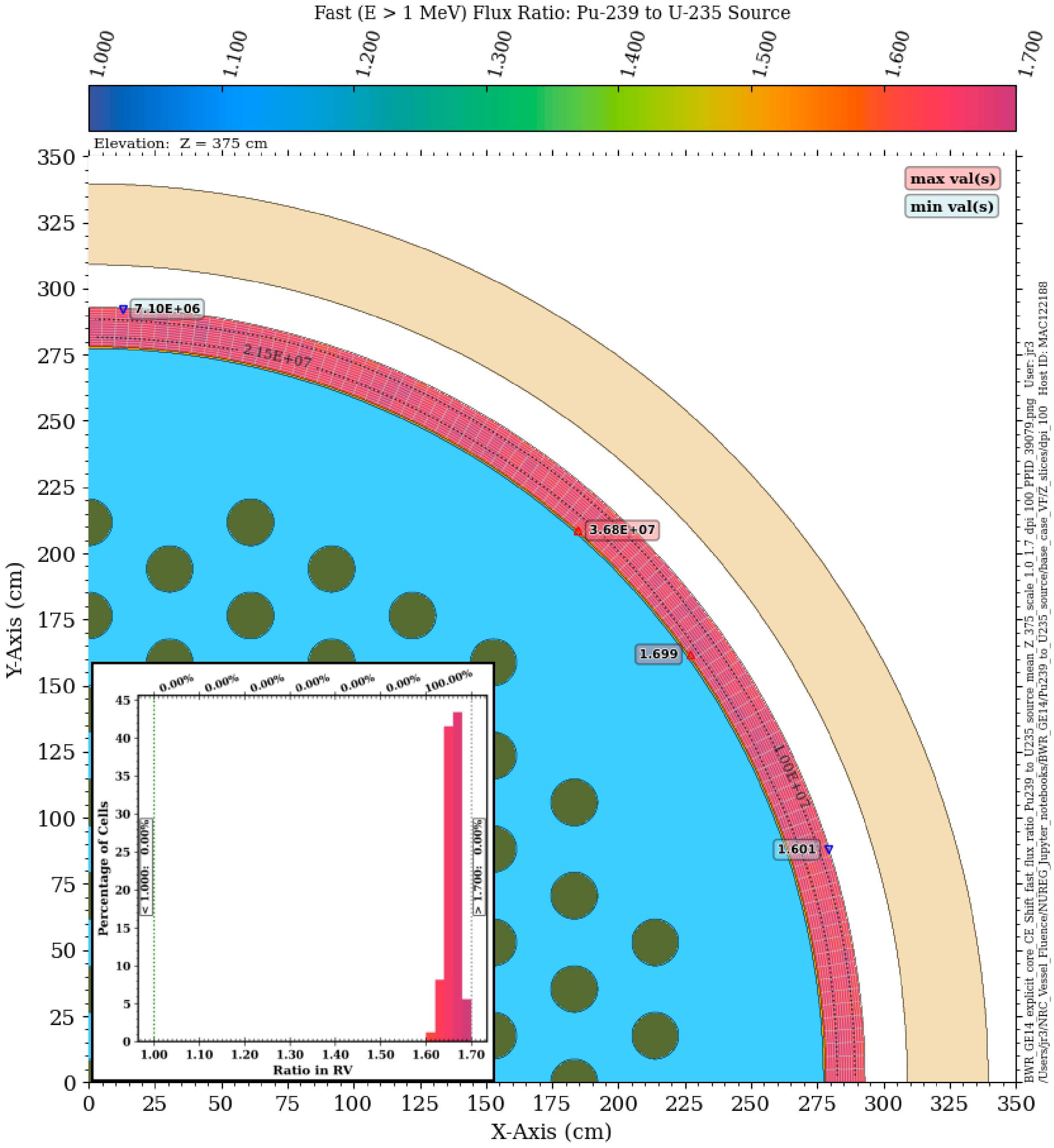


Figure 5-30 **Fast neutron flux ratio for a ^{239}Pu source relative to a ^{235}U source in the BWR model. Plan view above the core shroud at Z = 375 cm. The dashed contour lines represent the fast flux with a ^{235}U source**

5.3 Coolant temperature variations in the PWR model

Changes in the coolant temperature at any location within the RPV will affect neutron attenuation due to changes in the density of the coolant and hence the macroscopic cross section. Coolant temperature increases lead to reduced attenuation and hence higher fast flux levels incident to the RPV; the reverse is true for reductions in temperature. The primary question with regard to fast fluence levels in the extended beltline region is this: are coolant temperature changes likely to affect fast fluence in the extended beltline region differently than they do within the traditional beltline region?

While an exact analysis would require detailed knowledge of the coolant temperature at numerous locations within the RPV, a reasonable assessment of temperature changes can be made by simply adjusting all coolant temperatures by a specified amount. Calculations with the PWR model were run with temperature changes of 5, 10, and 15 °F above and below the baseline temperature. The source in each case was a spatially uniform pinwise distribution with a ^{235}U fission spectrum. Results for the -10 °F and +10 °F models are presented in this section. Results for the 5 °F and 15 °F temperature changes show the same overall trend.

The effect of reducing the coolant temperatures by 10°F is shown in Figure 5-31, Figure 5-32, and Figure 5-33. Within the traditional beltline region, the fast neutron flux in the RPV has a reduction of ~5% to ~10%. In the extended beltline region there is a greater reduction in the fast flux in the inner portion of the RPV. This increased effect is due to the greater amount of water through which the neutrons are transported to reach the inner portion of the RPV in the extended beltline region. There is little change in the fast flux ratio in the outer portion of the RPV at all elevations, as the flux levels in the outer RPV in the extended beltline region are dominated by cavity streaming neutrons.

The effect of increasing the coolant temperatures by 10 °F is shown in Figure 5-34, Figure 5-35, and Figure 5-36. Here the behavior is essentially the inverse of that seen for the coolant temperature reduction. The fast neutron flux increases by ~5 to 10% through the thickness of the RPV in the traditional beltline region. In the extended beltline region, the increased amount of water through which the fast neutrons are transported to the inner surface of the RPV results in a greater increase in the fast neutron flux. In the outer portion of the RPV, the fast flux change is again nearly constant through the extended beltline region, as well as the traditional beltline region.

The results of the parametric water temperature study indicate that the impact of changes in coolant temperature with respect to RPV fluence levels is more significant for locations outside the traditional beltline region. This sensitivity suggests that accurate modeling of coolant temperatures throughout the RPV and the nozzles as a function of the plant operating history (including power uprates or other changes that may affect coolant pressure and temperature) is particularly important for extended beltline fluence calculations.

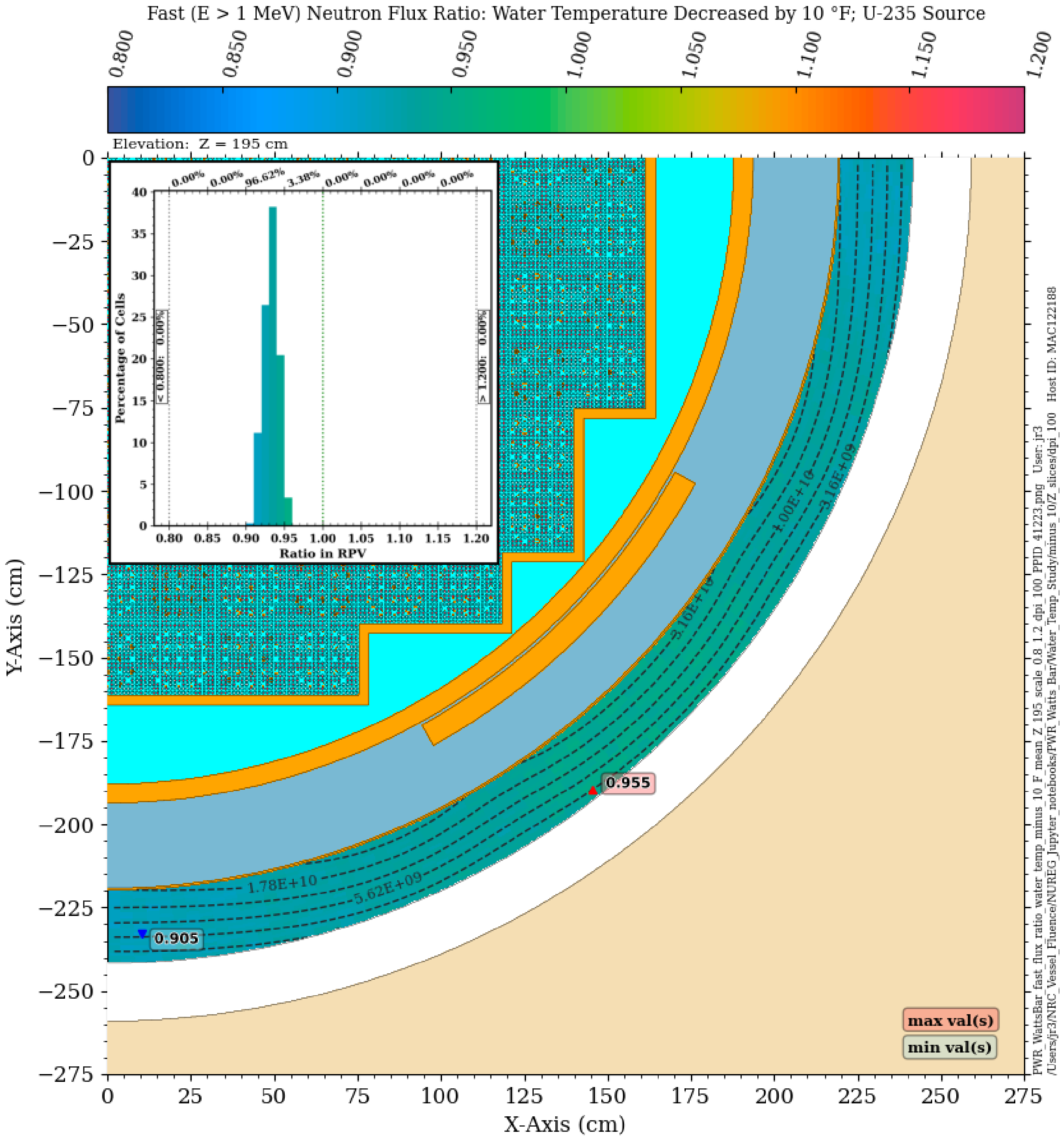


Figure 5-31 Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are decreased by 10°F relative to the base case model. Plan view at an elevation of Z = 195 cm. The contour lines show the fast neutron flux for the base case model

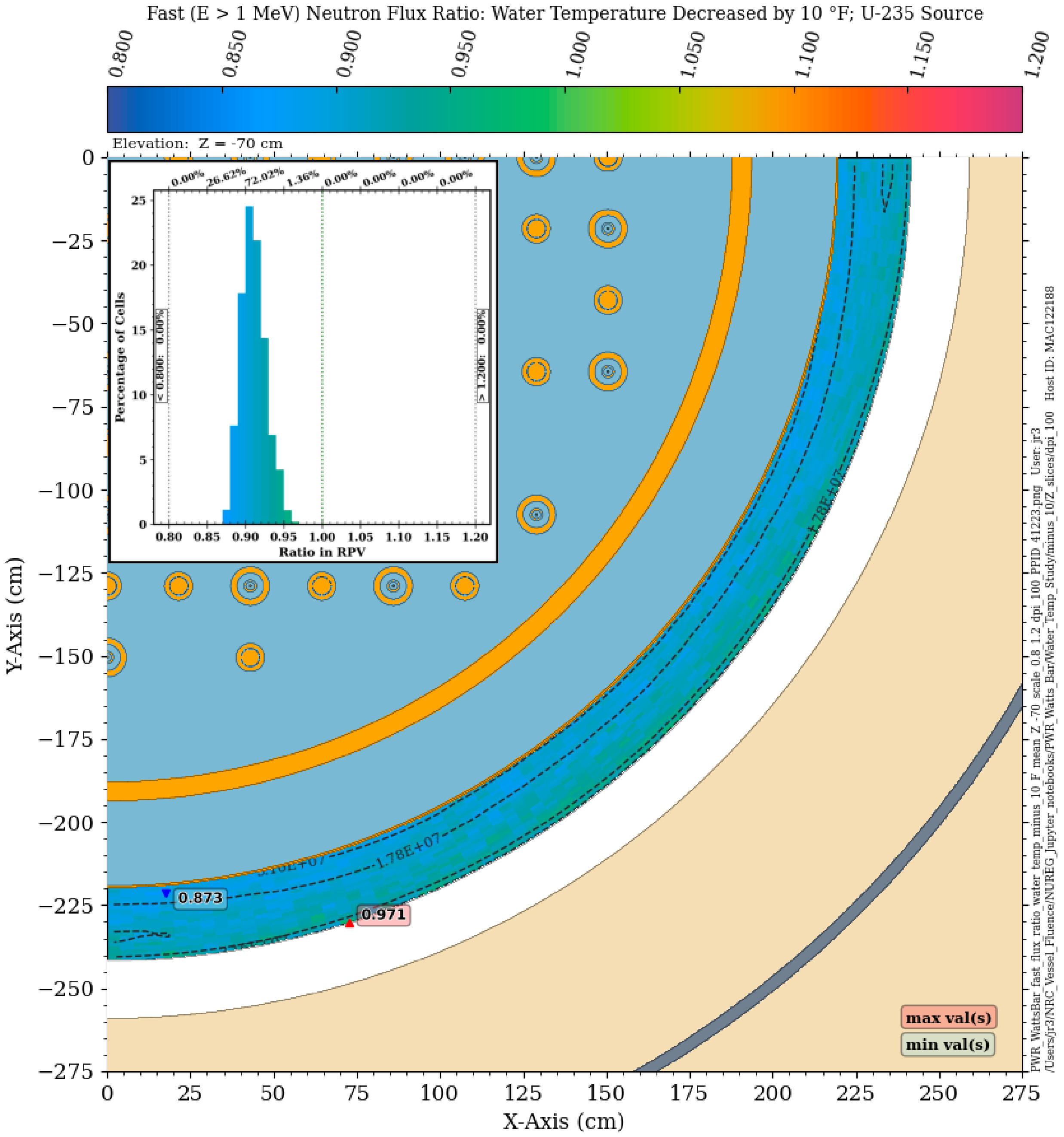


Figure 5-32 Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are decreased by 10°F relative to the base case model. Plan view at an elevation of Z = -70 cm. The contour lines show the fast neutron flux for the base case model

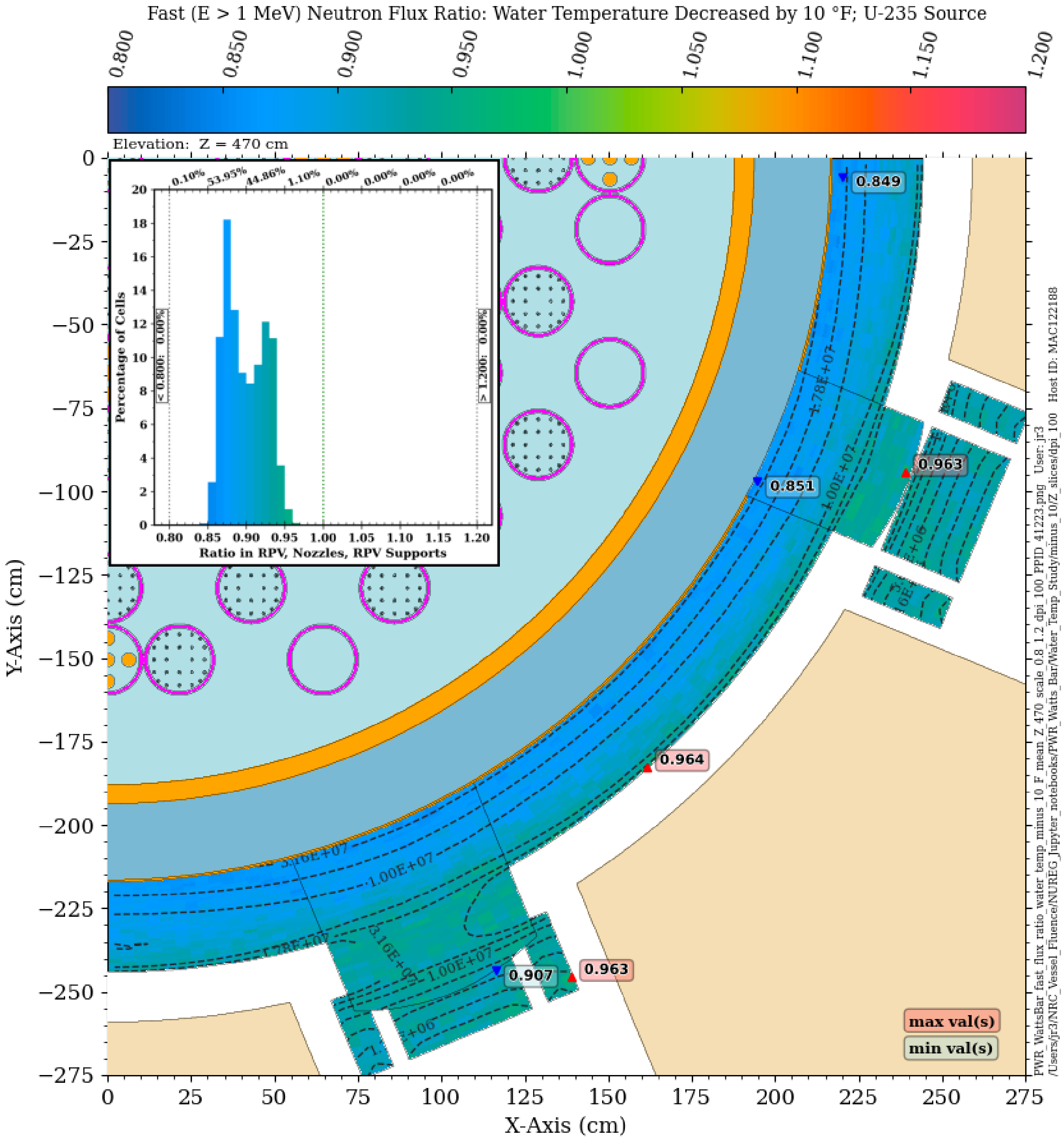


Figure 5-33 Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are decreased by 10°F relative to the base case model. Plan view at an elevation of Z = 470 cm. The contour lines show the fast neutron flux for the base case model

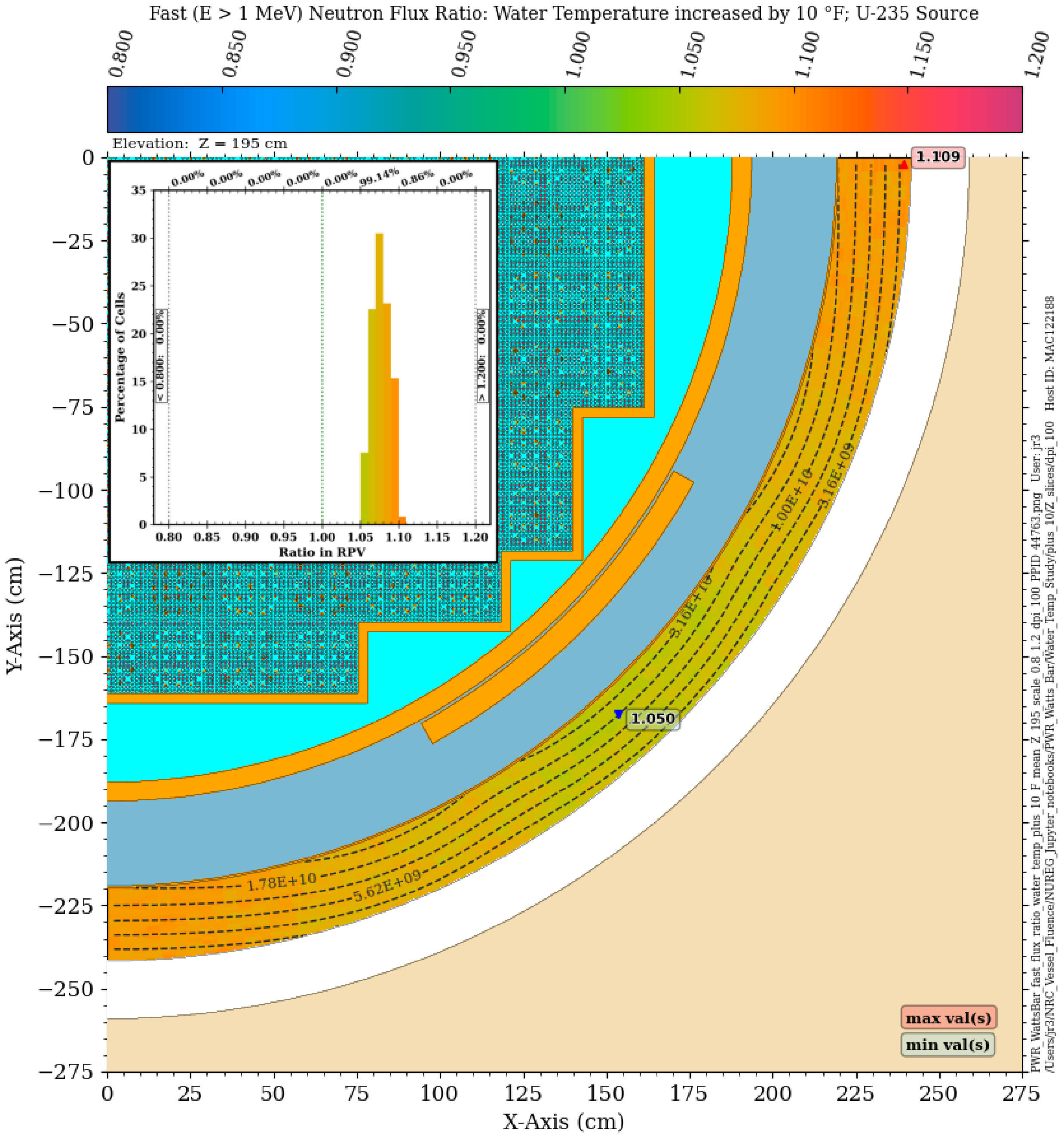


Figure 5-34 **Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are increased by 10°F relative to the base case model. Plan view at an elevation of Z = 195 cm. The contour lines show the fast neutron flux for the base case model**

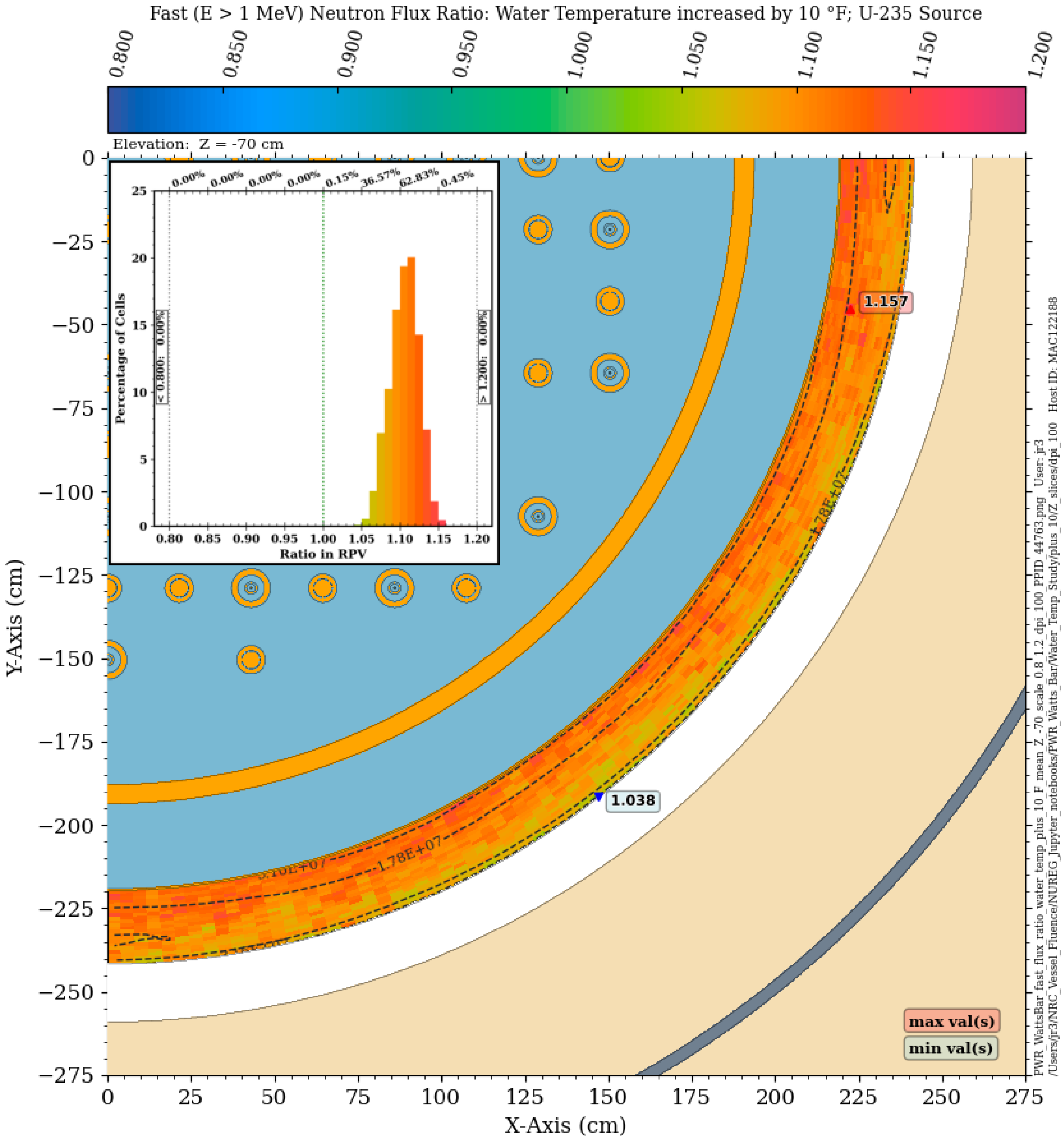


Figure 5-35 Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are increased by 10°F relative to the base case model. Plan view at an elevation of Z = -70 cm. The contour lines show the fast neutron flux for the base case model

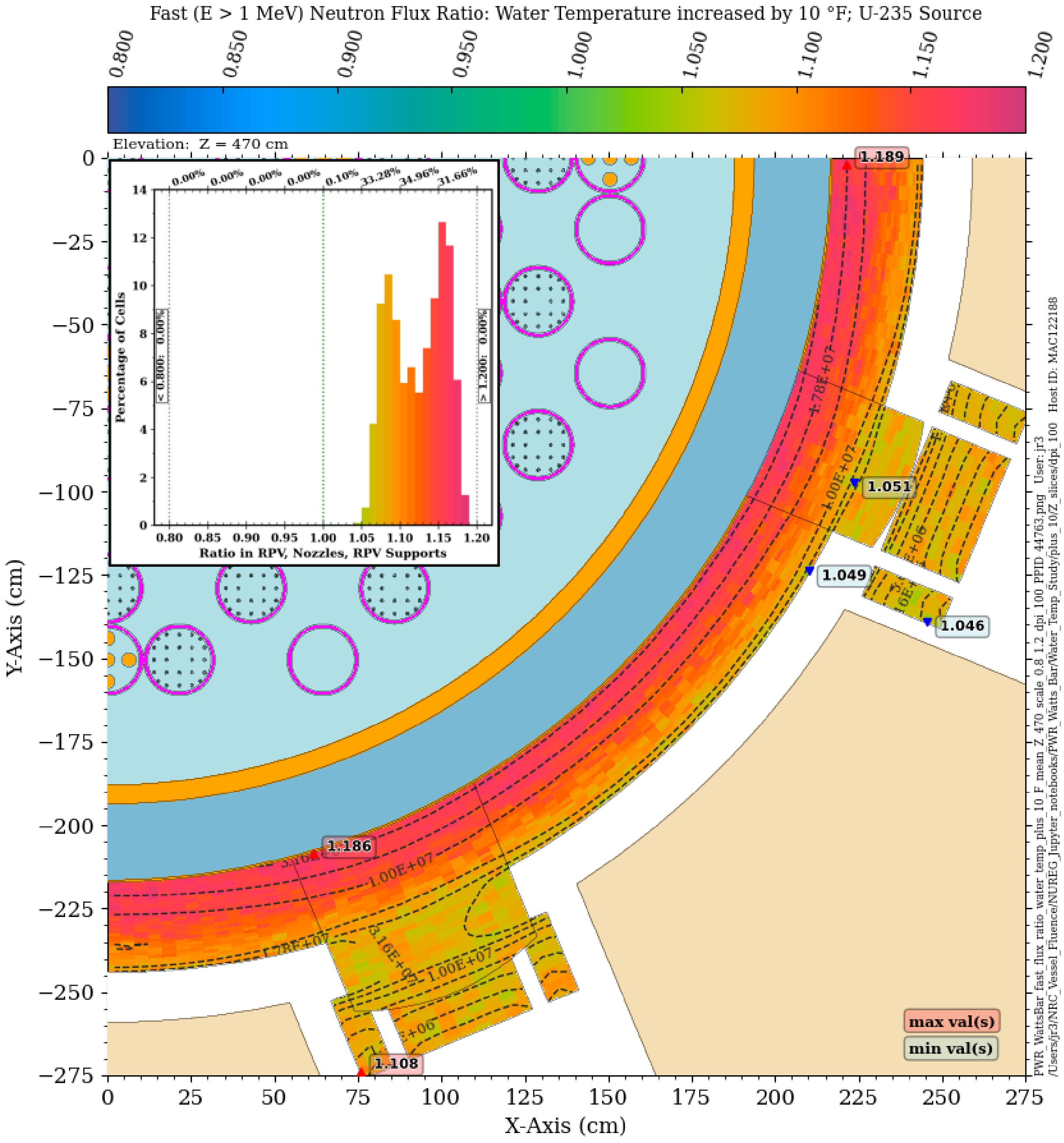


Figure 5-36 Ratio of the fast neutron flux in the RPV of the PWR when all water temperatures are increased by 10°F relative to the base case model. Plan view at an elevation of Z = 470 cm. The contour lines show the fast neutron flux for the base case model

5.4 Void fractions in the BWR model

Variations in the void fraction (VF) in a BWR core can affect the fast flux levels in the RPV due to two effects: changes in neutron attenuation within the fuel assemblies, and changes in the neutron spectrum as the isotopic fission fractions change due to hardening or softening of the neutron spectrum. In general, both of these effects will only be significant in the outer assemblies. Changes in the RPV fast flux levels due to fission spectrum changes are addressed in Section 5.2. The effects of VF changes are addressed in this section.

The VFs in each of the seven axial zones in the BWR model were obtained based on data in NUREG/CR-7224 [73]. In general, the VFs are highest near the center of the core and lower near the periphery. Data from Figure 4.1 of [73] were used to determine minimum, maximum, and average VF values based on assembly locations in the outermost part of the core. These values are shown in Table 5-5. The base case model has coolant densities based on the average VF values. Those densities are consistent with the BWR material data provided in Table 4-4 above. The source in each case was a spatially uniform pinwise distribution with a ^{235}U fission spectrum.

Table 5-5 VFs by axial zone in the BWR GE14 fuel assembly modeling

Axial zone (Figure 4-6)	Minimum VF (%)	Maximum VF (%)	Average VF (%)
NAT	2	2	2
PSZ	3	7	5
DOM	7	30	18.5
PLE	25	50	37.5
VAN	35	60	47.5
N-V	44	70	57
N-T	44	70	57

5.4.1 Minimum void fraction

Use of the minimum VF values by axial zone in place of the average values results in higher coolant densities for all zones above the NAT zone (Figure 4-6 and Table 5-5). Because the density differences in the NAT and PSZ zones are relatively small (or zero) when going from the average to the minimum VF, fast flux differences in the RPV at elevations near or below the lower portion of the core are likely to be small, whereas differences around the core midplane and above should be more pronounced.

This behavior can be seen in Figure 5-37 through Figure 5-40. At the core midplane, the reduction in the fast flux in the RPV is predominantly between 2.5 and 5%. At an elevation through the recirculation outlet nozzles at -250 cm, where the fast flux is approximately two orders of magnitude lower than at the core midplane, the differences in the fast flux are less than 2.5% in 98% of the mesh tally cells. At an elevation of 375 cm, where the fast flux is approximately two orders of magnitude lower than at the core midplane, the fast flux ratios are largely clustered between 0.925 and 0.95 (i.e., reductions of 5 to 7.5%). These results are consistent with expectations.

5.4.2 Maximum void fraction

Use of the maximum VF values results in coolant densities that are lower than the average case in all axial zones above the NAT zone. As with the minimum VF model, the density differences are minor in the two lower zones of the fuel assemblies, so fast flux differences in the RPV should be relatively minor for locations near or below the lower part of the core, and higher at the core midplane and above.

This behavior is confirmed in Figure 5-41 through Figure 5-44. At the core midplane, the fast flux in the RPV increases by 3–6%. At an elevation of -250 cm, the differences are less than 2.5% in nearly 99% of the mesh tally cells. At an elevation of 375 cm, the fast flux values in the RPV with the maximum VF model are 5–10% greater than the base case. As with the minimum VF case, these results are consistent with expectations.

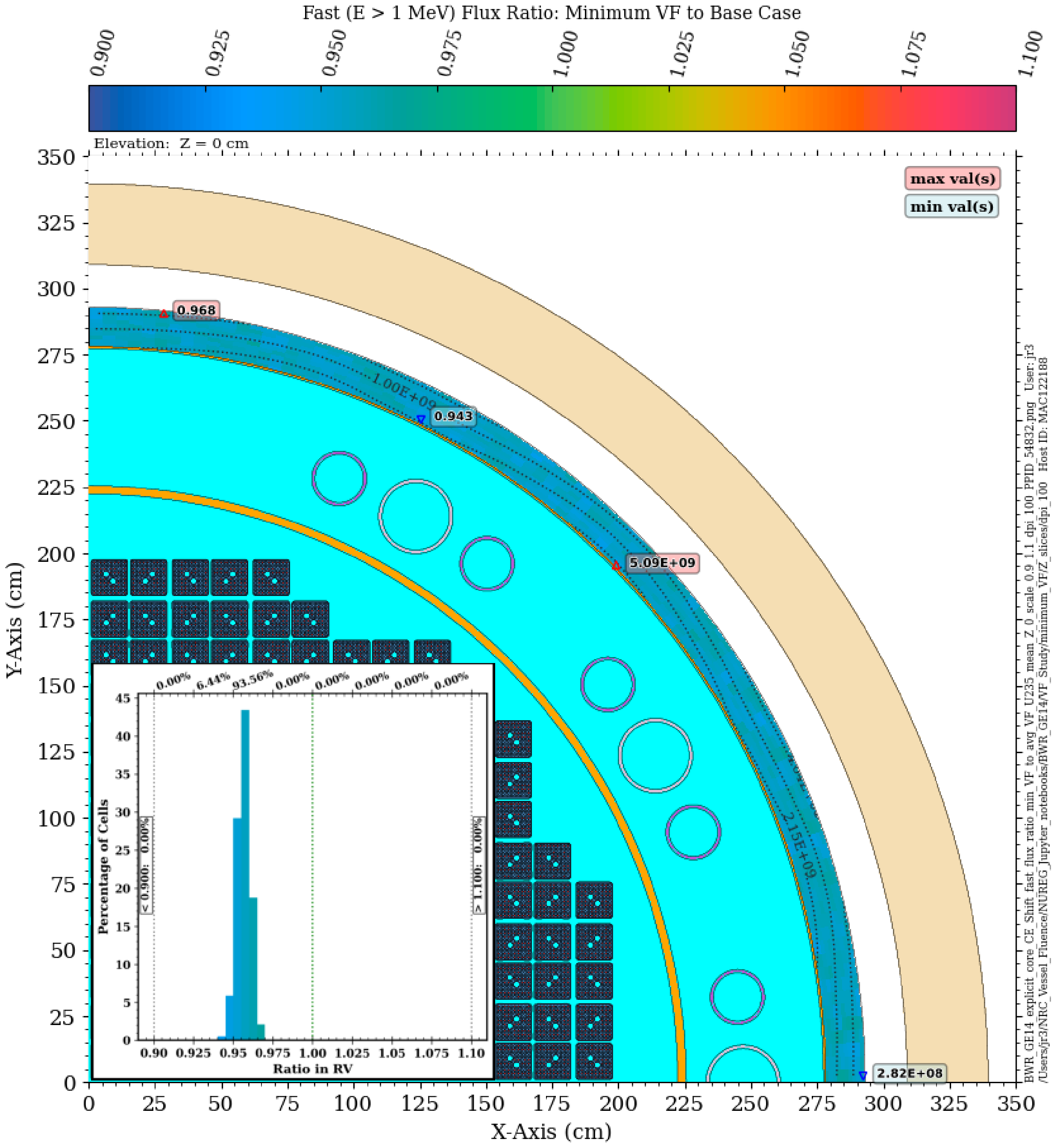


Figure 5-37 Fast neutron flux ratio for the minimum VF model to the base case BWR model. Plan view at the core midplane elevation

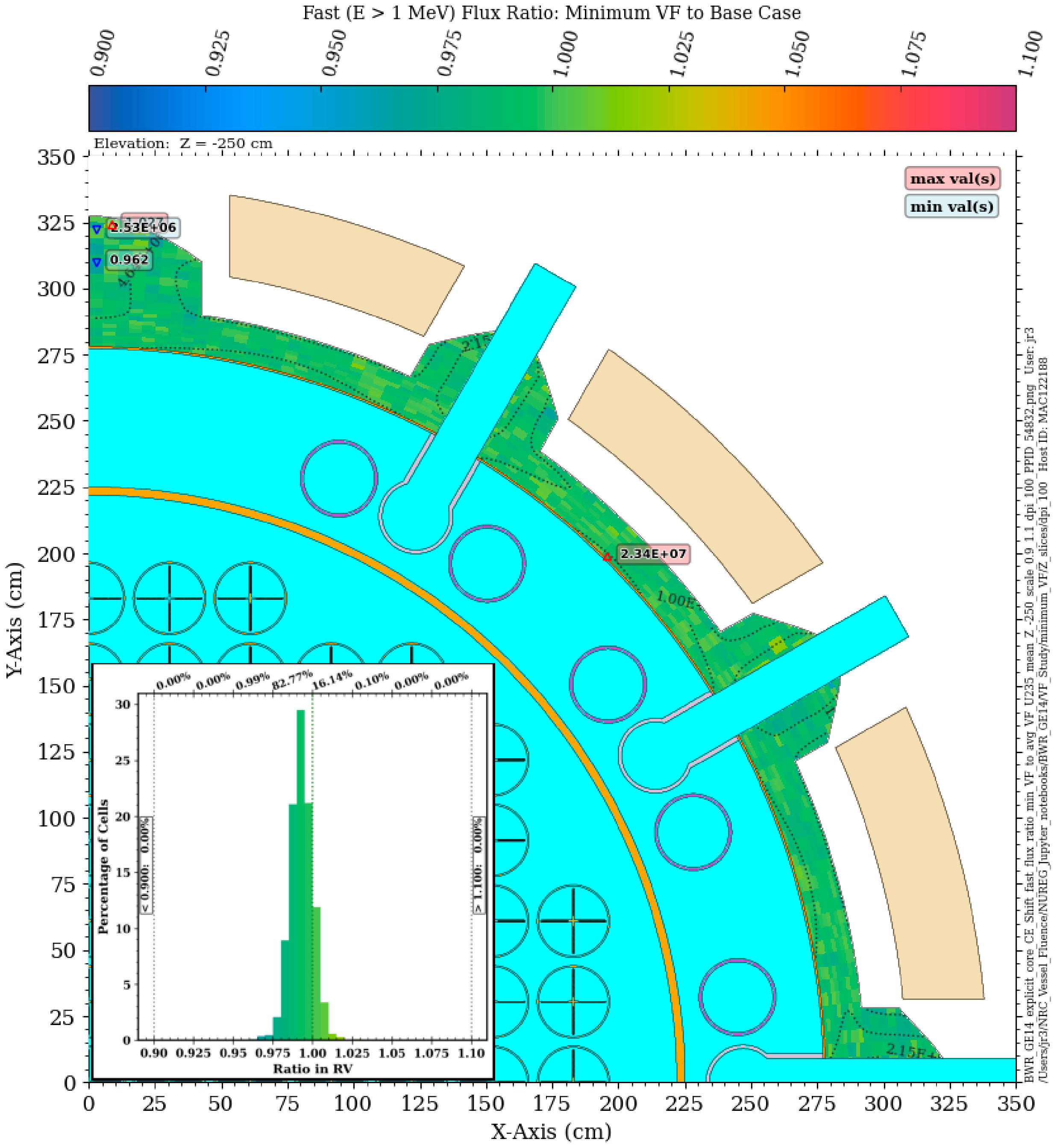


Figure 5-38 **Fast neutron flux ratio for the minimum VF model to the base case BWR model. Plan view at an elevation through the recirculation outlet nozzles**

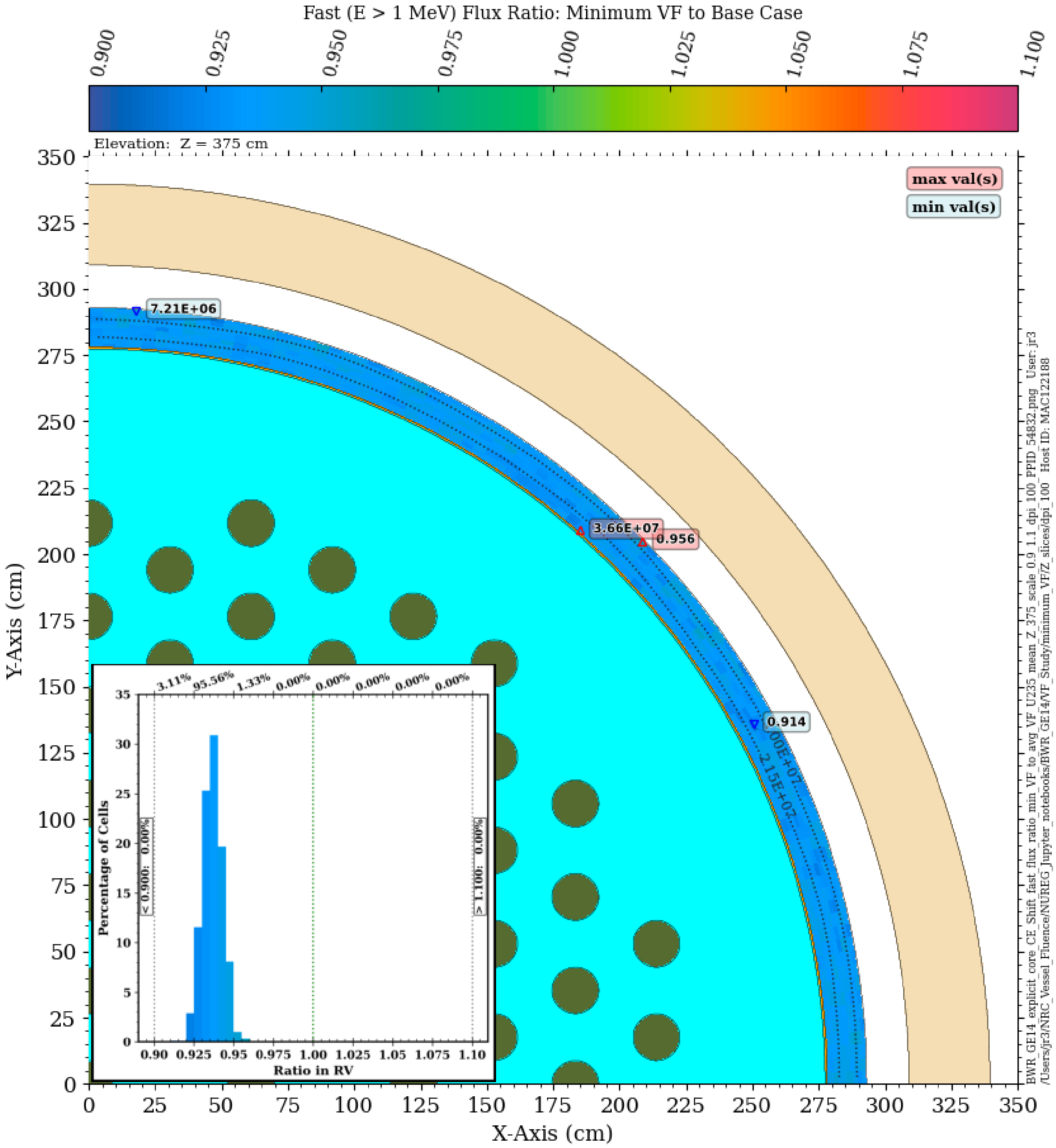


Figure 5-39 Fast neutron flux ratio for the minimum VF model to the base case BWR model. Plan view at an elevation above the core shroud

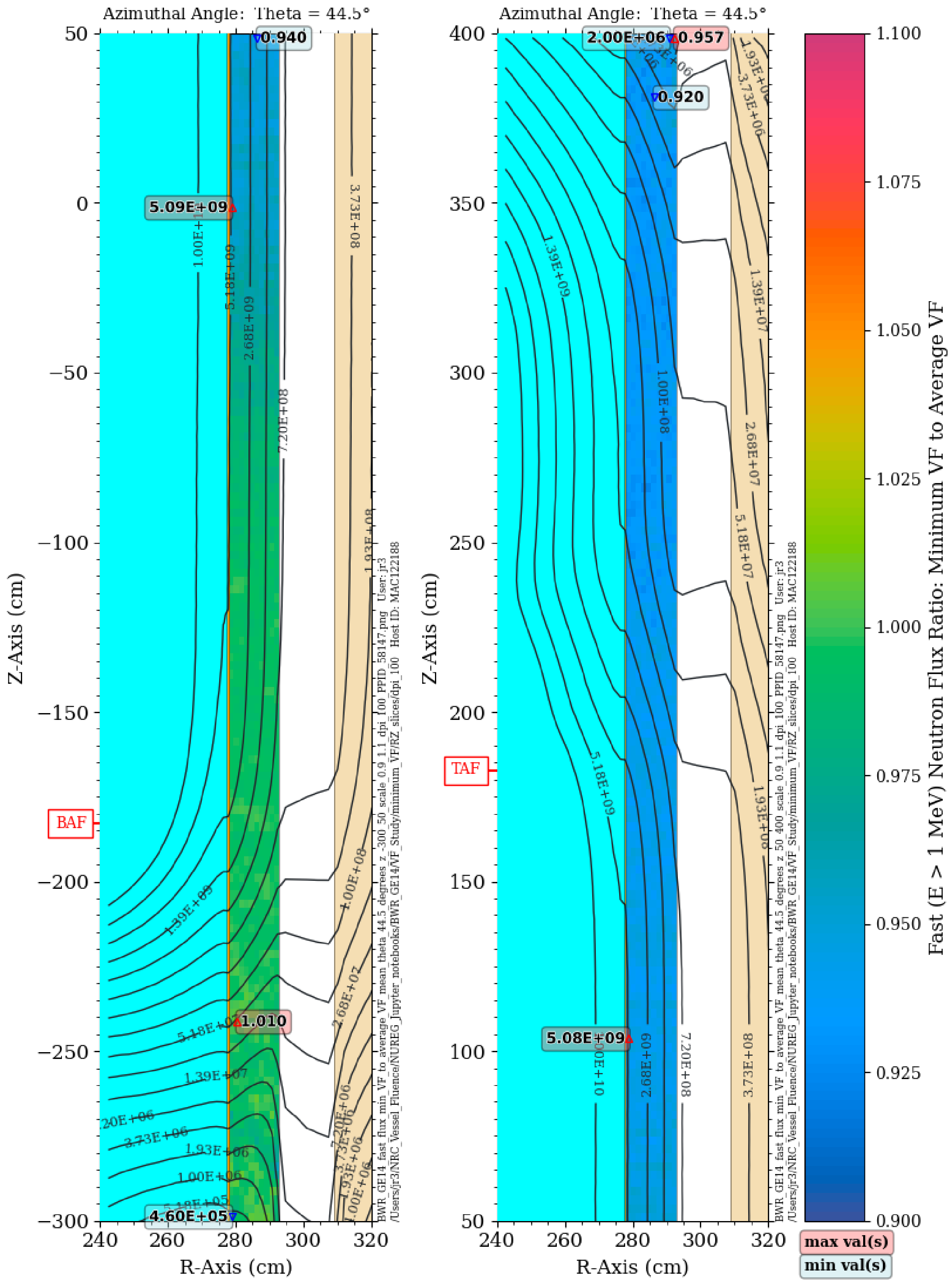


Figure 5-40 Fast neutron flux ratio for the minimum VF model to the base case BWR model at an azimuthal angle of 44.5°

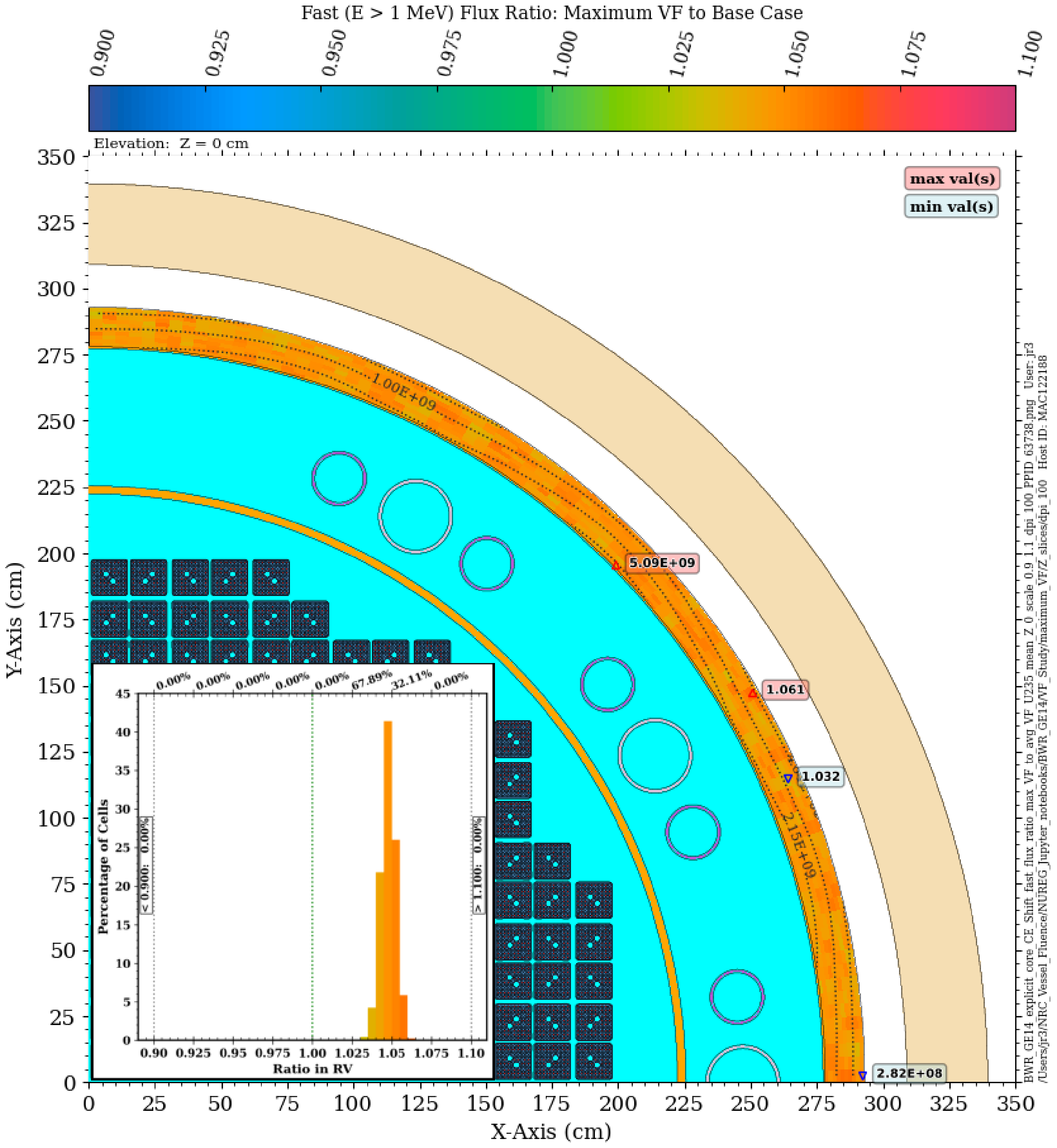


Figure 5-41 Fast neutron flux ratio for the maximum VF model to the base case BWR model at the core midplane elevation

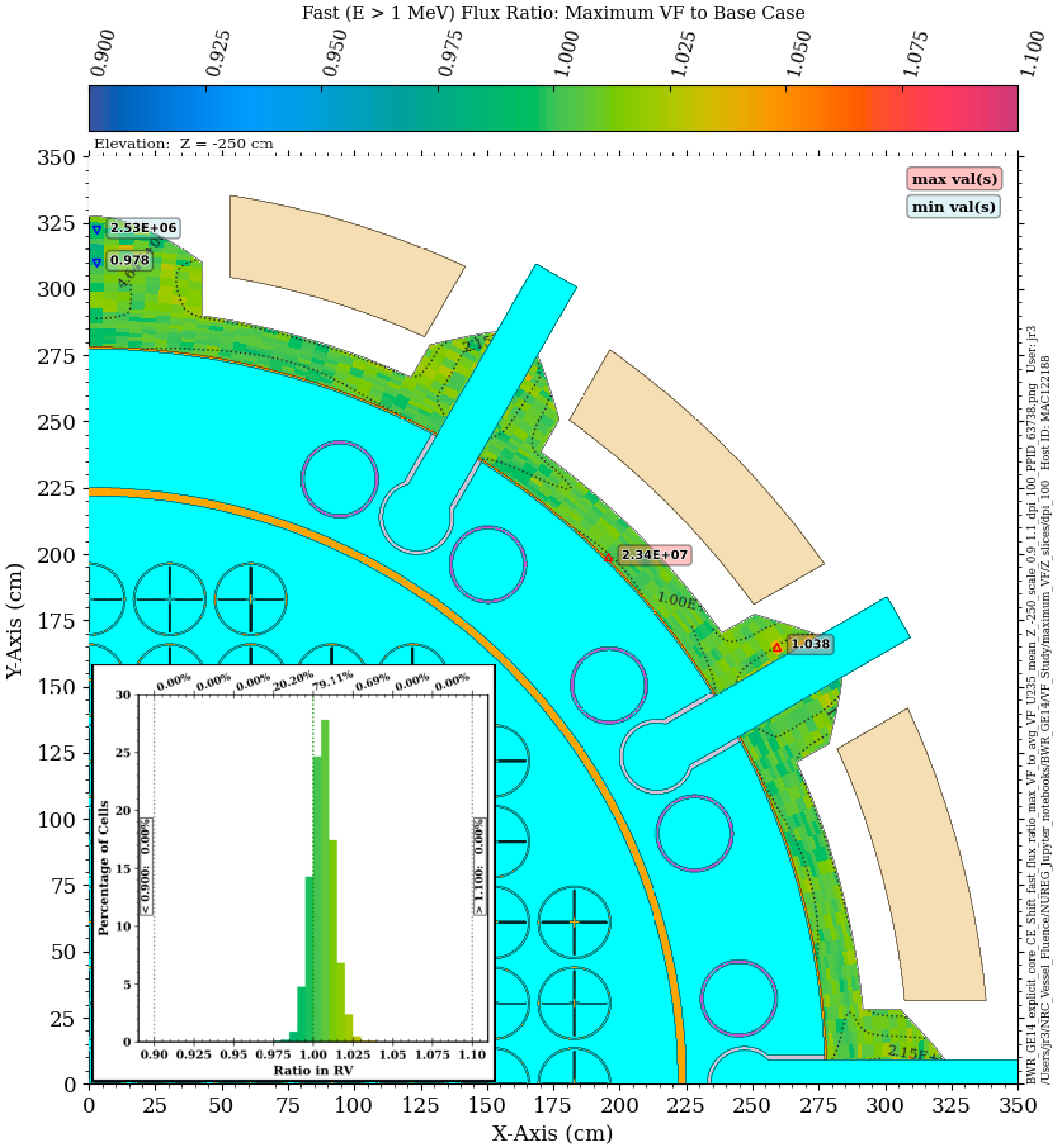


Figure 5-42 Fast neutron flux ratio for the maximum VF model to the base case BWR model. Plan view at an elevation through the recirculation outlet nozzles

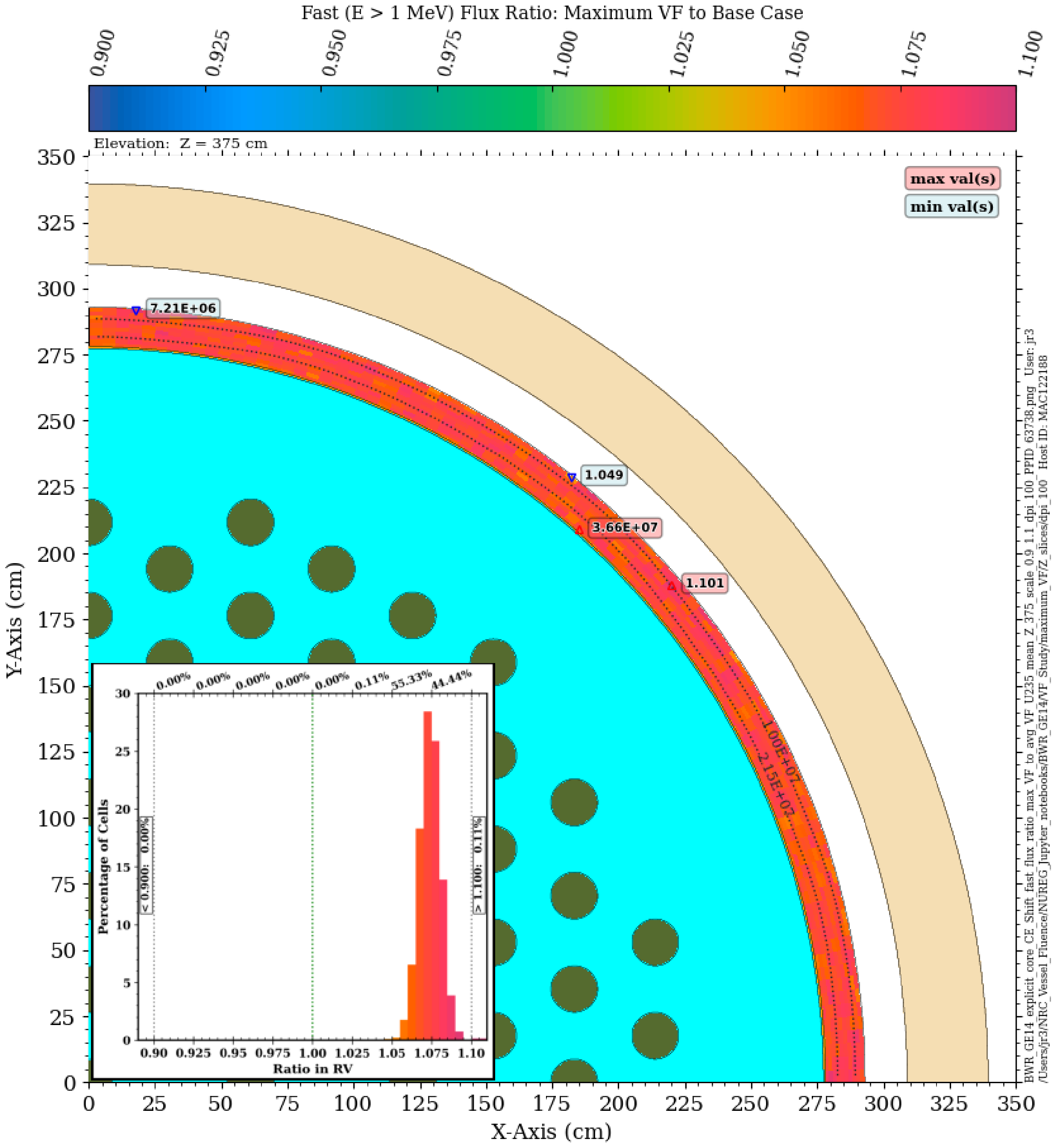


Figure 5-43 **Fast neutron flux ratio for the maximum VF model to the base case BWR model. Plan view at an elevation above the core shroud**

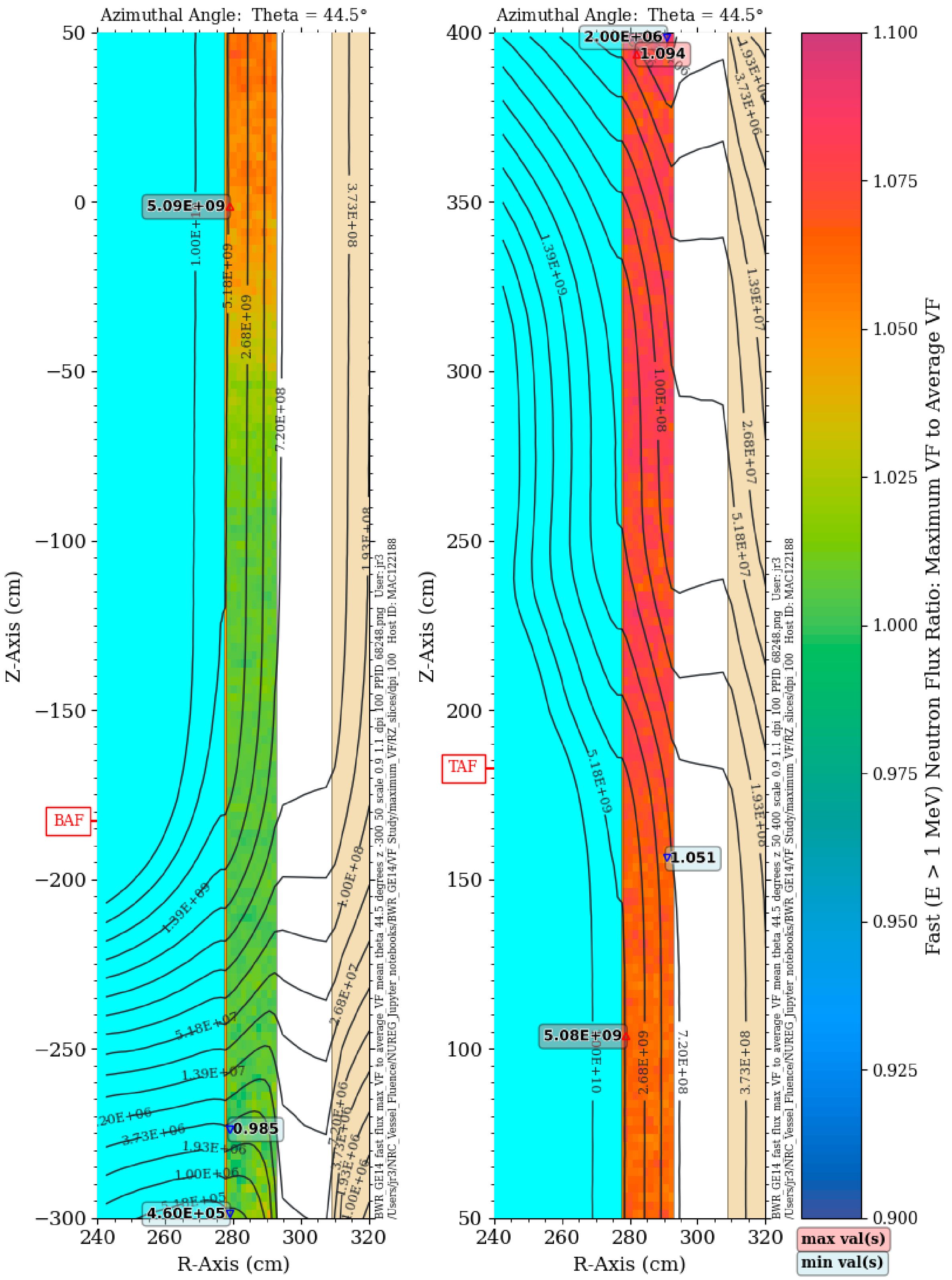


Figure 5-44 Fast neutron flux ratio for the maximum VF model to the base case BWR model. Elevation view at an azimuthal angle of 44.5°

5.5 Reactor cavity gap

For RPV locations above and below the traditional beltline region, the effect of neutrons that scatter in the bioshield and stream upwards and downwards in the gap between the RPV and the bioshield becomes progressively more important at increasing distances. As shown in Section 5.1, at sufficient distances from the core, these gap streaming neutrons are capable of not only reducing the rate of attenuation of the fast flux through the RPV, but they can also result in the peak flux being on the outer surface of the RPV. This behavior has been noted in previous studies, including work by Jones [76]. The effect of cavity streaming is particularly important with regard to the methods used for fluence predictions in the extended beltline region.

Because cavity gap streaming has been shown to be more significant in PWRs than BWRs in this work and in [76], the parametric study for the effect of the cavity gap width on fast neutron flux levels in the extended beltline region was carried out for the PWR model only. The cavity gap width in the baseline PWR model is 17.38 cm at elevations below Z = 402.59 cm, and the cavity gap width is 14.75 cm above 402.59 cm (Table 4-1). Three variant models were constructed having gap width increases of 10, 20, and 30 cm. The source in each case was a spatially uniform pinwise distribution with a ^{235}U fission spectrum.

No changes were made to the modeling of the nozzles or vessel supports. Consequently, flux levels in the vicinity of the nozzles and supports will not be realistic, as the bioshield inner surface is moved to radii beyond the location of the vessel supports. However, at an azimuthal angle of 315.5°, which is midway between the inlet and outlet nozzles, the fast flux levels are relatively unperturbed by the nozzle structures. This is also the azimuthal location where the fast flux levels in the RPV are greatest.

Ratio plots for the three increased-gap models at the azimuthal angle of 315.5° are shown in Figure 5-45 through Figure 5-50. The following observations can be made from these results:

1. Within the traditional beltline region, there is very little change in the fast flux in the RPV as the cavity width increases. The fast flux at the outer surface of the RPV shows a slight decrease (ranging from ~2.5% for the gap-plus-10-cm model to ~4.5% for the gap-plus-30-cm model) as the gap width increases. This is due to a decrease in the contribution of neutrons which scatter from the bioshield back to the RPV. As the gap width increases, the fast flux incident to the inner surface of the bioshield is reduced as a result of distance fall-off effects, so the contribution of neutrons that scatter from the bioshield back to the RPV diminishes.
2. At elevations in the extended beltline region, there is an increase in the fast flux level in the outer portion of the RPV as the cavity gap width is increased. The fast flux increase becomes greater with increasing distance from the core, and it also extends further into the RPV because of the dominant contribution of cavity streaming neutrons at these elevations.

Ratio plots for the increased gap models at an azimuthal angle of 270.5°—which is also midway between inlet and outlet nozzles—show behavior that is very similar to the results for the location at 315.5°. The magnitude of the fast flux is lower at 270.5° (Section 5.1.1), but the flux ratios for the increased gap cases are very similar.

A more comprehensive evaluation of differing cavity gaps would require plant-specific modeling to address the specific geometry of the RPV, nozzles, and vessel supports, as well as the cavity gap. However, this limited parametric study does confirm that the impact of changes in cavity gap width could be much more significant in the extended beltline region than the traditional beltline region.

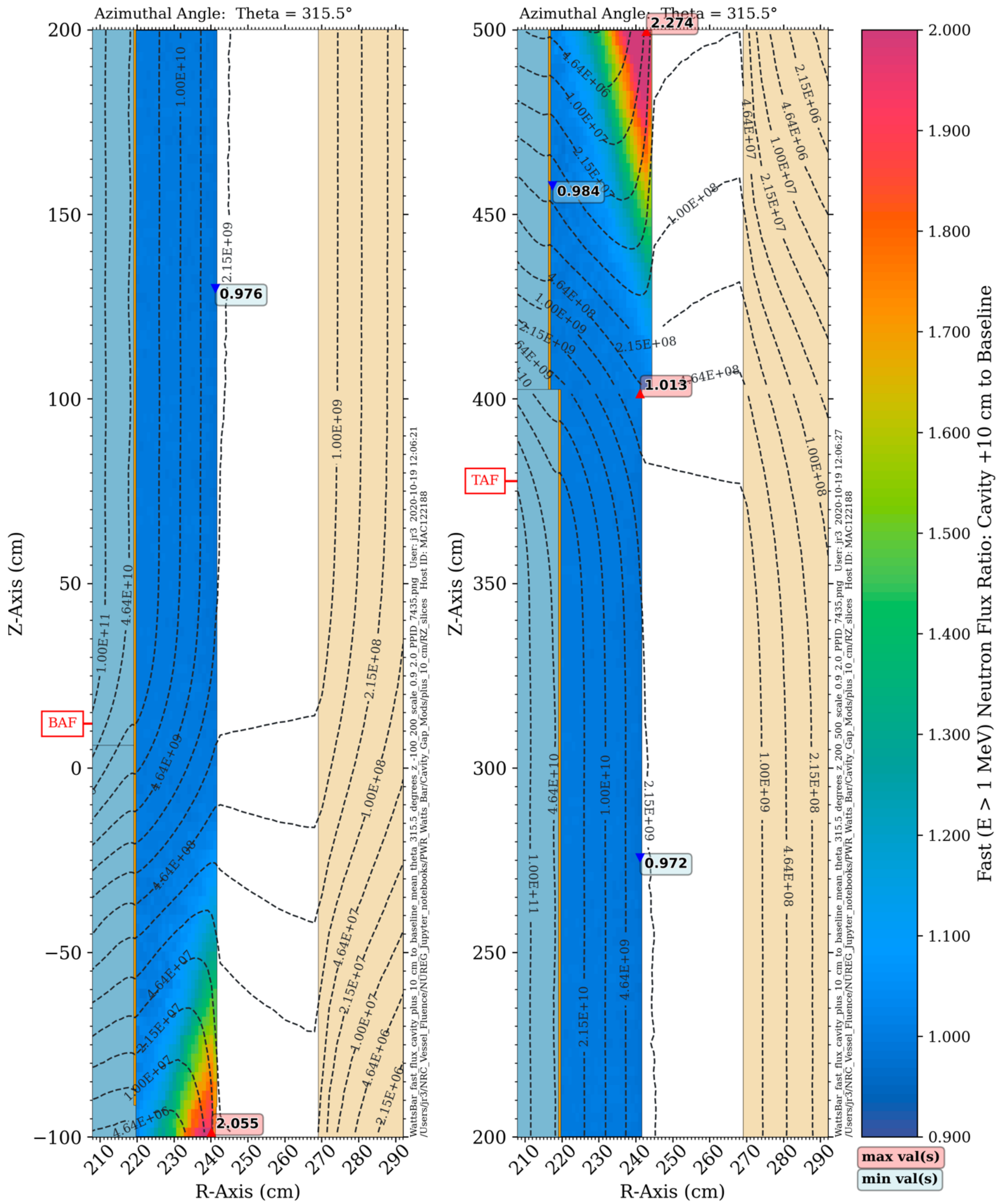


Figure 5-45 Fast neutron flux ratio in the PWR model with a cavity gap increase of 10 cm relative to the baseline gap width. Elevation view at an azimuthal angle of 315.5°. The contour lines represent the fast flux with the increased cavity gap

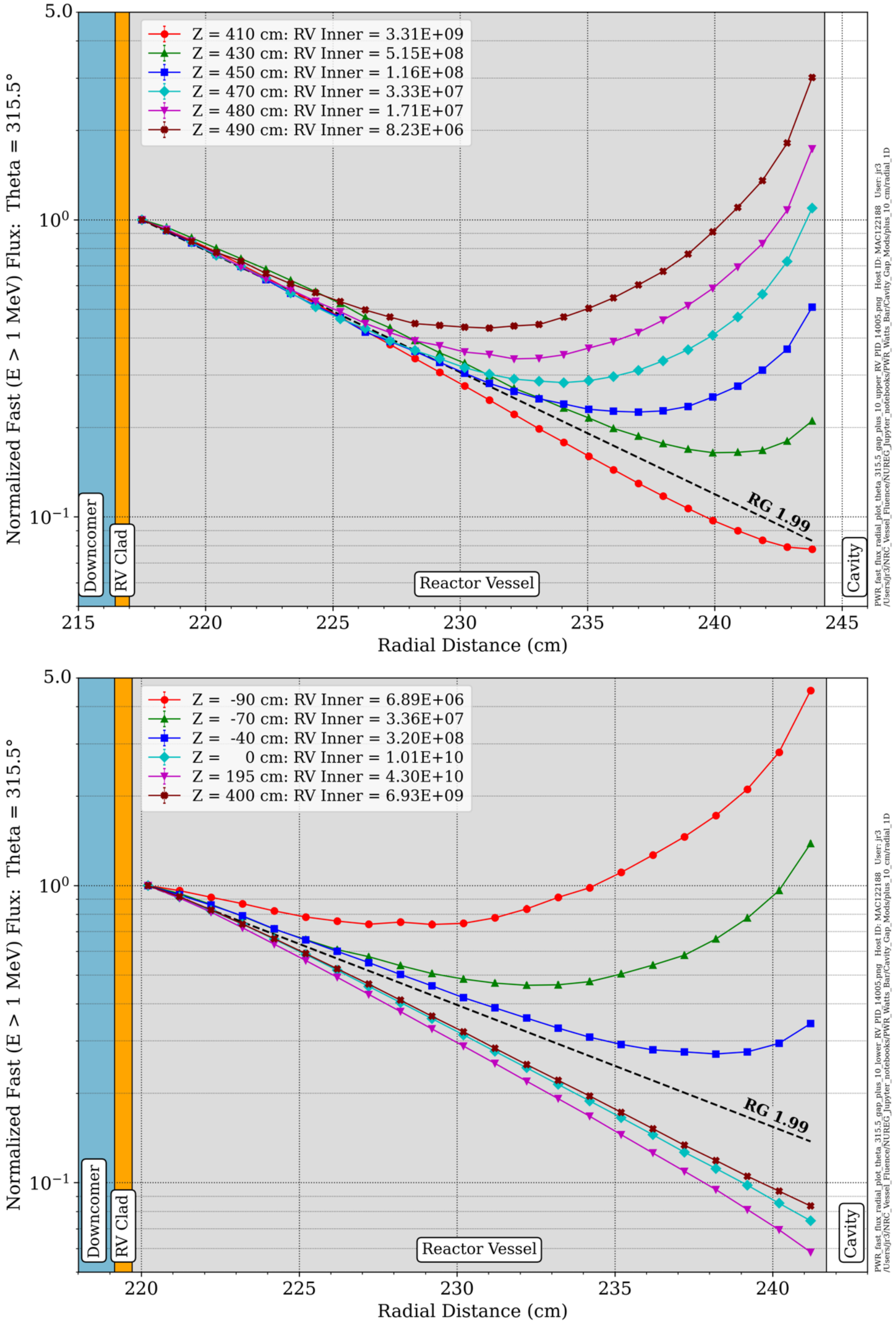


Figure 5-46 Normalized radial fast neutron flux profiles in the PWR model with a cavity gap increase of 10 cm. Azimuthal angle of 315.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation

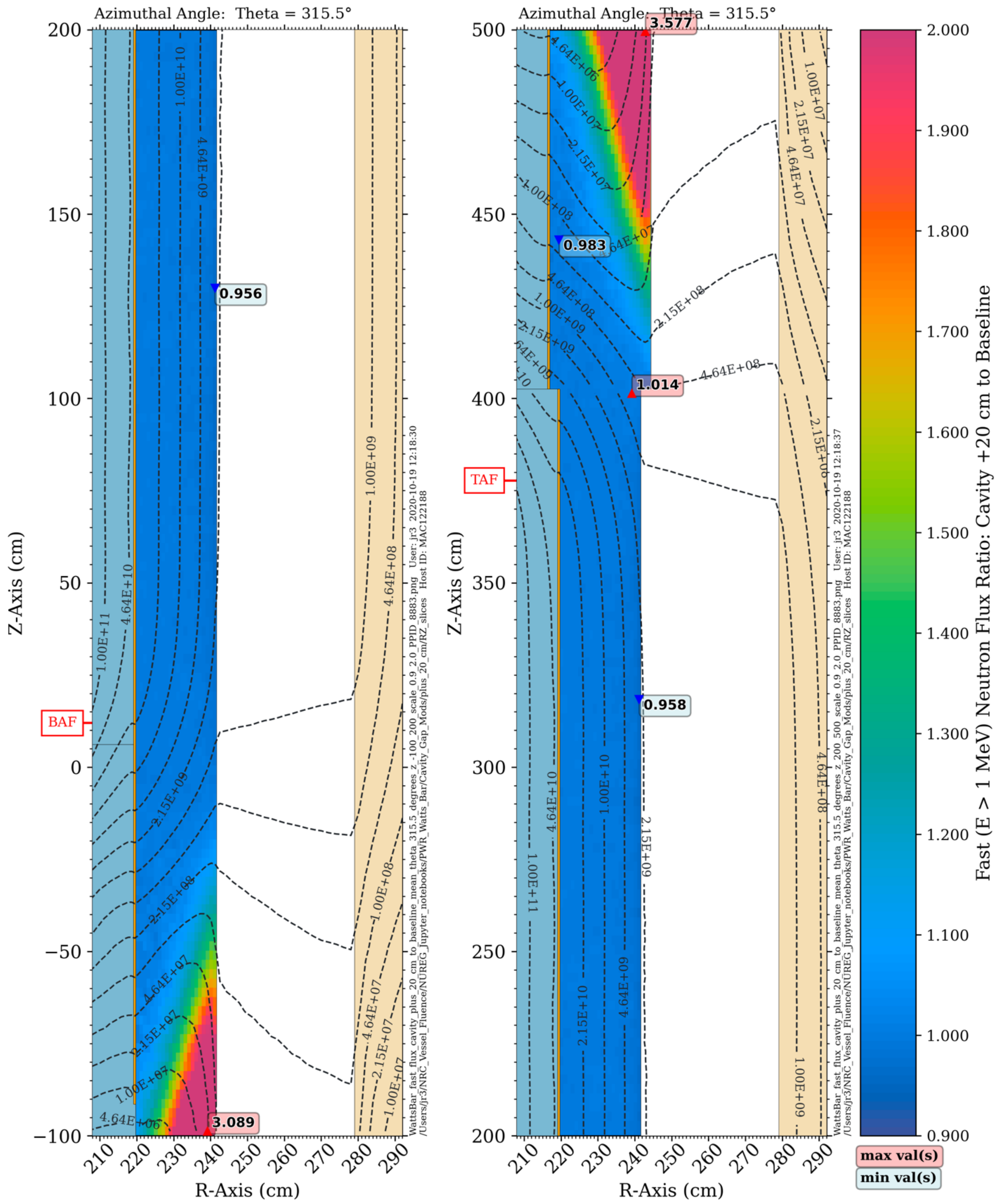


Figure 5-47 **Fast neutron flux ratio in the PWR model with a cavity gap increase of 20 cm. Elevation view at an azimuthal angle of 315.5°. The contour lines represent the fast flux with the increased cavity gap**

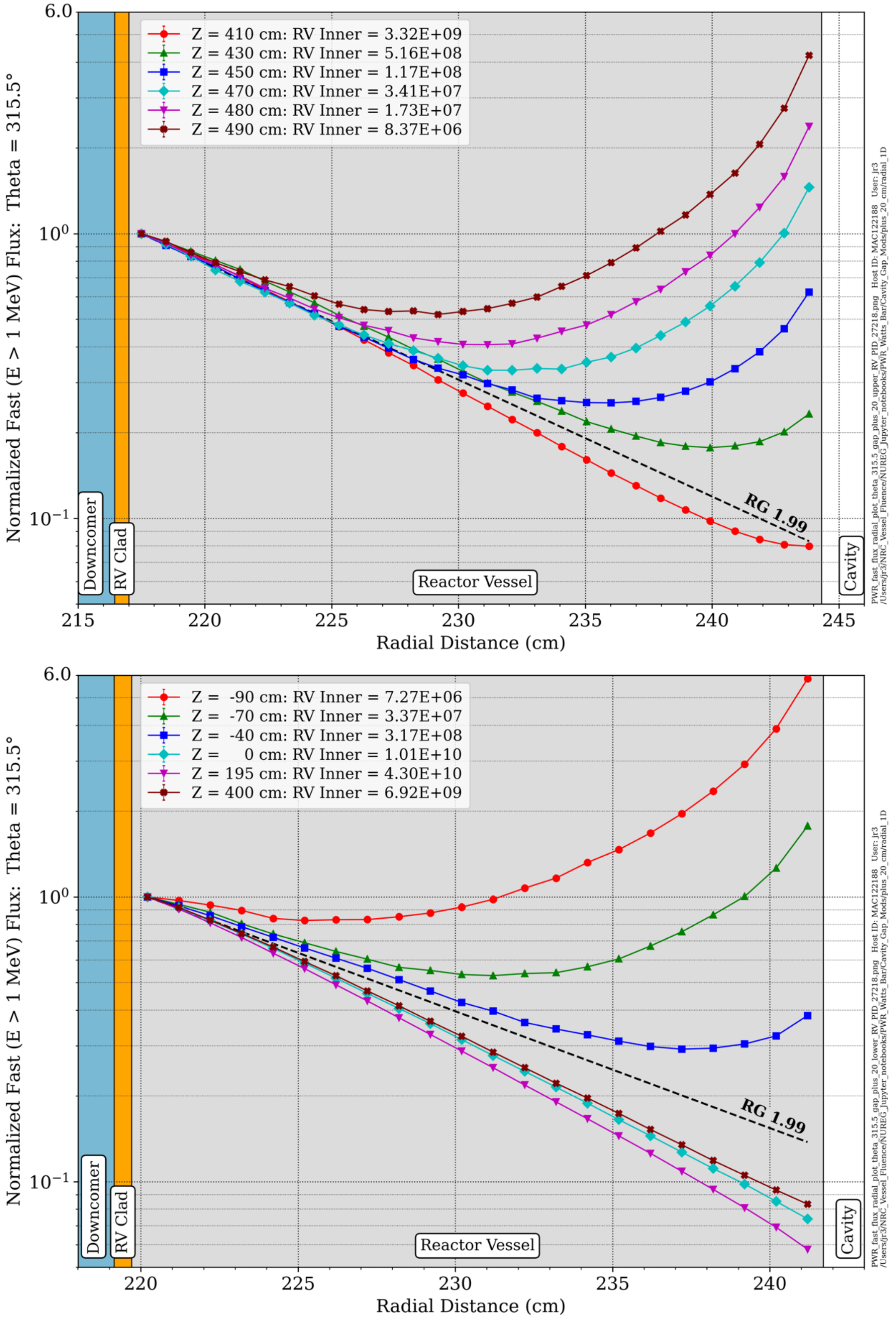


Figure 5-48 Normalized radial fast neutron flux profiles in the PWR model with a cavity gap increase of 20 cm. Azimuthal angle of 315.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation

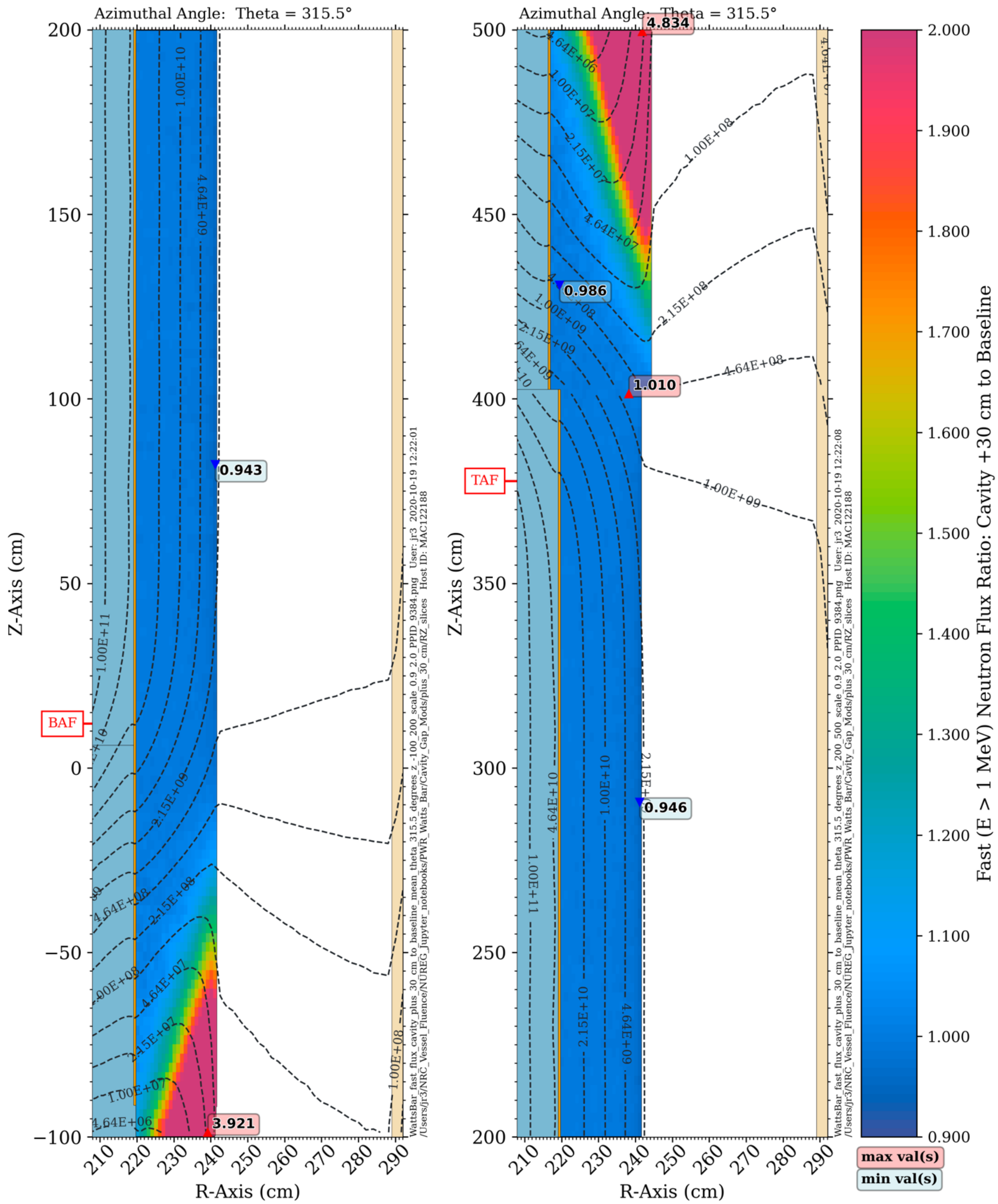


Figure 5-49 Fast neutron flux ratio in the PWR model with a cavity gap increase of 30 cm. Elevation view at an azimuthal angle of 315.5°. The contour lines represent the fast flux with the increased cavity gap

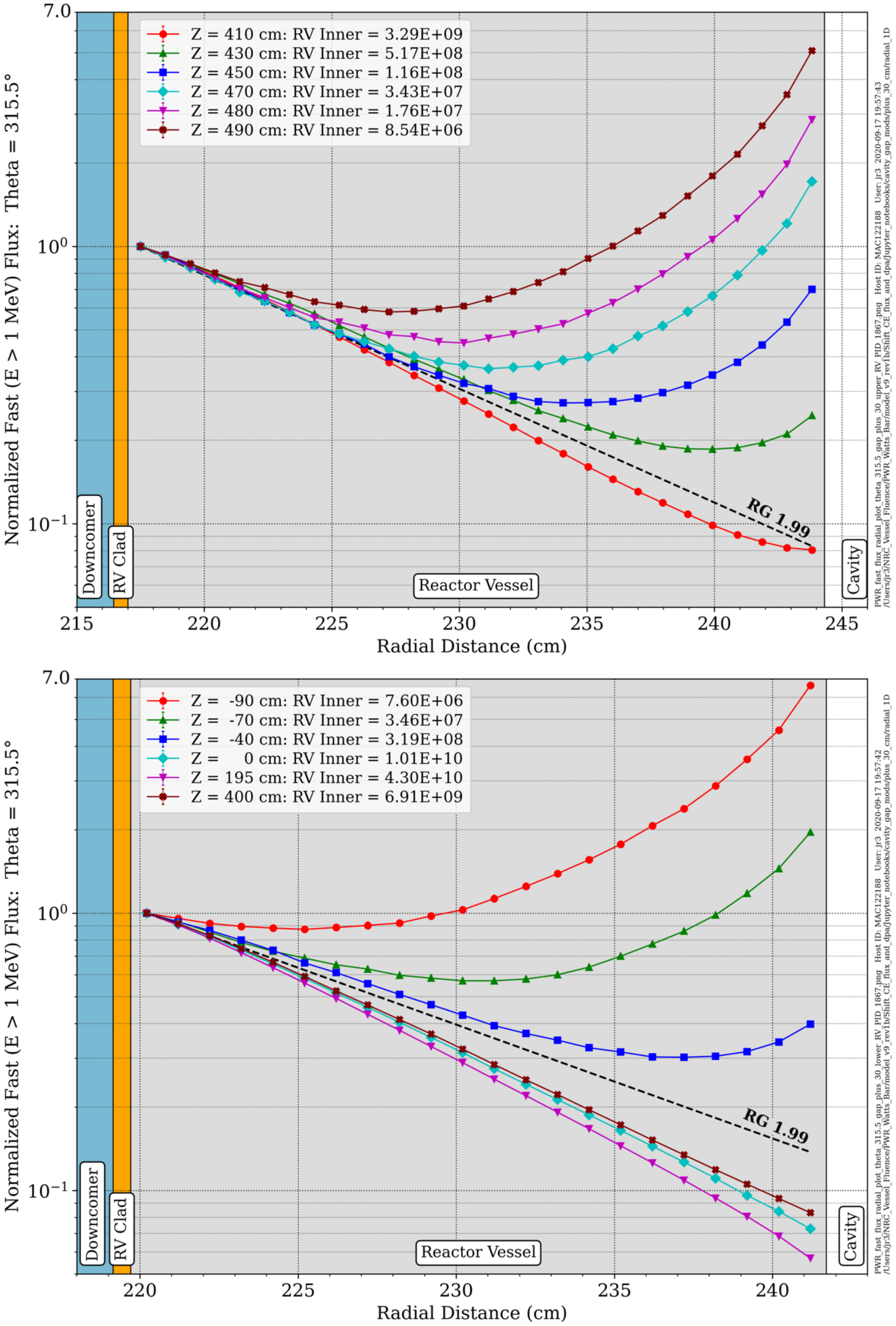


Figure 5-50 Normalized radial fast neutron flux profiles in the PWR model with a cavity gap increase of 30 cm. Azimuthal angle of 315.5°. The profiles are normalized to the flux at the RPV inner diameter at each elevation

5.6 Concrete composition

The discussion in Section 5.1 shows that the effect of neutron scattering from the concrete bioshield into the cavity gap has a significant effect on RPV fast flux levels at some elevations in the extended beltline region. Although this effect is minor for the BWR model, it can be quite pronounced for the PWR model, as shown in Figure 5-7 and Figure 5-8.

Because the compositions of different types of concrete vary significantly, the type of concrete can have a noticeable effect on RPV fast flux levels in regions where gap streaming is important. Of particular importance is the amount of hydrogen present in the concrete. This importance is due to two primary effects:

1. High-energy neutrons scatter from hydrogen with a forward-peaked angular distribution.[7] Thus, neutrons entering the concrete bioshield and scattering from hydrogen are more likely to be scattered further into the bioshield rather than backscattered into the cavity gap.
2. As noted in Section 5.2, the average energy loss from elastic scattering of neutrons by hydrogen is greater than the average energy loss for elastic scattering by any other element, so those neutrons that do scatter from hydrogen into the cavity gap are more likely to have an energy below the 1 MeV cutoff than neutrons that scatter from other elements in the bioshield back into the cavity gap.

To assess the impact of variations in concrete composition on cavity streaming neutron flux levels, three variants of the PWR reference model were constructed using three different concrete compositions. (Recall that the baseline models for the PWR and BWR have Type 04 concrete.) The amount of hydrogen in Type 04 concrete can be considered to be a typical value. Concretes with minimum and maximum hydrogen densities were chosen for the parameter study, along with a variant of Type 04 concrete in which the water density was reduced by 50%. The four concretes used in this parameter study are listed in Table 5-6. The isotopic compositions are listed in Table 5-7.

Table 5-6 Concrete types used in the concrete parameter study

Concrete type	Reference	Hydrogen density (g/cm^3)	Concrete density (g/cm^3)	Note
Type 01	ANL-6443 [77]	0.00484	2.33	Minimum hydrogen density
Hanford wet	PNNL-15870, Rev. 1 [78]	0.029	2.35	Maximum hydrogen density
Type 04	ANS-6.4-2006 (R2016) [79]	0.013	2.35	ANS-6.4-2006 (R2016) recommendation
Type 04 with 50% water reduction (Type 04 Mod)	ISRD10 Proceedings [80]	0.0065	2.29	Evaluates the effect of reducing the water content of Type 04 concrete by 50% while leaving all other constituents unchanged.

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[7] The scattering of neutrons from hydrogen is isotropic in the center-of-mass (CM) coordinate system, but forward peaked in the laboratory coordinate system.

Table 5-7 Composition of four concretes used in the PWR concrete parameter study

Isotopic number density (atoms/b cm)	Concrete type and density (g/cc)			
	Type 04 2.35	Type 04 Mod 2.29	Type 01 2.33	Hanford wet 2.35
1001	7.7679E-03	3.8840E-03	2.8936E-03	1.7284E-02
6000	----------------	----------------	6.5223E-03	----------------
8016	4.4081E-02	4.2142E-02	4.3275E-02	4.5414E-02
11023	1.0479E-03	1.0479E-03	----------------	1.2325E-04
12024	1.1744E-04	1.1744E-04	9.5178E-05	5.9842E-04
12025	1.4868E-05	1.4867E-05	1.2050E-05	7.5760E-05
12026	1.6370E-05	1.6370E-05	1.3266E-05	8.3412E-05
13027	2.3884E-03	2.3884E-03	2.6577E-04	3.3596E-03
14028	1.4675E-02	1.4674E-02	8.6667E-03	1.1906E-02
14029	7.4547E-04	7.4546E-04	4.4028E-04	6.0484E-04
14030	4.9199E-04	4.9199E-04	2.9057E-04	3.9918E-04
16032	5.3526E-05	5.3525E-05	3.4275E-05	----------------
16033	4.2261E-07	4.2261E-07	2.7062E-07	----------------
16034	2.3948E-06	2.3948E-06	1.5335E-06	----------------
16036	5.6349E-09	5.6348E-09	3.6083E-09	----------------
19039	6.4646E-04	6.4645E-04	----------------	4.0538E-04
19040	8.1103E-08	8.1103E-08	----------------	5.0859E-08
19041	4.6653E-05	4.6652E-05	----------------	2.9256E-05
20040	2.8262E-03	2.8262E-03	8.4684E-03	2.5352E-03
20042	1.8862E-05	1.8862E-05	5.6519E-05	1.6920E-05
20043	3.9357E-06	3.9357E-06	1.1793E-05	3.5304E-06
20044	6.0814E-05	6.0814E-05	1.8222E-04	5.4552E-05
20046	1.1661E-07	1.1661E-07	3.4943E-07	1.0461E-07
20048	5.4518E-06	5.4517E-06	1.6335E-05	4.8904E-06
26054	1.8281E-05	1.8281E-05	----------------	7.8567E-05
26056	2.8697E-04	2.8697E-04	----------------	1.2333E-03
26057	6.6274E-06	6.6273E-06	----------------	2.8483E-05
26058	8.8198E-07	8.8197E-07	----------------	3.7906E-06
28058	----------------	----------------	5.0743E-05	4.0538E-04
28060	----------------	----------------	1.9546E-05	----------------
28061	----------------	----------------	8.4964E-07	----------------
28062	----------------	----------------	2.7090E-06	----------------
28064	----------------	----------------	6.8991E-07	----------------

Ratio plots showing the change in the fast neutron flux at elevations from Z = -100 cm to Z = 500 cm in the PWR model for Type 01, Type 04 Mod, and Hanford wet concrete are shown in Figure 5-51 through Figure 5-57.

For the Type 01 concrete, which has the minimum amount of hydrogen, there is essentially no change in the RPV fast flux in the traditional beltline region. At elevations beginning at about 25–30 cm below the bottom of the core and above the top of the core, the effect of the reduced hydrogen content in the Type 01 concrete results in increases of up to ~20% in the fast neutron flux in the RPV's outer portion.

At the azimuthal location of the outlet nozzle (Figure 5-52), there is an additional impact of the minimum hydrogen content in the Type 01 concrete. The fast neutron flux levels in the vessel support below the outlet nozzle increase by up to 55%. This increase is partially due to the increased contribution of cavity streaming with the Type 01 concrete. In addition, as a result of the lower hydrogen content, neutrons which enter the bioshield at locations below the vessel support experience less attenuation as they are transported through the Type 01 concrete. This is shown more clearly in Figure 5-53, in which the separation between the flux contours of the two solutions (Type 04 and Type 01) is relatively uniform in the nozzle region directly above the cavity gap, but it increases in the bioshield and in the lower part of the nozzle support. In these regions, the flux is dominated by fast neutrons that have penetrated the bioshield and entered the nozzle support.

The differences in the fast flux levels with the Type 04 Mod concrete model (Figure 5-54 and Figure 5-55) are very similar to those for the Type 01 concrete. This behavior is of particular significance for potential changes in the concrete water content over the operating lifetime of an NPP. Radiation transport calculations that include evaluations of the neutron fluence in the extended beltline region, and in the nozzle support regions, in particular, may need to employ transport models in which the concrete composition is modified throughout the operating history.

The results with the Hanford wet concrete (Figure 5-56 and Figure 5-57) are consistent with expectations. For this concrete, which has a hydrogen content more than twice that of Type 04 concrete, cavity streaming effects are reduced. Because there is a higher probability of neutron scattering from hydrogen in the Hanford wet concrete, the level of backscattering from the bioshield into the cavity gap is reduced. In addition, neutrons which penetrate the bioshield before entering the vessel support experience more attenuation, resulting in further depression of the fast flux in the vessel supports.

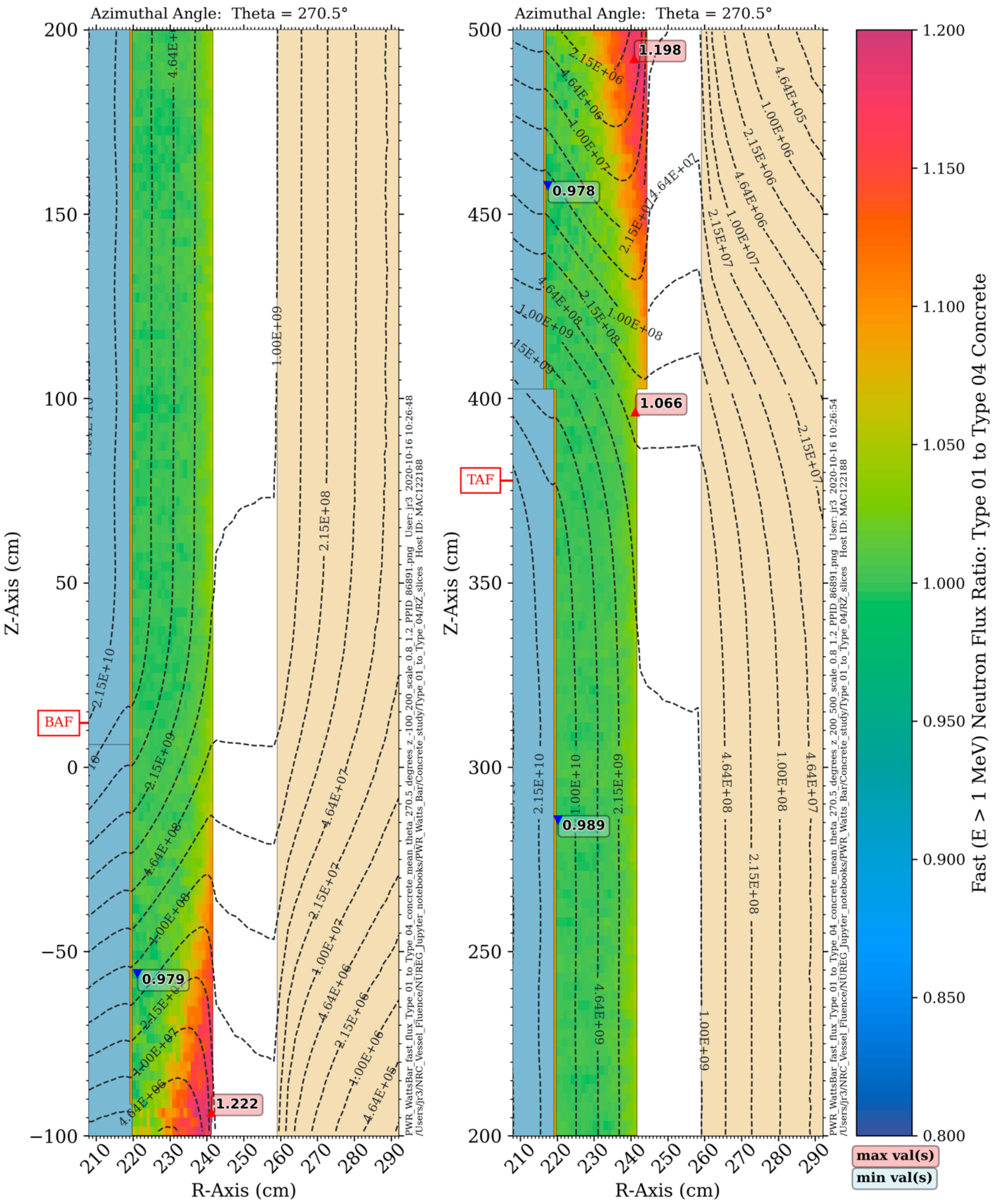

Figure 5-51 Fast neutron flux ratio in the PWR model with Type 01 concrete in the bioshield relative to Type 04 concrete. Elevation view at an azimuthal angle of 270.5°. The contour lines represent the fast flux for the Type 04 model. A spatially uniform pinwise ^{235}U source is used in both cases

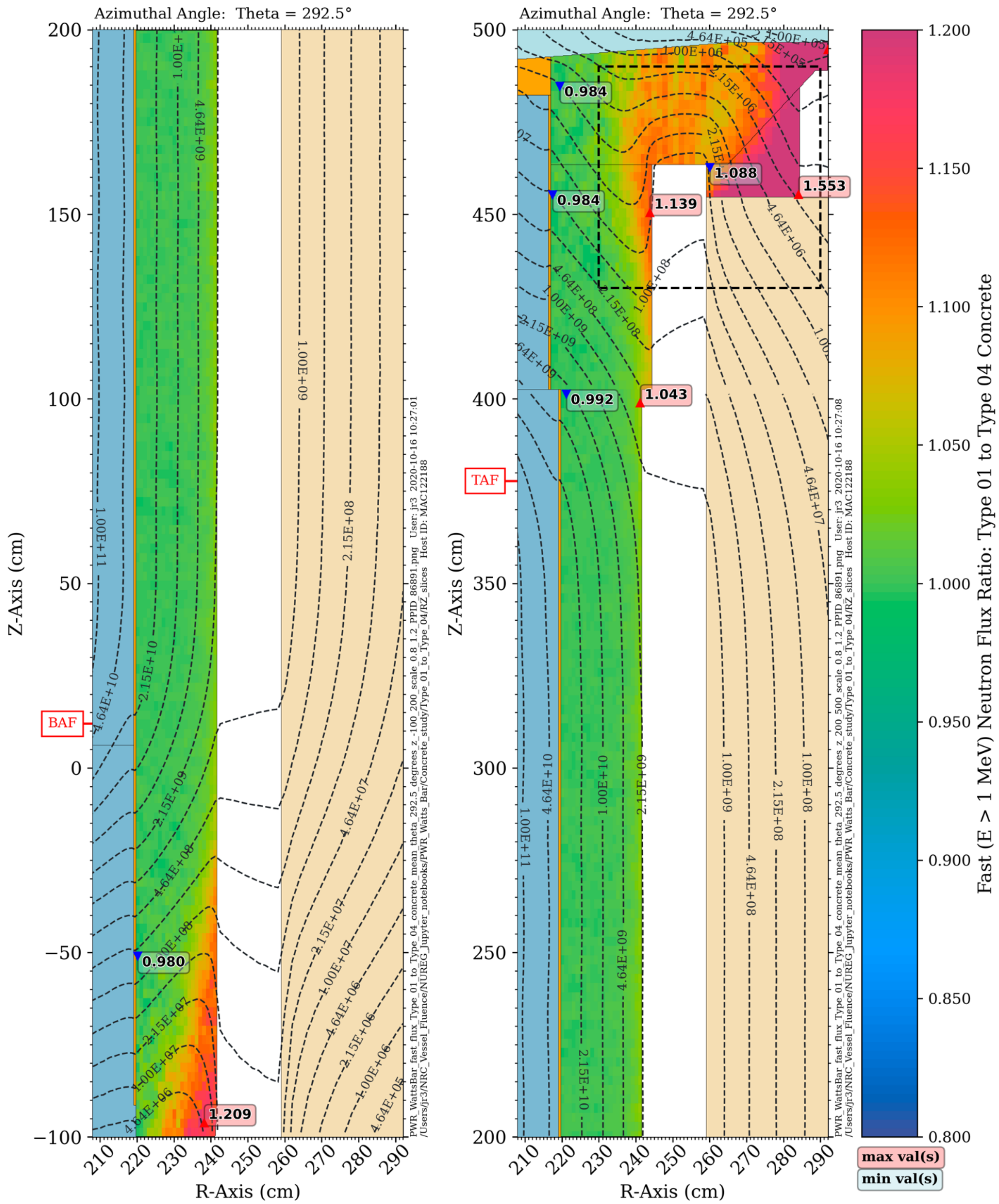

Figure 5-52 Fast neutron flux ratio in the PWR model with Type 01 concrete in the bioshield relative to Type 04 concrete. Elevation view at an azimuthal angle of 292.5°. The contour lines represent the fast flux for the Type 04 model. A spatially uniform pinwise ^{235}U source is used in both cases. The dashed box in the vicinity of the outlet nozzle vessel support corresponds to the plot area of Figure 5-53

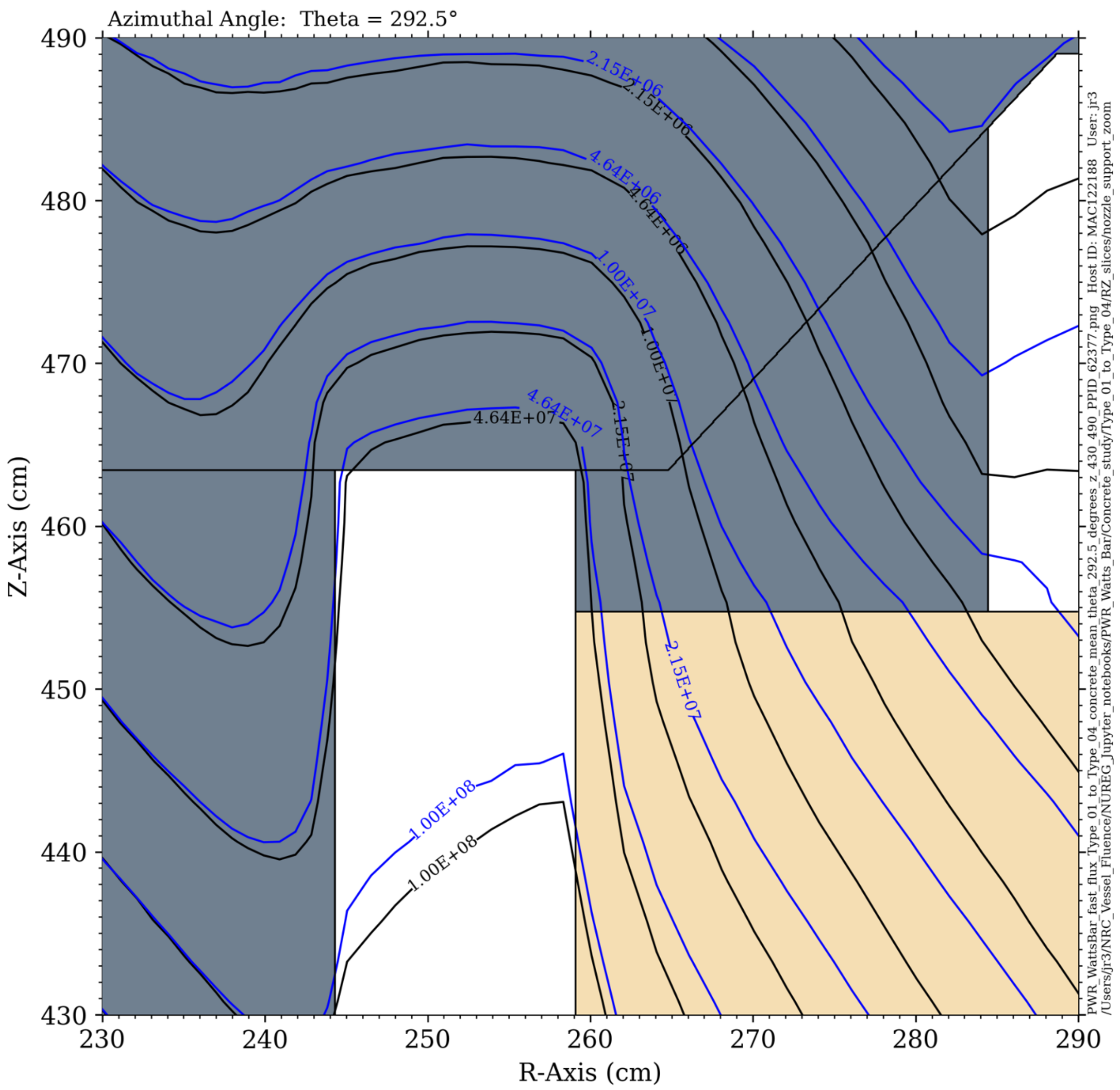

Figure 5-53 Fast neutron flux levels in the vicinity of the outlet nozzle vessel support. Elevation view at an azimuthal angle of 292.5°. The black contour lines represent the fast flux for the Type 04 concrete model, and the blue lines represent the fast flux for the Type 01 concrete model. Note the increasing separation in the two solutions that occurs with increasing depth into the concrete, with the Type 01 concrete providing less attenuation

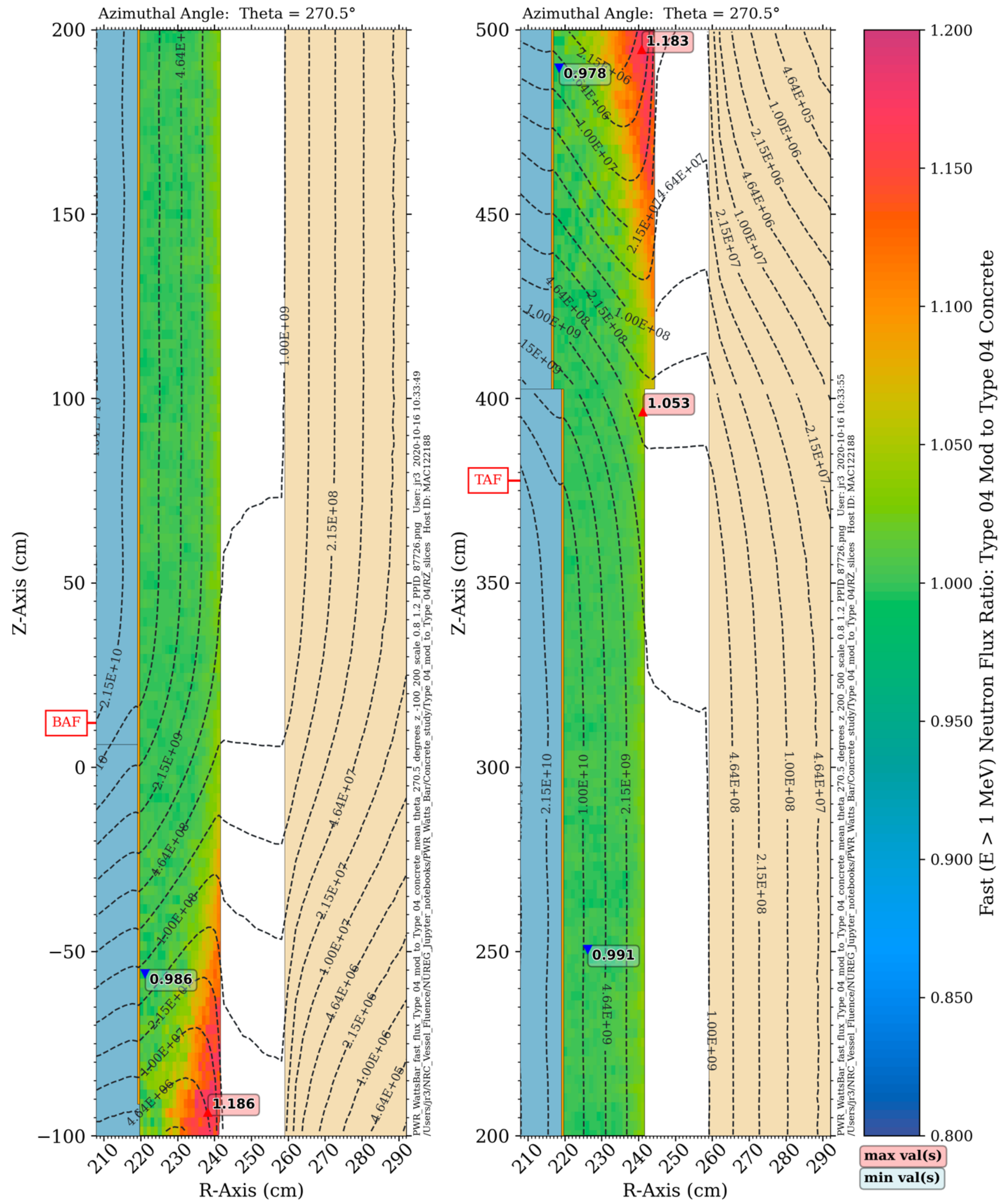

Figure 5-54 **Fast neutron flux ratio in the PWR model with Type 04 Mod concrete in the bioshield relative to Type 04 concrete. Elevation view at an azimuthal angle of 270.5°. The contour lines represent the fast flux for the Type 04 model. A spatially uniform pinwise ^{235}U source was used in both cases**

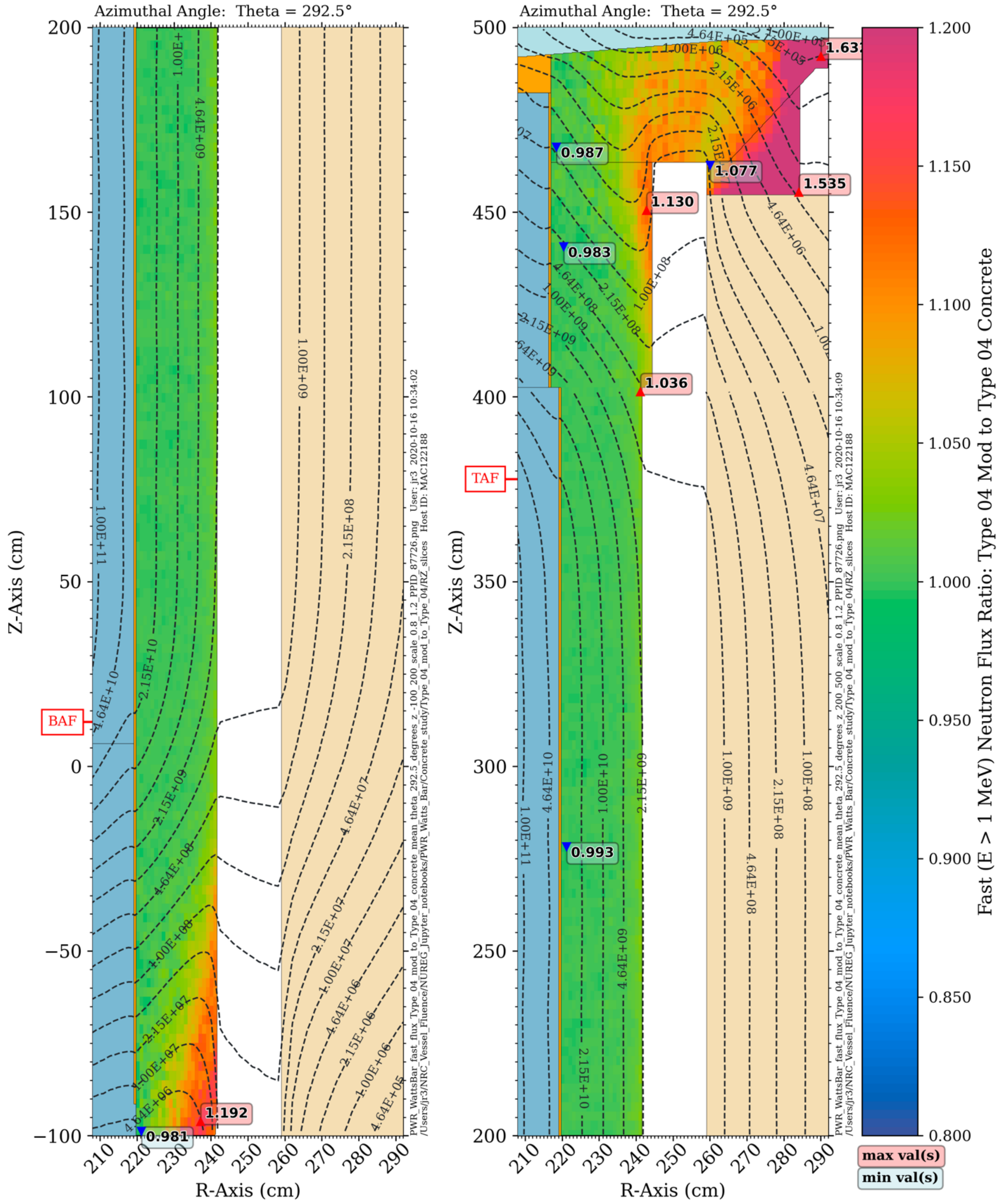

Figure 5-55 **Fast neutron flux ratio in the PWR model with Type 04 Mod concrete in the bioshield instead of Type 04 concrete. Elevation view at an azimuthal angle of 292.5°. The contour lines represent the fast flux for the Type 04 model. A spatially uniform pinwise ^{235}U source was used in both cases**

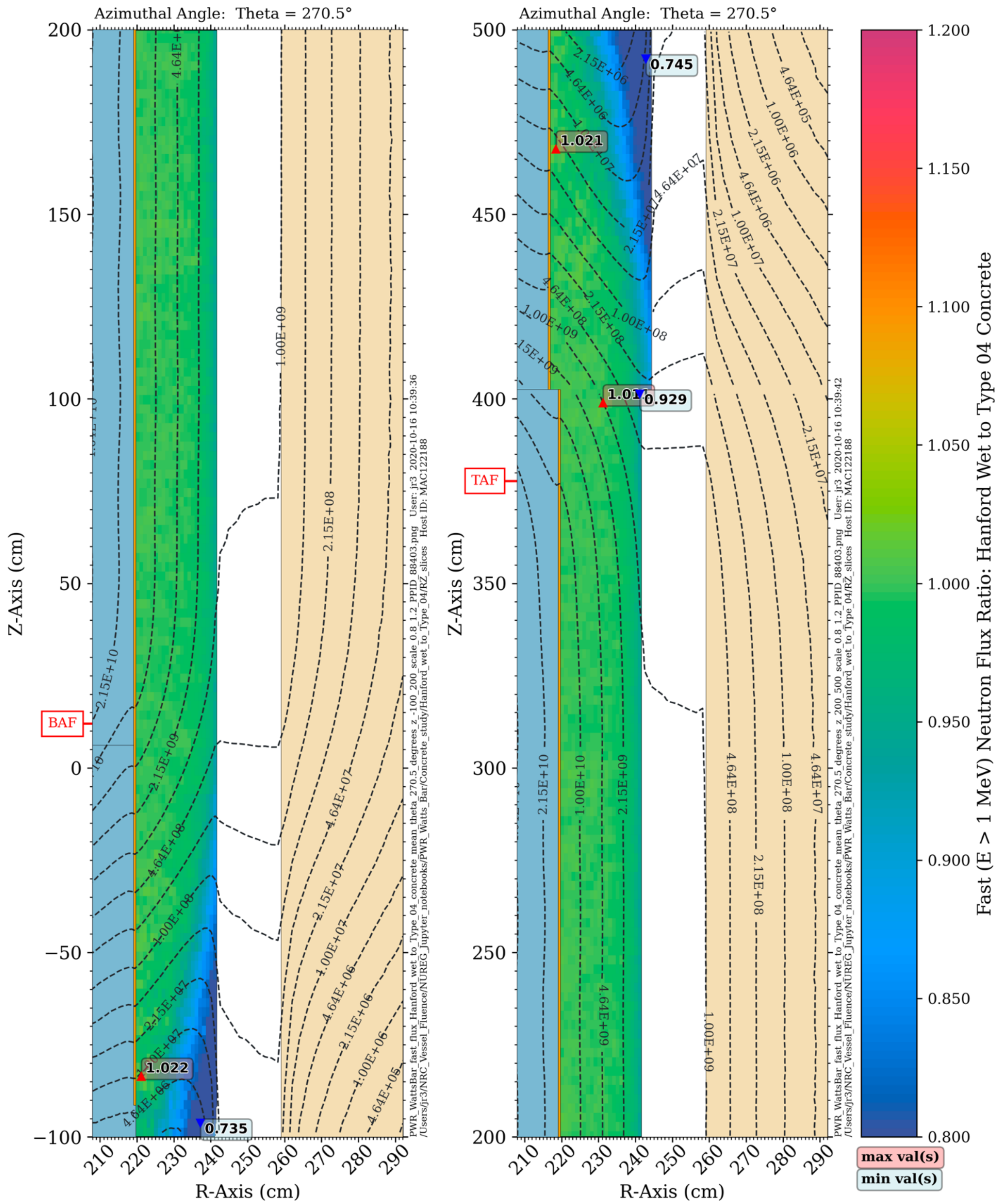

Figure 5-56 Fast neutron flux ratio in the PWR model with Hanford wet concrete in the bioshield relative to Type 04 concrete. Elevation view at an azimuthal angle of 270.5°. The contour lines represent the fast flux for the Type 04 model. A spatially uniform pinwise ^{235}U source was used in both cases

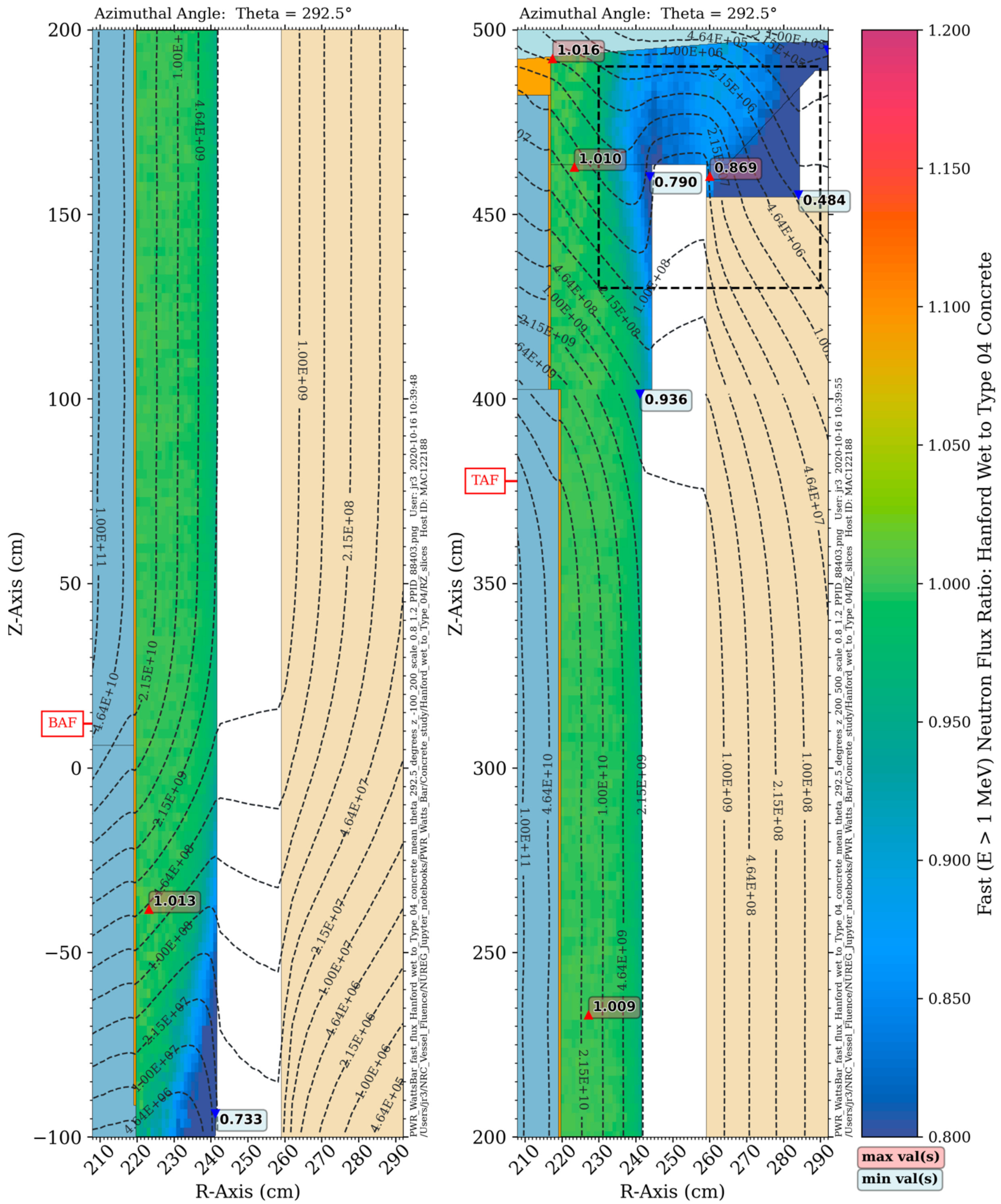

Figure 5-57 Fast neutron flux ratio in the PWR model with Hanford wet concrete in the bioshield relative to Type 04 concrete. Elevation view at an azimuthal angle of 292.5°. The contour lines represent the fast flux for the Type 04 model. The dashed box in the vicinity of the outlet nozzle vessel support corresponds to the plot area of Figure 5-58

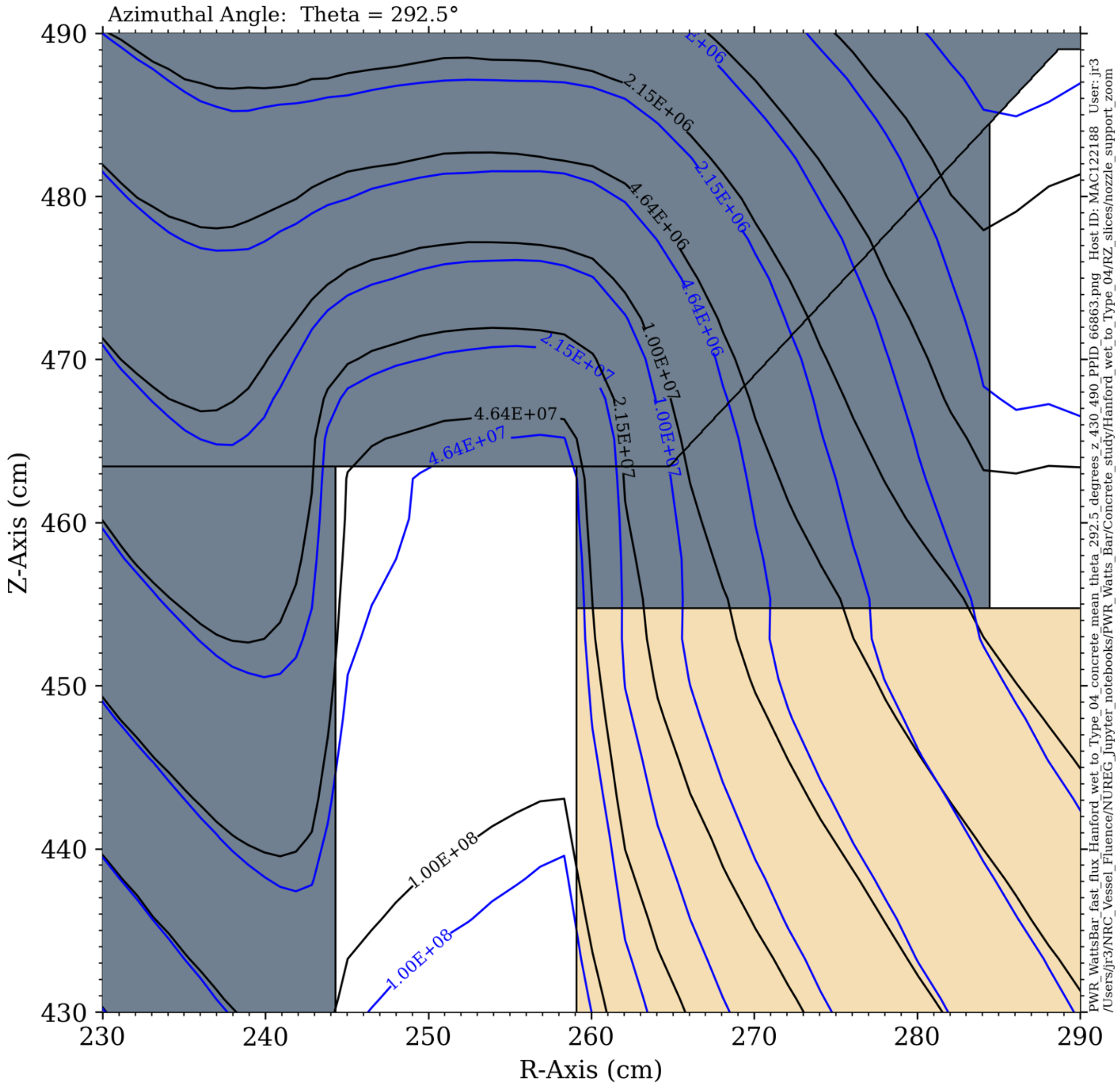

Figure 5-58 Fast neutron flux levels in the vicinity of the outlet nozzle vessel support. Elevation view at an azimuthal angle of 292.5°. The black contour lines represent the fast flux for the Type 04 concrete model; the blue lines represent the fast flux for the Hanford wet concrete model. Note the increasing separation in the two solutions with increasing depth into the concrete, with the Hanford wet concrete providing significantly more attenuation

5.7 Steel bioshield liner

The bioshield in the baseline PWR and BWR models is constructed of Type 04 concrete with no liner on the inner surface of the concrete, which is the cylindrical surface facing the RPV. Some reactor plant designs include a steel liner. The presence of a liner will have an effect on scattering from the bioshield into the cavity gap, as the angular distribution and average energy loss of scattered neutrons are different in steel than they are in the lighter elements that are the dominant constituents of the concrete.

Because the effect of a bioshield liner will primarily be on cavity-streaming neutrons, the PWR model was selected for a liner sensitivity study. (As shown in Section 5.1.2, cavity streaming neutrons in the BWR model are only significant at elevations where the magnitude of the fast flux is approximately three orders of magnitude lower than the peak fast flux in the beltline region.) To assess the impact of a steel liner on fluence levels in the reactor vessel, a 6.35 mm Type 304 stainless steel liner was added to the concrete bioshield in the PWR model below an elevation of 630.48 cm, where the inner radius of the concrete bioshield increases near the vessel flange (Figure 4-1).

The effect of the stainless steel liner is illustrated in Figure 5-59 through Figure 5-62. In Figure 5-59 and Figure 5-60 it can be seen that the effect of the liner on the fast neutron flux at the outer surface of the RPV is less than 3% at locations away from the nozzles.[8] The most notable effect is the reduction in the fast flux in the vessel support, which is also shown in Figure 5-61. Section 5.6 shows that the fast flux in the vessel support is dominated by neutrons that are transported through the concrete into the vessel support. This is also shown in Figure 5-62. At each location in the concrete region below the vessel support, the fast neutron flux when a stainless steel liner is present ‘lags’ the fast neutron flux with no liner. This difference is due to the reduced energy of fast neutrons that scatter in the liner before entering the concrete.

Calculations were also performed using a carbon steel liner with the same thickness (6.35 mm) as that of the stainless steel liner. The effect of a carbon steel liner is nearly equivalent to that of a stainless steel liner, although the changes in the fast flux (whether increases on the outer surface of the RPV or decreases in the vessel supports) are slightly less pronounced.

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[8] Only the outlet nozzle location at an azimuthal angle of 292.5° is shown. The liner–to–no-liner ratio at the location of the inlet nozzle (337.5°) is consistent with that at the outlet nozzle.

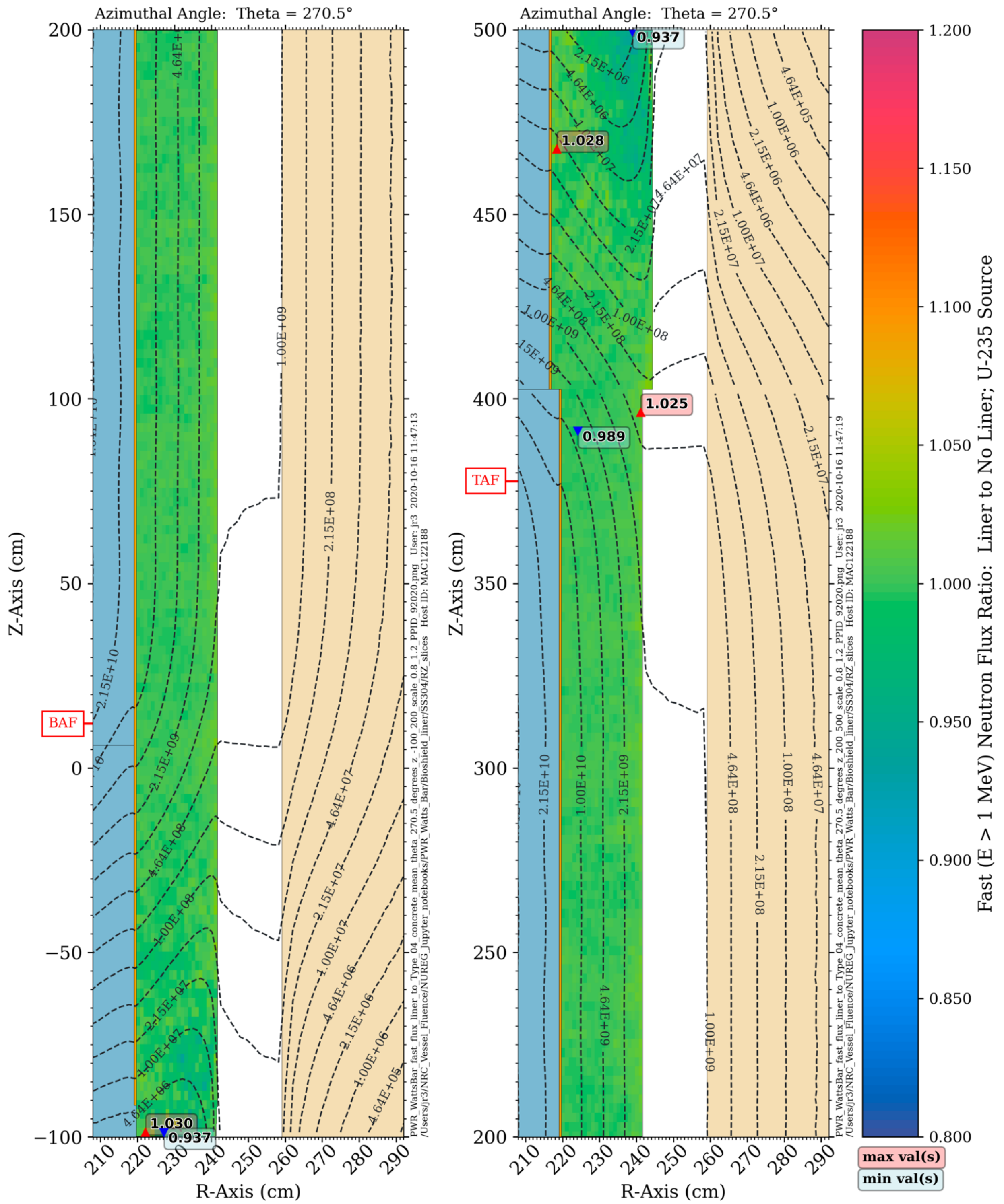

Figure 5-59 **Ratio of the fast neutron flux in the RPV of the PWR model with a 6.35 mm stainless steel bioshield liner relative to the base case with no liner. Elevation view at an azimuthal location of 270.5°. The contour lines show the fast neutron flux for the base case model**

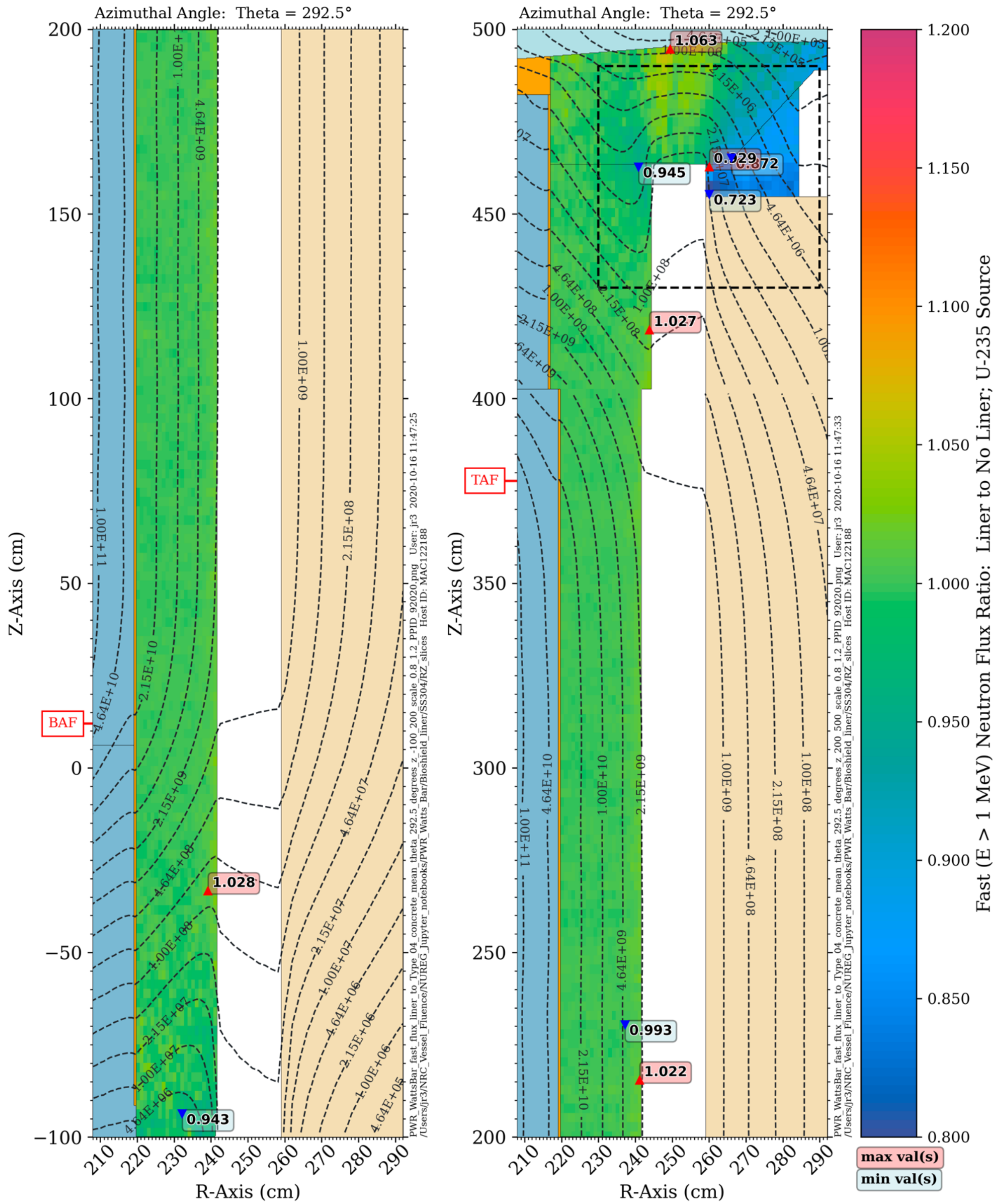

Figure 5-60 Ratio of the fast neutron flux in the RPV of the PWR model with a 6.35 mm stainless steel bioshield liner relative to the base case with no liner. Elevation view at an azimuthal location of 292.5°. The contour lines show the fast neutron flux for the base case model. The dashed box in the vicinity of the outlet nozzle vessel support corresponds to the plot area of Figure 5-61

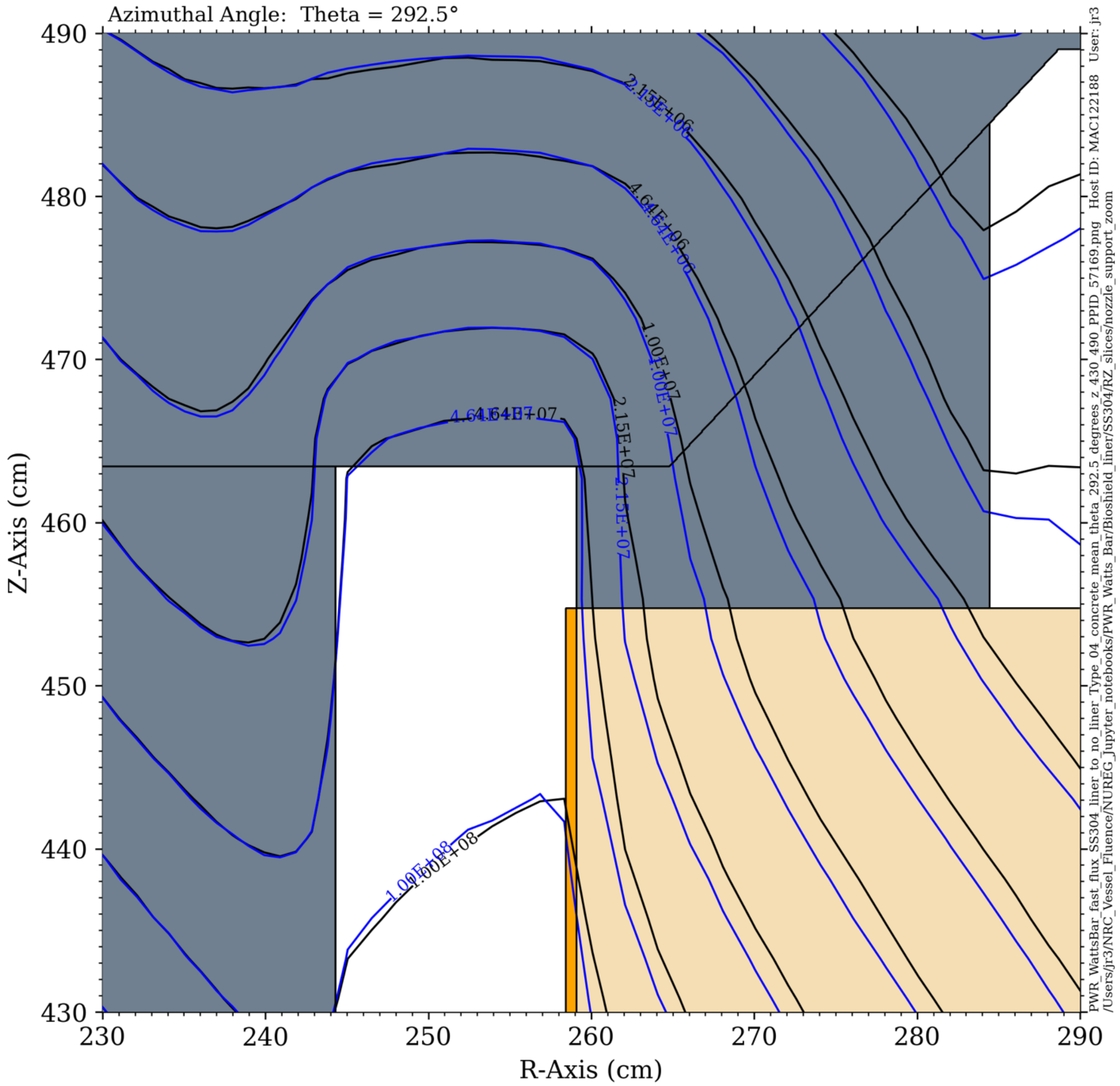

Figure 5-61 Fast neutron flux levels in the vicinity of the outlet nozzle vessel support. Elevation view at an azimuthal angle of 292.5°. The black contour lines represent the fast flux for the baseline model with no bioshield liner; the blue lines represent the fast flux for the model that includes at 6.35 mm SS304 bioshield liner

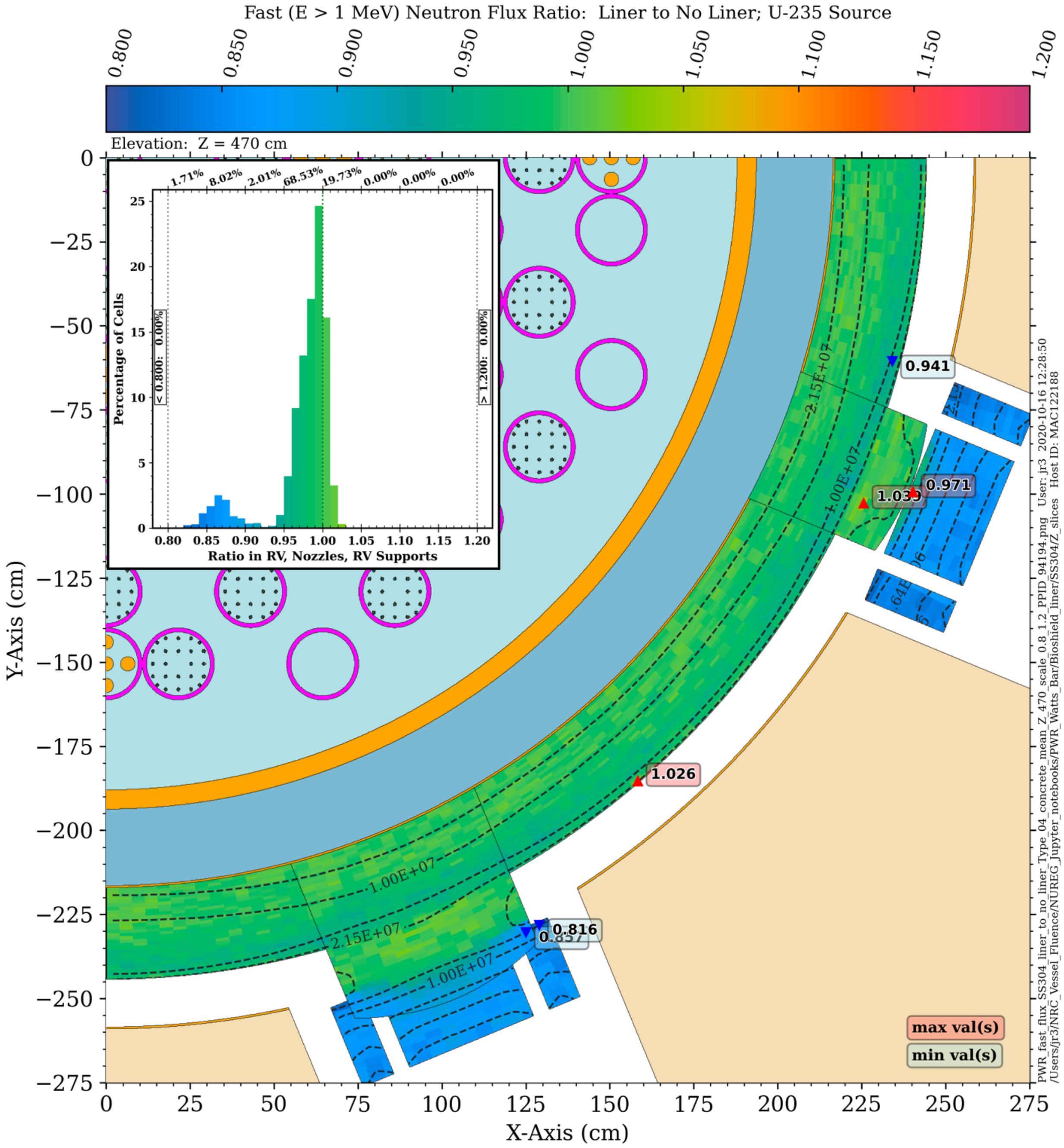

Figure 5-62 Ratio of the fast neutron flux in the RPV of the PWR model with a 6.35 mm stainless steel bioshield liner relative to the base case with no liner. Plan view at Z = 470 cm. The dashed contour lines show the fast neutron flux for the base case model

5.8 Thermal insulation

RPVs for PWR and BWR designs typically have a layer of thermal insulation between the RPV's outer radius and the inner radius of the concrete bioshield. This insulation layer has a minor effect on fast flux levels in the RPV in the beltline region, where it can cause very slight increases in the fast flux levels at the outer surface of the RPV due to backscatter of neutrons from the insulation. At locations in the extended beltline region, the presence of thermal insulation can reduce fast flux levels in the RPV due to attenuation of the cavity-streaming neutron flux by the thermal insulation.

To assess the effect of insulation, a reflective metallic insulation layer was modeled using a 7.62 cm (3 in.) layer composed of air with a 3% volume fraction of stainless steel. The thickness and material composition are based on data provided in Table 1.1 of NUREG/CR-6453 [81]. The axial extent of the insulation is from the bottom of the lower cylindrical portion of the RPV to the elevation of the nozzle tunnels. Note that changes in the thickness, axial extent, and/or the insulation composition could affect the neutron transport behavior in the cavity gap.

The effect of the insulation is shown at two azimuthal locations in Figure 5-63 and Figure 5-64 and at an elevation of $Z = 470$ cm in Figure 5-65. While there is no significant effect within the traditional beltline region, the decreased cavity streaming due to the presence of the insulation leads to a reduction of up to ~15–20% in the fast flux in the RPV at locations where cavity streaming dominates the fast flux profile in the RPV.

It should be noted that even though the reflective metallic insulation has an effective density of only 0.24 g/cm^3, neutrons that scatter from the bioshield back into the cavity gap and subsequently enter the outer surface of the RPV will typically pass through the insulation twice (once before and once after scattering in the bioshield), and they may have steep, slanted paths through the insulation that further increase the attenuation provided by the insulation.

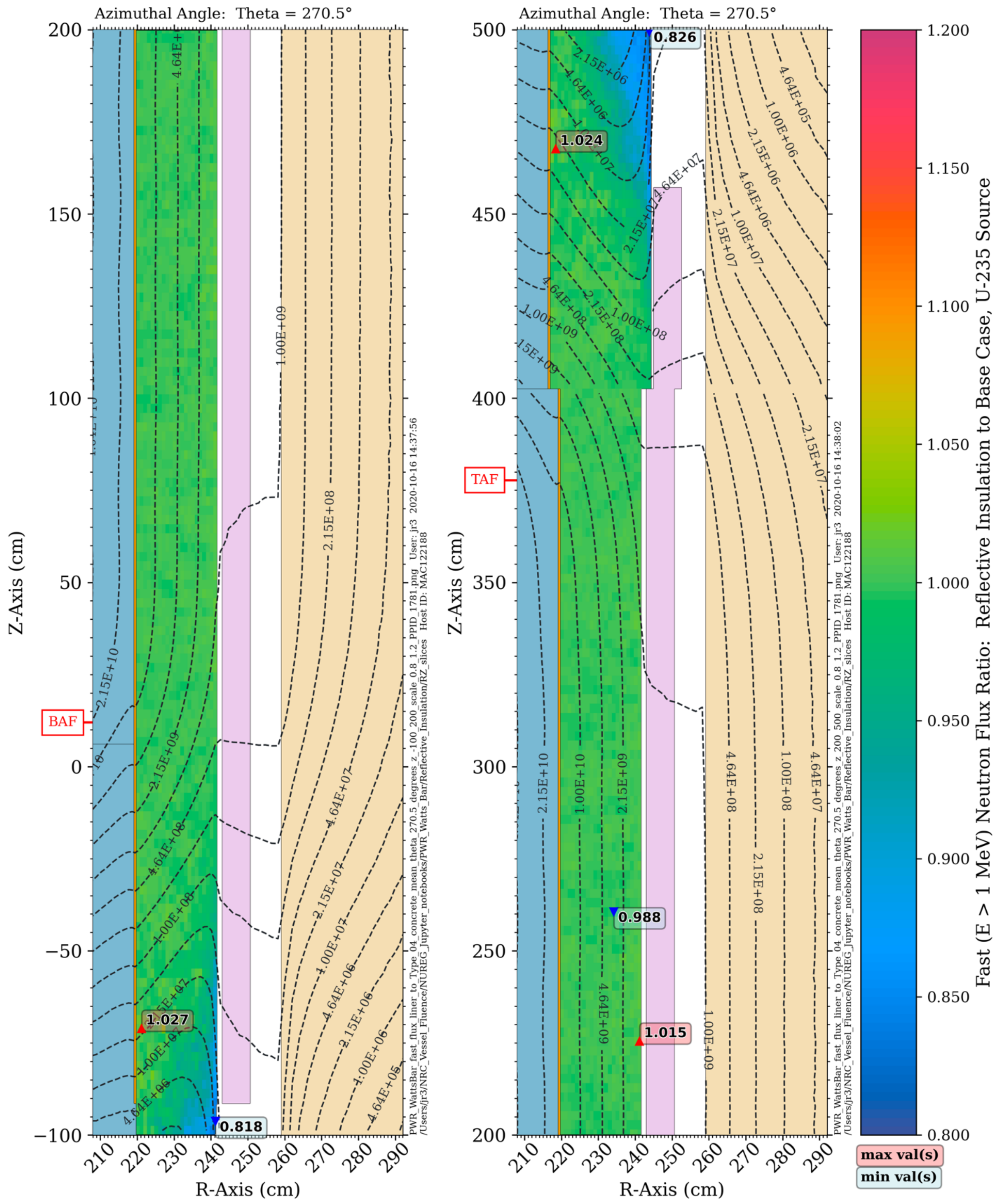

Figure 5-63 Fast neutron flux ratio for a PWR model with reflective metallic thermal insulation relative to the base case model with no thermal insulation. Elevation view at an azimuthal angle of 270.5°. The contour lines represent the fast flux for the base case model

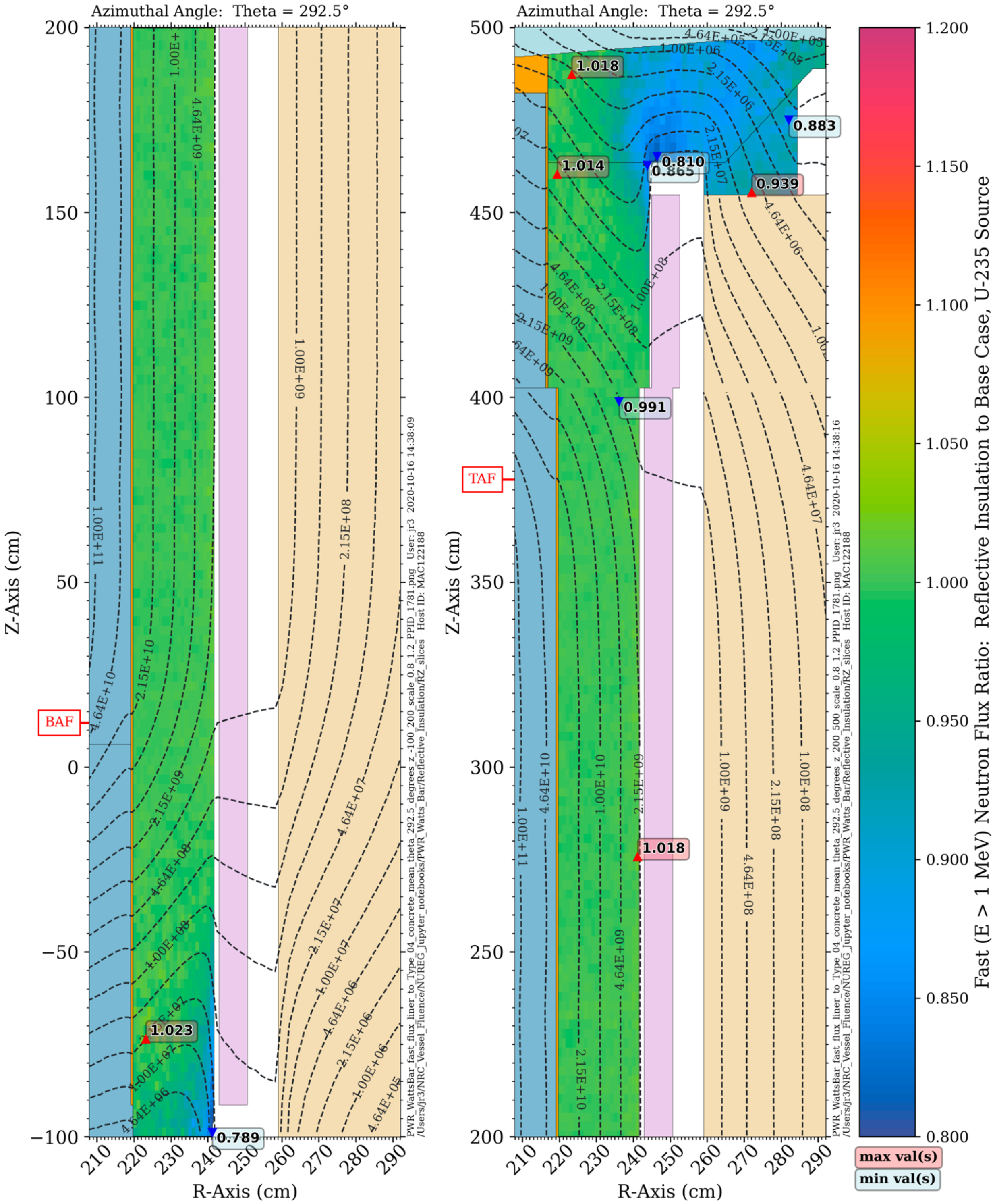

Figure 5-64 Fast neutron flux ratio for a PWR model with reflective metallic thermal insulation relative to the base case model with no thermal insulation. Elevation view at an azimuthal angle of 292.5°. The contour lines represent the fast flux for the base case model

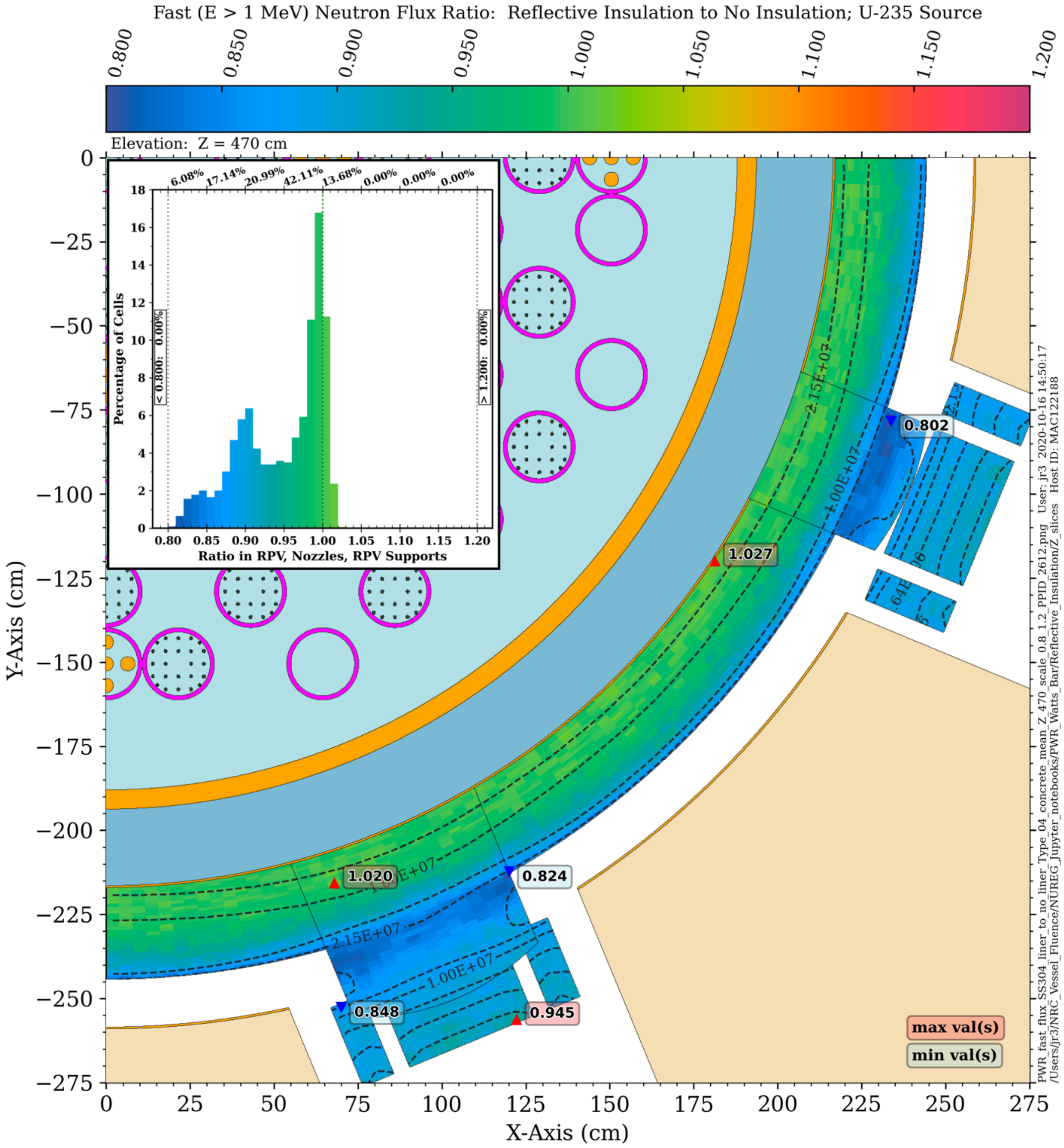

Figure 5-65 Fast neutron flux ratio for a PWR model with reflective metallic thermal insulation relative to the base case model with no thermal insulation. Plan view at an elevation of Z = 470 cm. The contour lines represent the fast flux for the base case model

5.9 Core homogenization

A common approximation in RPV fluence calculations is to homogenize the materials within the fuel assemblies into a set of mixtures rather than having the explicit geometry representation of the fuel pins, control assemblies, guide tubes, and other components. This is a reasonable modeling approximation for fast neutron flux calculations, as the neutron transport characteristics of the homogenized fuel assemblies are essentially identical to those of explicitly modeled assemblies when calculating fast neutron flux levels in the RPV.

To verify the validity of this modeling approximation and assess whether it is appropriate for fluence calculations in the extended beltline region, homogenized versions of the PWR and BWR reference models were created by calculating the mass fractions of each element in specific axial regions of each assembly and creating homogenized assembly models in which a single mixed material fills each assembly region. These homogenized models are also used in the Denovo deterministic calculations discussed in Section 6. Using transport models with homogenized fuel assemblies for deterministic calculations with Denovo—or with any code that uses material mixing for spatially discretized models—is significantly more memory efficient. Using an explicit core model with Denovo would produce mixed materials for every spatial mesh cell that has a unique material mixture. This could result in a very large number of mixed materials, which would substantially increase the amount of memory needed to store the cross-section data.

5.9.1 PWR model

Calculations of the fast flux levels in the RPV with the homogenized PWR model were compared with explicit core calculations for a spatially uniform ^{235}U source. Plan views of the fast flux ratio for the homogenized core model relative to the explicit core model are shown in Figure 5-66, Figure 5-67, and Figure 5-68. At the core midplane elevation (Figure 5-66), the fast flux with the homogenized core model agrees with the explicit core model within 1.5% in all mesh tally voxels. At an elevation ~80 cm below the bottom of the core (Figure 5-67), the fast flux in the RPV with the homogenized core is up to ~4% higher on average than the explicit core model. However, the maximum fast flux in the RPV at this elevation is more than three orders of magnitude lower than the core midplane value, so this difference is not significant. At an elevation ~90 cm above the top of the core (Figure 5-68), the homogenized core solution typically agrees with the explicit core solution within 2%.

5.9.2 BWR model

Calculations of the fast flux levels in the RPV with the homogenized BWR model were compared with explicit core calculations for a spatially uniform ^{235}U source. Average VFs (Section 5.4) were used in both the explicit and homogenized models. Plan views of the fast flux ratio for the homogenized core model compared to the explicit core model are shown in Figure 5-69, Figure 5-70, and Figure 5-71. At the core midplane elevation, the fast flux with the homogenized core model agrees with the explicit core model within 2% in all mesh tally voxels. At the lower ($Z = -250$ cm) and upper ($Z = 375$ cm) elevations, the agreement is within 2% in nearly all the mesh tally voxels.

5.9.3 Summary

The results presented in Section 5.9.1 and Section 5.9.2 suggest that homogenization of the core in both the PWR and BWR models is appropriate not only for RPV fluence calculations within the traditional beltline region, but also within the extended beltline region. The primary purpose of this parameter study is to confirm that the use of homogenized fuel assembly models is appropriate in discrete ordinates calculations of RPV vessel fluence.

There is little if any benefit to using a homogenized core model for Monte Carlo transport calculations. There may be a slight reduction in the average time per history because there are fewer surface crossings when transporting neutrons through a homogenized core, but those savings are typically not significant. Use of an explicit core model also provides the benefit of having a single model that can be used for core eigenvalue and power distribution calculations, as well as RPV fluence calculations.

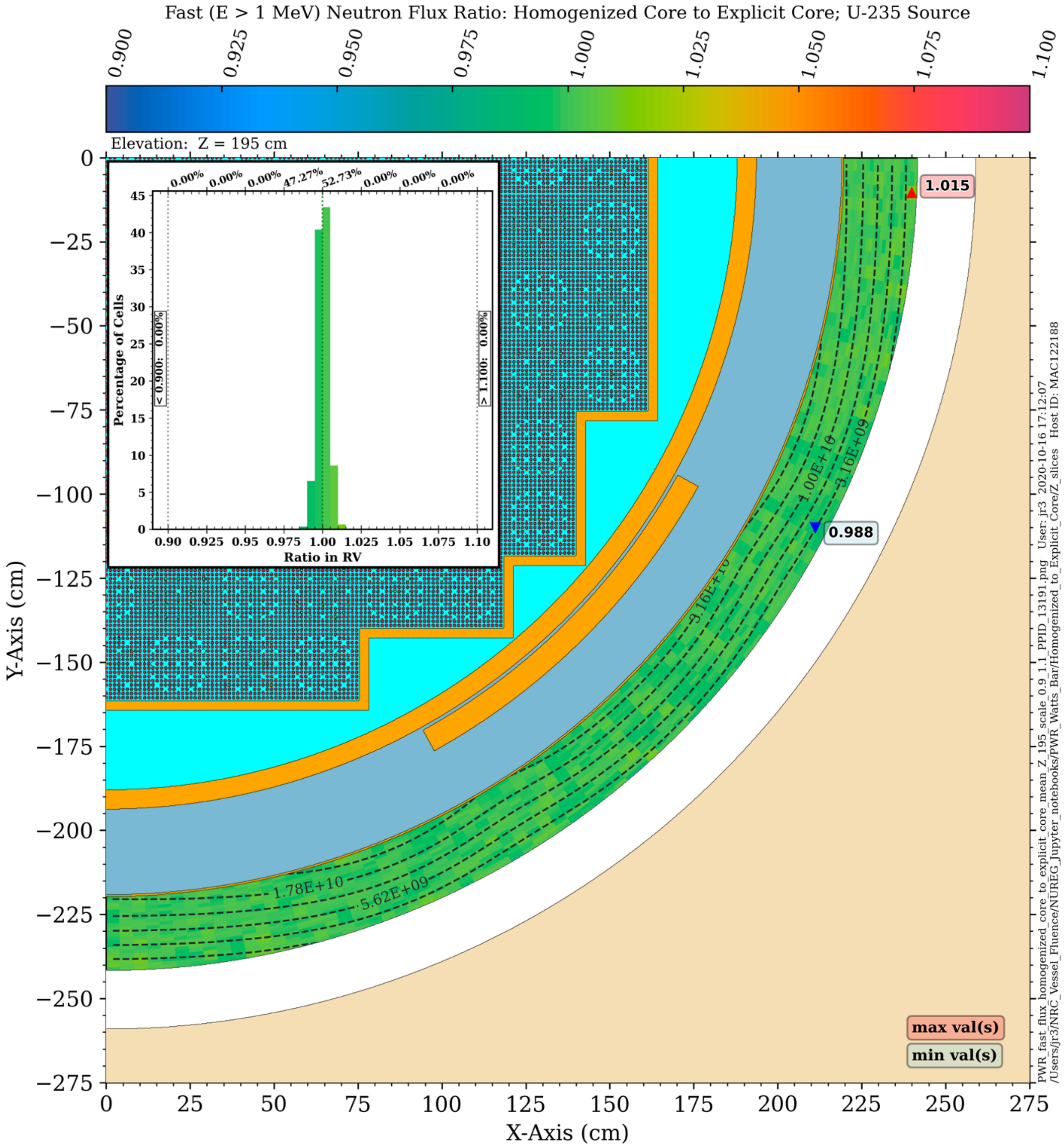

Figure 5-66 Fast neutron flux ratio for the homogenized PWR core model relative to the explicit core model. Plan view at Z = 195 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model

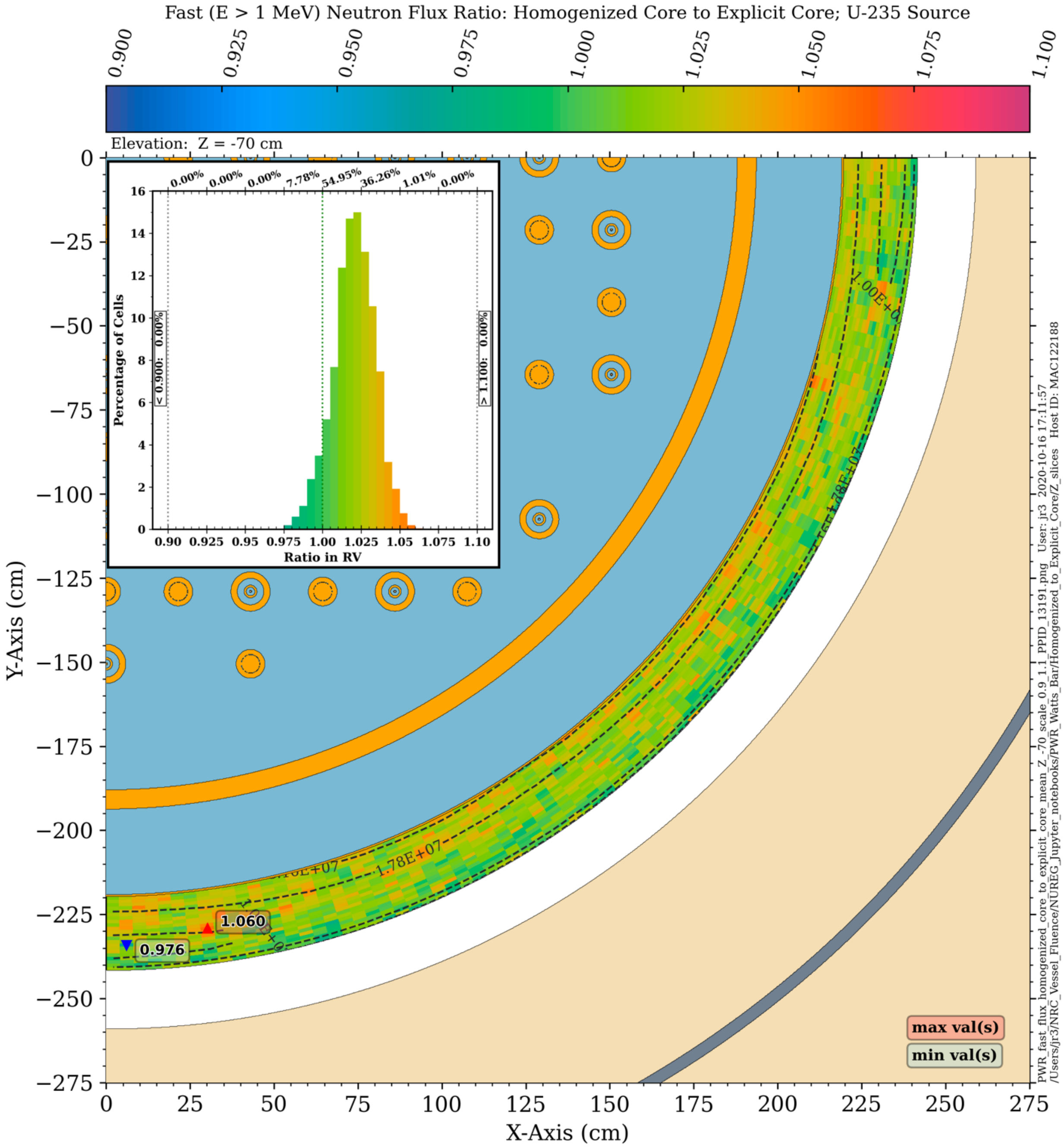

Figure 5-67 **Fast neutron flux ratio for the homogenized PWR core model relative to the explicit core model. Plan view at Z = -70 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model**

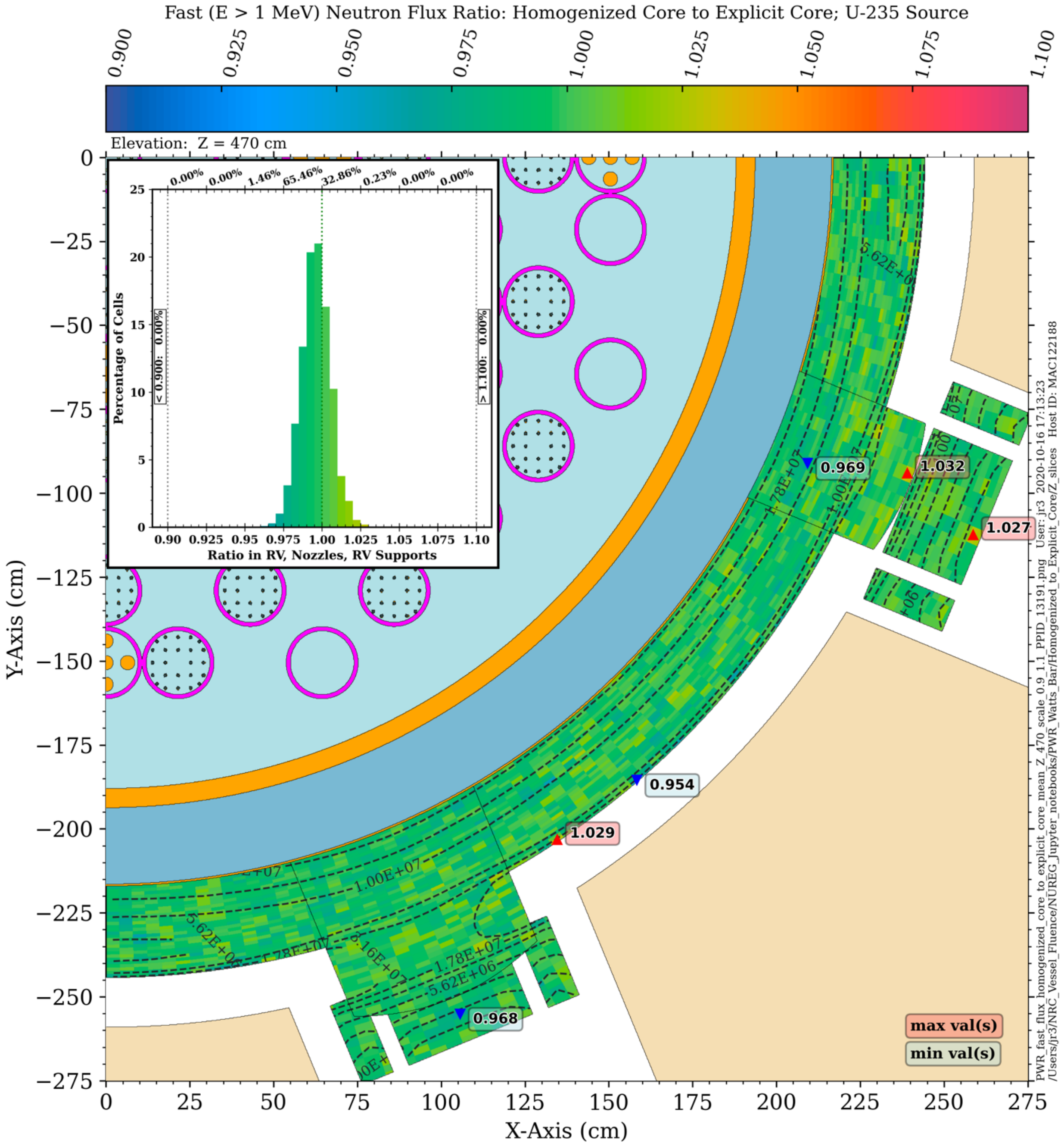

Figure 5-68 Fast neutron flux ratio for the homogenized PWR core model relative to the explicit core model. Plan view at Z = 470 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model

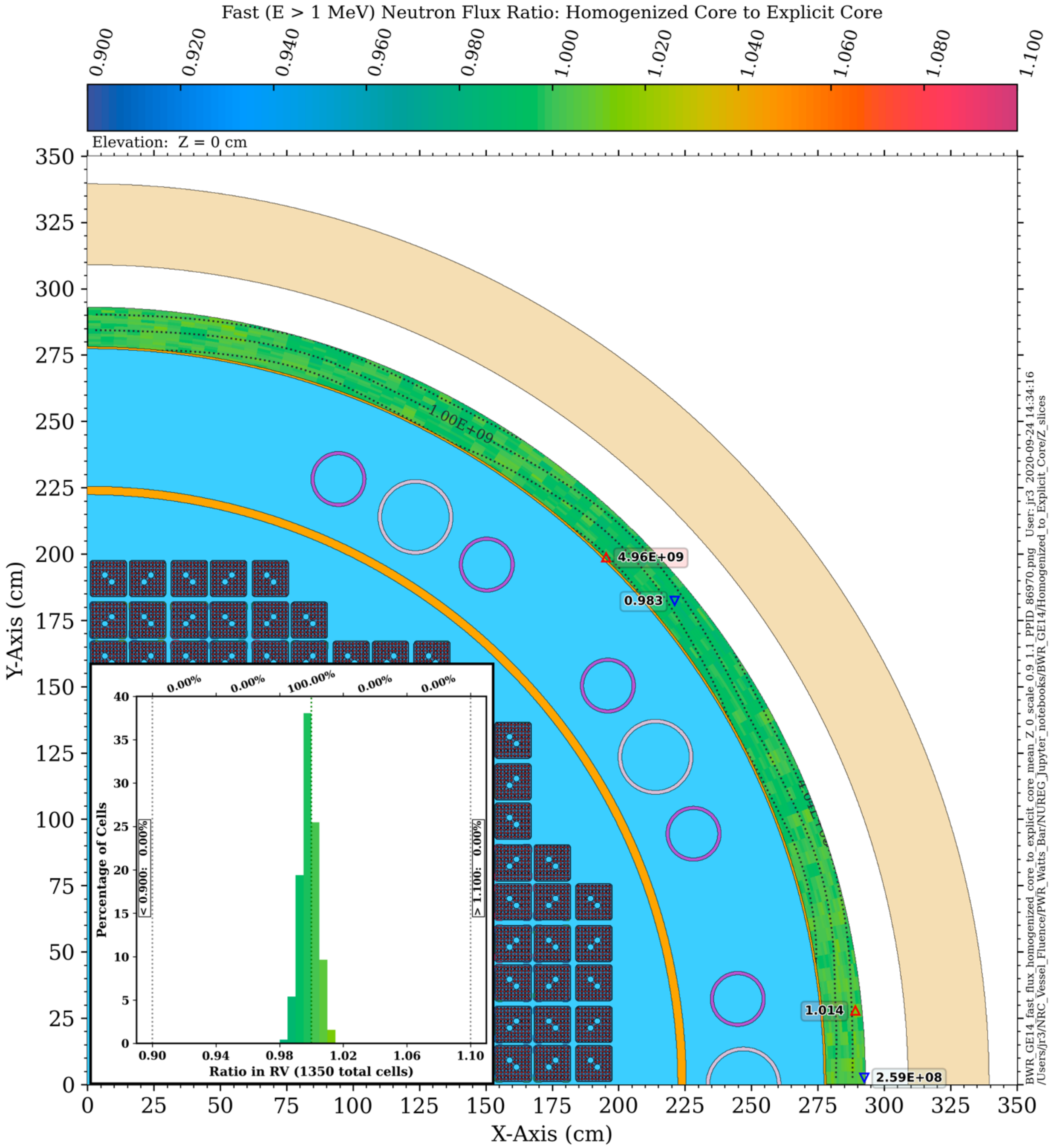

Figure 5-69 Fast neutron flux ratio for the homogenized BWR core model relative to the explicit core model. Plan view at Z = 195 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model

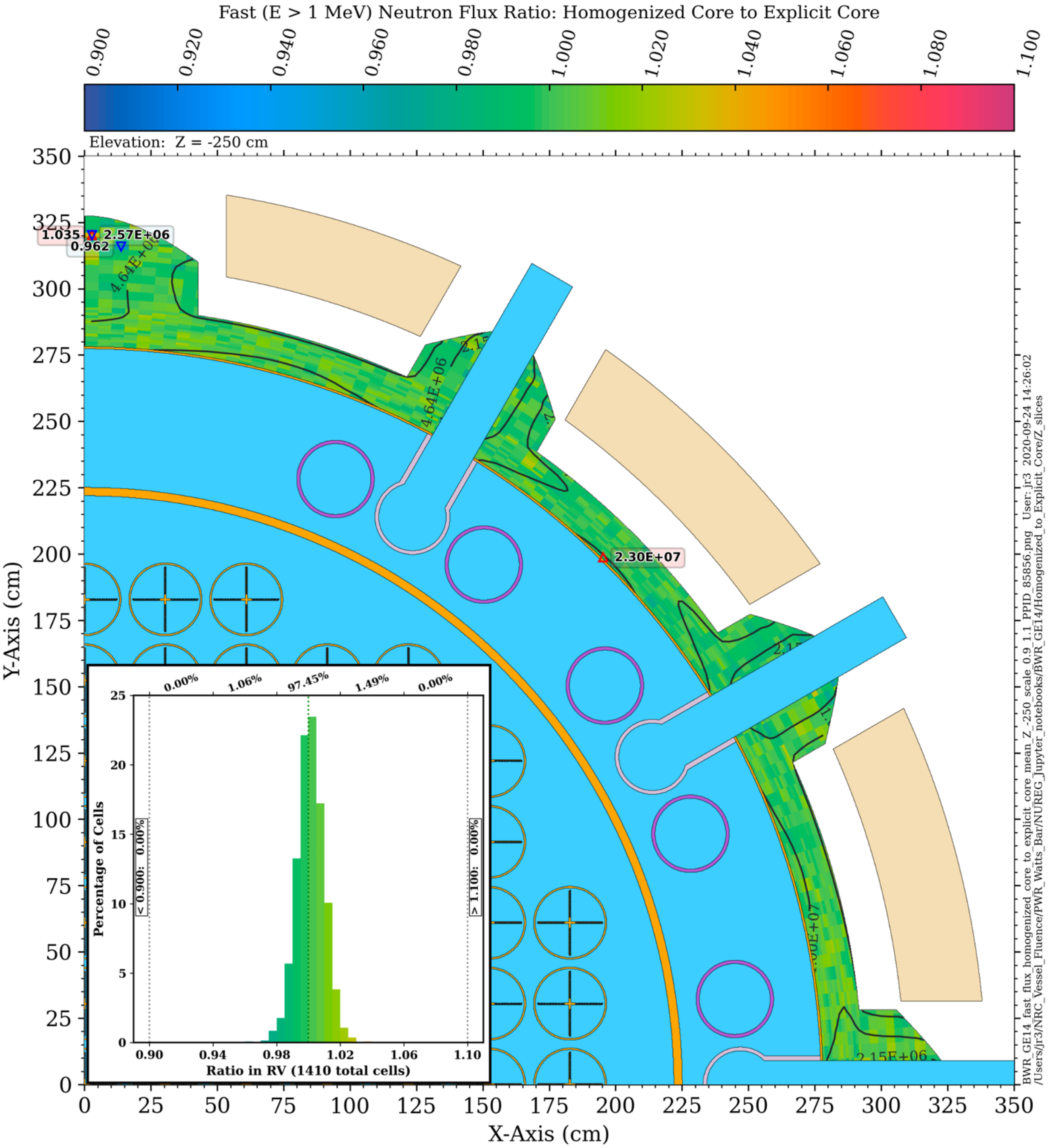

Figure 5-70 **Fast neutron flux ratio for the homogenized BWR core model relative to the explicit core model. Plan view at Z = -250 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model**

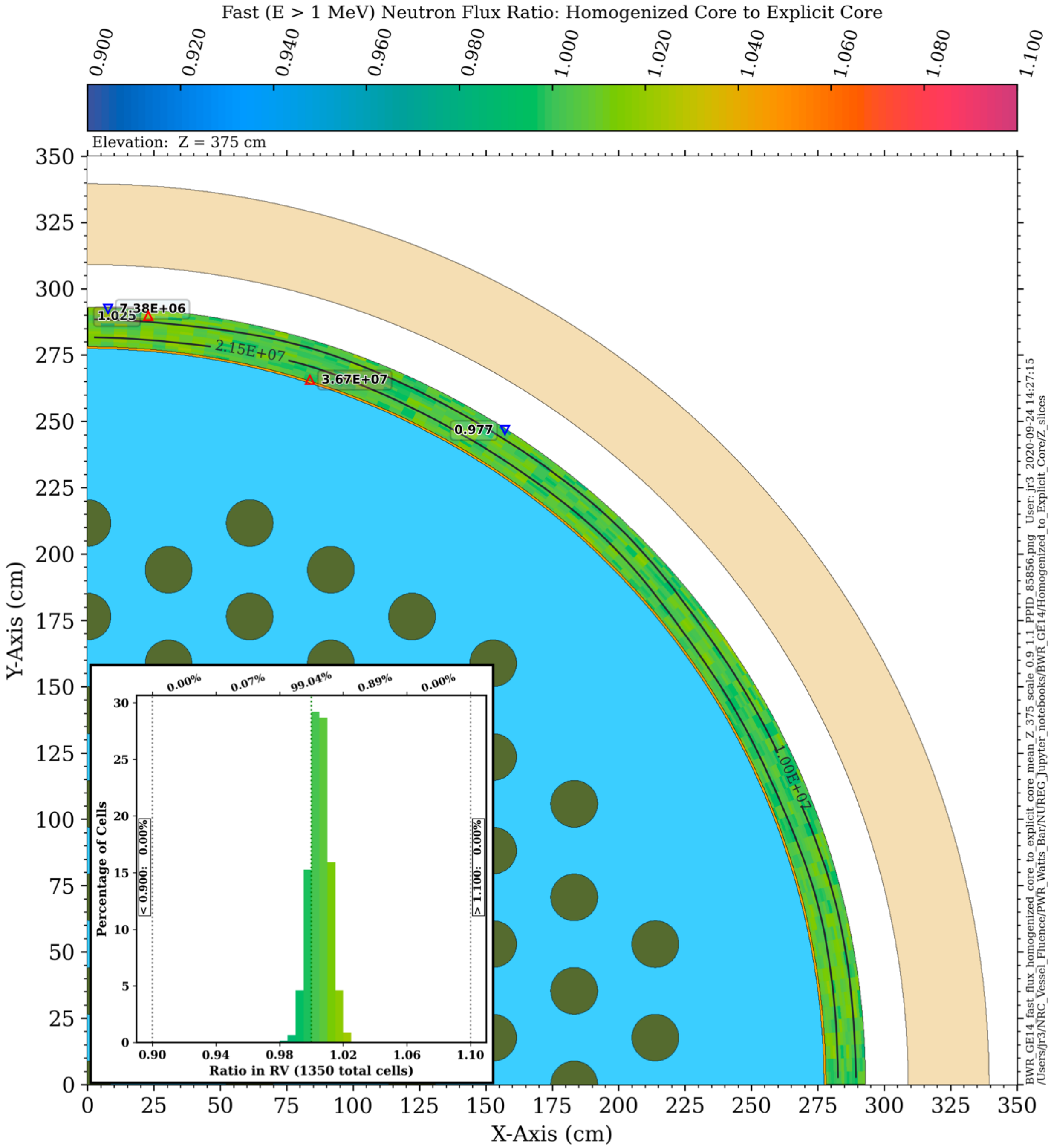

Figure 5-71 Fast neutron flux ratio for the homogenized BWR core model relative to the explicit core model. Plan view at Z = -250 cm. The source is spatially uniform with a ^{235}U fission spectrum. The contour lines represent the fast flux from the explicit core model

6 DISCRETE ORDINATES QUADRATURE SENSITIVITY IN THE EXTENDED BELTLINE REGION

As noted in Section 2.1.3, the accuracy that can be obtained with discrete ordinates transport calculations is dependent on the angular quadrature that is used, among other factors. In examining the effect of quadrature selection on RPV fluence calculations, three questions should be considered:

1. Are fluence calculations in the extended beltline region more sensitive to angular quadrature relative to calculations within the beltline region?
2. Ray effects tend to be more pronounced at high energies and to diminish at lower energies. Are discrete ordinates calculations of dosimetry reaction rates for reactions with a high threshold energy, such as the ^{27}Al (n,α) reaction, more sensitive to quadrature than fast fluence calculations? This question is important with regard to calculations used to establish calculated-to-measured (C/M) ratios using measured dosimetry reaction data.
3. If there are significant solution differences with different quadrature sets, can one set be demonstrated to be more accurate than another?

The first and second questions are addressed in Sections 6.3 and 6.4 through comparisons of Denovo solutions in which the spatial mesh and MG libraries are kept constant and the quadrature set is changed. This type of comparison is often performed when demonstrating whether a discrete ordinates solution has converged with respect to quadrature (i.e., the point at which the solution changes by less than a specified amount as the quadrature is refined). Sections 6.3 and 6.4 also demonstrate that achieving convergence with respect to quadrature in the extended beltline region is very challenging.

The third question is considered in Section 6.5 by comparing Denovo calculations with Shift calculations that employ the same MG libraries as those used in Denovo. By using the same cross-section libraries and demonstrating that the Denovo solution has converged with respect to the mesh, it is possible to effectively isolate the effects of quadrature in the Denovo solutions and to demonstrate which quadratures are superior for a given application.

The observations and conclusions are summarized in Section 6.6.

6.1 Quadrature sets evaluated in this study

The quadrature sets evaluated in this study included level symmetric sets and QR sets. A brief overview of these two quadrature types is provided in Section 2.1.3. The following quadrature sets were used in this study:

- Level symmetric: S8, S16
- Quadruple range: QR4T, QR6T, QR8T, QR8x8, QR10T, QR10x10, QR12T, QR12x12, and QR16T

The notation QR*n*T refers to a QR set with *n* polar levels and a triangular arrangement of azimuthal angles (i.e., one angle on the polar level nearest the Z-axis, two angles on the next level, . . . , and *n* angles on the lowest level). The notation QR*n*×*m* refers to a QR set with *n* polar levels and *m* azimuthal angles on each level.

Figure 2-3 and Figure 2-4 show the quadrature ordinates and weights for the level symmetric S8 and S16 quadratures, which are widely used in RPV fluence calculations, and the QR8T set. Note that the QR8T quadrature has ordinates nearer each of the coordinate system axes, so it is likely to be more appropriate than S16 for problems in which particle streaming near the coordinate axes is a significant transport path.

Although many quadrature comparisons were performed, only selected results are presented in this section. Comments about additional quadrature sets (i.e., sets that are not included in the comparisons of Sections 6.3 and 6.4) are provided in Section 6.6.

6.2 Overview of quadrature sensitivity plots

The quadrature sensitivity plots in Sections 6.3 through 6.5 include 2D and 3D ratio plots. The 3D plots illustrate the areas of the RPV (and the shroud for the BWR model) that are most susceptible to quadrature sensitivity. The 2D plots focus on specific elevations and azimuthal angles and provide a more quantitative assessment of the quadrature sensitivity in key locations.

In each of the 2D ratio plots, the maximum and minimum values are indicated for the RPV and for the nozzles and vessel supports where appropriate. Maximum and minimum values are also indicated in the cavity gap to show the quadrature sensitivity in locations where ex-vessel dosimetry may be used. The inset histogram plots show the magnitude of the solution differences in those regions, and they also indicate whether there is bias between the two solutions.

6.3 Denovo quadrature sensitivity: PWR model

The sensitivity of discrete ordinates solutions to the angular quadrature selection was first examined using the PWR reference model with a homogenized core representation (Section 5.9.1). The discrete ordinates calculations were run using Denovo with the following parameters:

- BUGLE-B7 [82] MG cross-section library with P_3 scattering expansions
- Uniform 1 cm mesh in X, Y, and Z with X- and Y-extents of 320 cm and a Z-extent of 750 cm (76.8 million cells)
- Linear discontinuous (LD) differencing scheme
- Source iteration (SI) solver with a convergence criterion of 1E-6

The mesh spacing was selected based on a parametric study which indicated that the Denovo solution converges with respect to mesh with 1 cm intervals in X, Y, and Z. The adequacy of this mesh is also confirmed using the Denovo/Shift comparisons presented in Section 6.5.

6.3.1 Denovo solutions: S8 vs S16

The most commonly used quadratures for RPV fluence calculations are the level symmetric S8 and S16 sets. RG 1.190 prescribes a minimum quadrature order of S8 for RPV fluence calculations and notes that higher-order quadratures may be needed in reactor cavity fluence calculations. This section considers the differences between S8 and S16 Denovo solutions in the extended beltline region of the PWR model, and it also address differences within the traditional beltline region, where S8 quadrature is routinely used.

Ratios of the PWR S8 Denovo solution to the S16 Denovo solution are shown in Figure 6-1 through Figure 6-10. Figure 6-1 illustrates the fast neutron flux ratio on the inner and outer surfaces of the RPV. The axial extent of these plots is from Z = -100 cm to Z = 650 cm. This axial range, which extends beyond the upper Z limit of 500 cm used in the 2D RZ plots in this and other sections, helps illustrate how quadrature sensitivity is particularly significant in the vicinity of the inlet and outlet nozzles. A notable feature of Figure 6-1 is the appearance of horizontal and vertical bands on the inner and (to a lesser extent) outer surfaces of the RPV. These bands are evidence of ray effects, which are an artifact of the discrete ordinates approximation in which the continuous angular variable is represented by a set of discrete directions.

Figure 6-2 shows the S8/S16 fast flux ratio at the core midplane. There is clear evidence of ray effects associated with the outer corners of the core and (to a lesser extent) the neutron pad. It is particularly noteworthy that the minimum and maximum values of the S8/S16 ratio in the RPV occur at locations separated by only ~2° in the azimuthal angle. (Because the model has octant symmetry over the height of the core, the flux solutions, and hence the ratios, are essentially identical at 292.5° and 337.5° and at 294.5° and 335.5°.) Over this narrow interval, the S8/S16 ratio changes by nearly 9%. This rapid variation over a small azimuthal extent suggests that S8 quadrature may not be appropriate for accurate calculations of fast fluence and dosimetry reaction rates, even within the traditional beltline region. A slight uncertainty in the azimuthal location of a surveillance capsule could result in significant changes in C/M values.

At elevations of Z = 390 cm (Figure 6-3) and Z = 400 cm (Figure 6-4), there are still indications of the assembly corner effects noted in Figure 6-2. There is also a significant change in the S8/S16 ratio throughout the RPV at these two elevations, with the general S8/S16 trend being greater than 1.0 at Z = 390 cm and less than 1.0 at Z = 400 cm. This can also be seen on the RPV inner surface in Figure 6-1.

At elevations of Z = -70 cm (Figure 6-5) and Z = 470 cm (Figure 6-6), there are significant differences in the S8/S16 fast flux ratio in the RPV, nozzles, and vessel supports. Of particular note are the locations in the nozzles and nozzle supports, where the S8 solution is lower than the S16 solution by 30% or more.

Figure 6-7 and Figure 6-8 provide insights into the horizontal bands seen on the inner surface of the RPV (Figure 6-1). Figure 6-7 is at an azimuthal angle, with the maximum distance between the former plates and the core barrel, and Figure 6-8 is at an azimuthal angle, with the minimum distance between the former plates and the core barrel. In Figure 6-7, pronounced ray effects originate near the former plates and propagate through the RPV. These ray effects correspond to the horizontal bands in Figure 6-1. In Figure 6-8, the effect of the former plates is very minor. As shown in Figure 6-1, the horizontal bands on the RPV inner surface at elevations within the height of the core are most prominent near 270° and 360° and are minimized near 310° and 320°.

Figure 6-9 and Figure 6-10 show the pronounced variation over a narrow azimuthal range (335.5° to 337.5°) in the S8/S16 fast flux ratio that was noted above in the discussion of Figure 6-2.

Further discussion of the S8 and S16 Denovo solutions is provided in Sections 6.5 and 6.6.

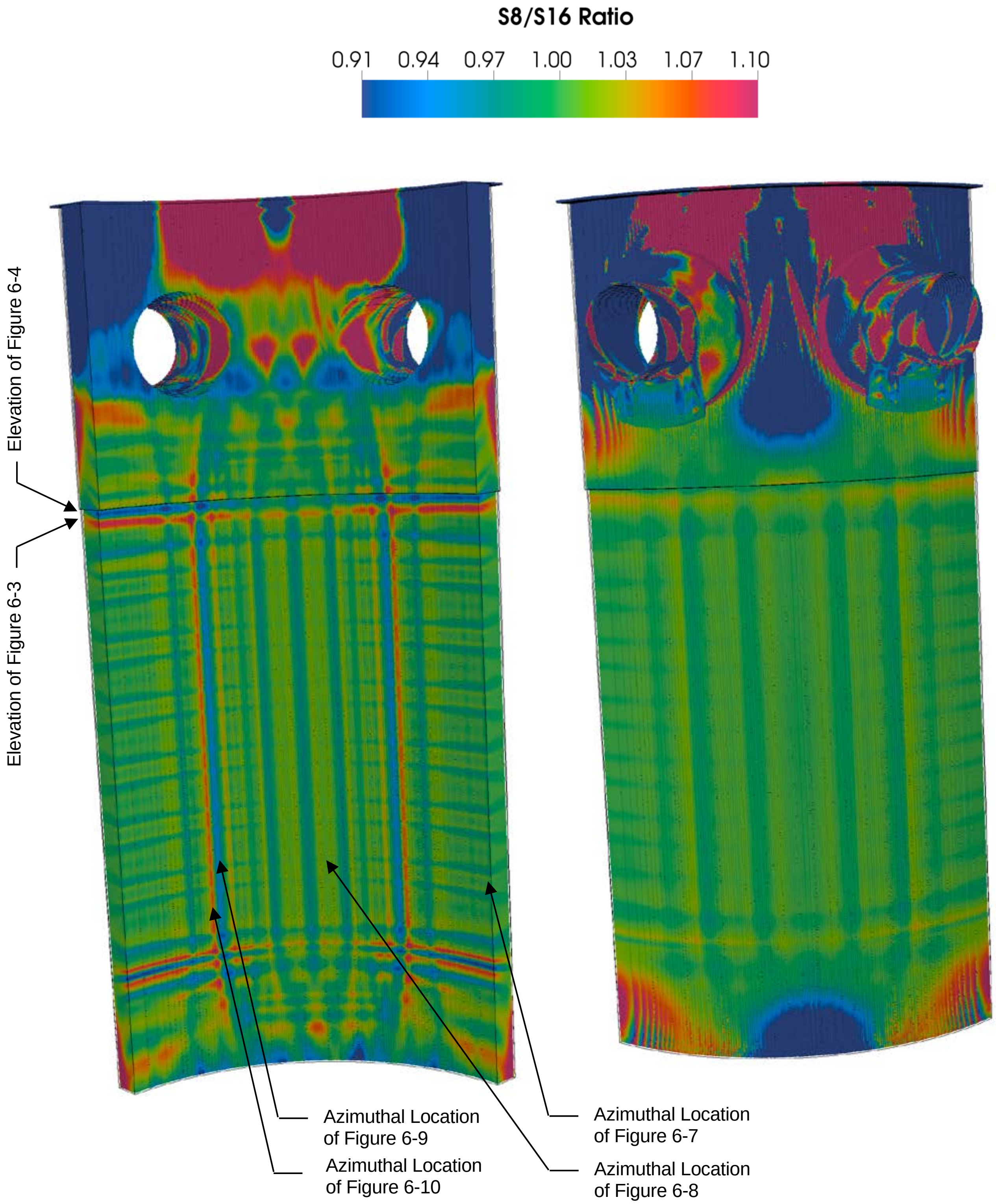

Figure 6-1 **Fast neutron flux ratio on the inner and outer surfaces of the RPV for the PWR reference model. The ratio is for a Denovo S8 solution relative to an S16 solution**

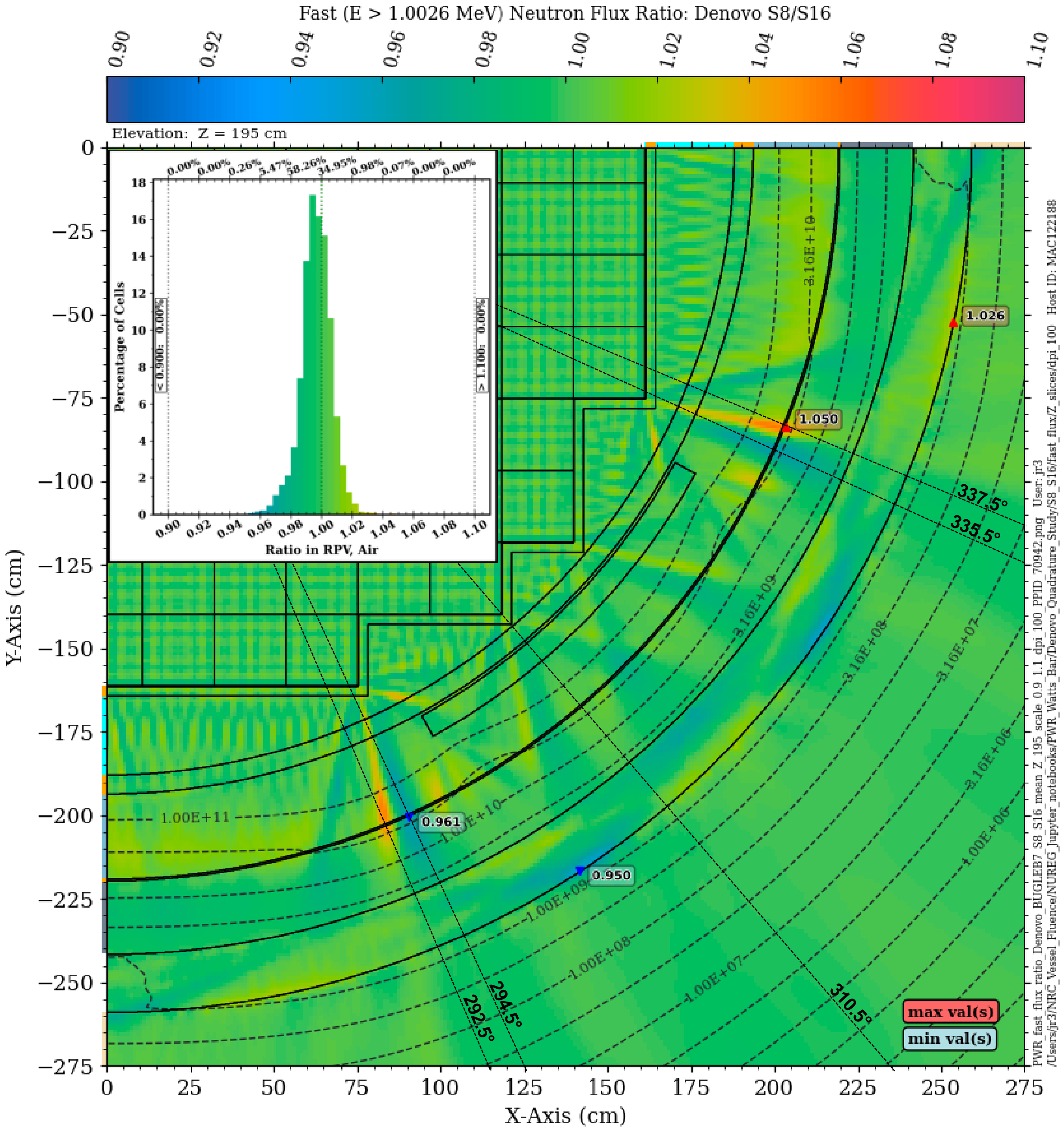

Figure 6-2 Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 195 cm. The contour lines are the fast flux from the S16 solution. The azimuthal lines at 310.5°, 335.5°, and 337.5° are the locations for Figure 6-8, Figure 6-9, and Figure 6-10, respectively

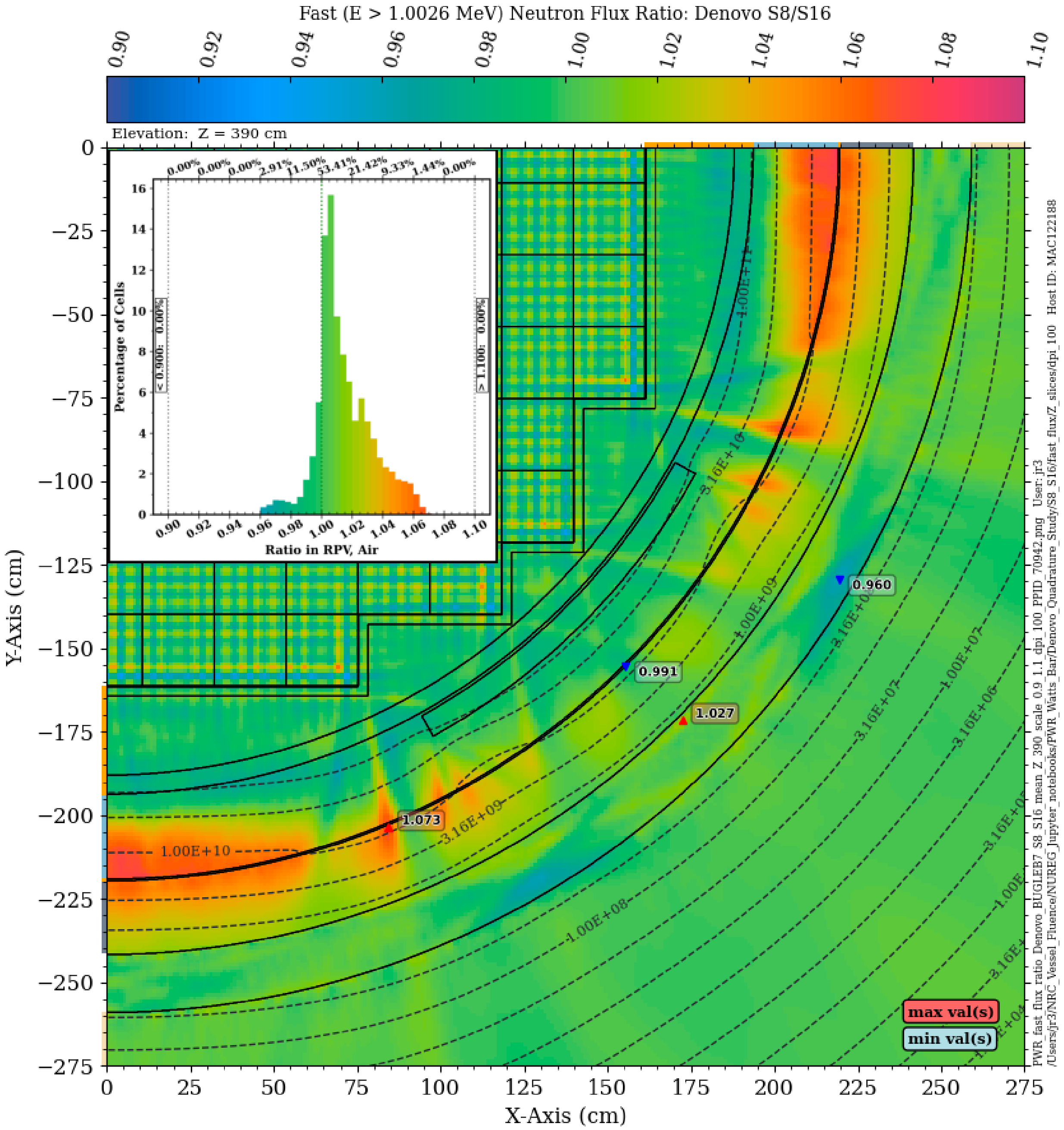

Figure 6-3 Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 390 cm. The contour lines are the fast flux from the S16 solution

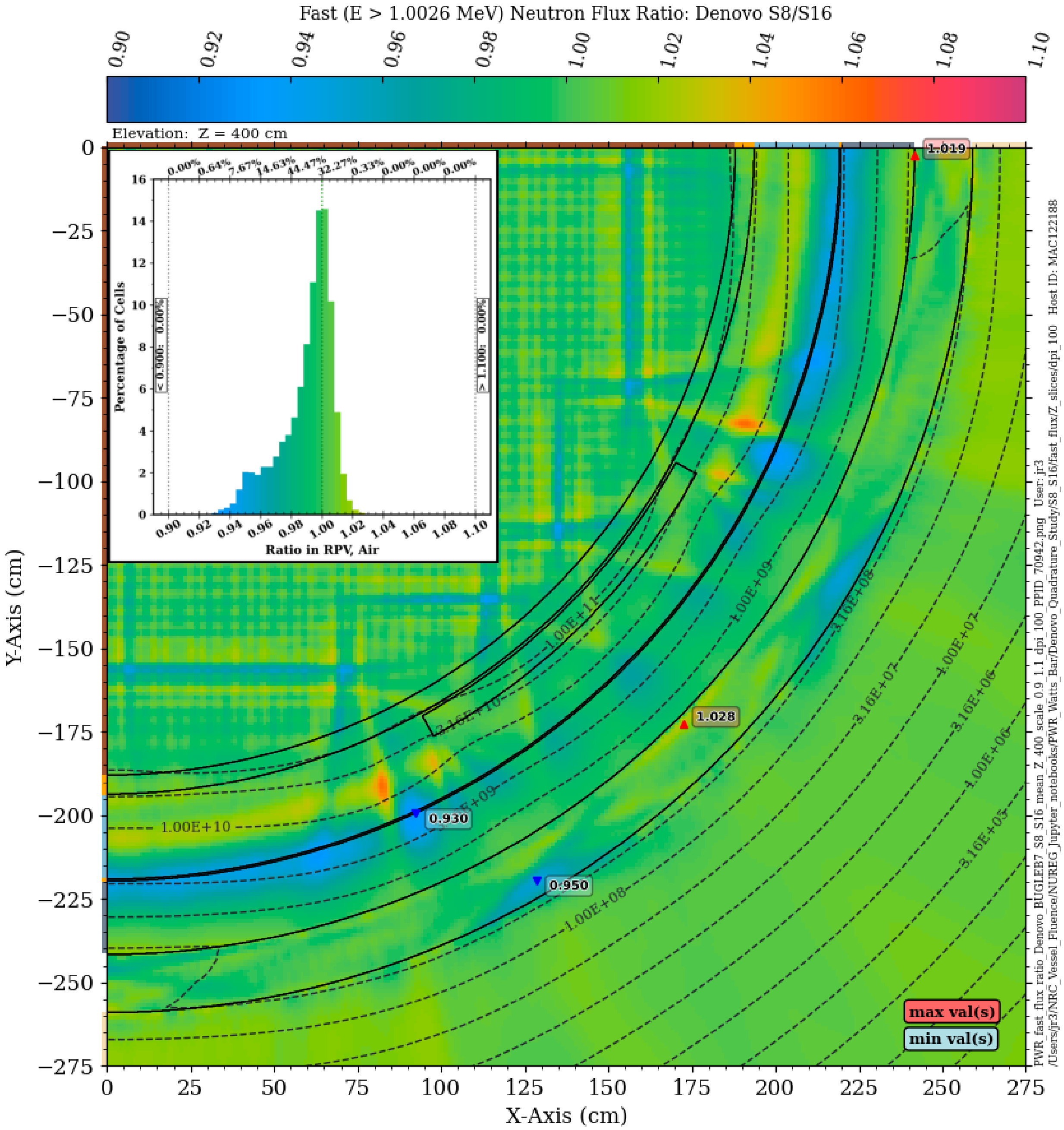

Figure 6-4 **Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 400 cm. The contour lines are the fast flux from the S16 solution**

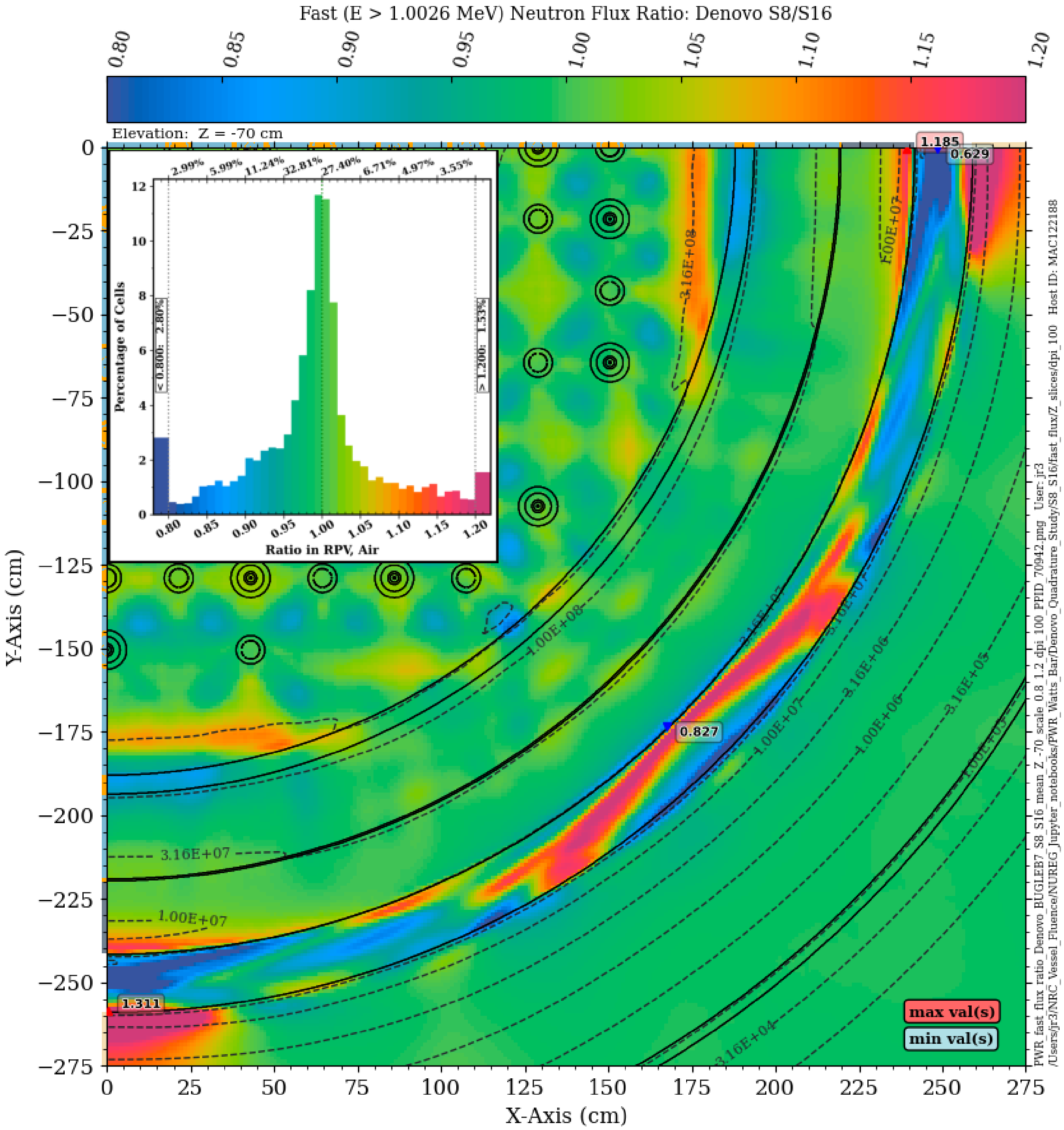

Figure 6-5 Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = -70 cm. The contour lines are the fast flux from the S16 solution. Note the change in scale relative to the previous three figures

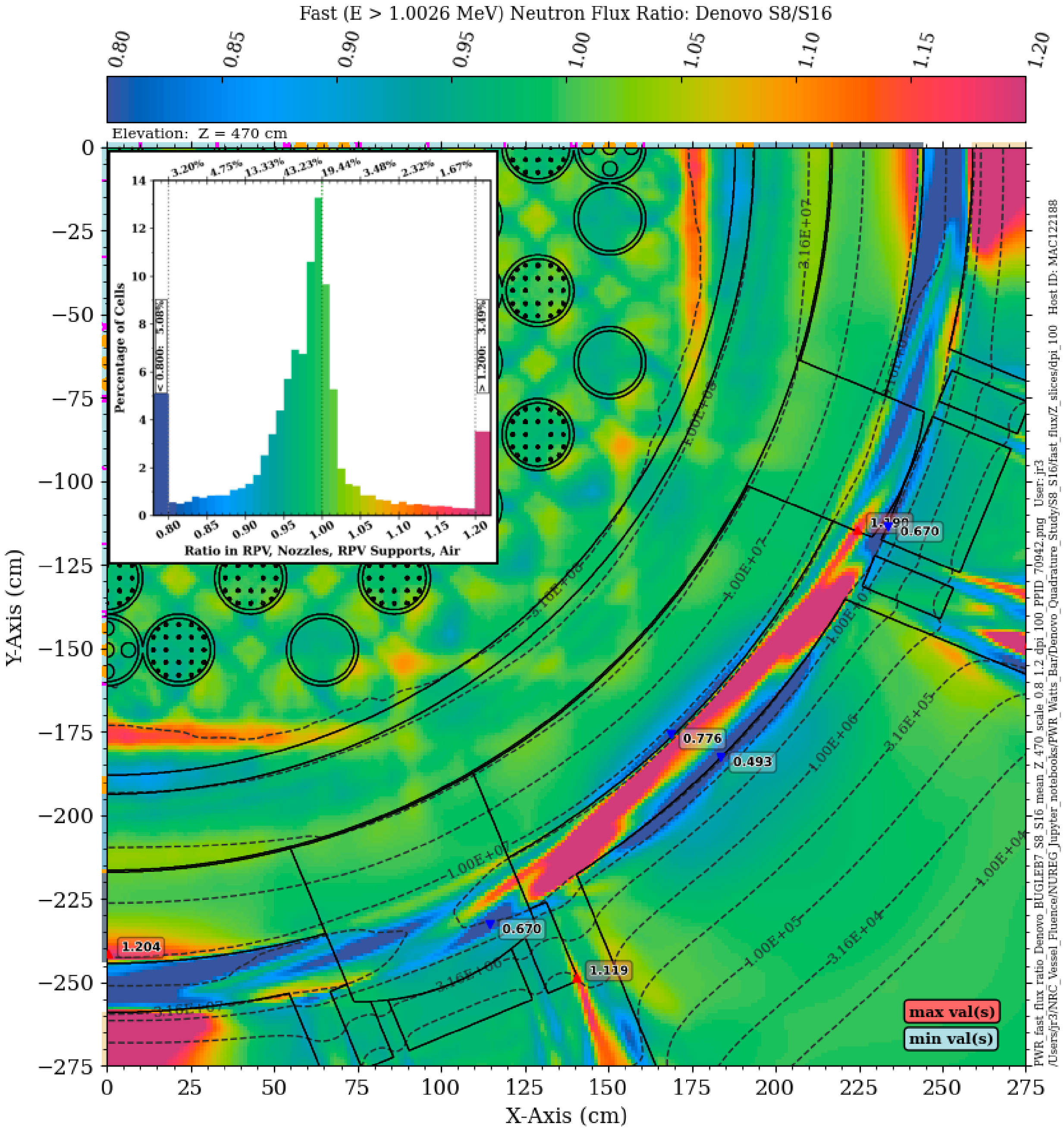

Figure 6-6 Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 470 cm. The contour lines are the fast flux from the S16 solution

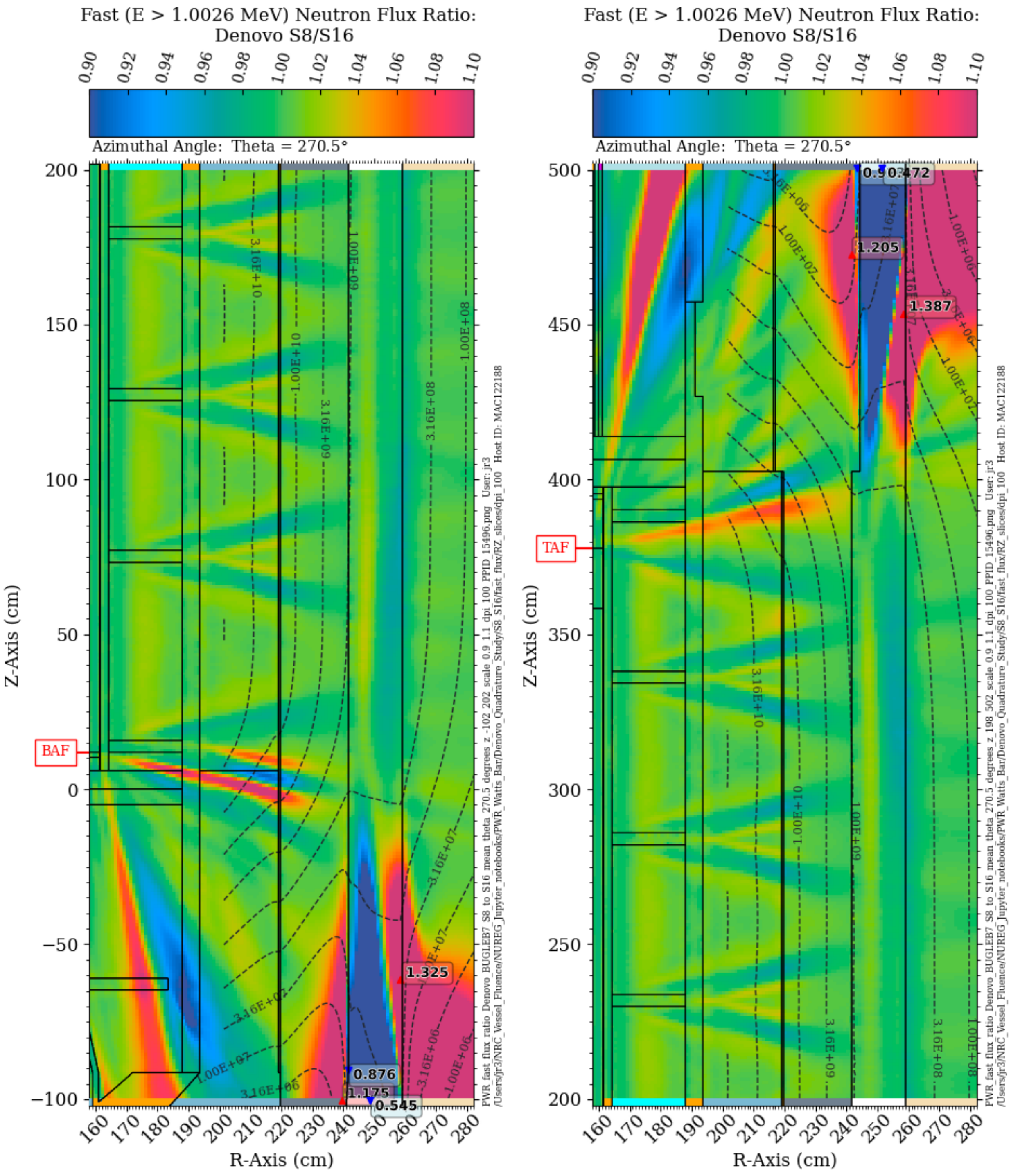

Figure 6-7 **Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 270.5°. The contour lines are the fast flux from the S16 solution**

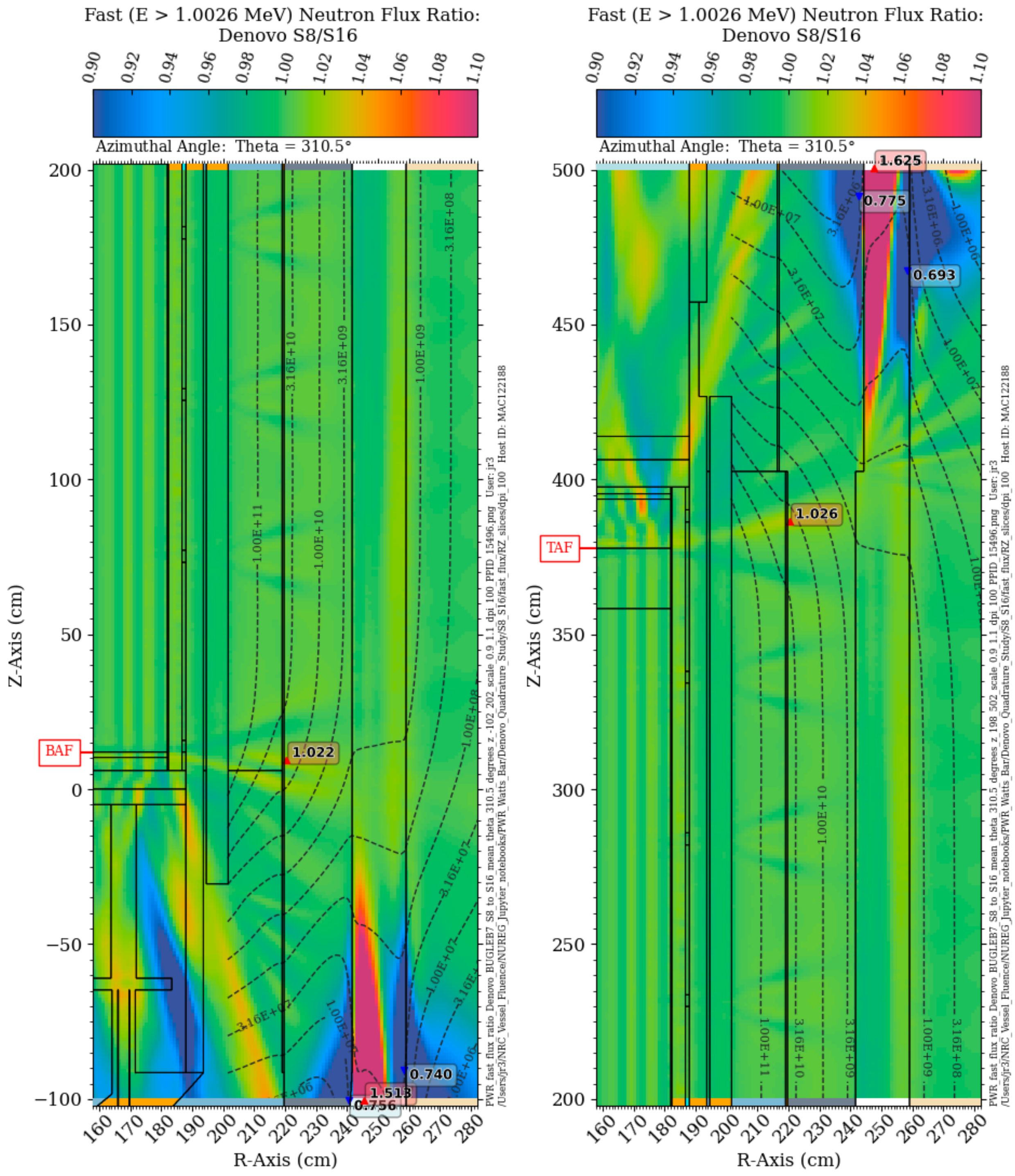

Figure 6-8 **Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 310.5°. The contour lines are the fast flux from the S16 solution**

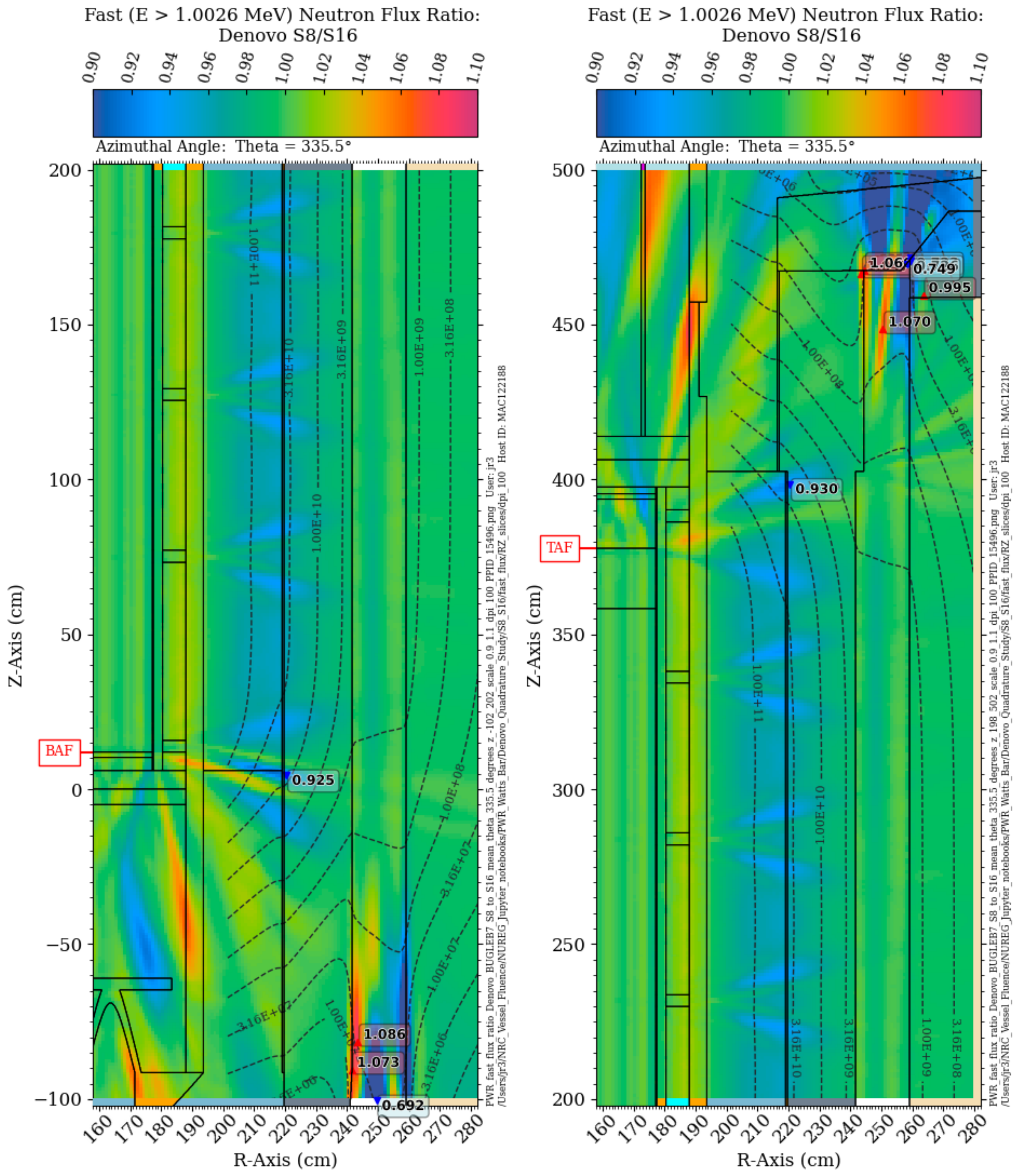

Figure 6-9 **Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 335.5°. The contour lines are the fast flux from the S16 solution**

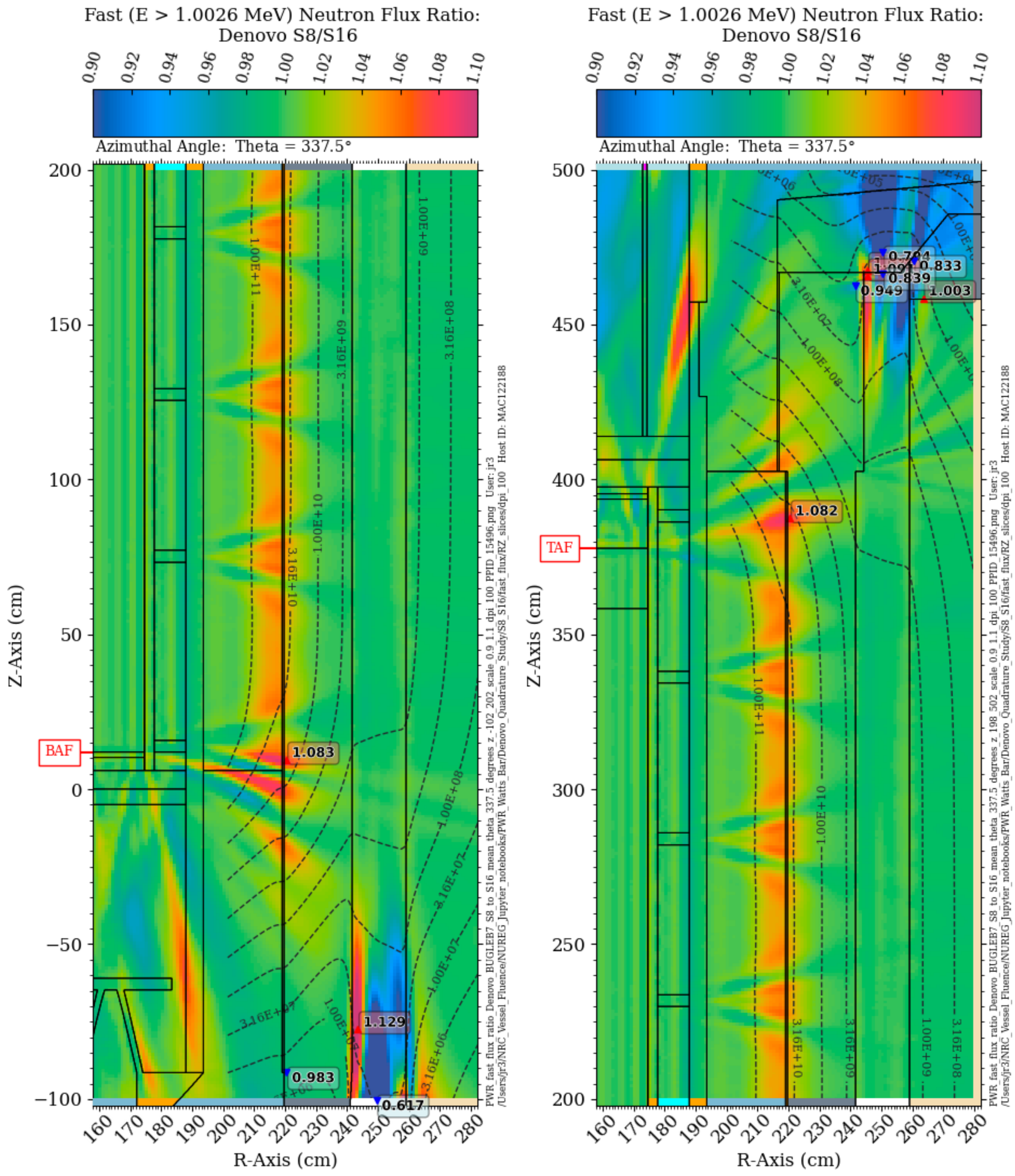

Figure 6-10 Fast neutron flux ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 337.5°. The contour lines are the fast flux from the S16 solution

6.3.2 Denovo solutions: S16 vs QR8T

The S16 and QR8T quadratures each have 36 angles per octant on 8 polar levels with a triangular arrangement of azimuthal angles by polar level (Figure 2-4). As noted in Section 2.1.3, the QR8T set may be expected to provide more accurate discrete ordinates solutions for problems with material interfaces along any of the coordinate axes or with significant streaming paths along any of the coordinate axes.

Figure 6-11 through Figure 6-14 illustrate the S16/QR8T fast flux ratio on the inner and outer surfaces of the RPV and at elevations of Z = 195 cm, Z = -70 cm, and Z = 470 cm. The differences in the S16 and QR8T solutions at the core midplane are less than 2%, although there are still indications of ray effects. At the extended beltline elevations of Z = -70 cm and Z = 470 cm, differences between the S8 and QR16T solutions exceed 20% in some locations, particularly in the cavity gap region.

The question of which of these two quadrature sets provides a more accurate solution is addressed in Section 6.5.

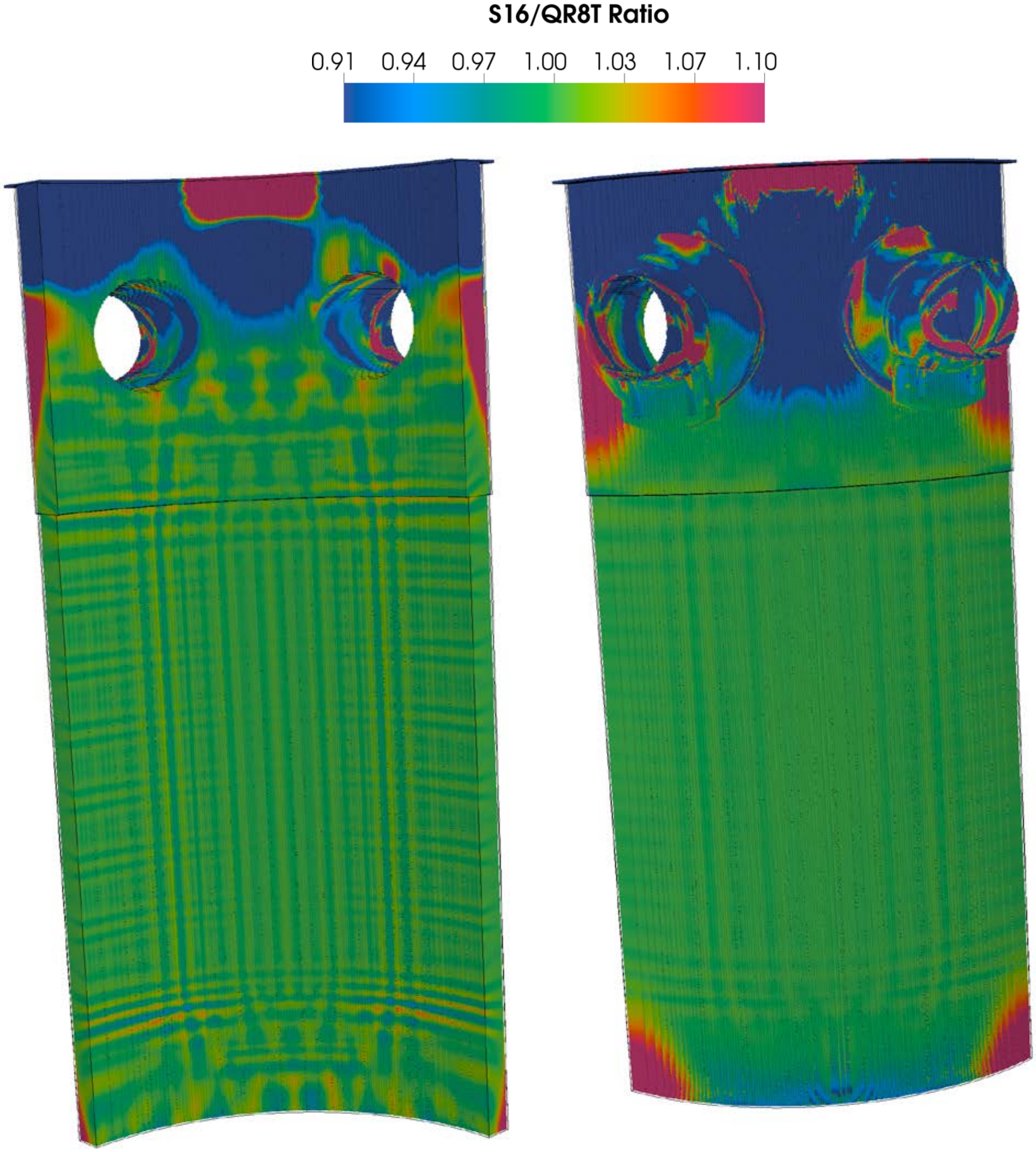

Figure 6-11 **Fast neutron flux ratio on the inner and outer surfaces of the RPV for the PWR reference model. The ratio is for a Denovo S16 solution relative to a QR8T solution**

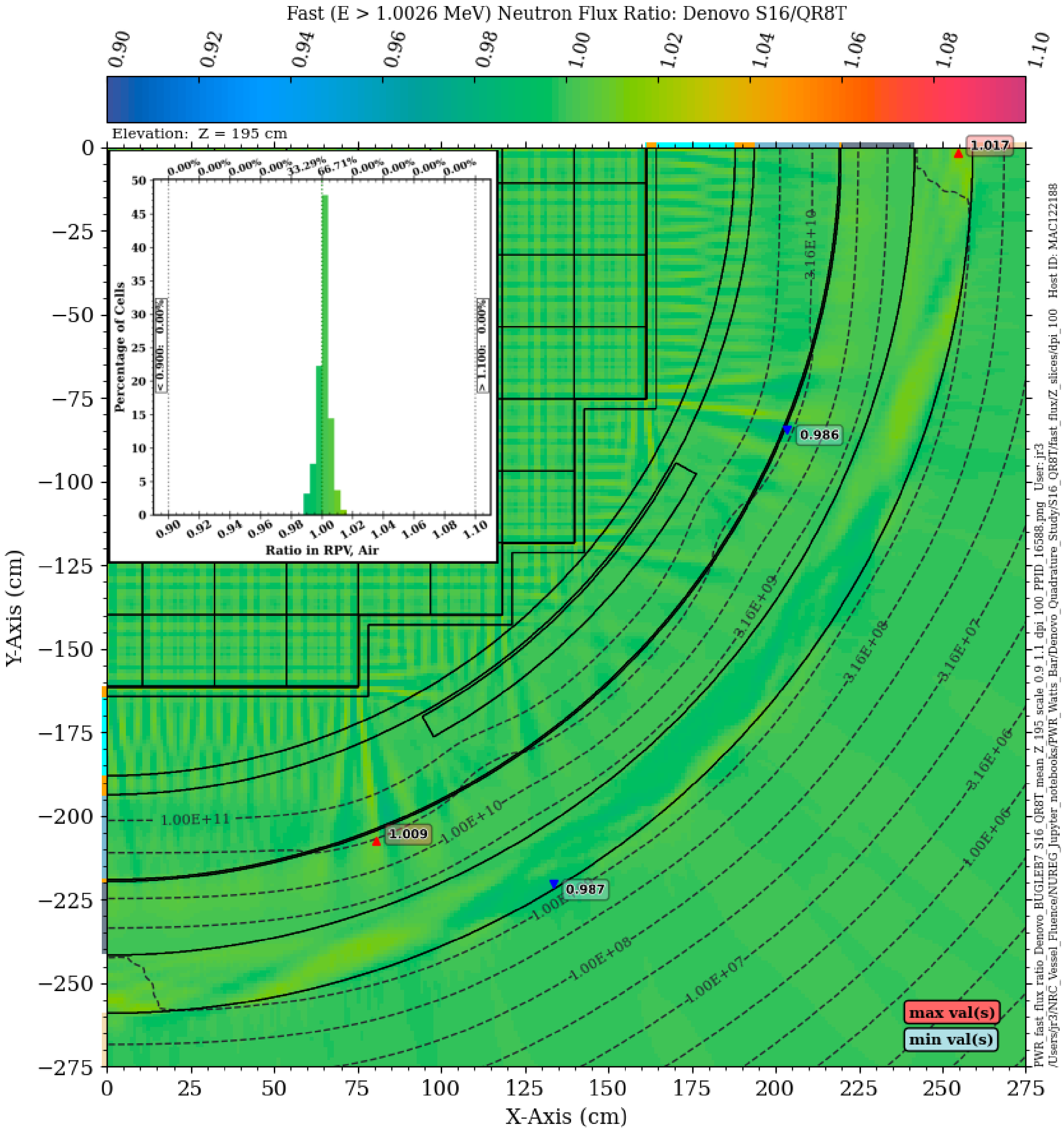

Figure 6-12 Fast neutron flux ratio in the PWR reference model: S16/QR8T quadrature. Plan view at Z = 195 cm. The contour lines are the fast flux from the QR8T solution

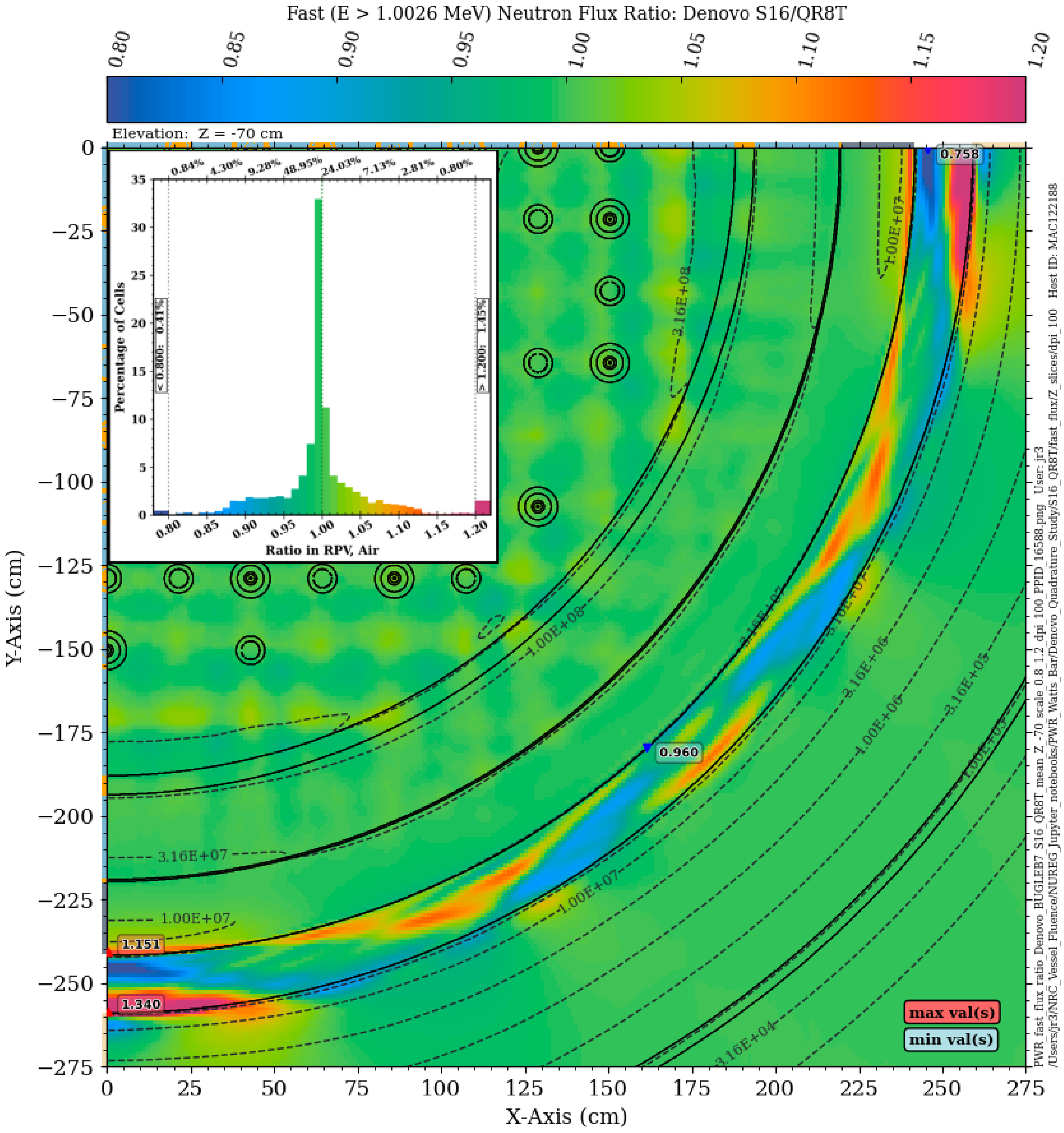

Figure 6-13 Fast neutron flux ratio in the PWR reference model: S16/QR8T quadrature. Plan view at Z = -70 cm. The contour lines are the fast flux from the QR8T solution. Note the change in scale relative to Figure 6-12

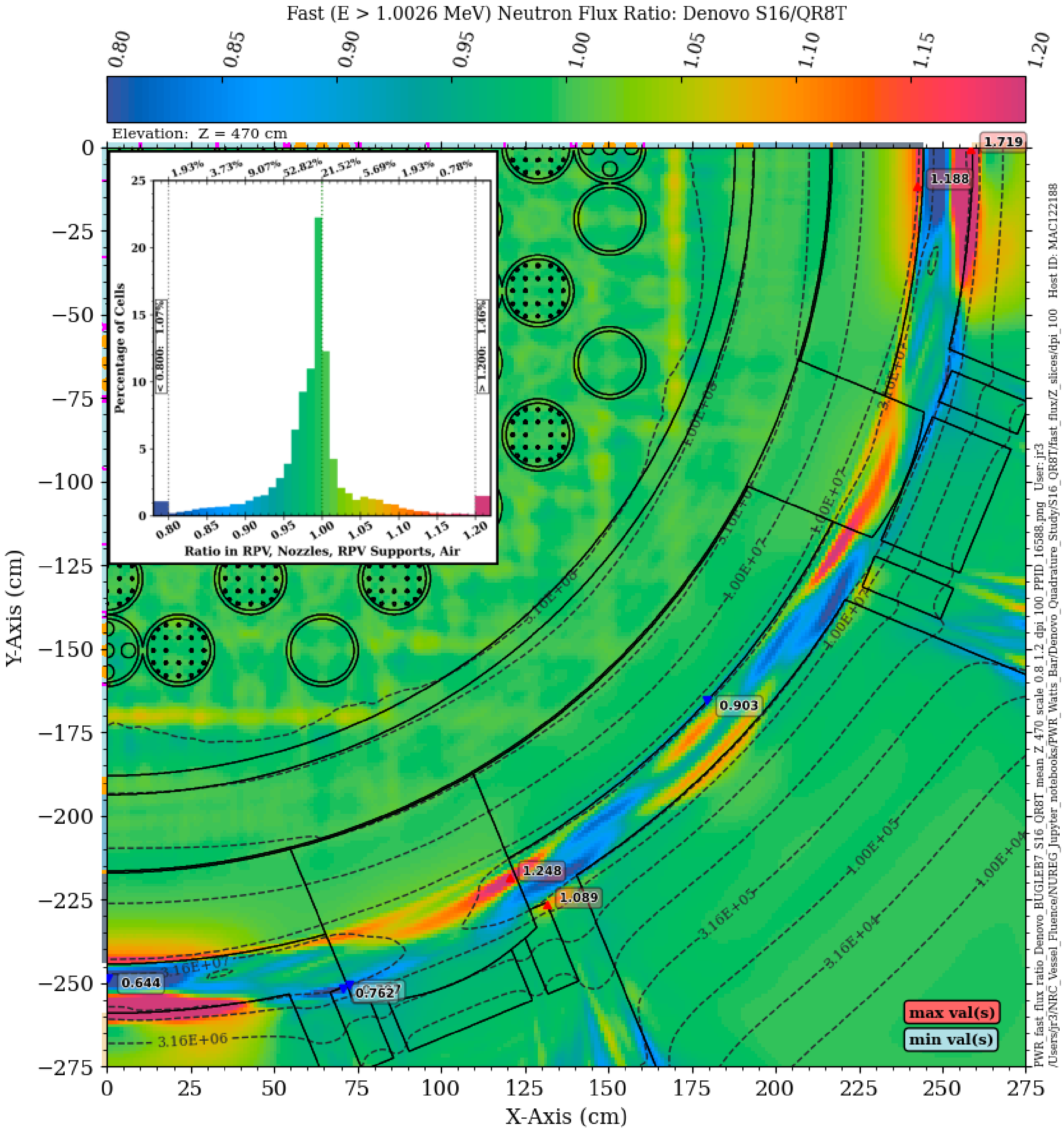

Figure 6-14 **Fast neutron flux ratio in the PWR reference model: S16/QR8T quadrature. Plan view at Z = 470 cm. The contour lines are the fast flux from the QR8T solution**

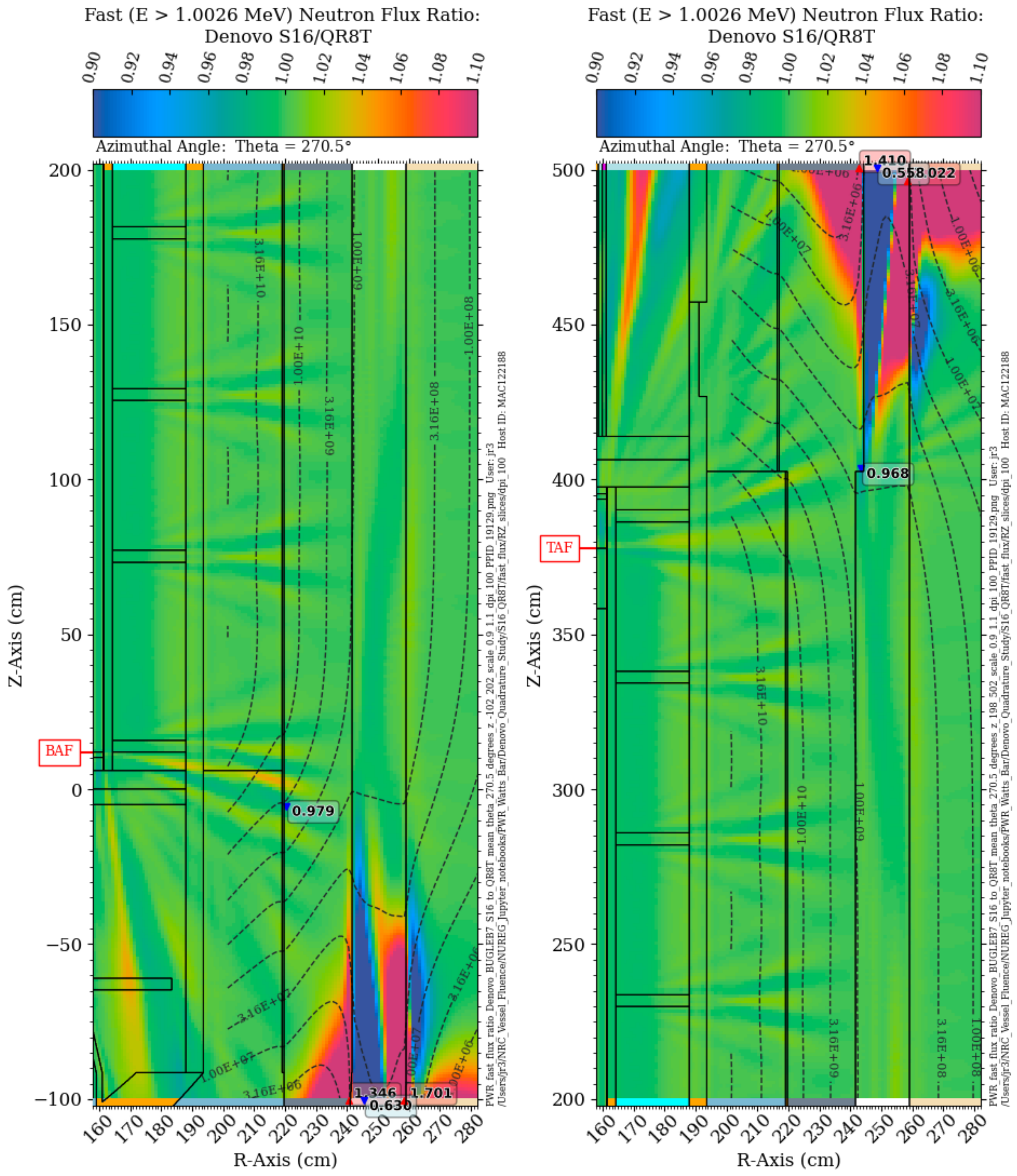

Figure 6-15 Fast neutron flux ratio in the PWR reference model: S16/QR8T quadrature. Elevation view at an azimuthal angle of 270.5°. The contour lines are the fast flux from the QR8T solution

6.3.3 Denovo solutions: QR8T vs QR16T

It was noted in Section 2.1.3 that level symmetric SN quadrature sets have negative weights for orders greater than S20. The QR sets do not suffer from negative weights and allow higher-order quadratures to be used. The final Denovo solution comparison for the PWR model uses a QR8T solution and a QR16T solution. Note that a QR16T quadrature set has 136 angles per octant, which would be equivalent to an S32 set.

Figure 6-16 through Figure 6-19 illustrate the QR8T/QR16T fast flux ratio on the inner and outer surfaces of the RPV and at elevations of Z = 195 cm, Z = -70 cm, and Z = 470 cm. At the core midplane, the two solutions agree within 1% within the RPV and within less than 2% in the cavity gap. At the extended beltline elevations of Z = -70 cm (Figure 6-18) and Z = 470 cm (Figure 6-19), the differences are reduced relative to the S16/QR8T ratios in Figure 6-13 and Figure 6-14. While the QR8T and QR16T solutions at Z = -70 cm agree within 5% in the RPV and typically within 10% in the cavity gap, the differences are again more pronounced at Z = 470 cm, where differences can exceed 20% in the cavity gap.

Comparisons of the QR8T and QR16T with an MG Shift calculation using the BUGLE-B7 library are provided in Section 6.5.

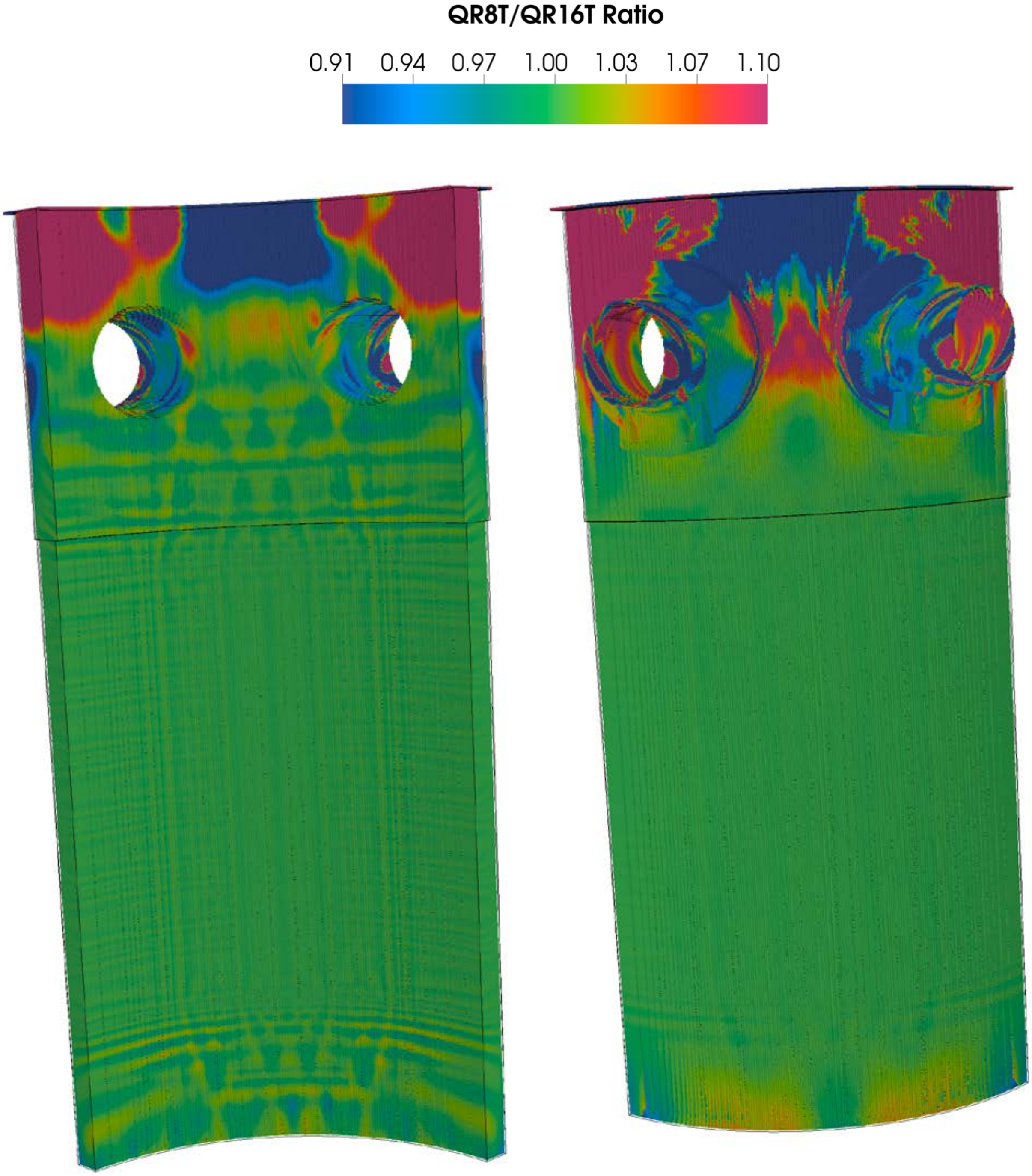

Figure 6-16 Fast neutron flux ratio on the inner and outer surfaces of the RPV for the PWR reference model. The ratio is for a Denovo QR8T solution relative to a QR16T solution

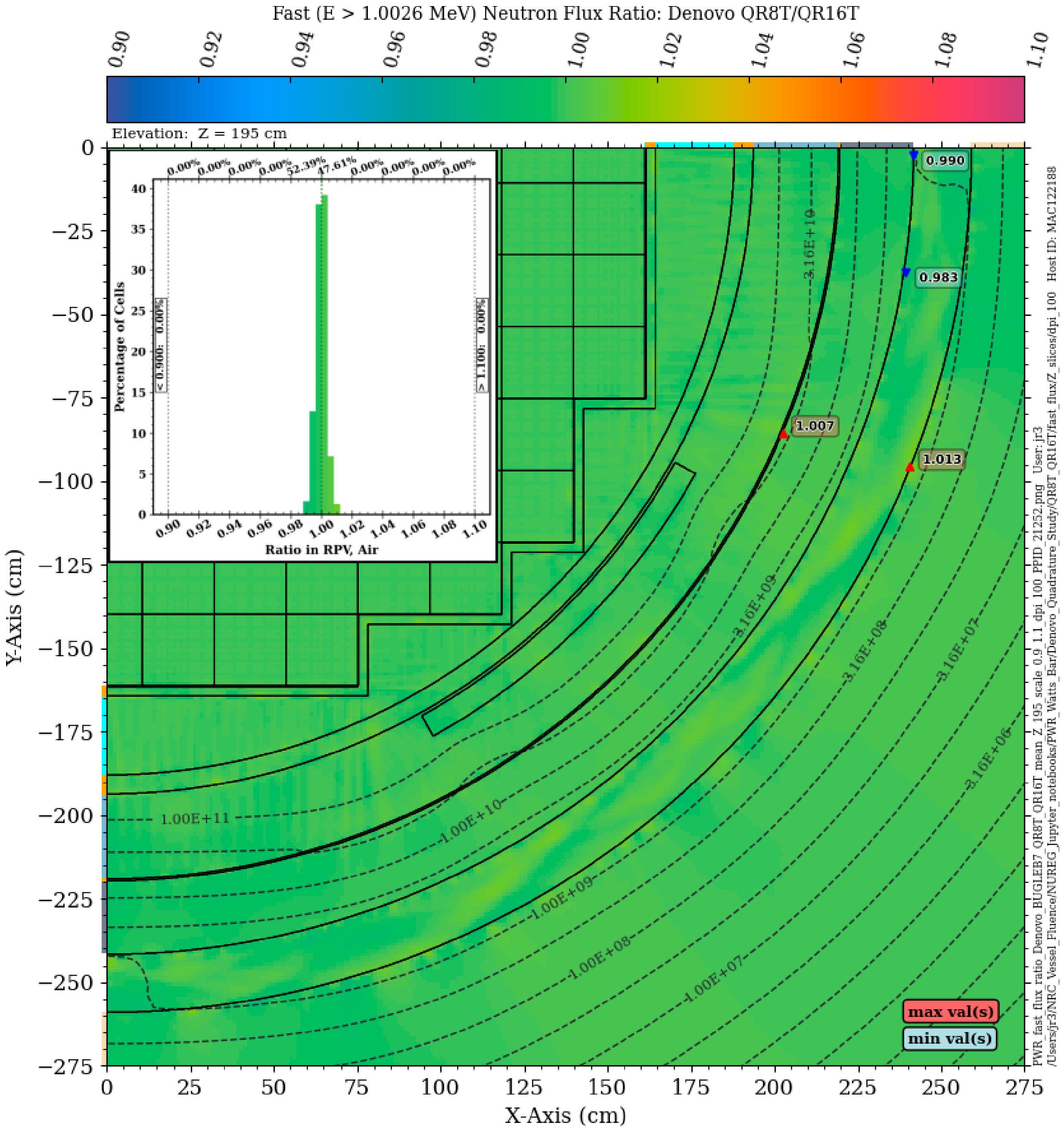

Figure 6-17 Fast neutron flux ratio in the PWR reference model: QR8T/QR16T quadrature. Plan view at Z = 195 cm. The contour lines are the fast flux from the QR16T solution

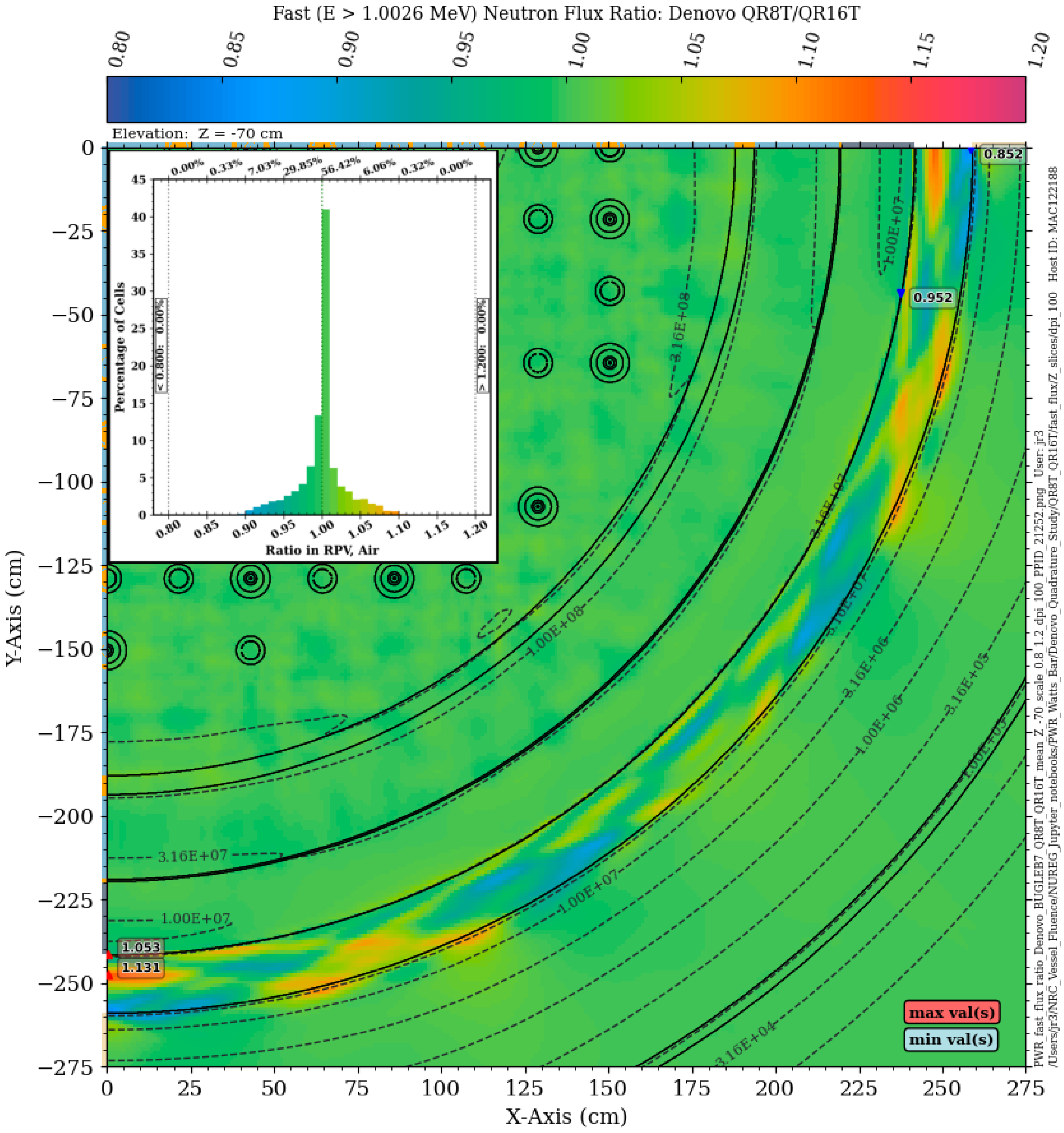

Figure 6-18 Fast neutron flux ratio in the PWR reference model: QR8T/QR16T quadrature. Plan view at Z = -70 cm. The contour lines are the fast flux from the QR16T solution. Note the change in scale relative to Figure 6-17

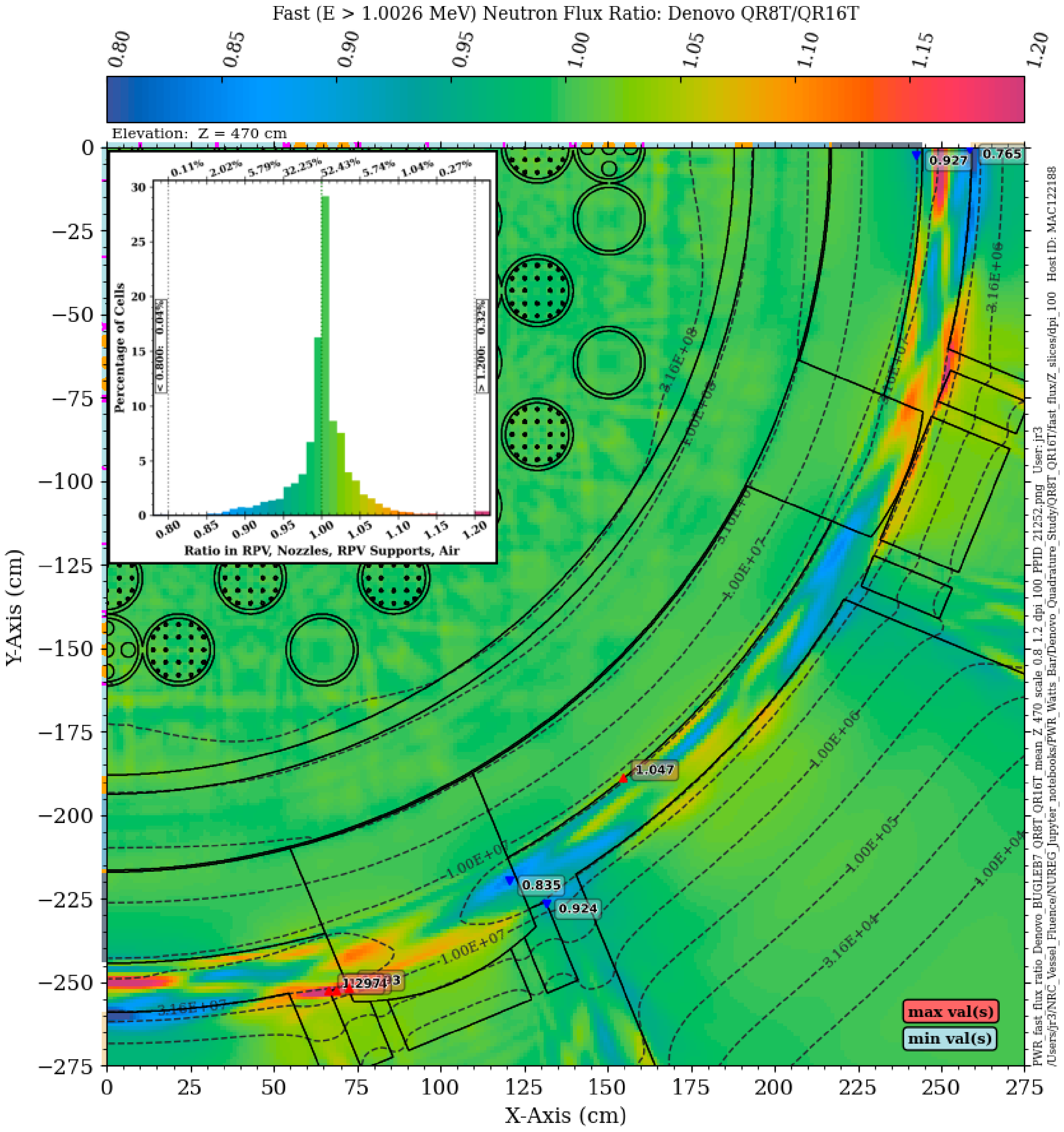

Figure 6-19 Fast neutron flux ratio in the PWR reference model: QR8T/QR16T quadrature. Plan view at Z = 470 cm. The contour lines are the fast flux from the QR16T solution

6.3.4 Quadrature sensitivity for the ^{27}Al (n,α) reaction rate

Because ray effects tend to be more pronounced in high-energy groups in discrete ordinates calculations, the discrete ordinates solution sensitivity to quadrature order was examined for the Al-27 (n,α) dosimetry reaction. This reaction has a threshold energy (i.e., minimum neutron energy required for the reaction to occur) of 3.25 MeV and an energy response range (i.e., the energy range over which 90% of the response occurs in a ^{235}U fission spectrum) of 6.45–11.9 MeV. Quadrature comparisons were made for the ^{27}Al (n,α) reaction rate for the S8, S16, QR8T, and QR16T quadratures.

The S8/S16 quadrature sensitivity for the ^{27}Al (n,α) reaction rate at the core midplane (Figure 6-20) is significantly more pronounced than the S8/S16 fast flux ratio at the core midplane (Figure 6-2). Similarly, the S8/S16 ^{27}Al (n,α) reaction rate ratios at Z = 390 cm (Figure 6-21) and at Z = 400 cm (Figure 6-22) show substantially more variation than the S8/S16 fast flux ratios at those elevations (Figure 6-3 and Figure 6-4). Note that the ratio range extends from [0.9, 1.1] for the fast flux ratios and [0.8, 1.2] for the ^{27}Al (n,α) reaction rate ratios in those figures.

The ray effects that originate near the former plates and propagate through the RPV are also much more significant for the ^{27}Al (n,α) reaction rate than for the fast flux, as shown by a comparison of Figure 6-7 and Figure 6-25.

The differences in the ^{27}Al (n,α) reaction rates in the S16 and QR8T are substantially less than for the S8/S16 comparison, but they are still more pronounced than the S16/QR8T fast flux ratios. This can be seen by comparison of Figure 6-29 with Figure 6-12, and Figure 6-30 with Figure 6-15.

Differences between the QR8T and QR16T Al-27 (n,α) reaction rates (not shown) do not deviate significantly from the QR8T/QR16T fast flux ratios.

Further discussion of the increased quadrature sensitivity for high-energy threshold reactions is provided in Section 6.6.

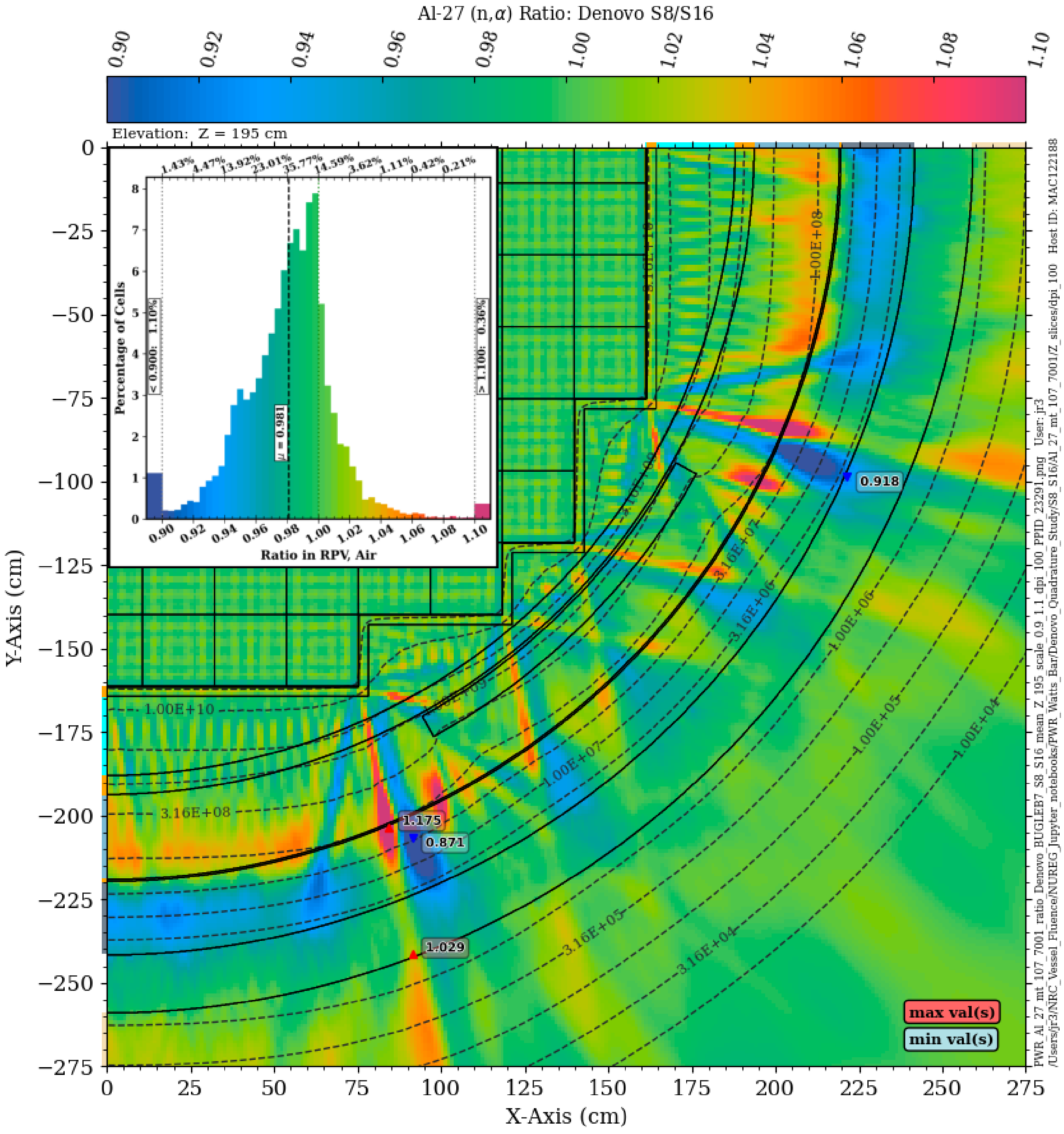

Figure 6-20 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 195 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

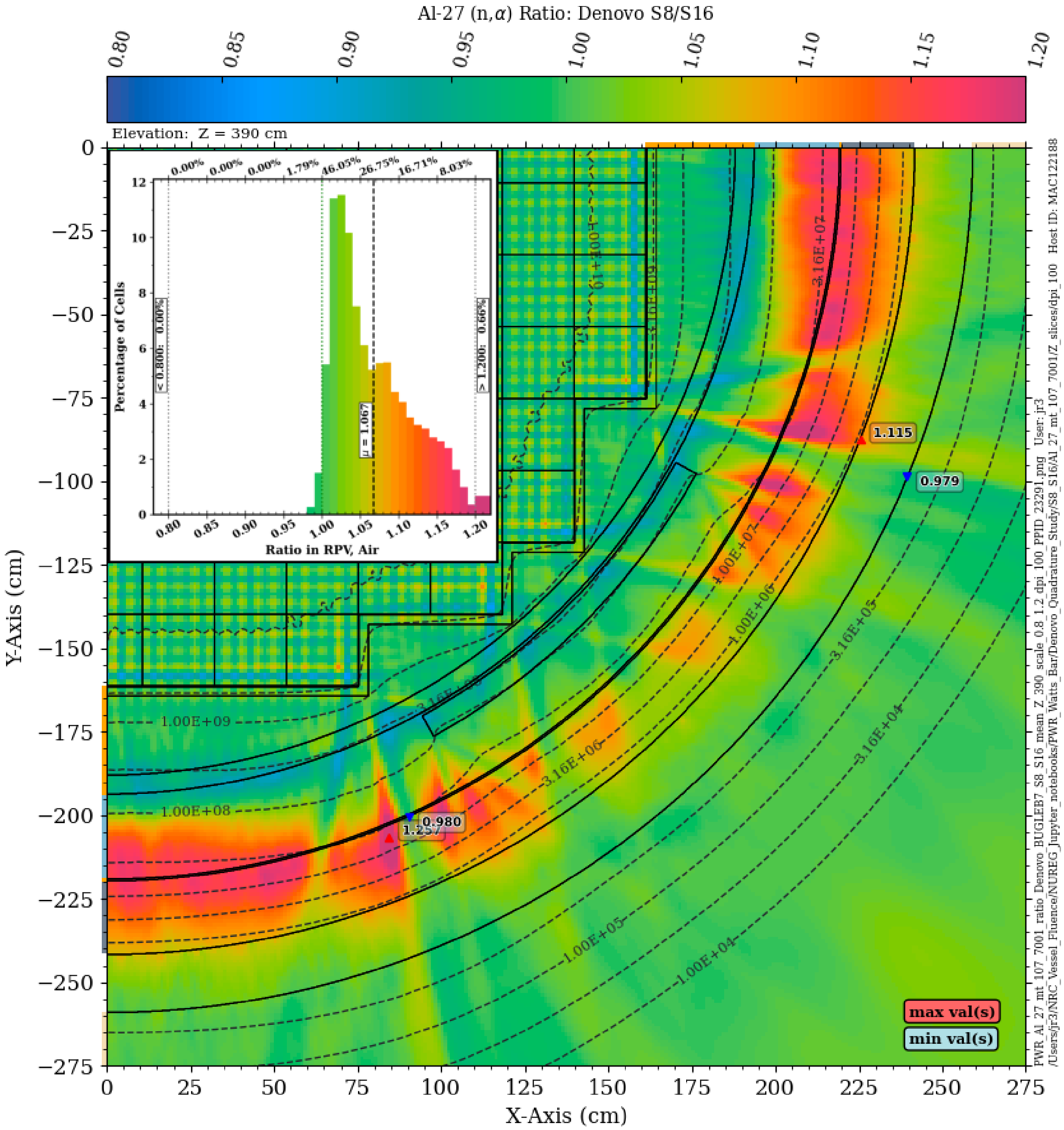

Figure 6-21 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 390 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

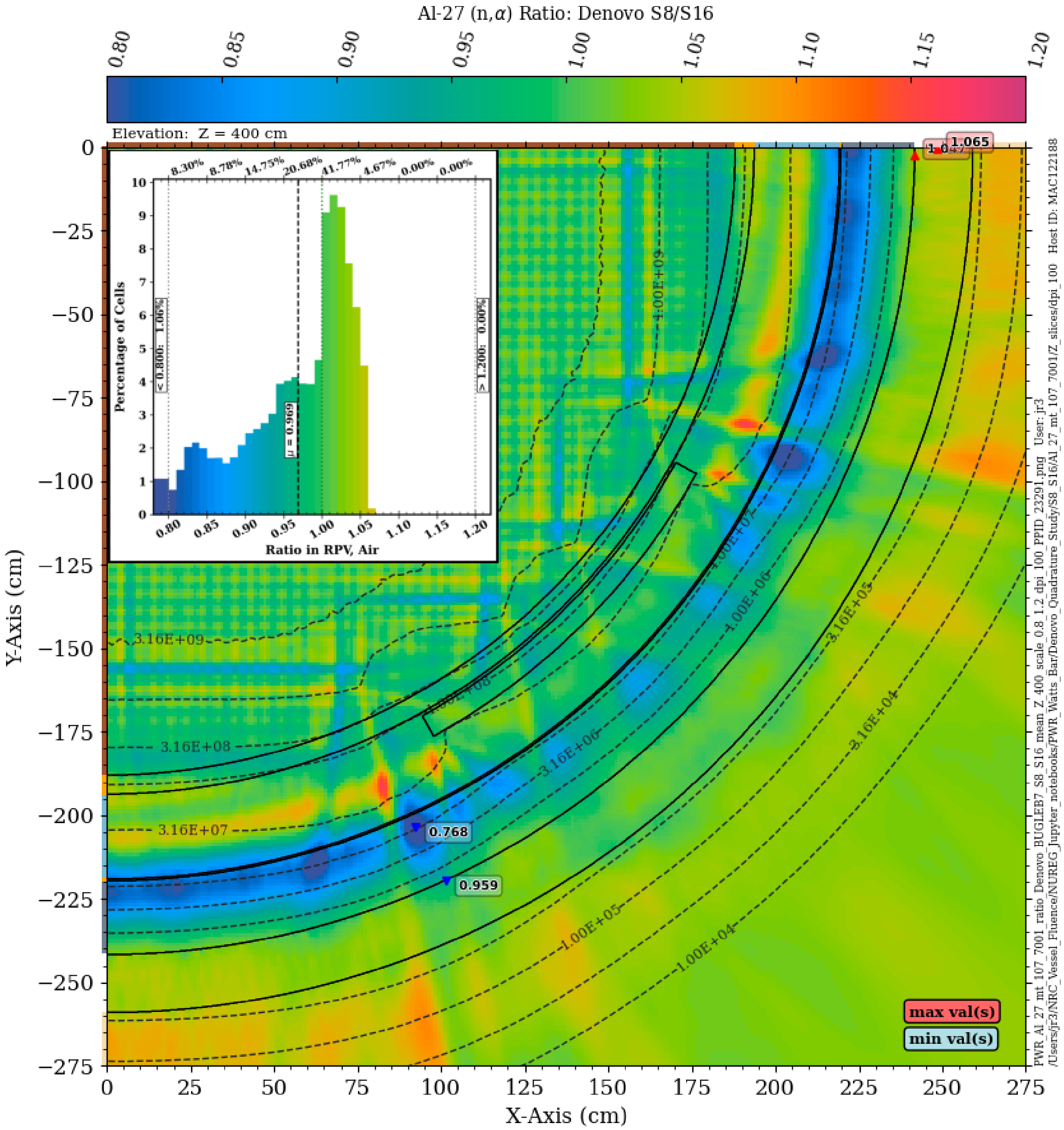

Figure 6-22 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 400 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

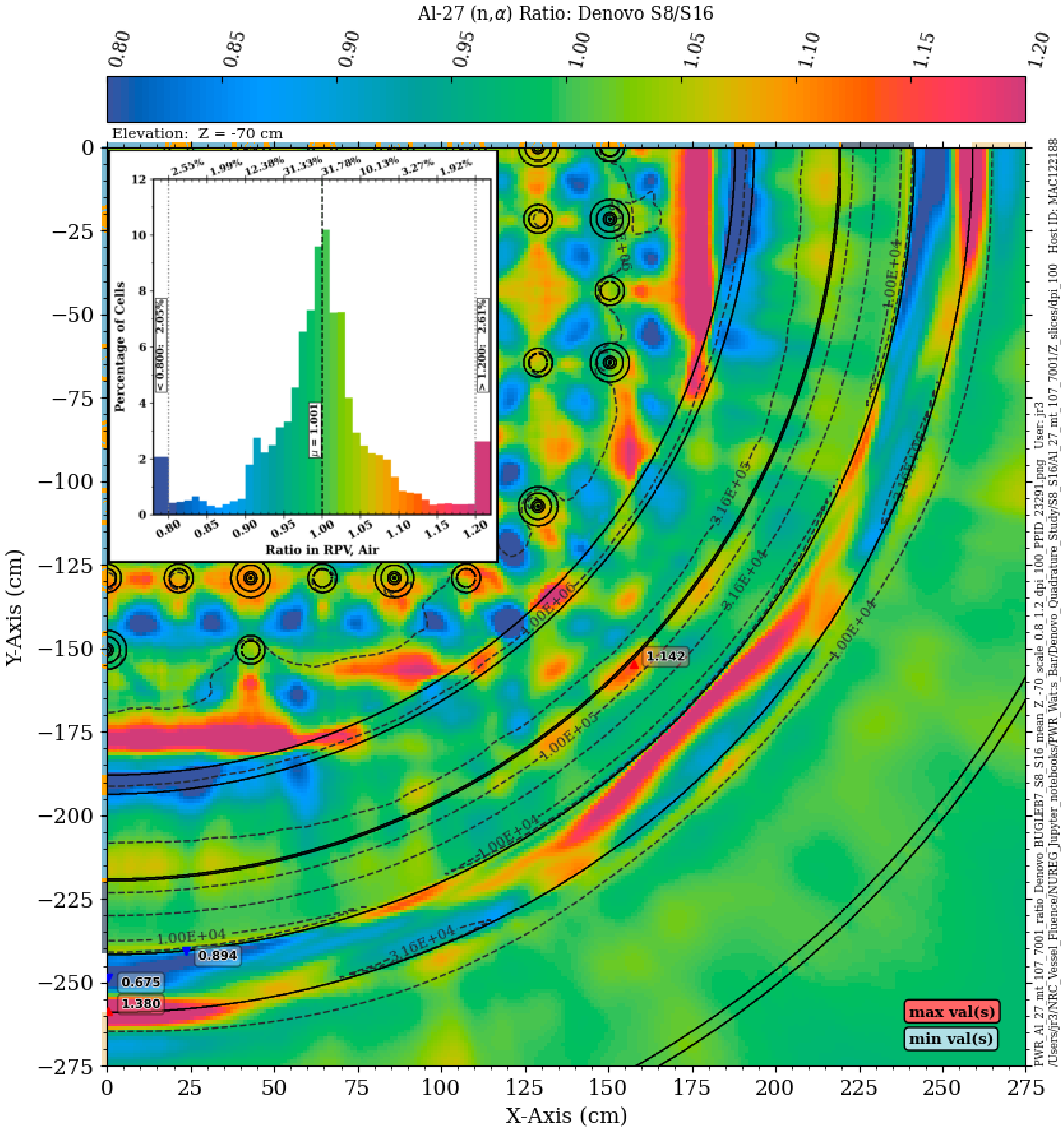

Figure 6-23 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = -70 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

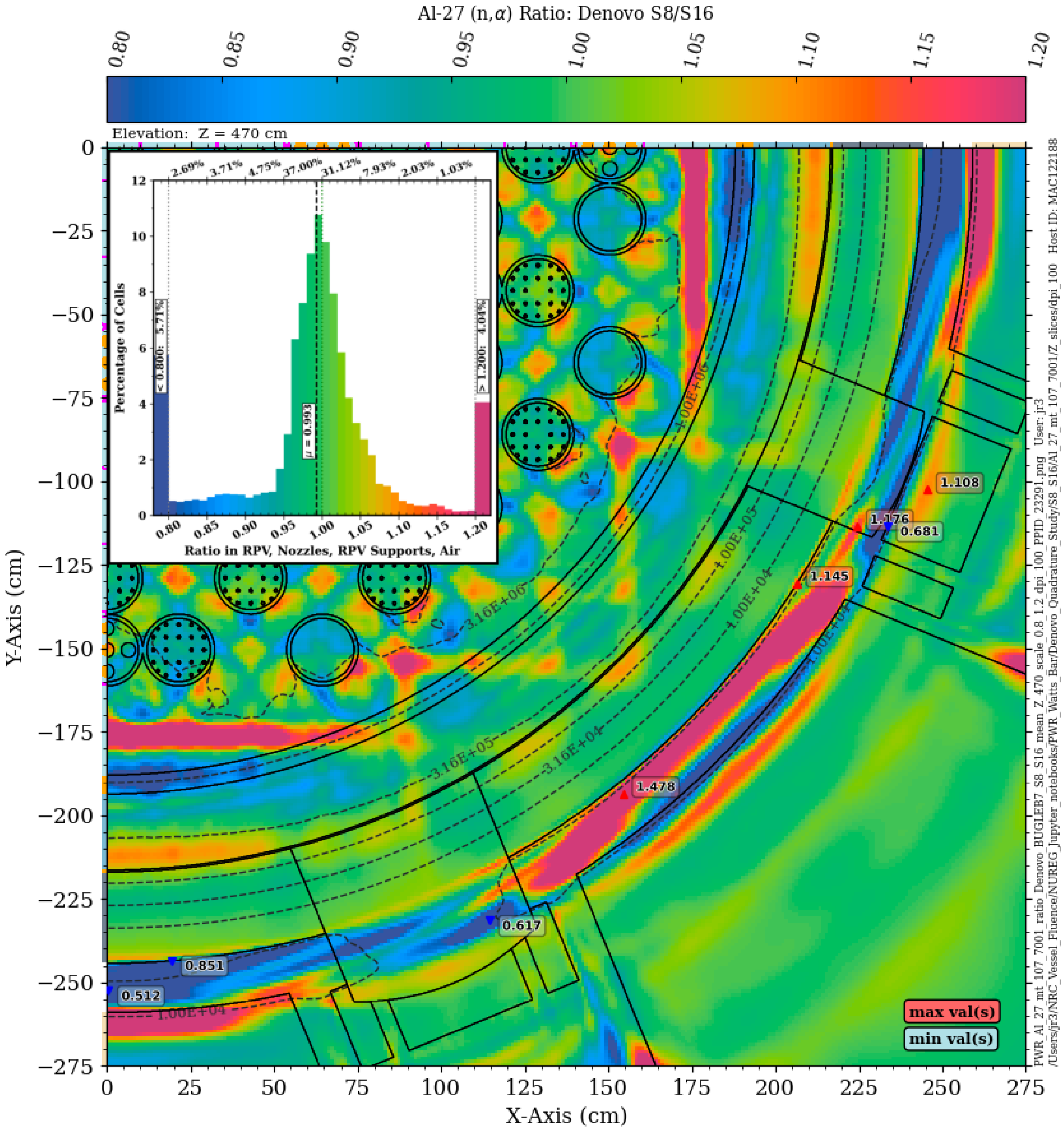

Figure 6-24 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Plan view at Z = 470 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

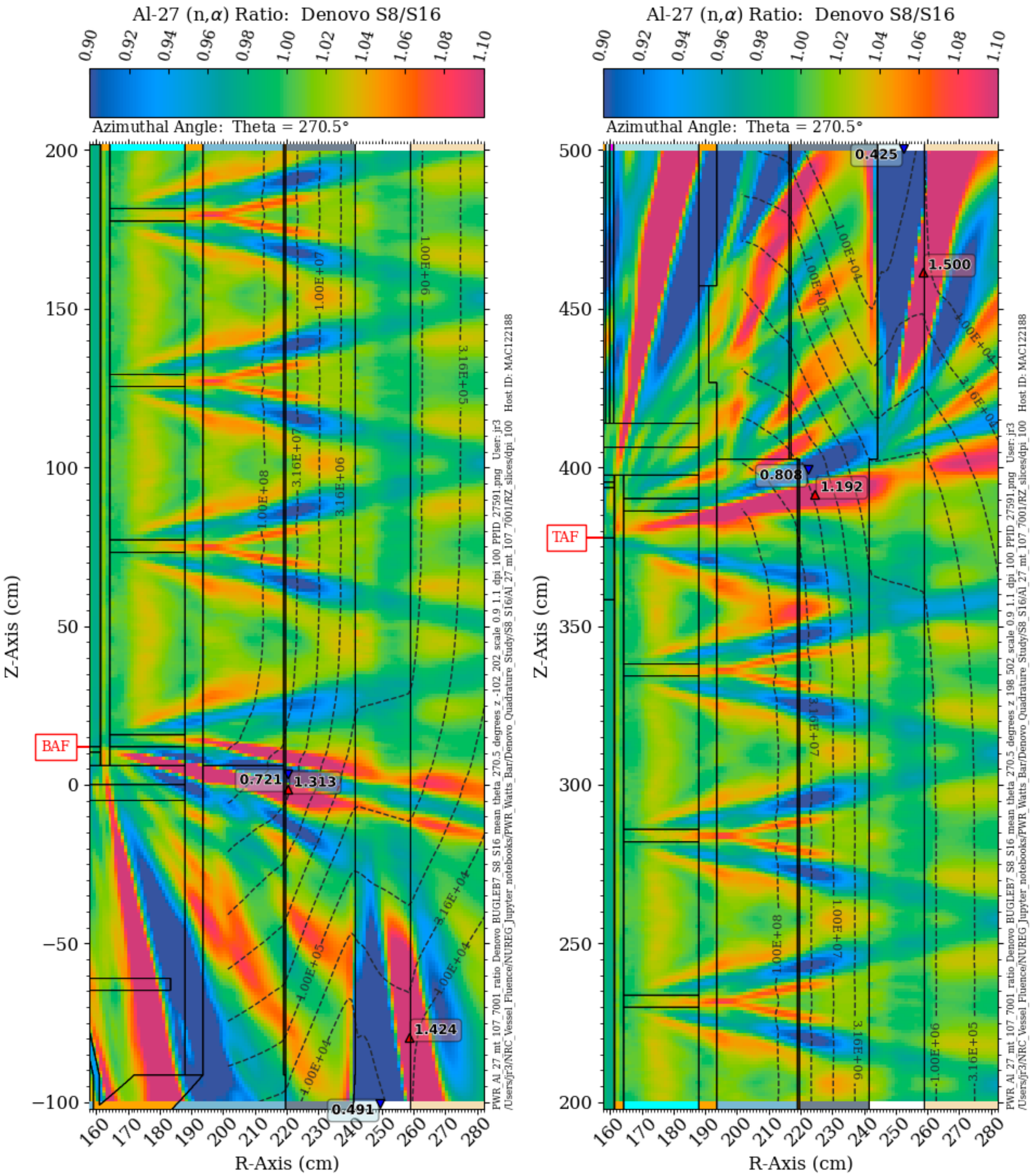

Figure 6-25 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 270.5°. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

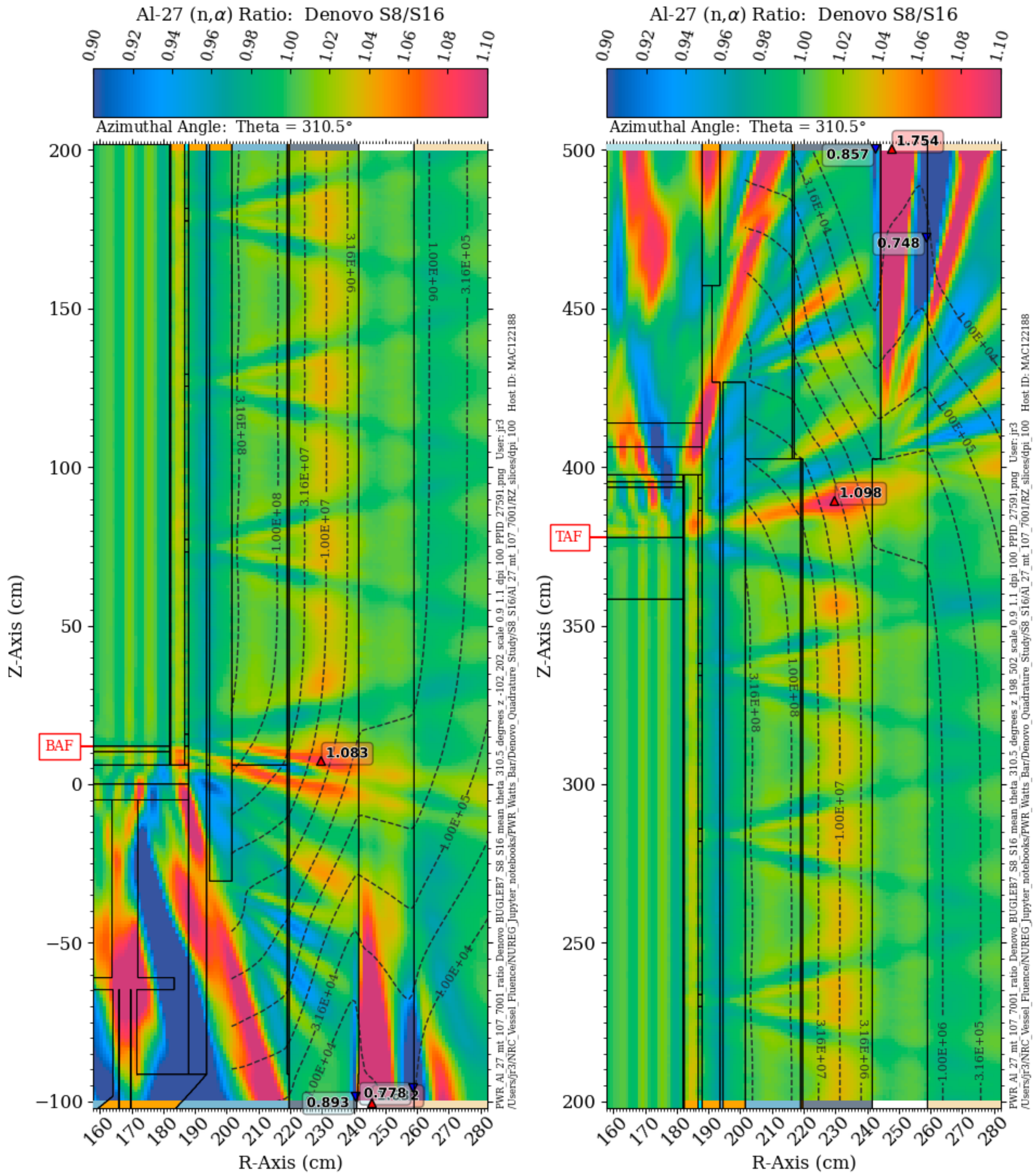

Figure 6-26 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 310.5°. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

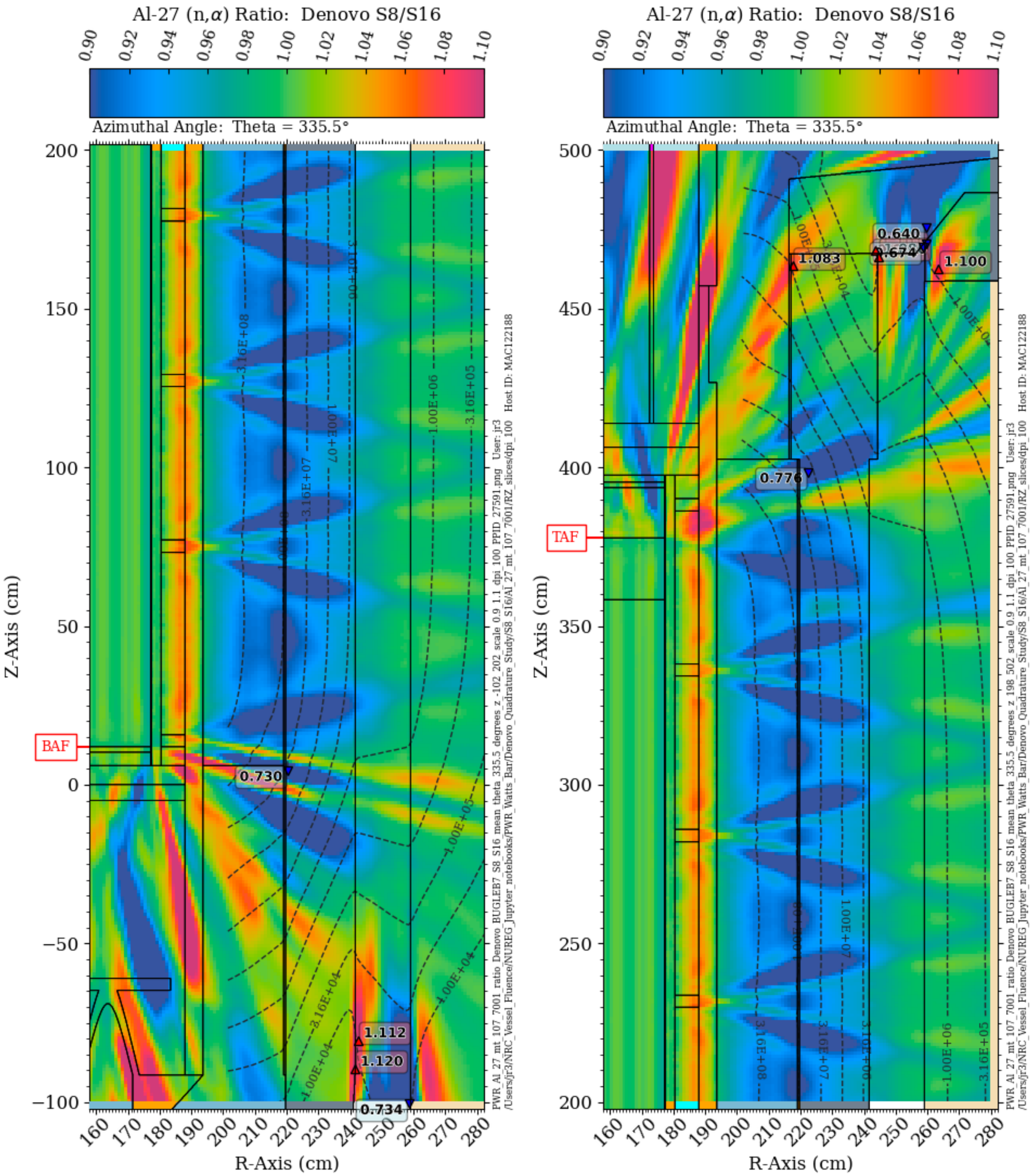

Figure 6-27 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 335.5°. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

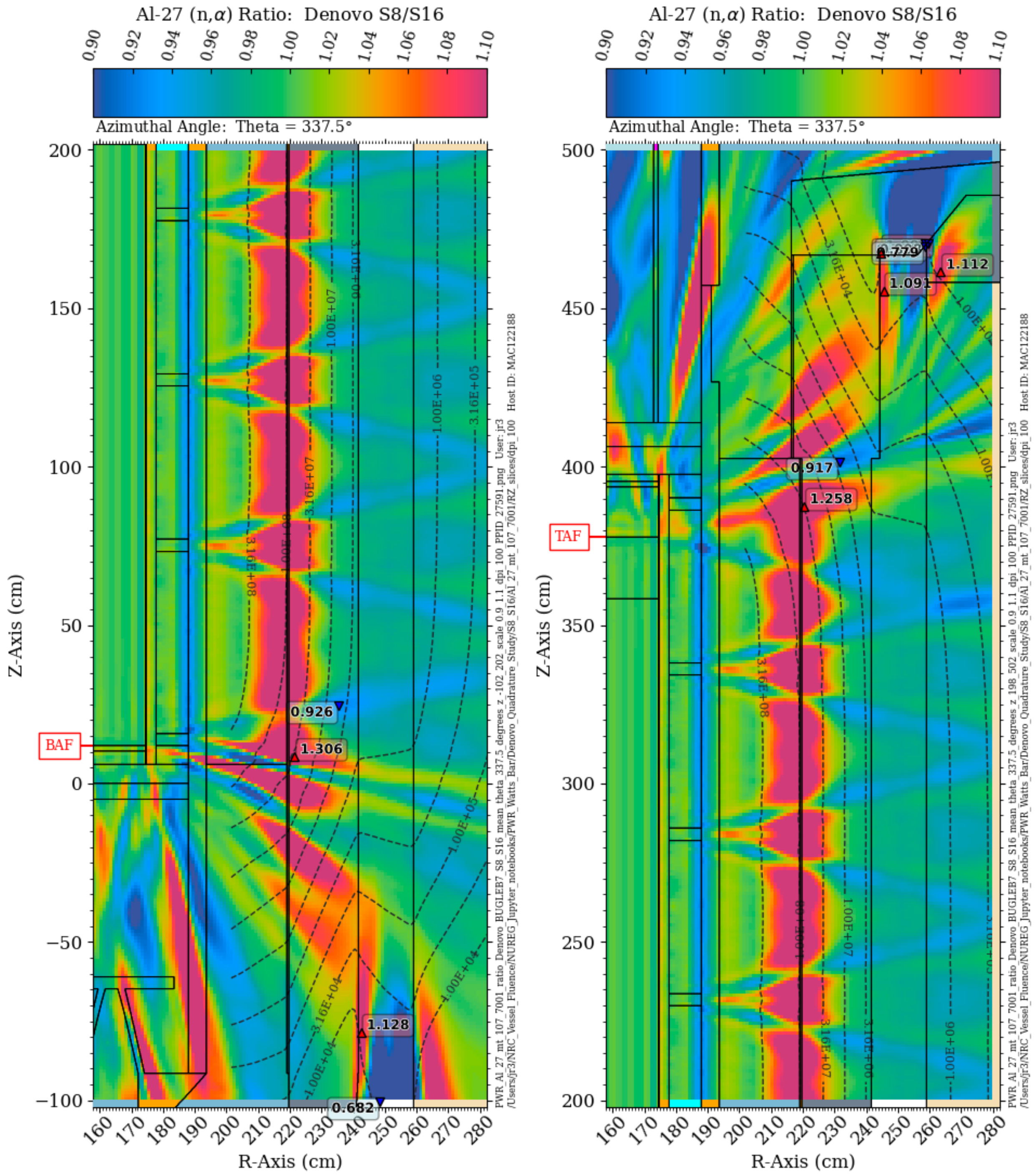

Figure 6-28 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S8/S16 quadrature. Elevation view at an azimuthal angle of 337.5°. The contour lines are the ^{27}Al (n,α) reaction rate from the S16 solution

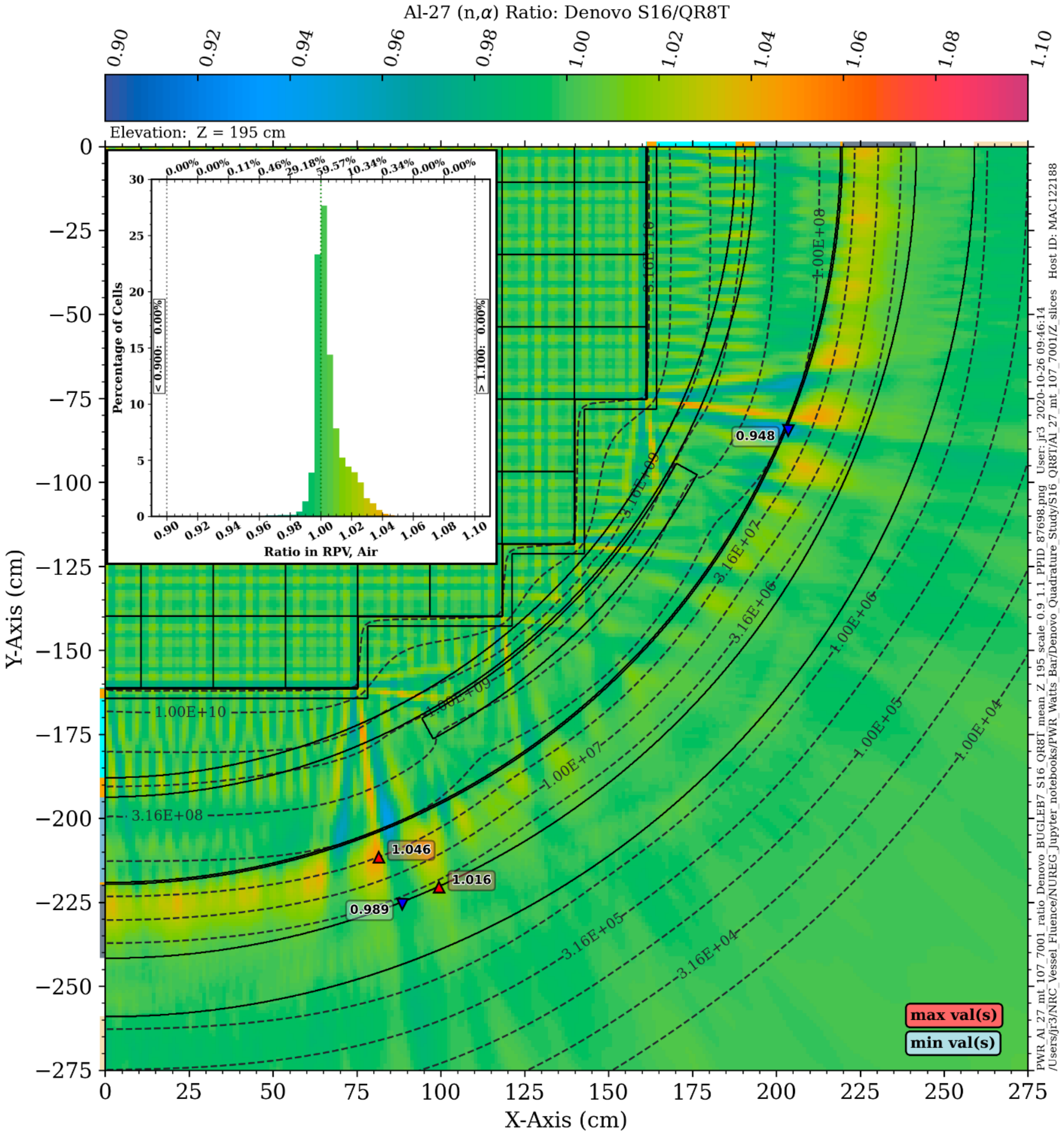

Figure 6-29 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S16/QR8T quadrature. Plan view at Z = 195 cm. The contour lines are the ^{27}Al (n,α) reaction rate from the QR8T solution

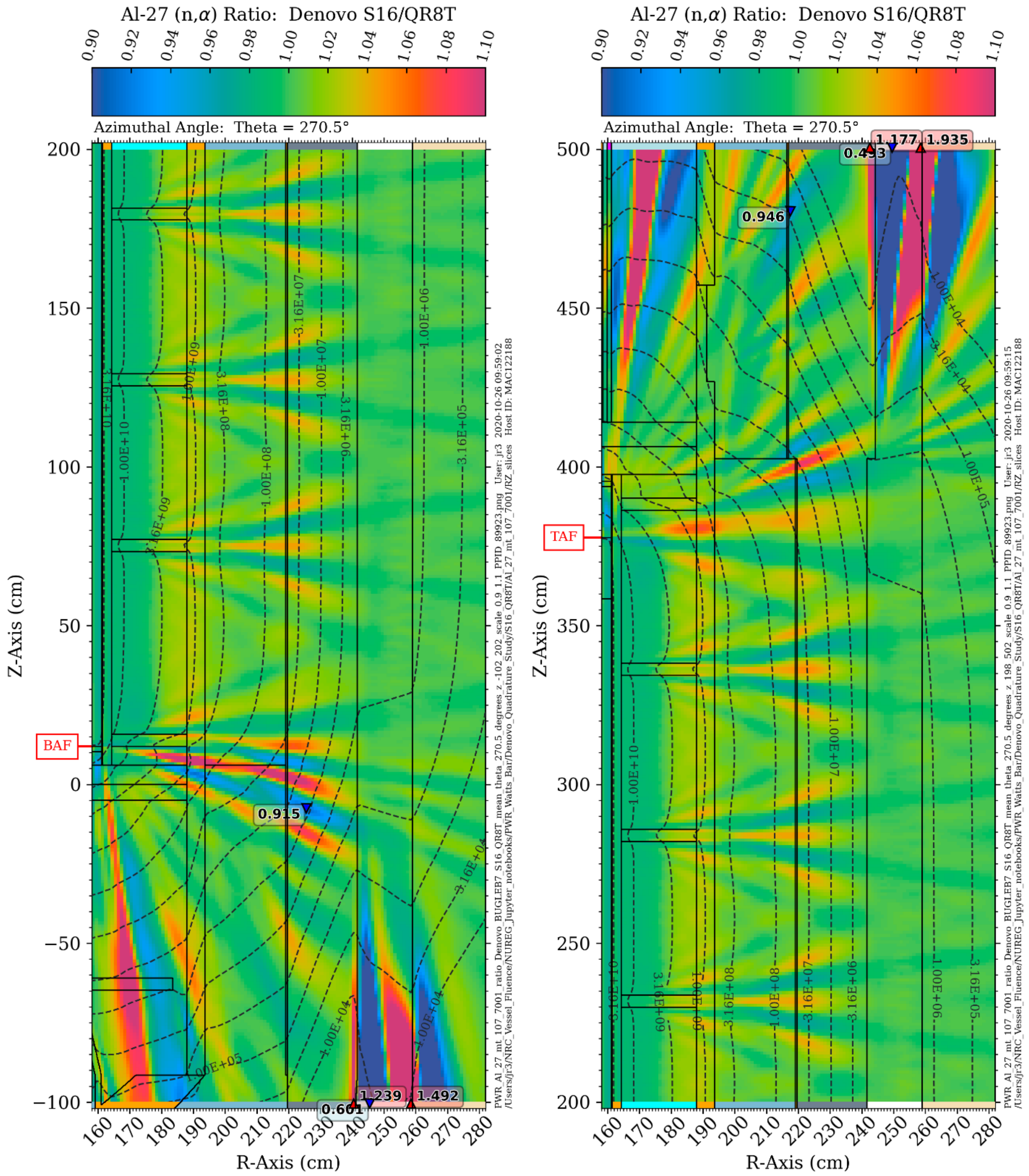

Figure 6-30 ^{27}Al (n,α) reaction rate ratio in the PWR reference model: S16/QR8T quadrature. Elevation view at an azimuthal angle of 270.5°. The contour lines are the ^{27}Al (n,α) reaction rate from the QR8T solution

6.4 Denovo quadrature sensitivity: BWR model

The sensitivity of discrete ordinates solutions to the angular quadrature selection for a BWR was examined using the BWR model with a homogenized core representation (Section 5.9.2). The discrete ordinates calculations were run using Denovo with the following parameters:

- BUGLE-B7 MG cross-section library with P_3 scattering expansions
- Uniform 1.25-cm mesh in X, Y, and Z with X- and Y-extents of 350 cm and a Z-extent of 800 cm (76.8 million cells)
- Linear discontinuous (LD) differencing scheme
- Source iteration (SI) solver with a convergence criterion of 1E-6

A parametric meshing study indicated that the Denovo solution converges with respect to mesh using the intervals noted above.

6.4.1 Denovo solutions: S8 vs S16

Ratios of the BWR S8 Denovo fast (E > 1.0026 MeV) flux to the S16 Denovo fast flux are provided in Figure 6-31 through Figure 6-37. Figure 6-31 and Figure 6-32 illustrate the fast neutron flux ratio on the inner and outer surfaces of the RPV and shroud, respectively. There is evidence of ray effects within the core height in the shroud and the RPV. More notable ray effects are apparent in the upper portion of the shroud, and at elevations below the bottom of the fuel.

Figure 6-33 shows the S8/S16 fast flux ratio at the core midplane. As with the PWR model, ray effects originate at each corner fuel assembly on the periphery of the core. The jet pump risers also produce ray effects. The S8/S16 fast flux ratio in the RPV varies by nearly 20%, with a maximum value of 1.109 and a minimum value of 0.927. As with the PWR model, these changes occur over a relatively small spatial scale.

Figure 6-34 and Figure 6-35 show the S8/S16 fast flux ratio at elevations of Z = 210 cm and Z = 225 cm, respectively. These elevations correspond to the horizontal red and blue bands above the top of the core in Figure 6-31. These plots demonstrate significant variations in the fast flux ratio in the RPV as a function of elevation and of azimuthal angle.

Figure 6-36 and Figure 6-37 show the S8/S16 fast flux ratio at elevations of Z = -250 cm and Z = 375 cm, which correspond to the upper and lower plan view elevations for the BWR model in Section 5. Note that the scale on these two plots is expanded from a range of [0.9, 1.1] to [0.8, 1.2]. Also, as shown in the histograms, there is a significant bias at elevations outside of the traditional beltline.

As with the PWR results in Section 6.3.1, these results suggest that the S8 quadrature set may not be optimal for RPV fluence calculations even within the traditional beltline region.

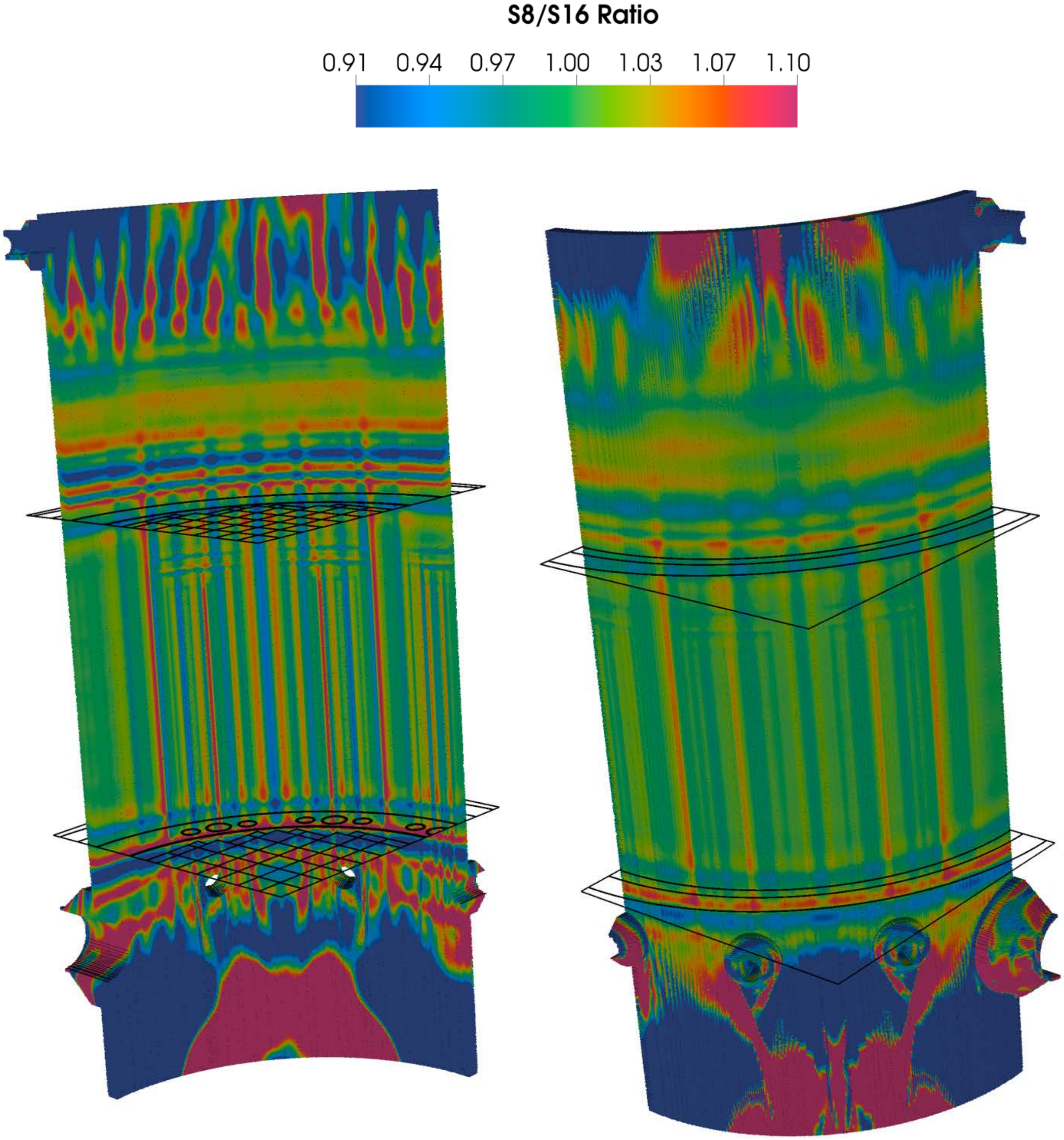

Figure 6-31 Fast neutron flux ratio on the inner and outer surfaces of the RPV for the BWR reference model. The ratio is for a Denovo S8 solution relative to an S16 solution. The boundary planes show the extent of the fuel assemblies

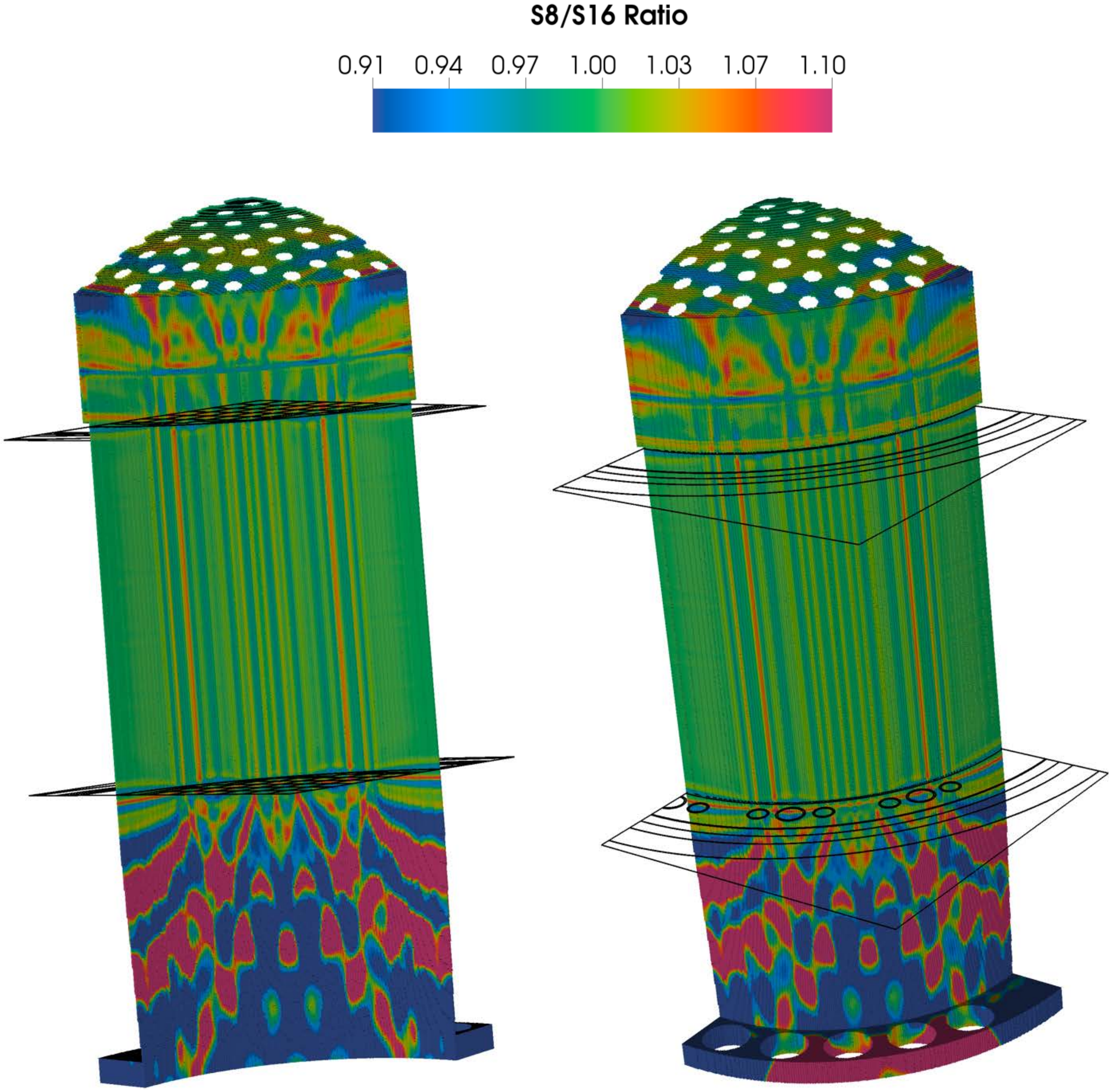

Figure 6-32 Fast neutron flux ratio on the inner and outer surfaces of the shroud for the BWR reference model. The ratio is for a Denovo S8 solution relative to an S16 solution. The boundary planes show the extent of the fuel assemblies

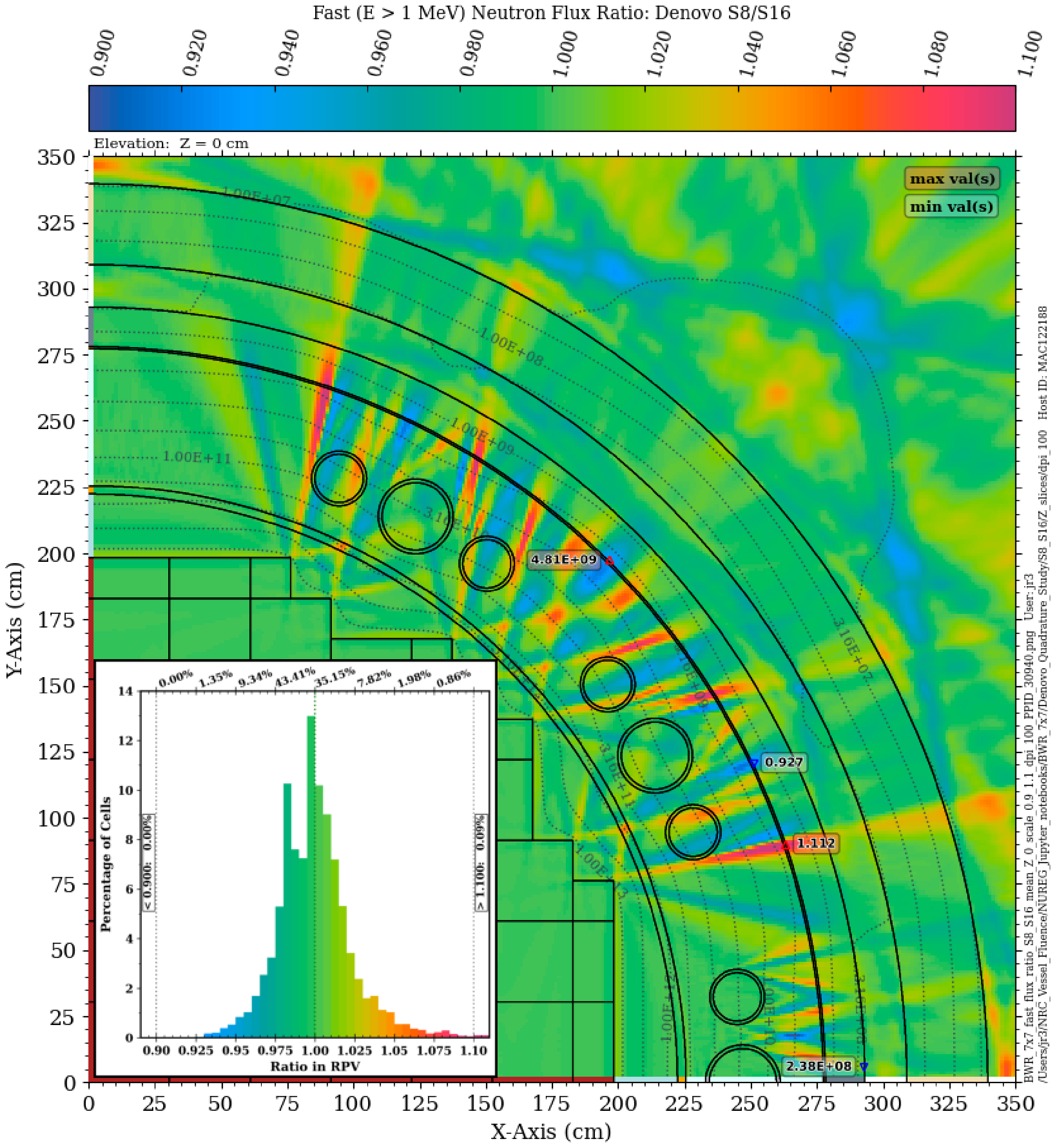

Figure 6-33 Fast neutron flux ratio in the BWR reference model: S8/S16 quadrature. Plan view at Z = 0 cm. The contour lines are the fast flux from the S16 solution. Minimum and maximum values within the RPV are shown for the S16 fast flux and the S8/S16 fast flux ratio

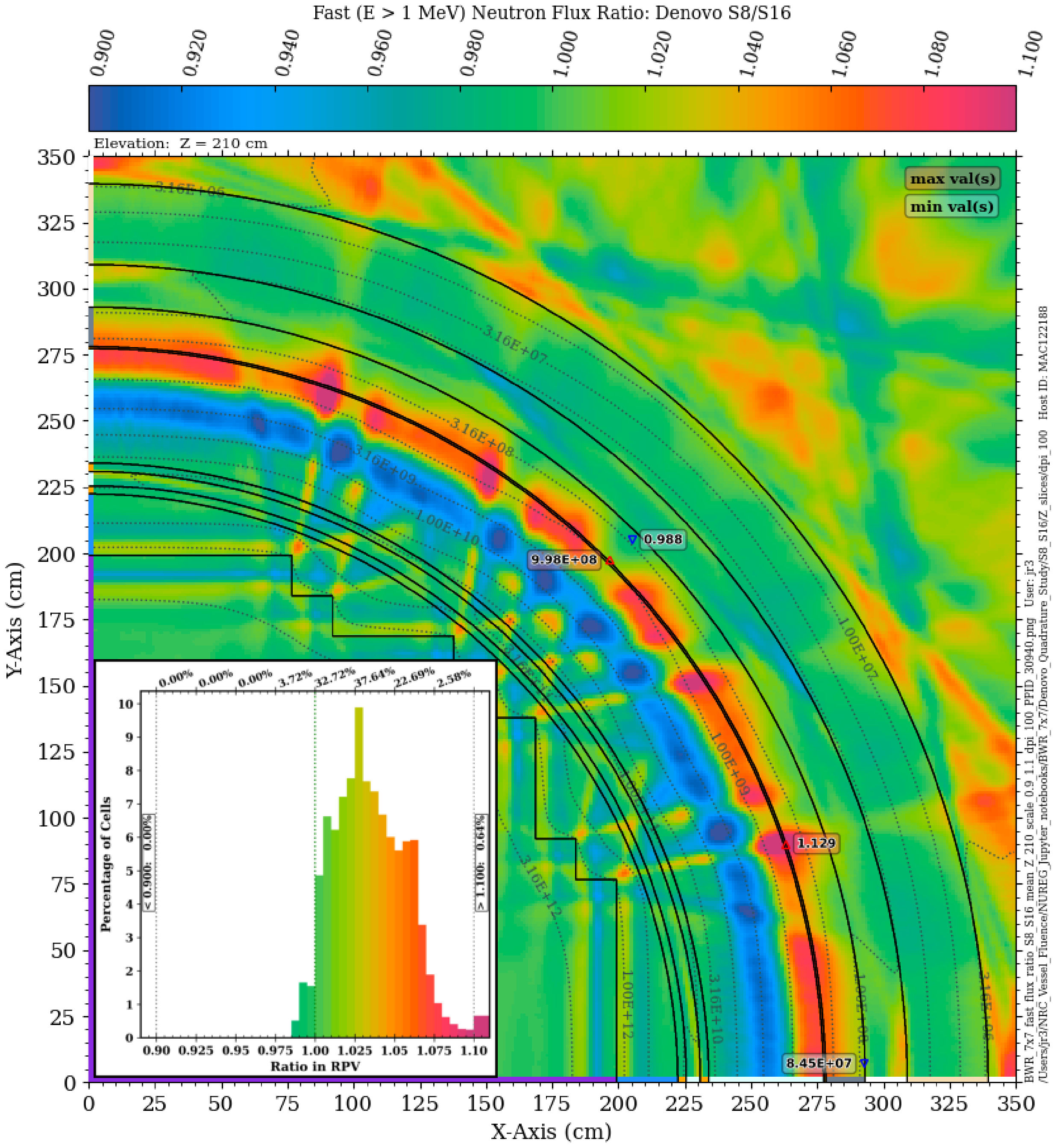

Figure 6-34 Fast neutron flux ratio in the BWR reference model: S8/S16 quadrature. Plan view at Z = 210 cm. The contour lines are the fast flux from the S16 solution. Minimum and maximum values within the RPV are shown for the S16 fast flux and the S8/S16 fast flux ratio

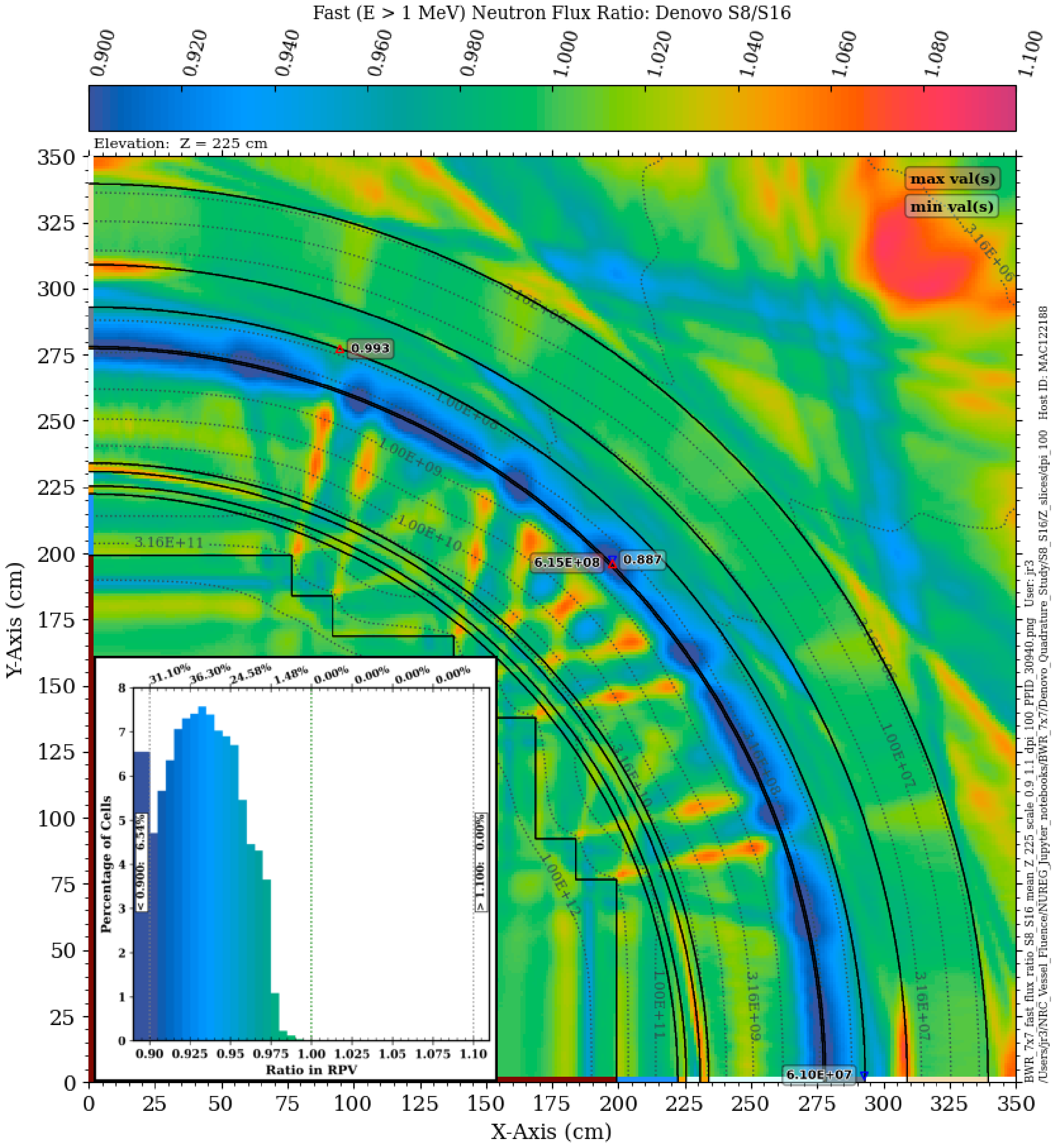

Figure 6-35 Fast neutron flux ratio in the BWR reference model: S8/S16 quadrature. Plan view at Z = 225 cm. The contour lines are the fast flux from the S16 solution. Minimum and maximum values within the RPV are shown for the S16 fast flux and the S8/S16 fast flux ratio

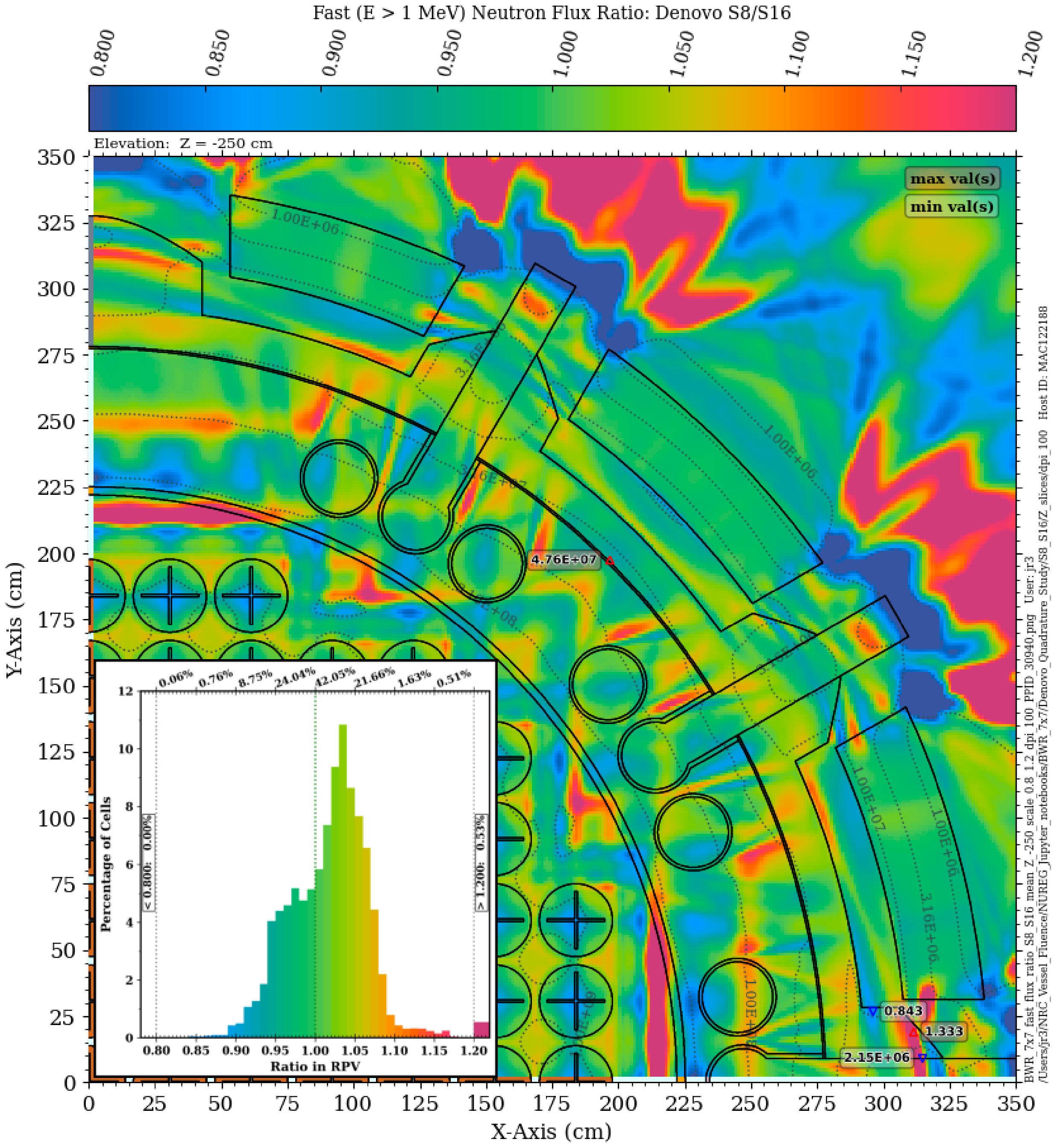

Figure 6-36 Fast neutron flux ratio in the BWR reference model: S8/S16 quadrature. Plan view at Z = -250 cm. The contour lines are the fast flux from the S16 solution. Minimum and maximum values within the RPV are shown for the S16 fast flux and the S8/S16 fast flux ratio. Note the change in scale relative to the three previous figures

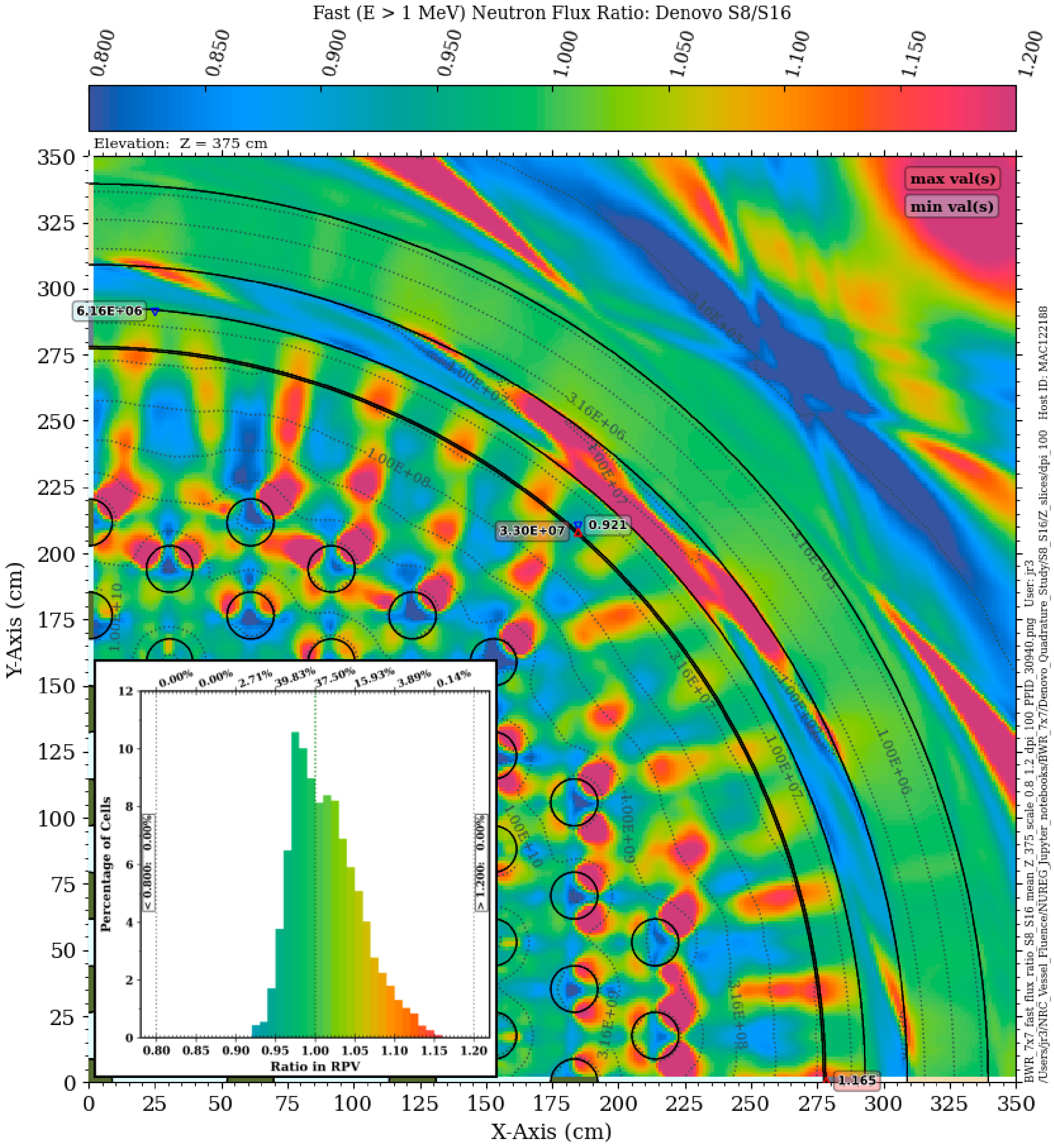

Figure 6-37 Fast neutron flux ratio in the BWR reference model: S8/S16 quadrature. Plan view at Z = 375 cm. The contour lines are the fast flux from the S16 solution. Minimum and maximum values within the RPV are shown for the S16 fast flux and the S8/S16 fast flux ratio

6.4.2 Denovo solutions: S16 vs QR8T

Ratios of the BWR fast flux for S16 and QR8T Denovo solutions are provided in Figure 6-38 through Figure 6-44. The S16/QR8T fast flux ratios on the inner and outer surfaces of both the RPV and the shroud (Figure 6-38 and Figure 6-39, respectively) show much less variation than the S8/S16 ratios, as expected. At the core midplane (Figure 6-40), the two solutions differ by more than 2.5% in less than 2% of the RPV cells. The appearance of ray effects due to axial and azimuthal variations at Z = 210 cm and Z = 225 cm (Figure 6-41 and Figure 6-42) is still present, although their magnitude is substantially attenuated. At the elevations of Z = -250 cm (Figure 6-43)and Z = 375 cm (Figure 6-44), the solution agreement is also markedly improved relative to the S8/S16 ratios.

6.4.3 Denovo solutions: QR8T vs QR16T

The final BWR comparison is between QR8T and QR16T solutions. The QR8T/QR16T fast flux ratios are plotted in Figure 6-45 through Figure 6-51. Within the axial range of interest (from Z = -250 cm to Z = 375 cm), the QR8T and QR16T solutions agree within 5% in nearly all the RPV cells.

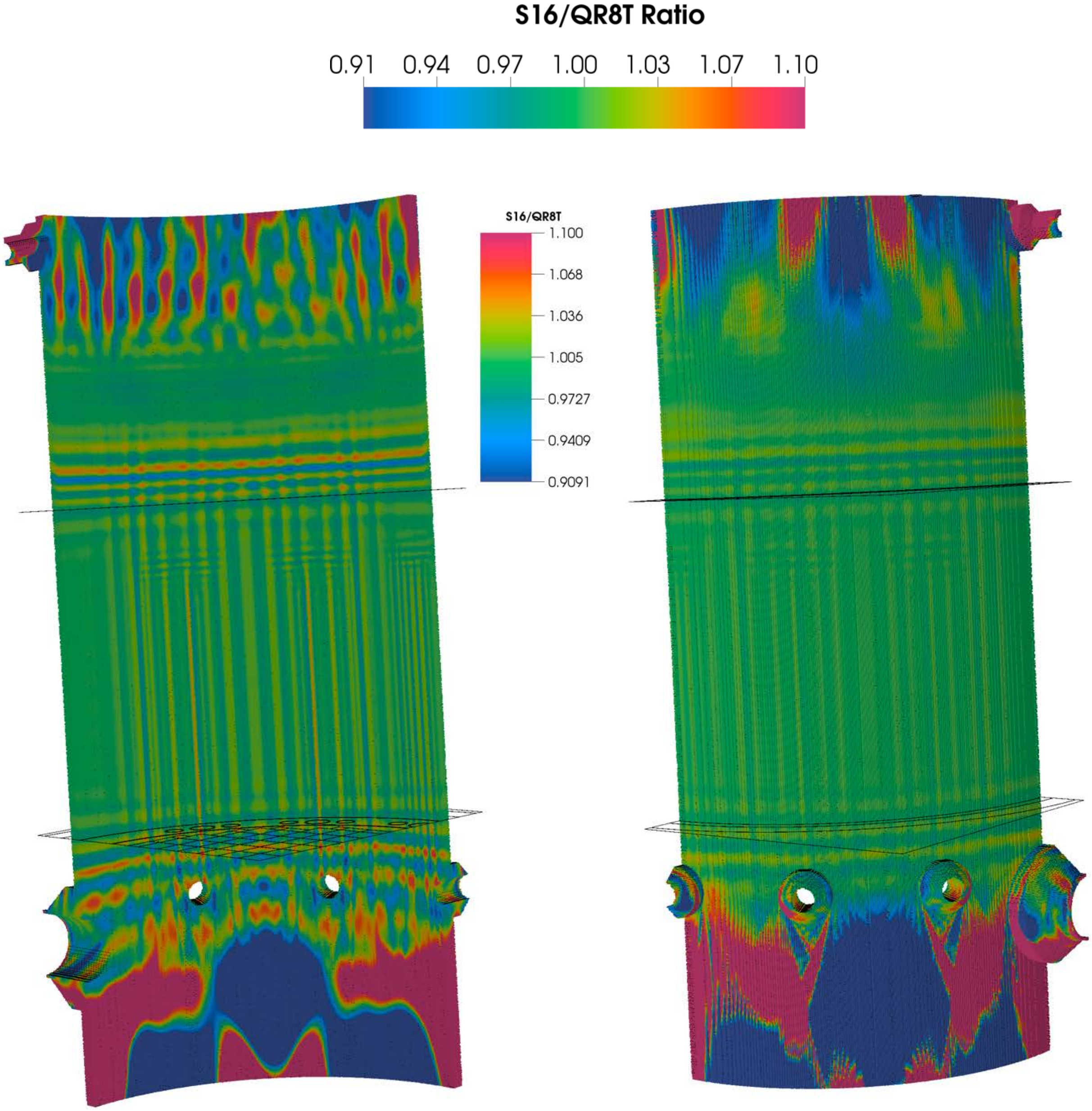

Figure 6-38 Fast neutron flux ratio on the inner and outer surfaces of the RPV for the BWR reference model. The ratio is for a Denovo S16 solution relative to a QR8T solution. The boundary planes show the extent of the fuel assemblies

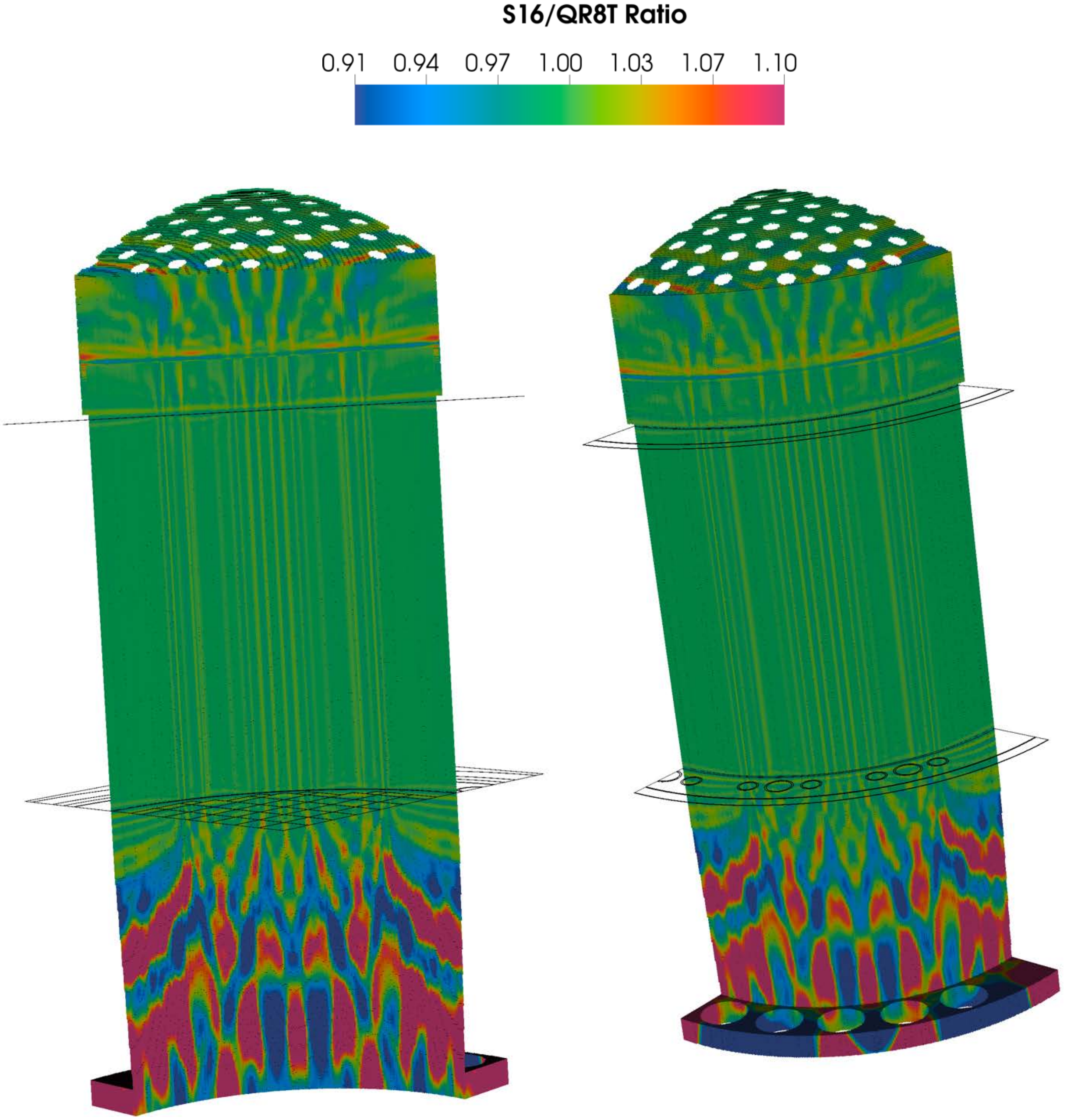

Figure 6-39 Fast neutron flux ratio on the inner and outer surfaces of the shroud for the BWR reference model. The ratio is for a Denovo S16 solution relative to a QR8T solution. The boundary planes show the extent of the fuel assemblies

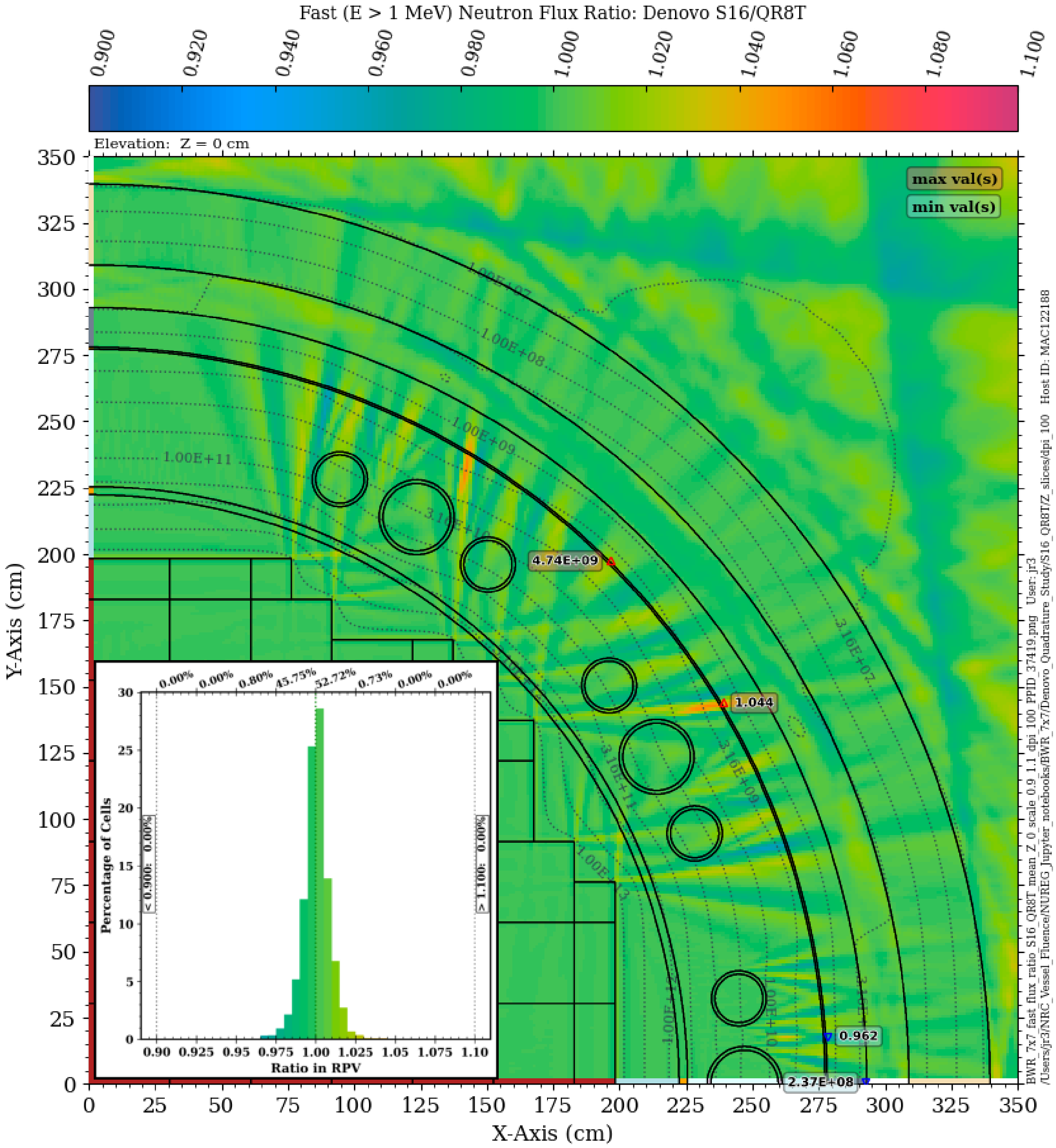

Figure 6-40 **Fast neutron flux ratio in the BWR reference model: S16/QR8T quadrature. Plan view at Z = 0 cm. The contour lines are the fast flux from the QR8T solution. Minimum and maximum values within the RPV are shown for the QR8T fast flux and the S16/QR8T fast flux ratio**

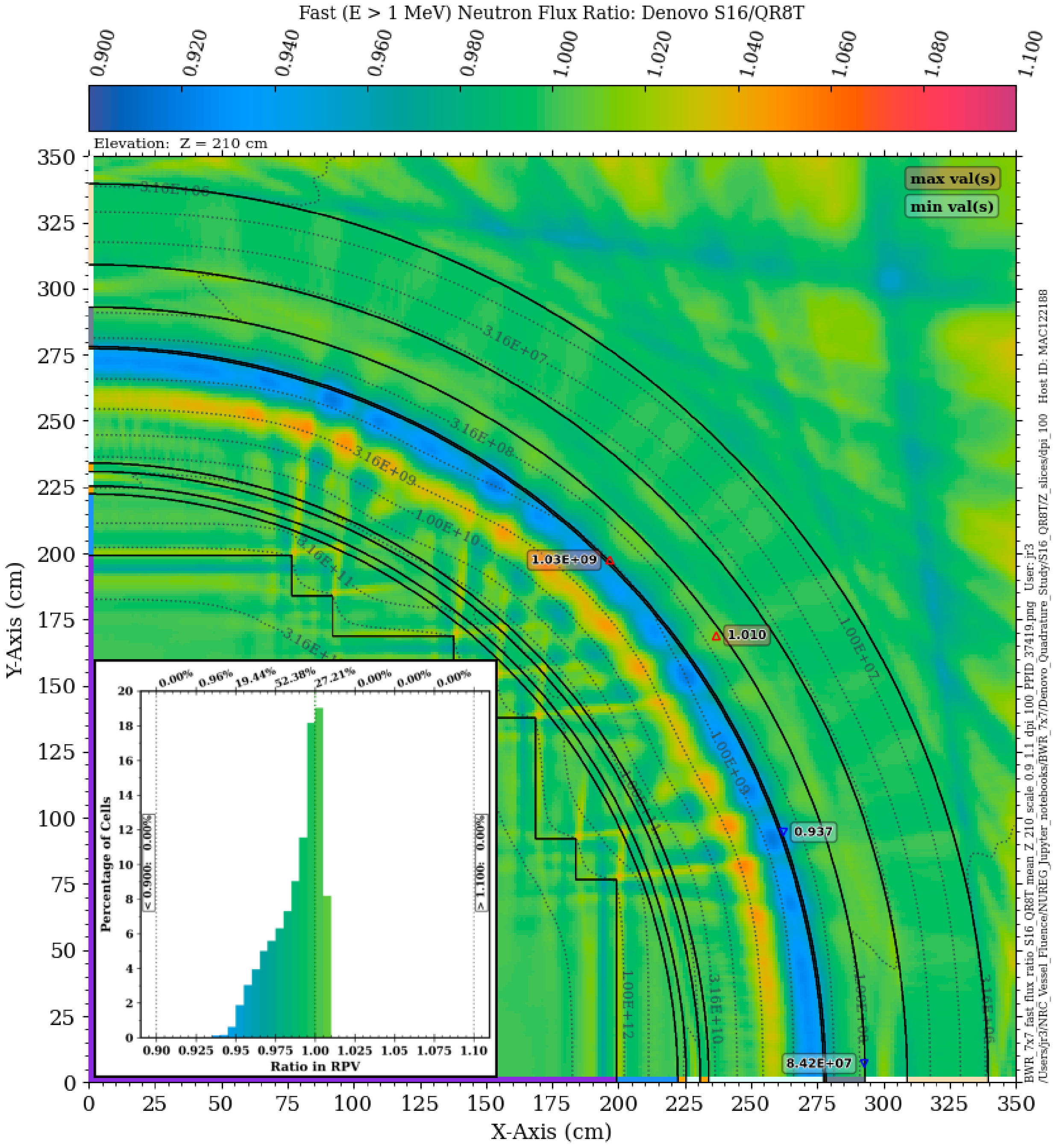

Figure 6-41 Fast neutron flux ratio in the BWR reference model: S16/QR8T quadrature. Plan view at Z = 210 cm. The contour lines are the fast flux from the QR8T solution. Minimum and maximum values within the RPV are shown for the QR8T fast flux and the S16/QR8T fast flux ratio

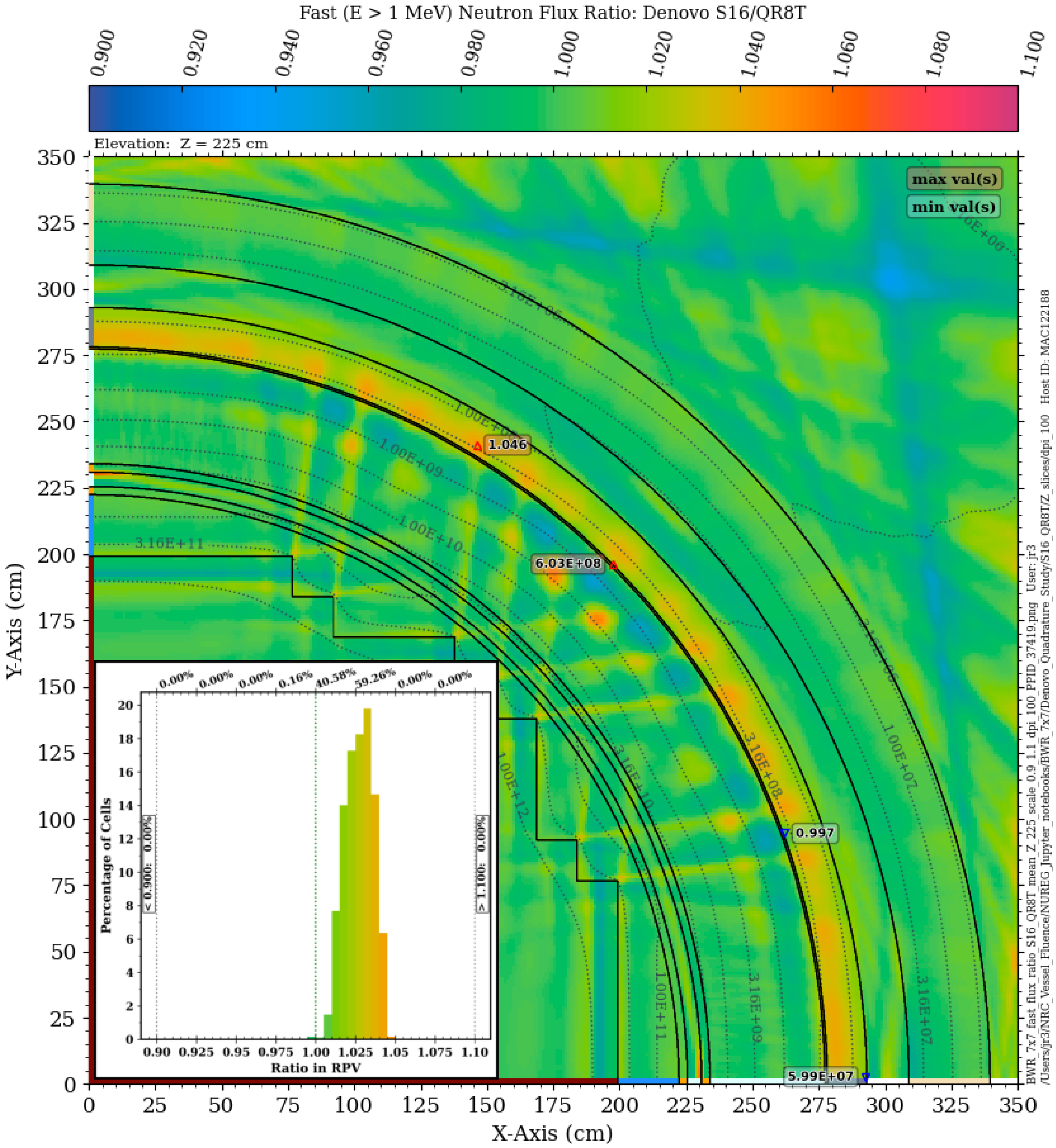

Figure 6-42 Fast neutron flux ratio in the BWR reference model: S16/QR8T quadrature. Plan view at Z = 225 cm. The contour lines are the fast flux from the QR8T solution. Minimum and maximum values within the RPV are shown for the QR8T fast flux and the S16/QR8T fast flux ratio

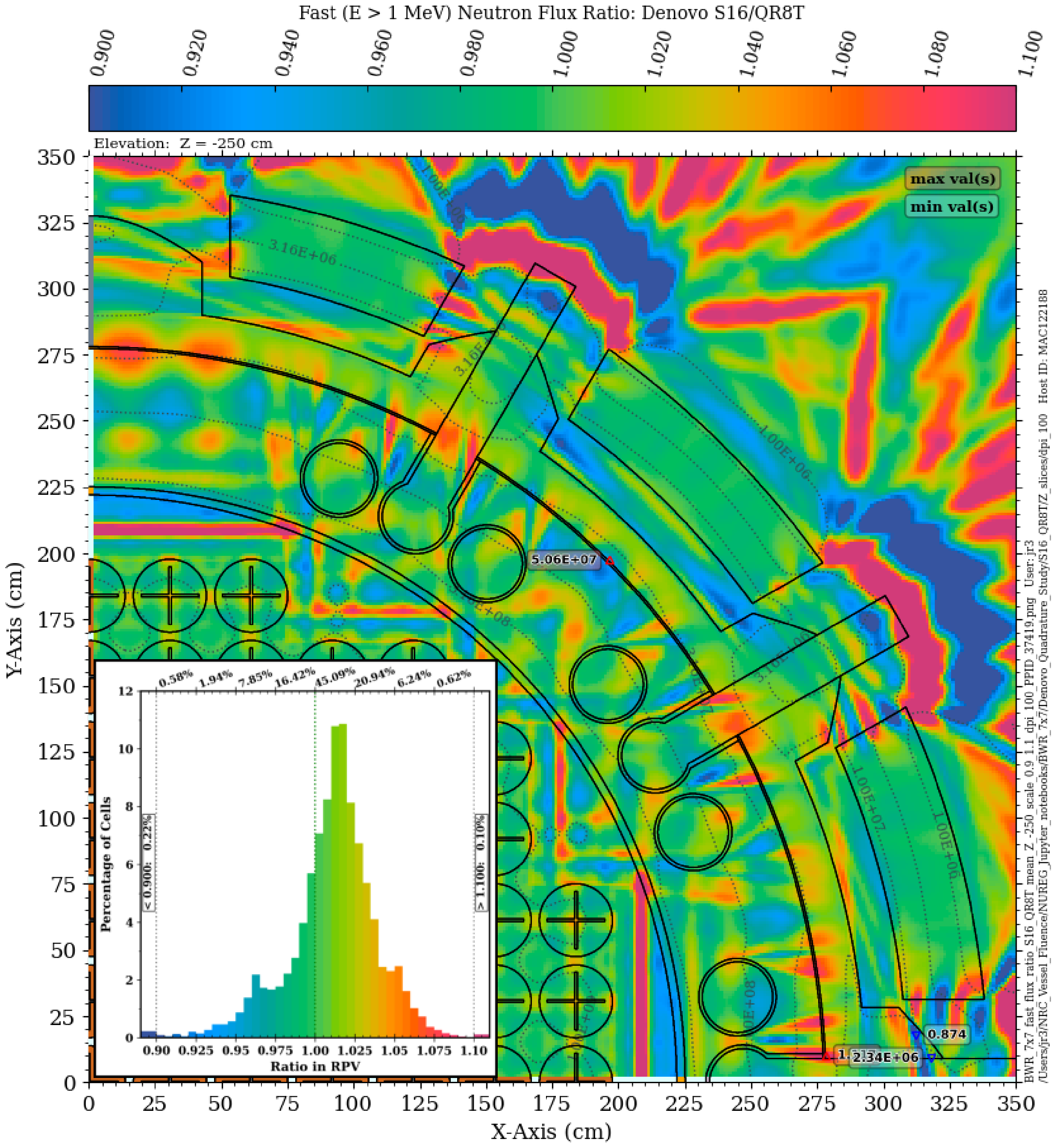

Figure 6-43 Fast neutron flux ratio in the BWR reference model: S16/QR8T quadrature. Plan view at Z = -250 cm. The contour lines are the fast flux from the QR8T solution. Minimum and maximum values within the RPV are shown for the QR8T fast flux and the S16/QR8T fast flux ratio

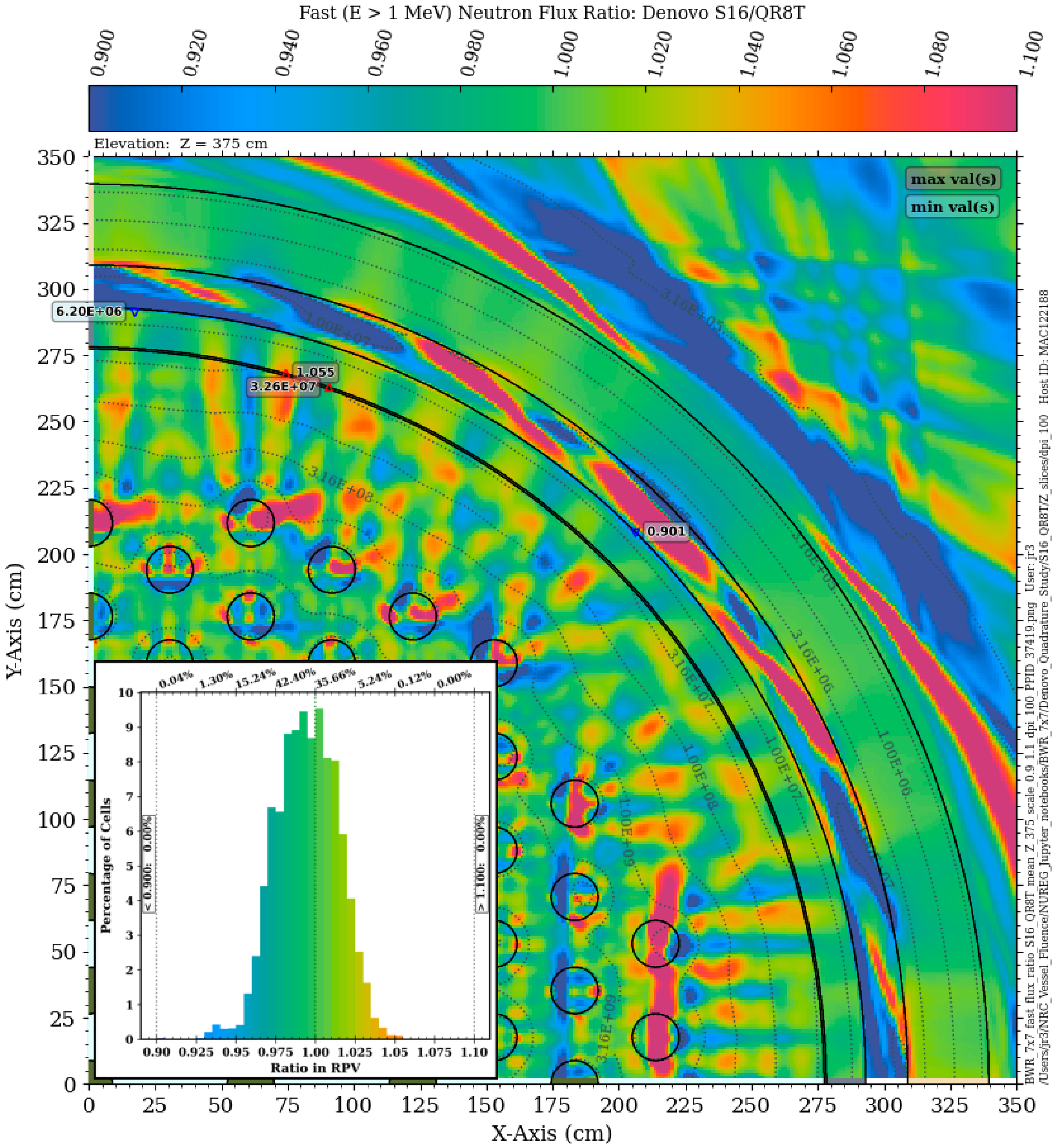

Figure 6-44 Fast neutron flux ratio in the BWR reference model: S16/QR8T quadrature. Plan view at Z = 375 cm. The contour lines are the fast flux from the QR8T solution. Minimum and maximum values within the RPV are shown for the QR8T fast flux and the S16/QR8T fast flux ratio

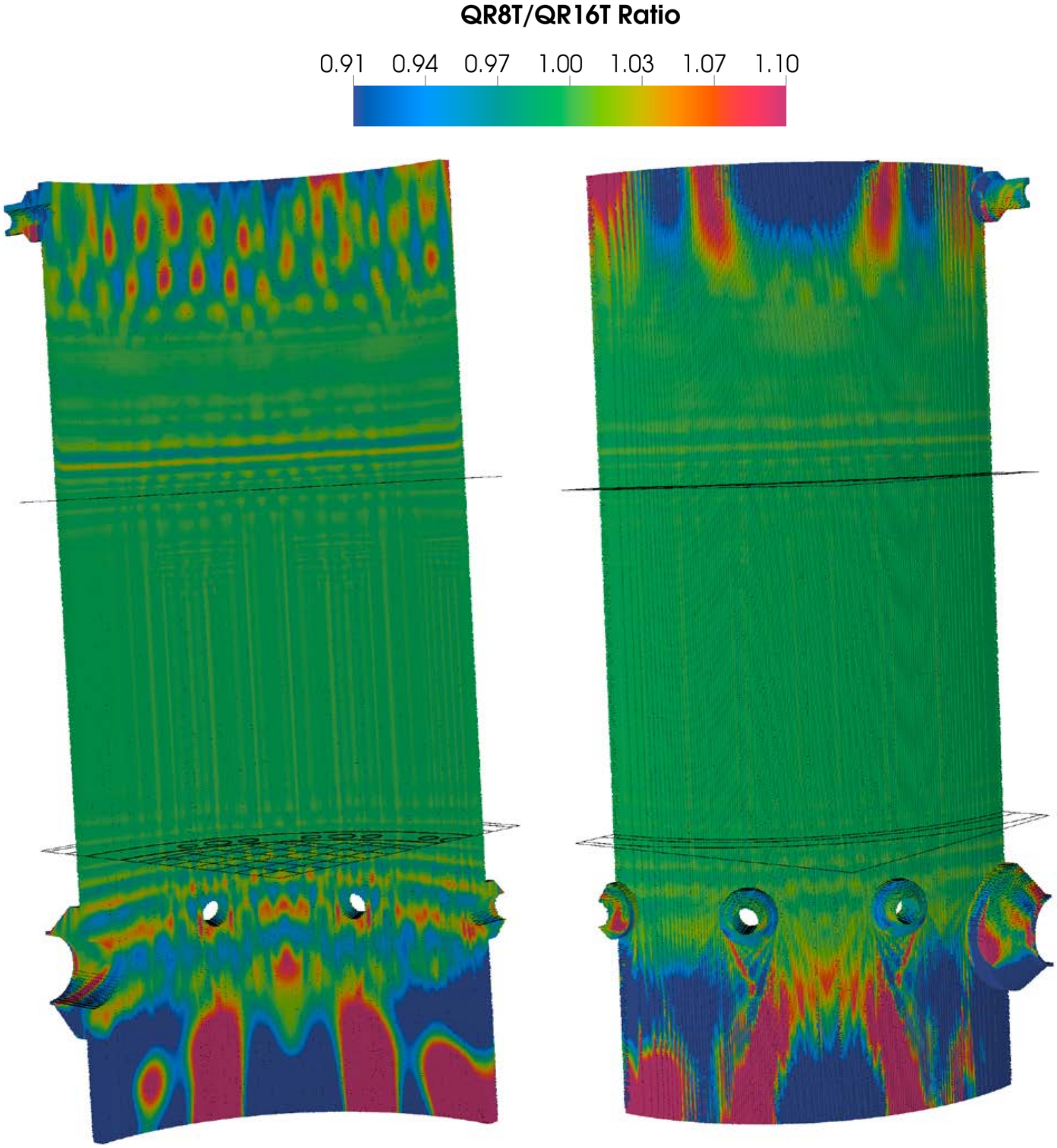

Figure 6-45 Fast neutron flux ratio on the inner and outer surfaces of the RPV for the BWR reference model. The ratio is for a Denovo QR8T solution relative to a QR16T solution. The boundary planes show the extent of the fuel assemblies

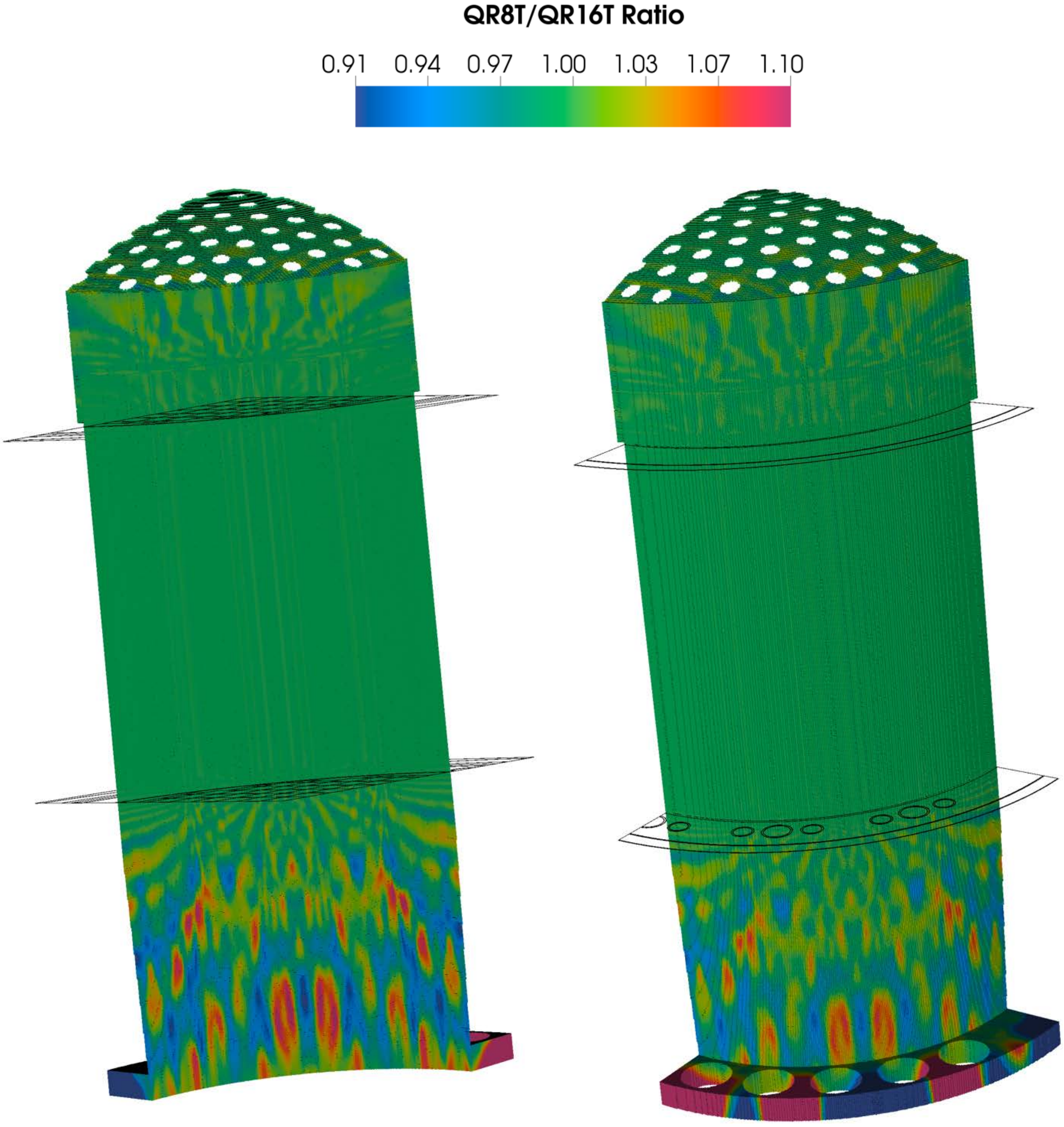

Figure 6-46 **Fast neutron flux ratio on the inner and outer surfaces of the shroud for the BWR reference model. The ratio is for a Denovo QR8T solution relative to a QR16T solution. The boundary planes show the extent of the fuel assemblies**

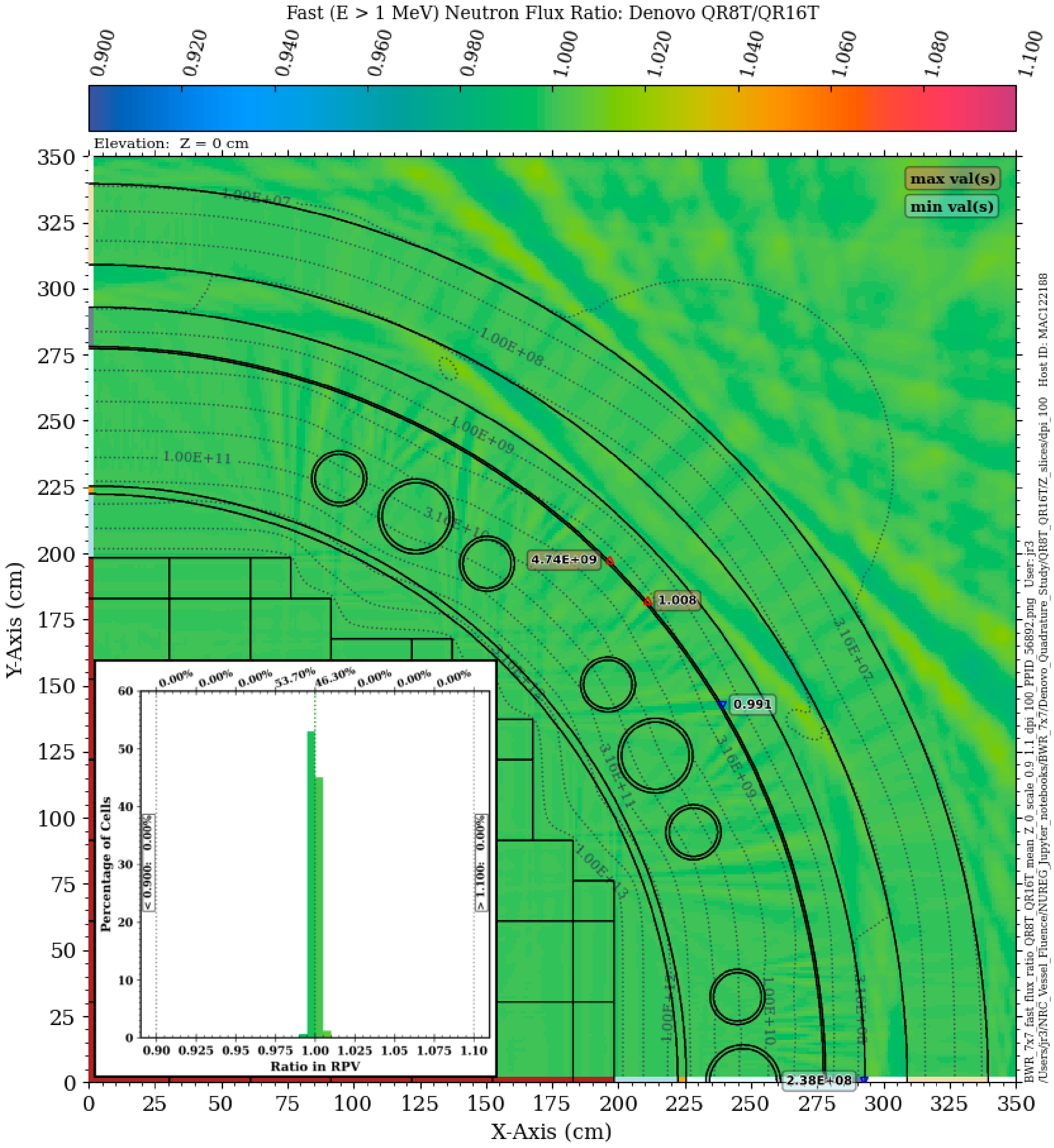

Figure 6-47 Fast neutron flux ratio in the BWR reference model: QR8T/QR16T quadrature. Plan view at Z = 0 cm. The contour lines are the fast flux from the QR16T solution. Minimum and maximum values within the RPV are shown for the QR16T fast flux and the QR8T/QR16T fast flux ratio

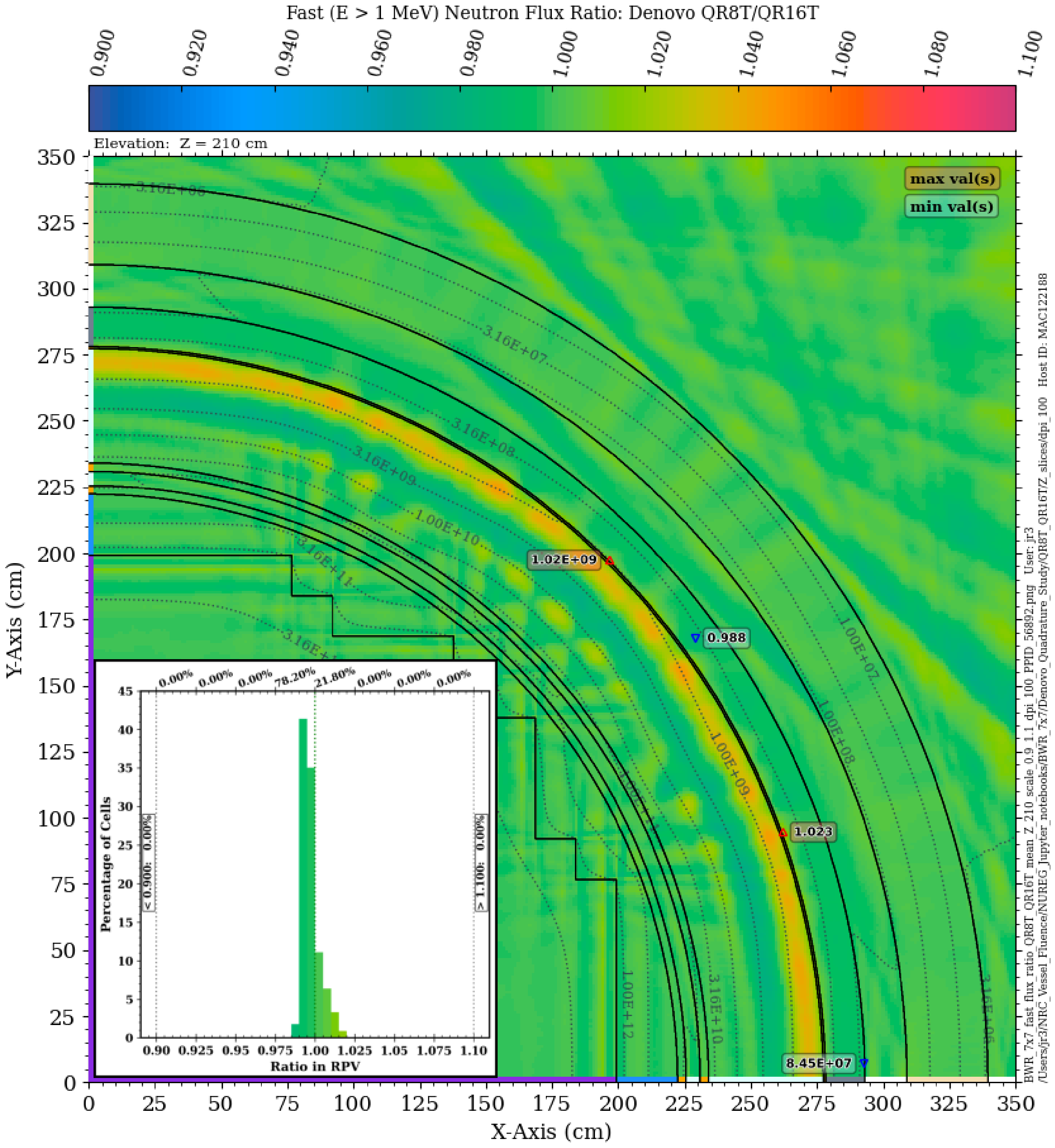

Figure 6-48 Fast neutron flux ratio in the BWR reference model: QR8T/QR16T quadrature. Plan view at Z = 210 cm. The contour lines are the fast flux from the QR16T solution. Minimum and maximum values within the RPV are shown for the QR16T fast flux and the QR8T/QR16T fast flux ratio

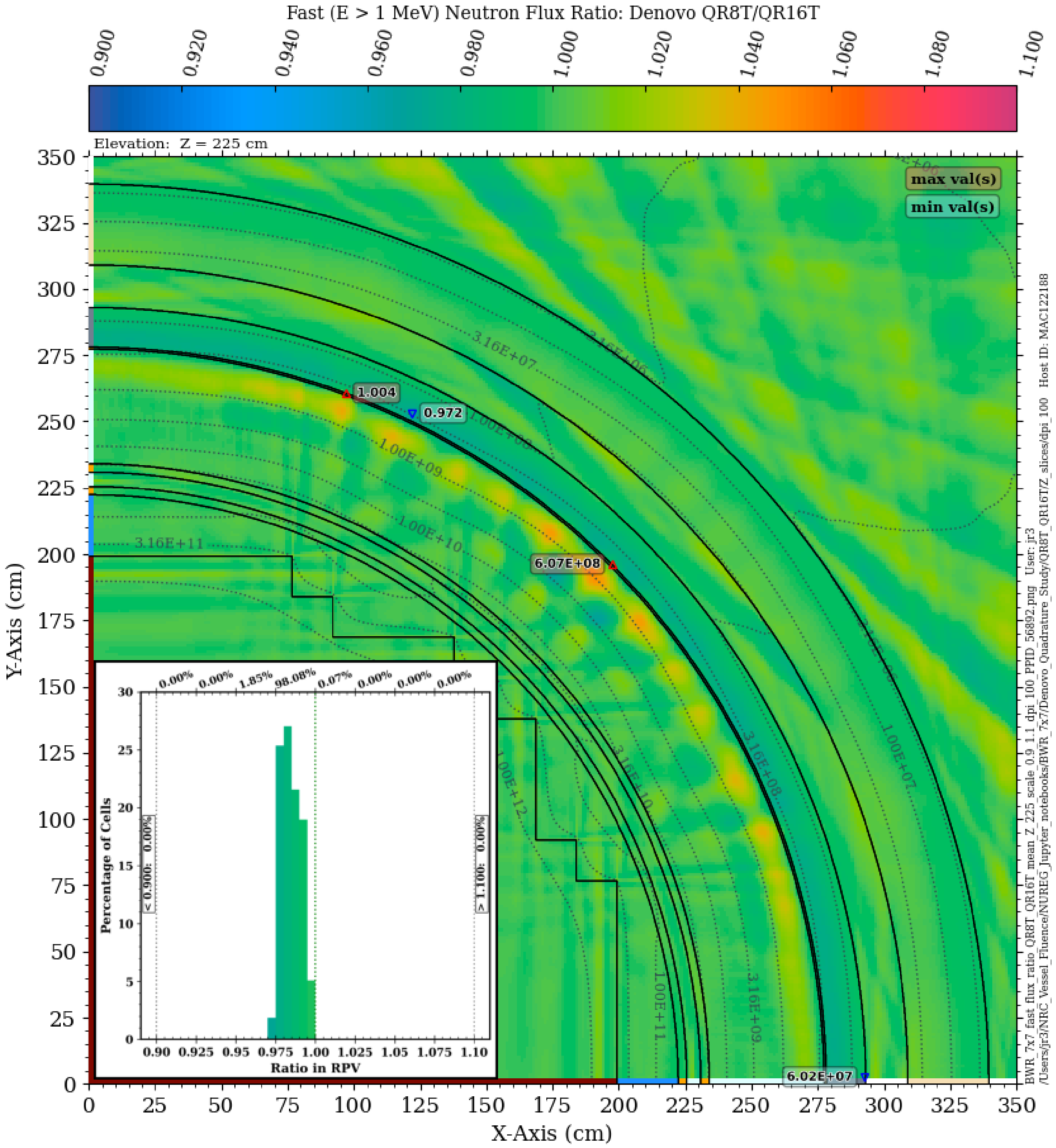

Figure 6-49 Fast neutron flux ratio in the BWR reference model: QR8T/QR16T quadrature. Plan view at Z = 225 cm. The contour lines are the fast flux from the QR16T solution. Minimum and maximum values within the RPV are shown for the QR16T fast flux and the QR8T/QR16T fast flux ratio

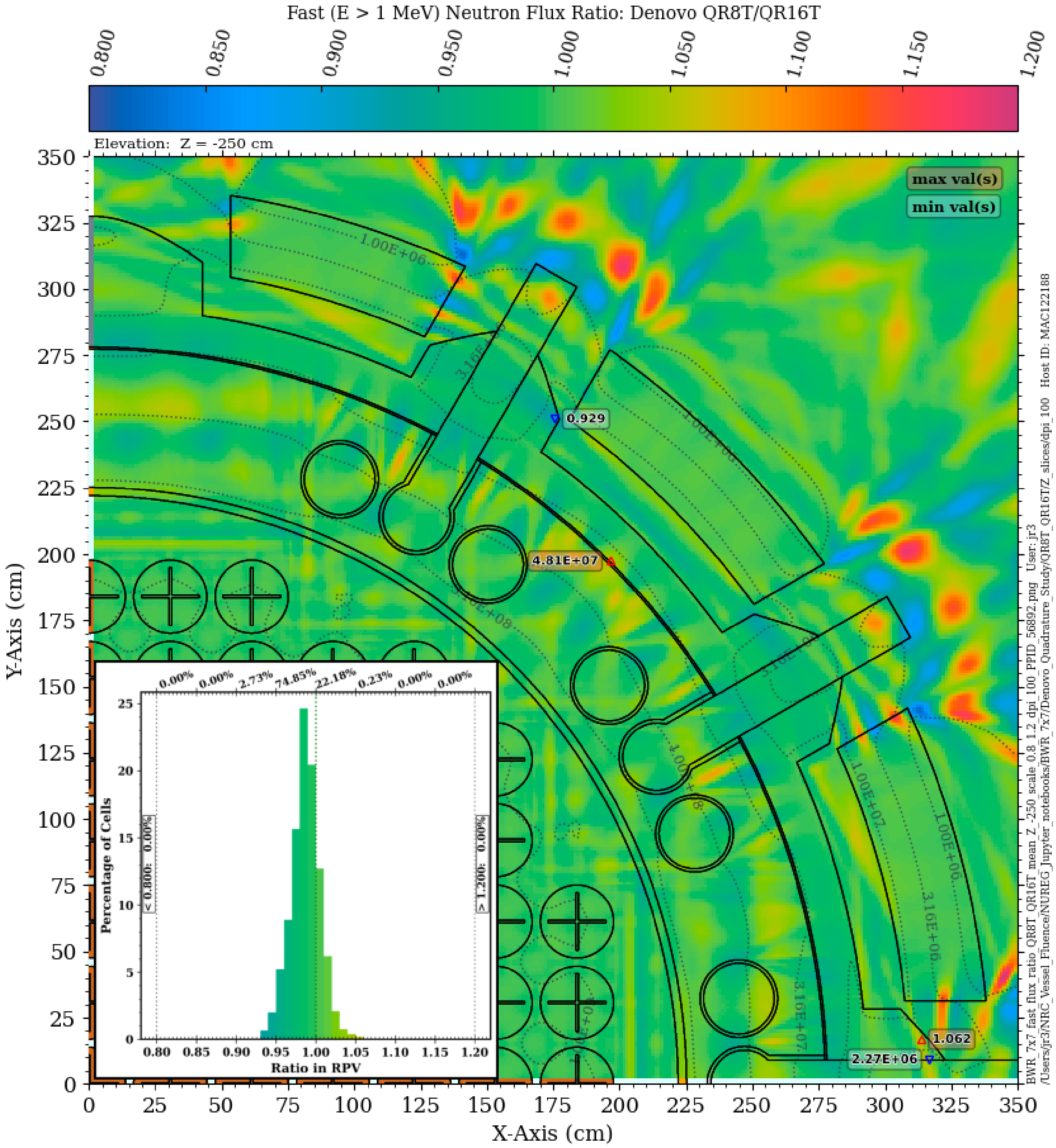

Figure 6-50 **Fast neutron flux ratio in the BWR reference model: QR8T/QR16T quadrature. Plan view at Z = -250 cm. The contour lines are the fast flux from the QR16T solution. Minimum and maximum values within the RPV are shown for the QR16T fast flux and the QR8T/QR16T fast flux ratio. Note the change in scale relative to the three previous plots**

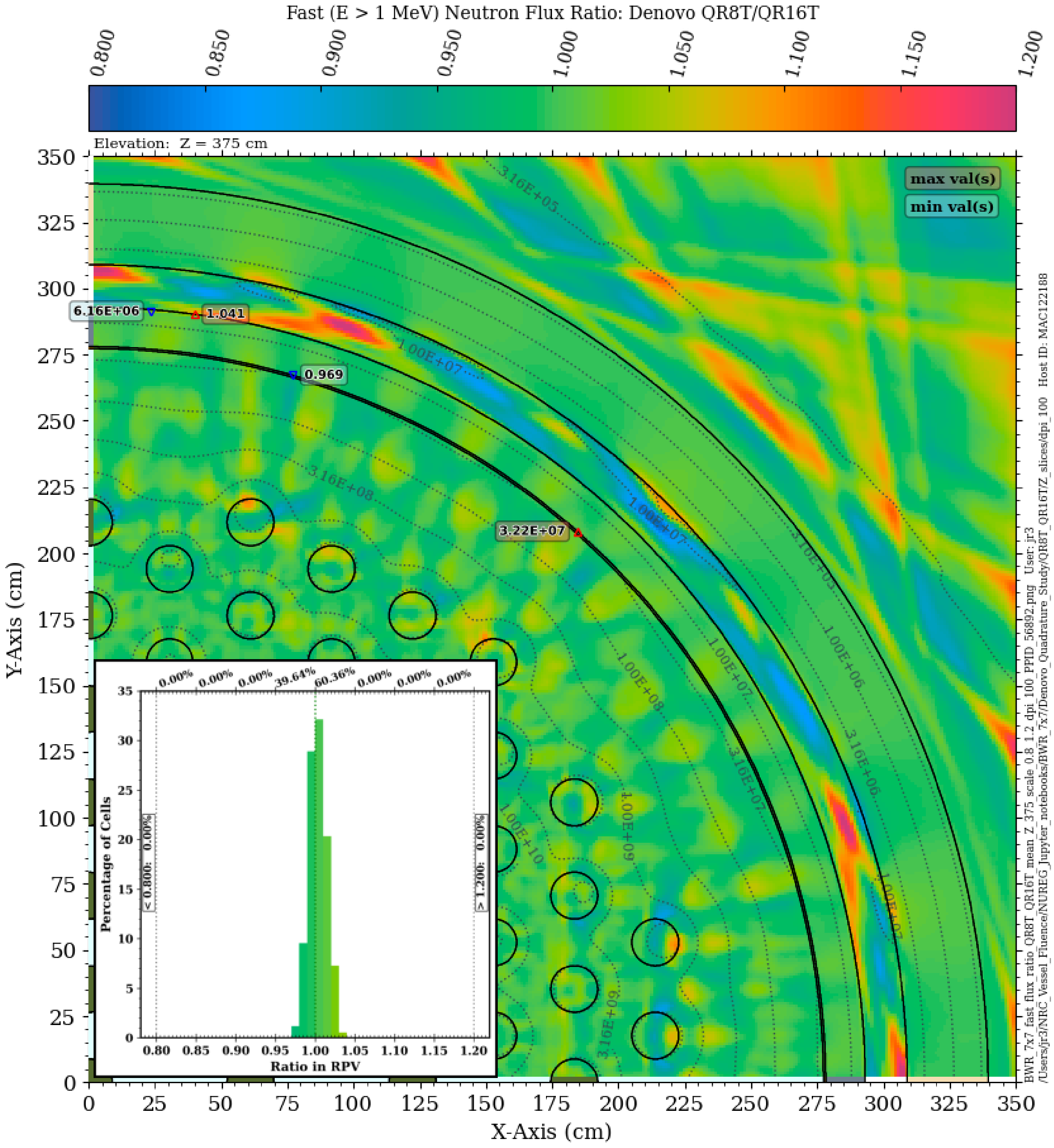

Figure 6-51 Fast neutron flux ratio in the BWR reference model: QR8T/QR16T quadrature. Plan view at Z = 375 cm. The contour lines are the fast flux from the QR16T solution. Minimum and maximum values within the RPV are shown for the QR16T fast flux and the QR8T/QR16T fast flux ratio

6.5 Comparison of Denovo solutions to multigroup Shift solutions: PWR model

The comparisons presented in Section 6.3 and Section 6.4 demonstrate that quadrature sensitivity (i.e., the variation between two discrete ordinates calculations in which the only parameter that is modified is the quadrature set) for RPV fluence calculations is more pronounced in the extended beltline region compared to the traditional beltline region.

As a means of determining the effect of selection of quadrature sets on RPV fluence calculations, a Shift hybrid Monte Carlo calculation was performed using the homogenized core PWR model and the BUGLE-B7 MG library. The Shift fast flux was tallied using a mesh tally with the uniform 1 cm mesh that was used in the Denovo calculations. Because the Shift and Denovo calculations employ the same model and same MG cross sections, and the Denovo mesh was demonstrated to provide convergence of the Denovo solution with respect to mesh, the ratios of the Denovo calculations to the Shift solution provide a means of effectively isolating the quadrature effect in the Denovo calculations.

The Denovo S8, S16, QR8T, and QR16T solutions are compared to the Shift solution in Figure 6-52 through Figure 6-67. The results of these comparisons can be summarized as follows.

Comparison of Figure 6-52 with Figure 6-2 confirms that the ray effects seen in Figure 6-2 are indeed an indication of (minor) deficiencies in the S8 quadrature, even within the traditional beltline region. As noted in Section 6.3.1, these ray effects are associated with the corner fuel assemblies along the periphery of the core. Figure 6-53 and Figure 6-54 confirm the presence of significant ray effects for the S8 solution in the cavity gap and adjacent portions of the RPV and bioshield. Figure 6-55 confirms the presence of ray effects due to the former plate material interfaces, which is consistent with Figure 6-1 and Figure 6-2. However, as shown in the histograms, there is little indication of overall bias until the distance from the core midplane is large (Figure 6-54).

Comparison of the S16 and QR8T quadrature results is instructive, as both sets contain 36 angles per octant and thus have equivalent computational footprints. At the core midplane, both the S16 and QR8T solutions show excellent agreement with the BUGLE-B7 Shift solution, although the S16 solution (Figure 6-56) shows slightly more evidence of ray effects emanating from the corner fuel assemblies compared with the QR8T solution (Figure 6-60). At the extended beltline elevations of Z = -70 cm (Figure 6-57 and Figure 6-61) and Z = 470 cm (Figure 6-58 and Figure 6-62), both Denovo solutions exhibit ray effects in the cavity gap region. At both of these elevations, the QR8T solution provides better agreement than the S16 solution when compared to the Shift solution.

The same is true for elevation plots at an azimuthal angle of 270.5°, as shown in Figure 6-59 (S16) and Figure 6-63 (QR8T). While both solutions exhibit ray effects in the cavity gap, the QR8T solution does not have the ray effects that are seen in the S16 solution emanating from the former plates.

Denovo/Shift ratios for the QR16T solution are presented in Figure 6-64 through Figure 6-67. The QR16T quadrature, with 136 angles per octant, substantially reduces the ray effects in the cavity gap, although there are still locations in the extended beltline region where the QR16T Denovo solution differs from the Shift solution my 15% or more.

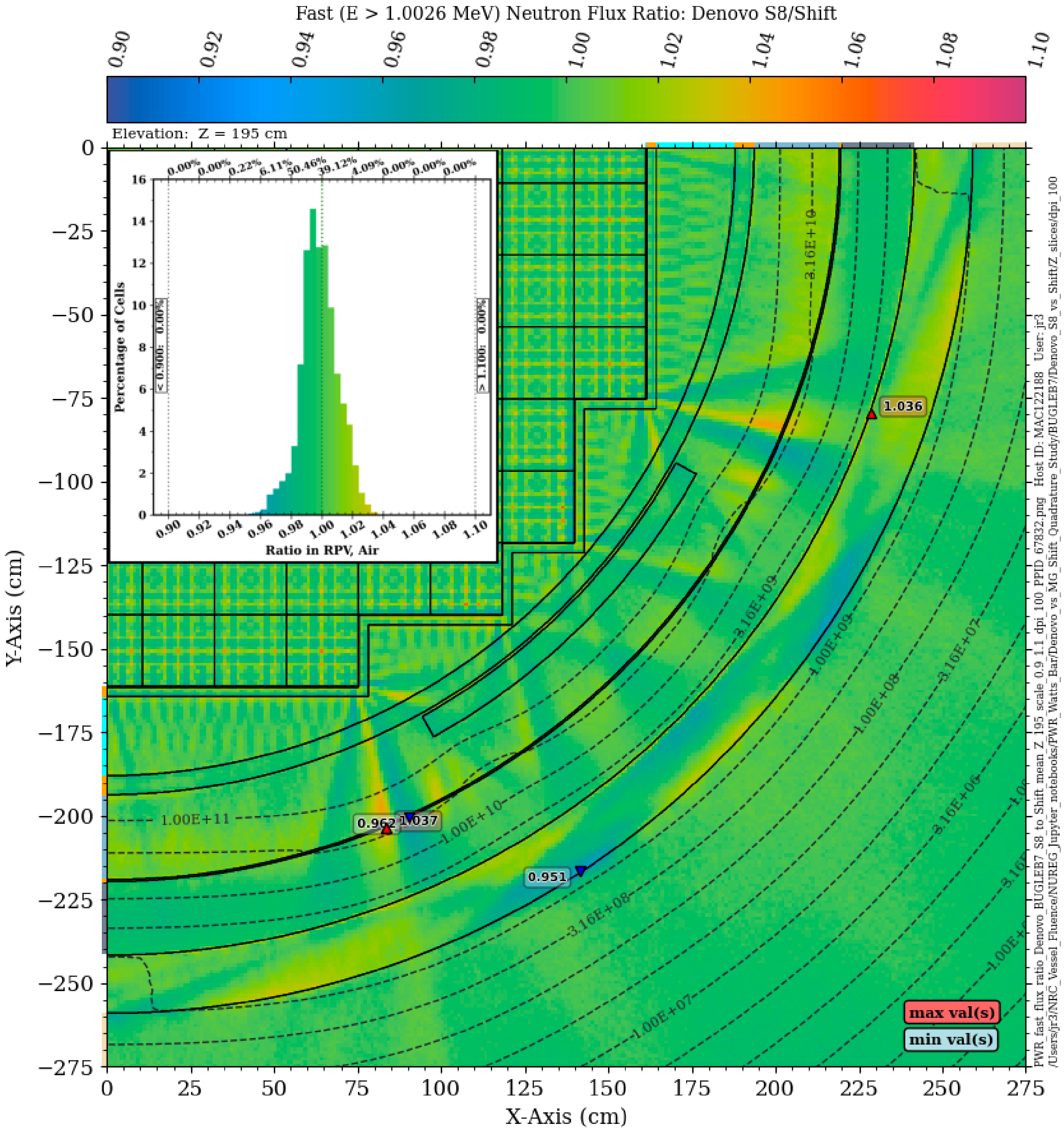

Figure 6-52 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S8 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 195 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap

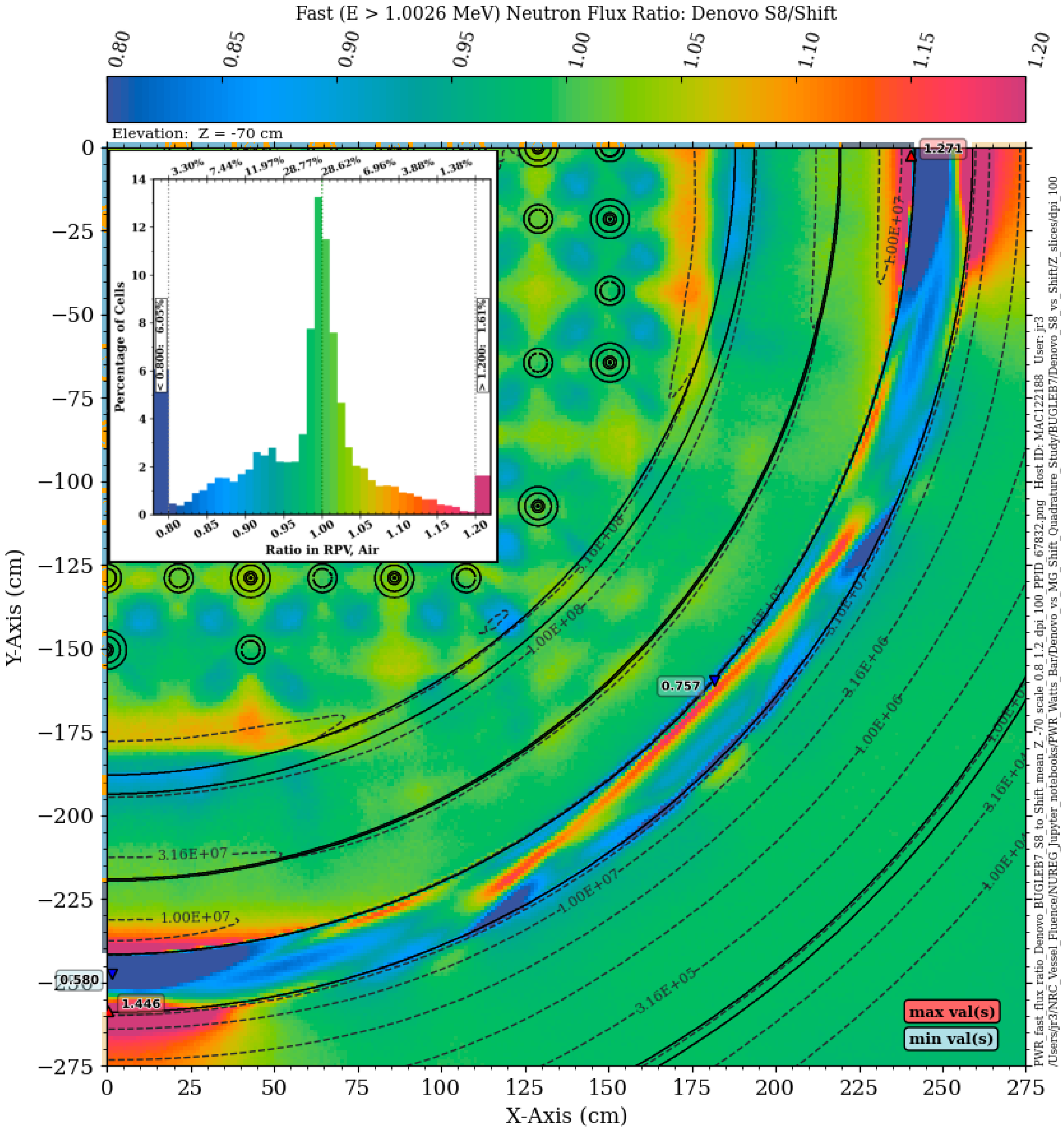

Figure 6-53 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S8 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = -70 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap. Note the change in scale relative to Figure 6-52

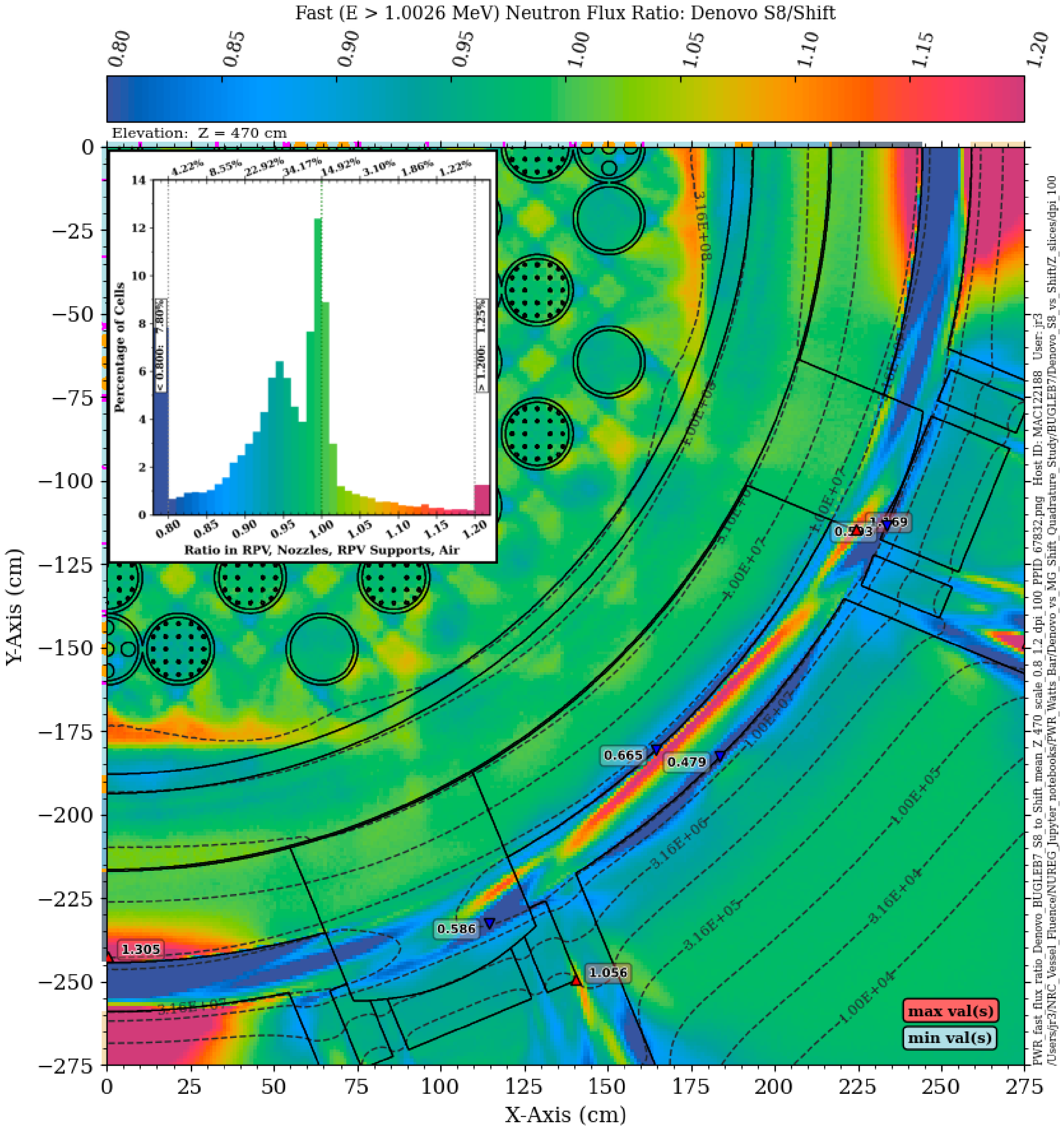

Figure 6-54 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S8 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 470 cm. The histogram plot shows the distribution of ratio values within the RPV, nozzle, vessel supports, and cavity gap

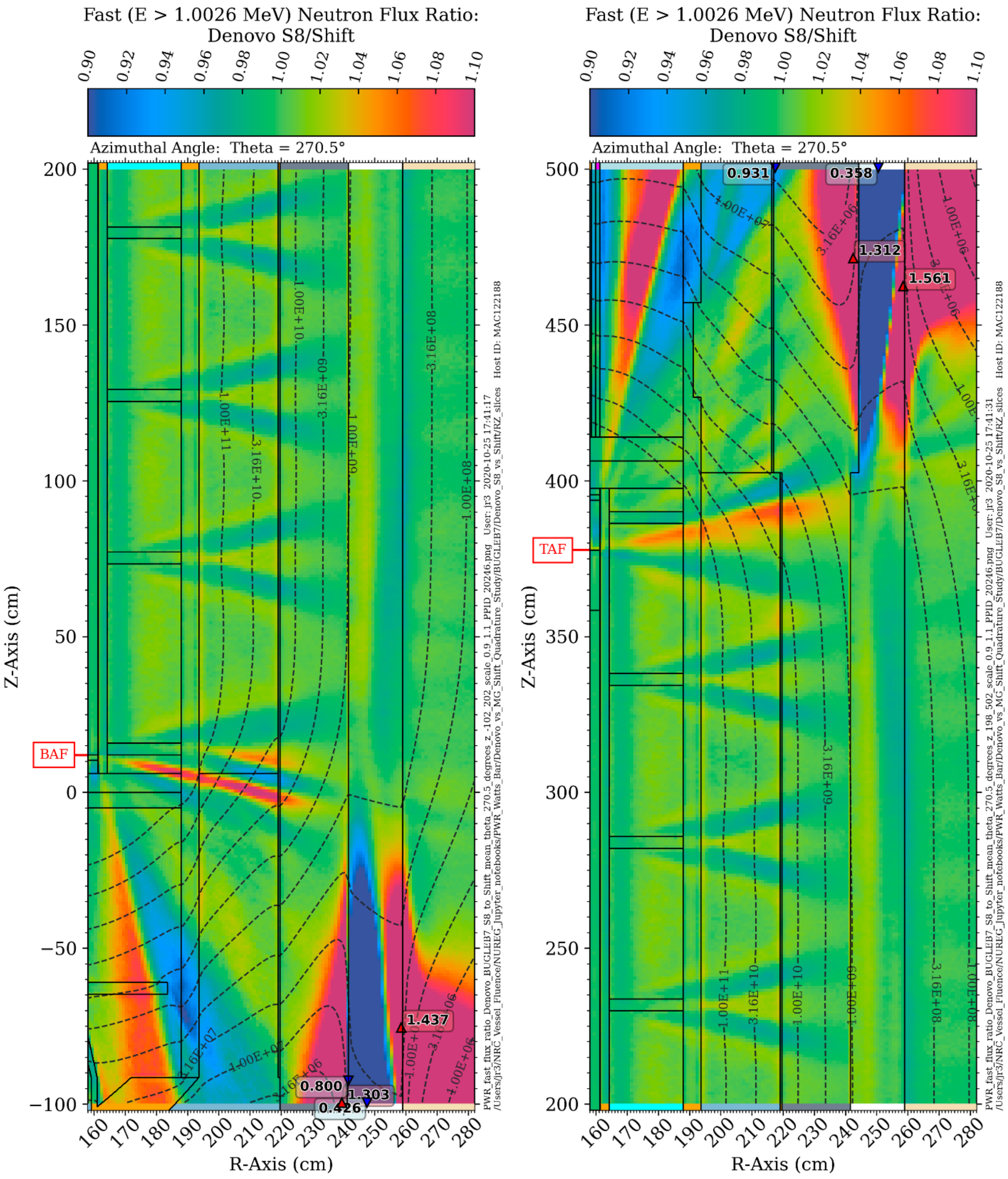

Figure 6-55 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S8 quadrature to a BUGLE-B7 Shift solution with the PWR model. Elevation view at an azimuthal angle of 270.5°

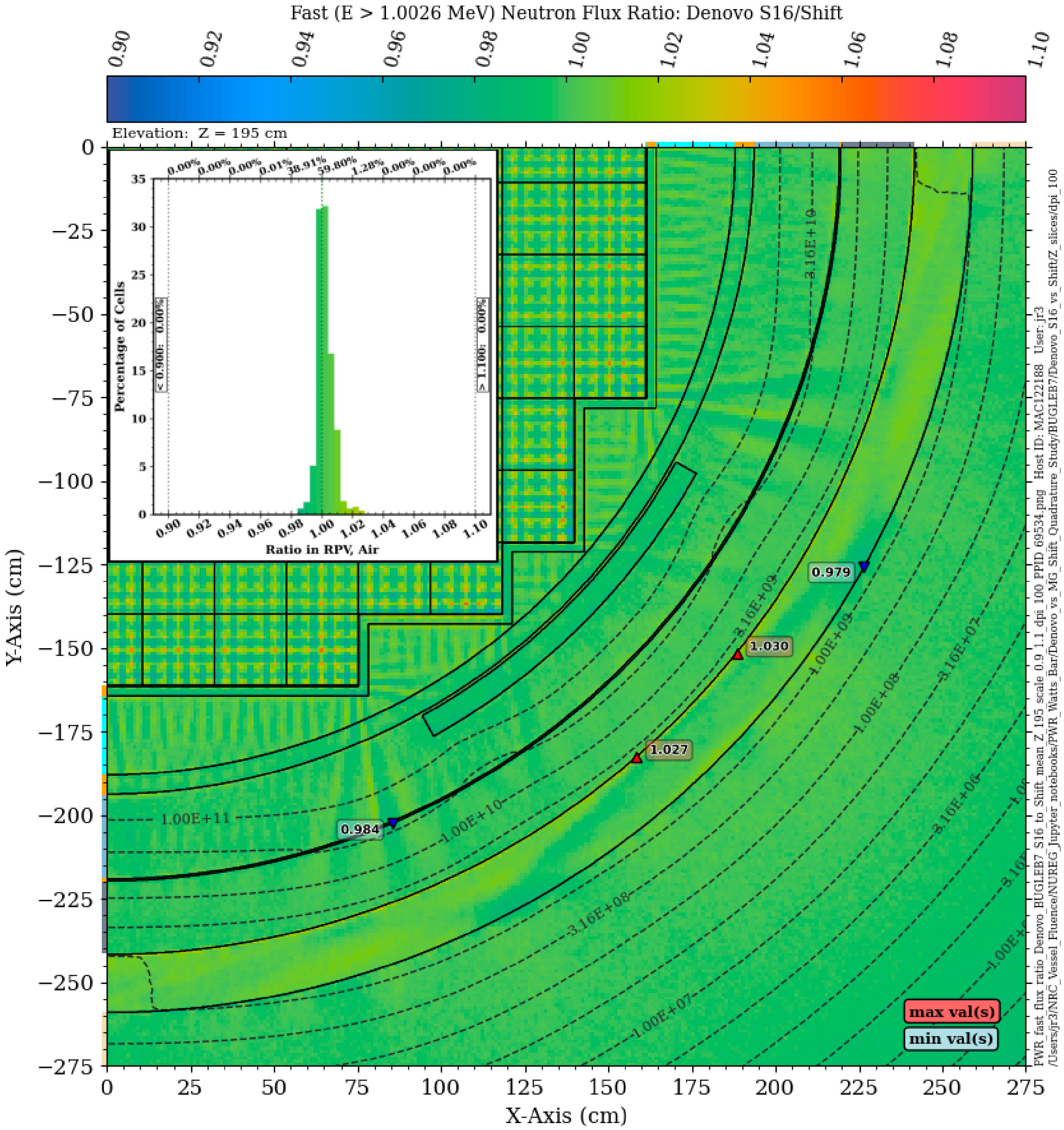

Figure 6-56 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S16 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 195 cm. The histogram plot shows the distribution of ratio values with the RPV and cavity gap

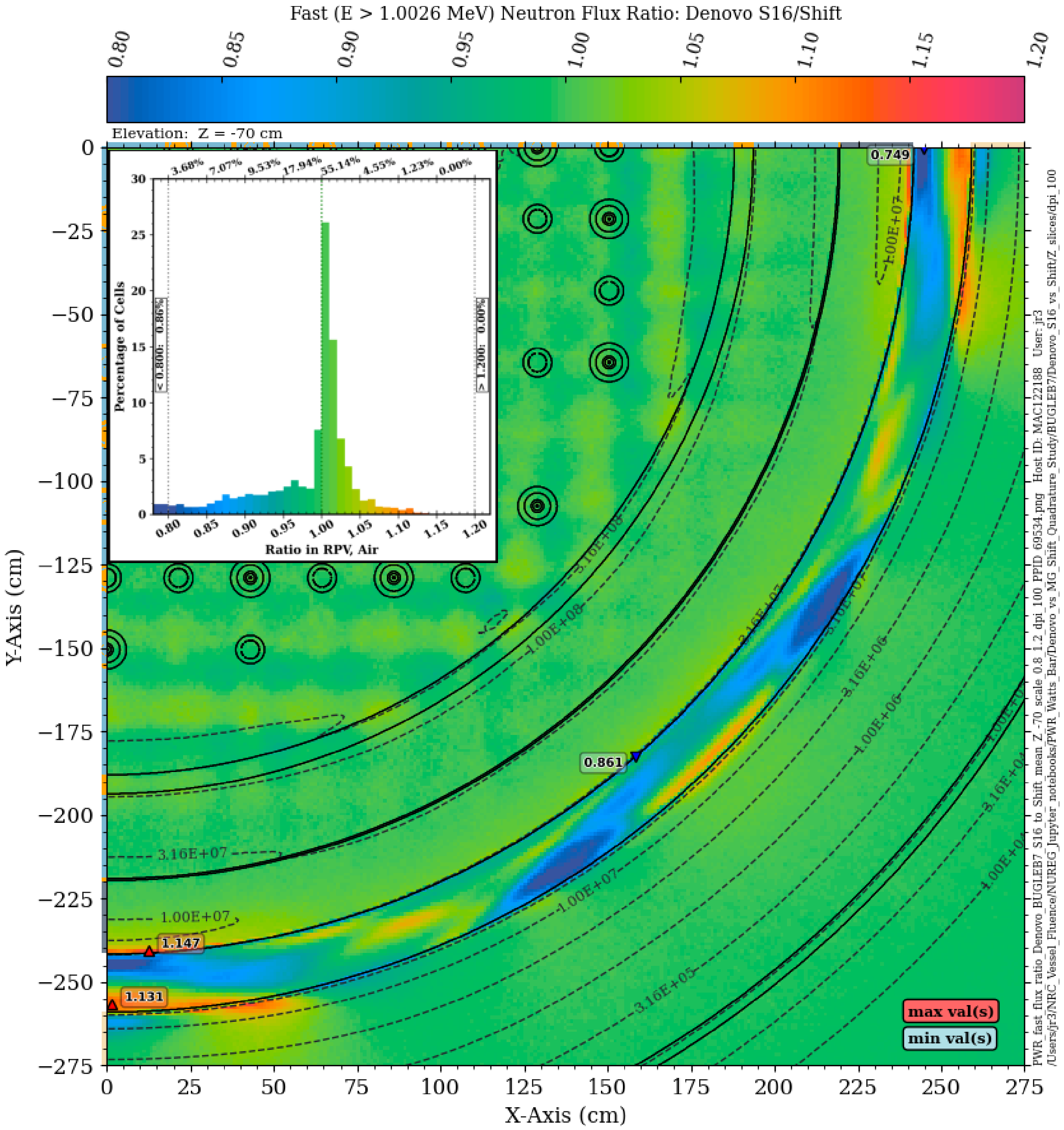

Figure 6-57 **Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S16 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = -70 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap. Note the change in scale relative to Figure 6-56**

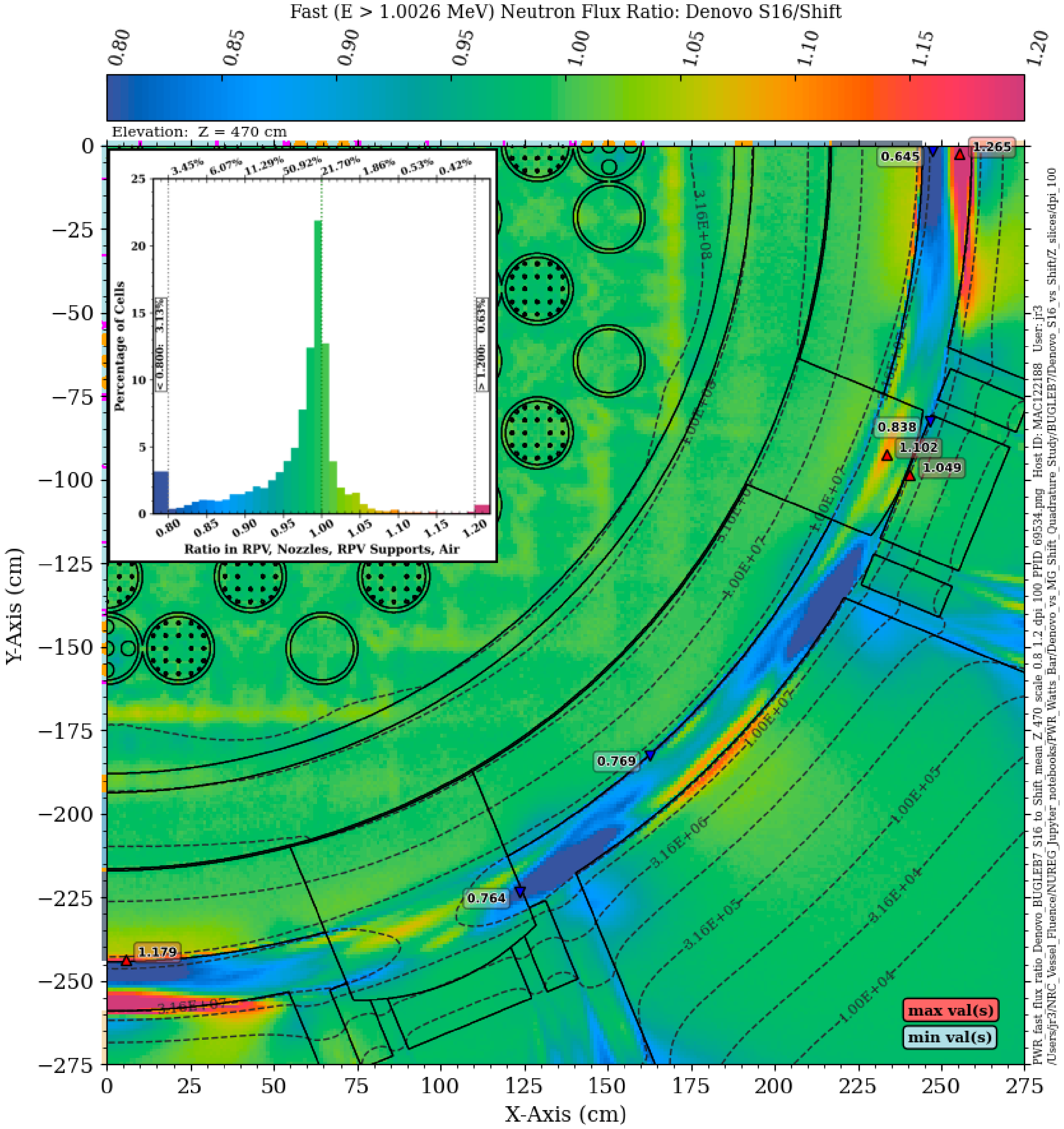

Figure 6-58 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S16 quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 470 cm. The histogram plot shows the distribution of ratio values within the RPV, nozzle, vessel supports, and cavity gap

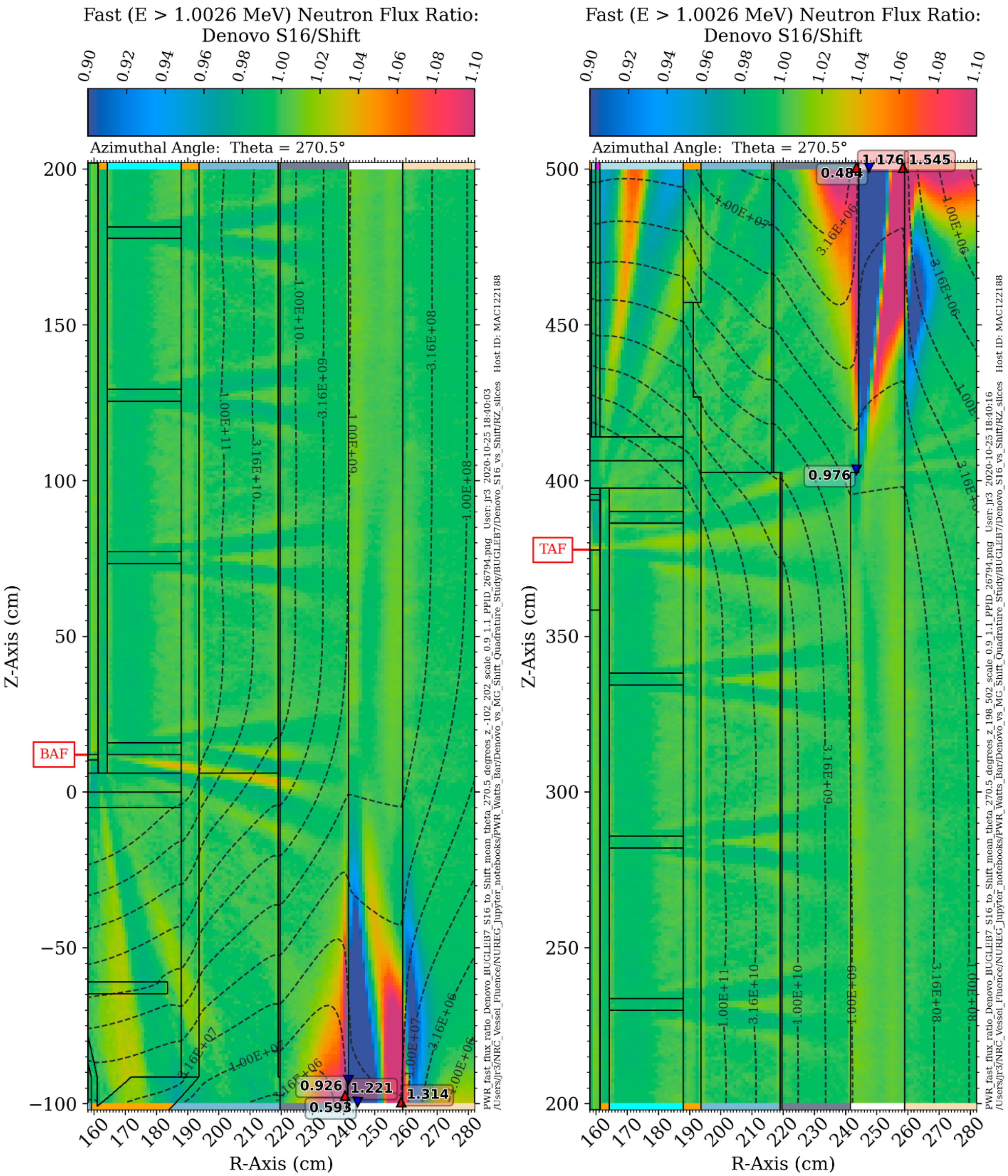

Figure 6-59 **Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with S16 quadrature to a BUGLE-B7 Shift solution with the PWR model. Elevation view at an azimuthal angle of 270.5°**

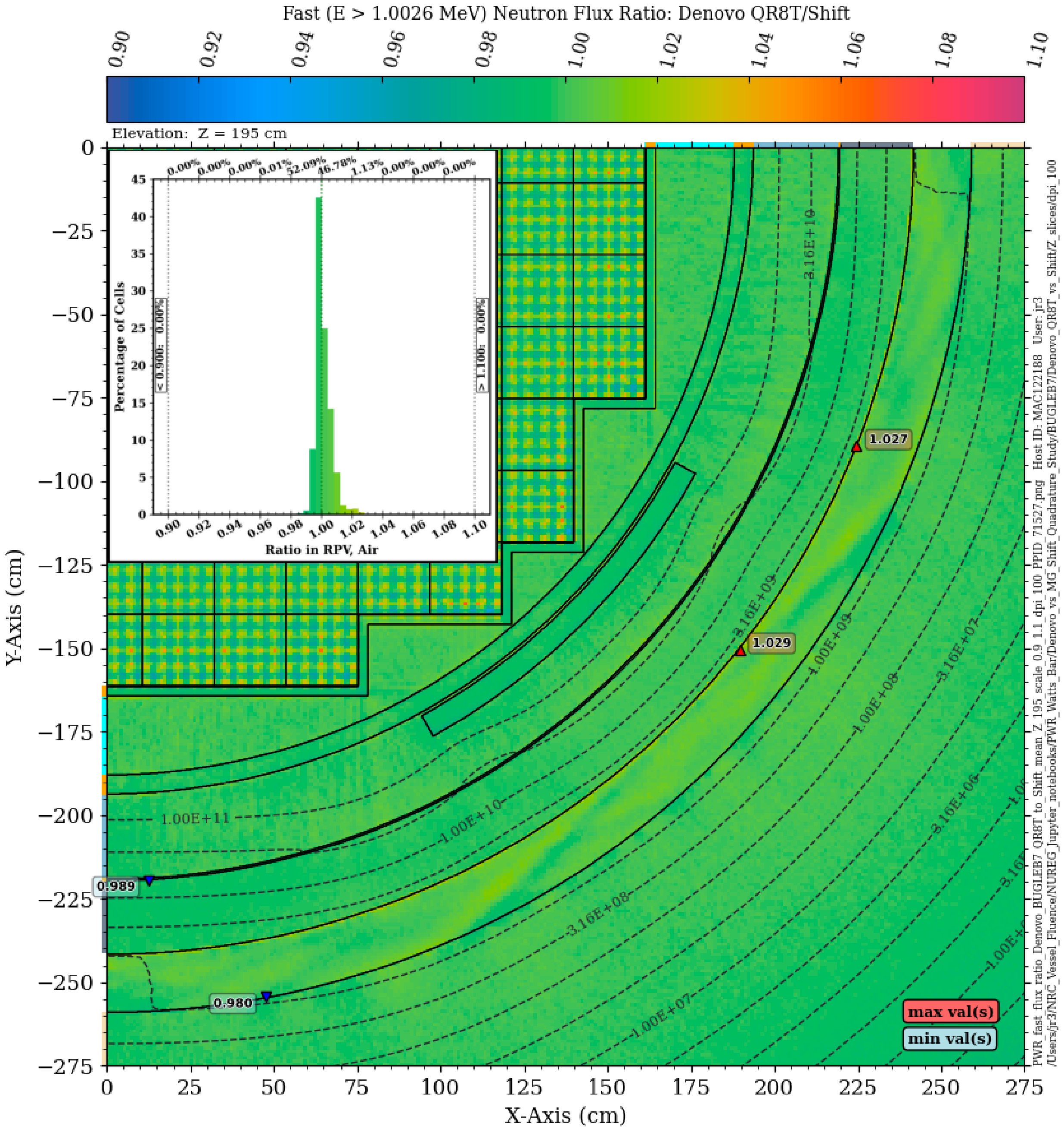

Figure 6-60 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR8T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 195 cm. The histogram plot shows the distribution of ratio values with the RPV and cavity gap

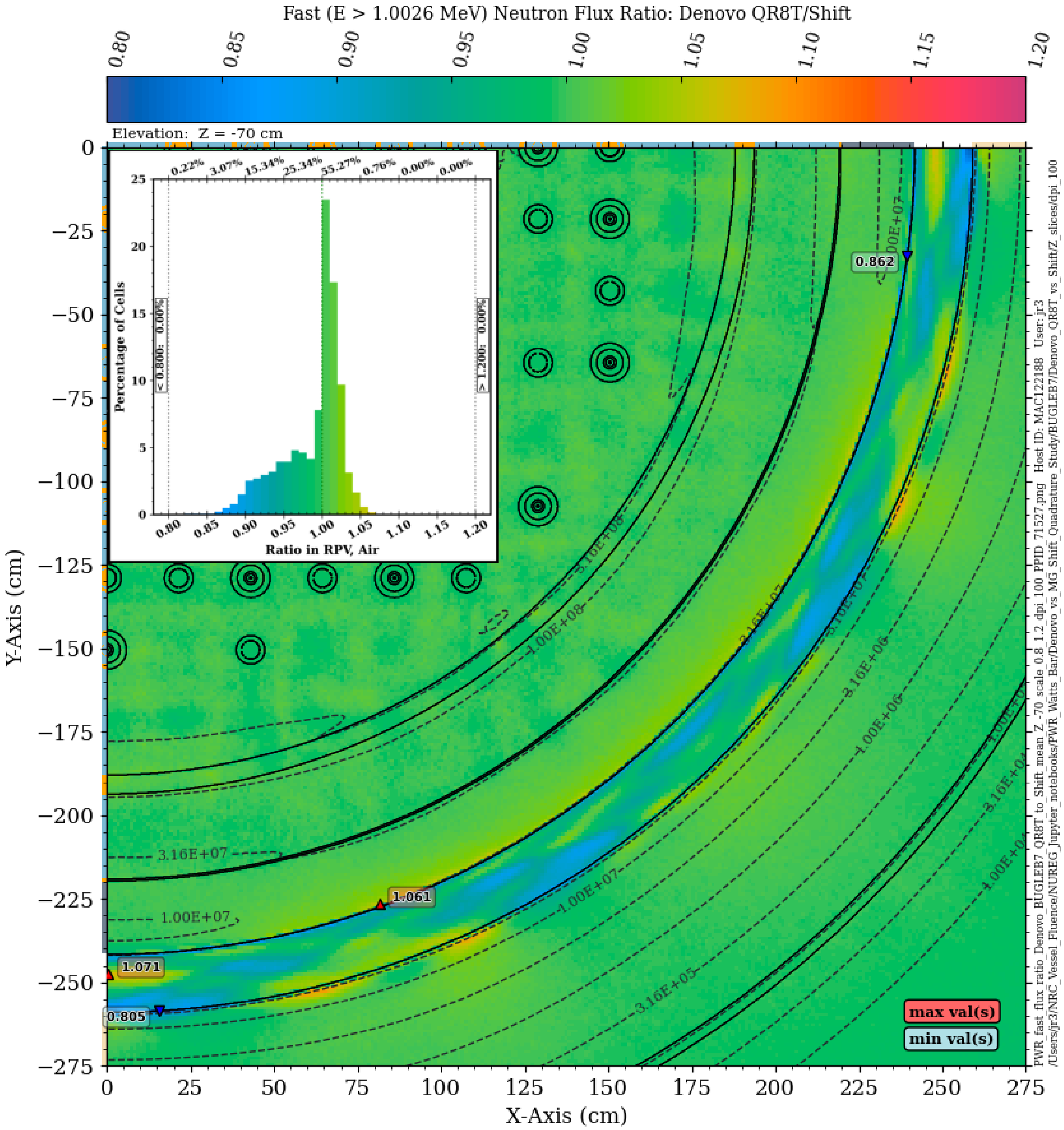

Figure 6-61 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR8T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = -70 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap. Note the change in scale relative to Figure 6-60

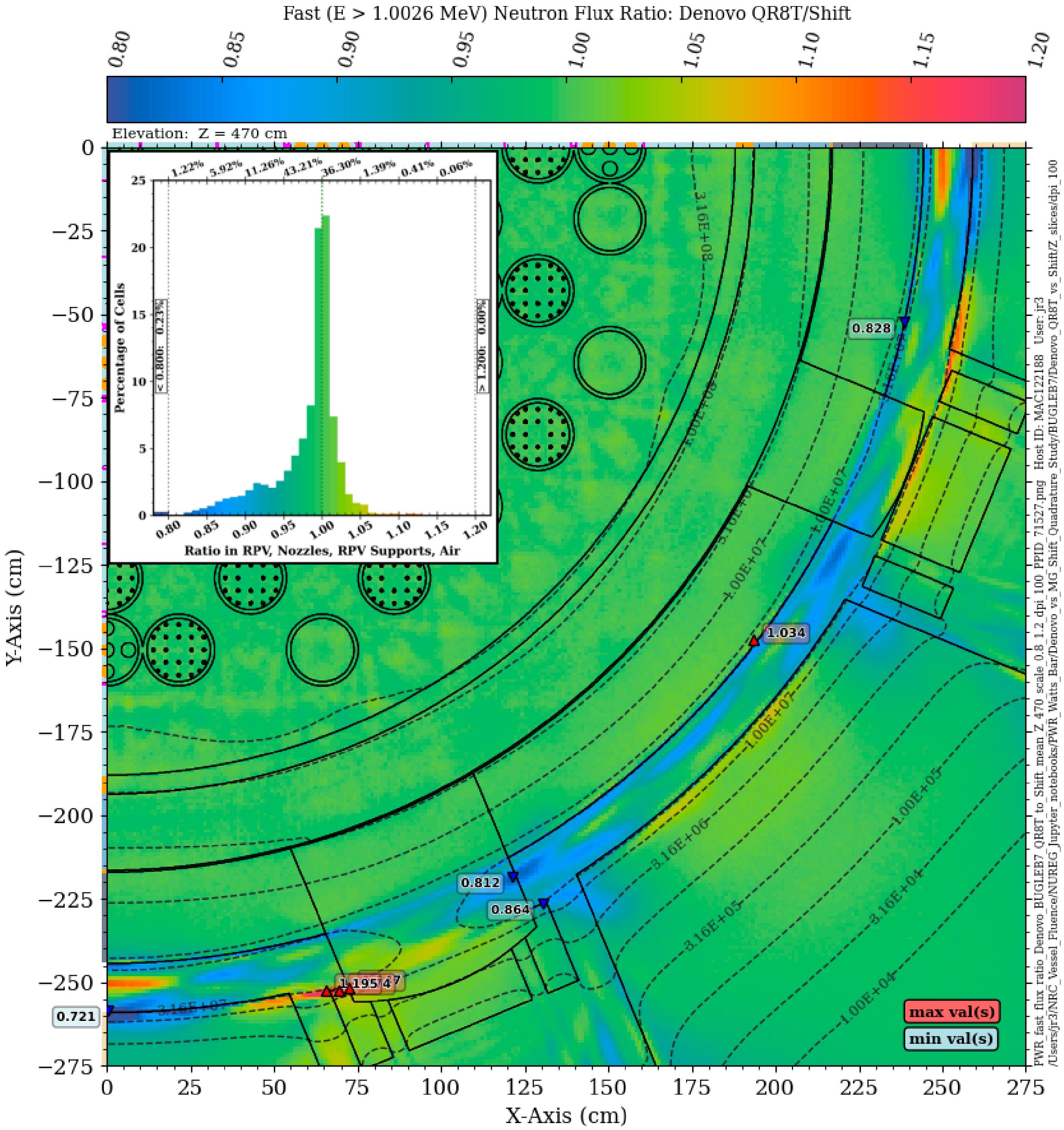

Figure 6-62 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR8T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 470 cm. The histogram plot shows the distribution of ratio values within the RPV, nozzle, vessel supports, and cavity gap

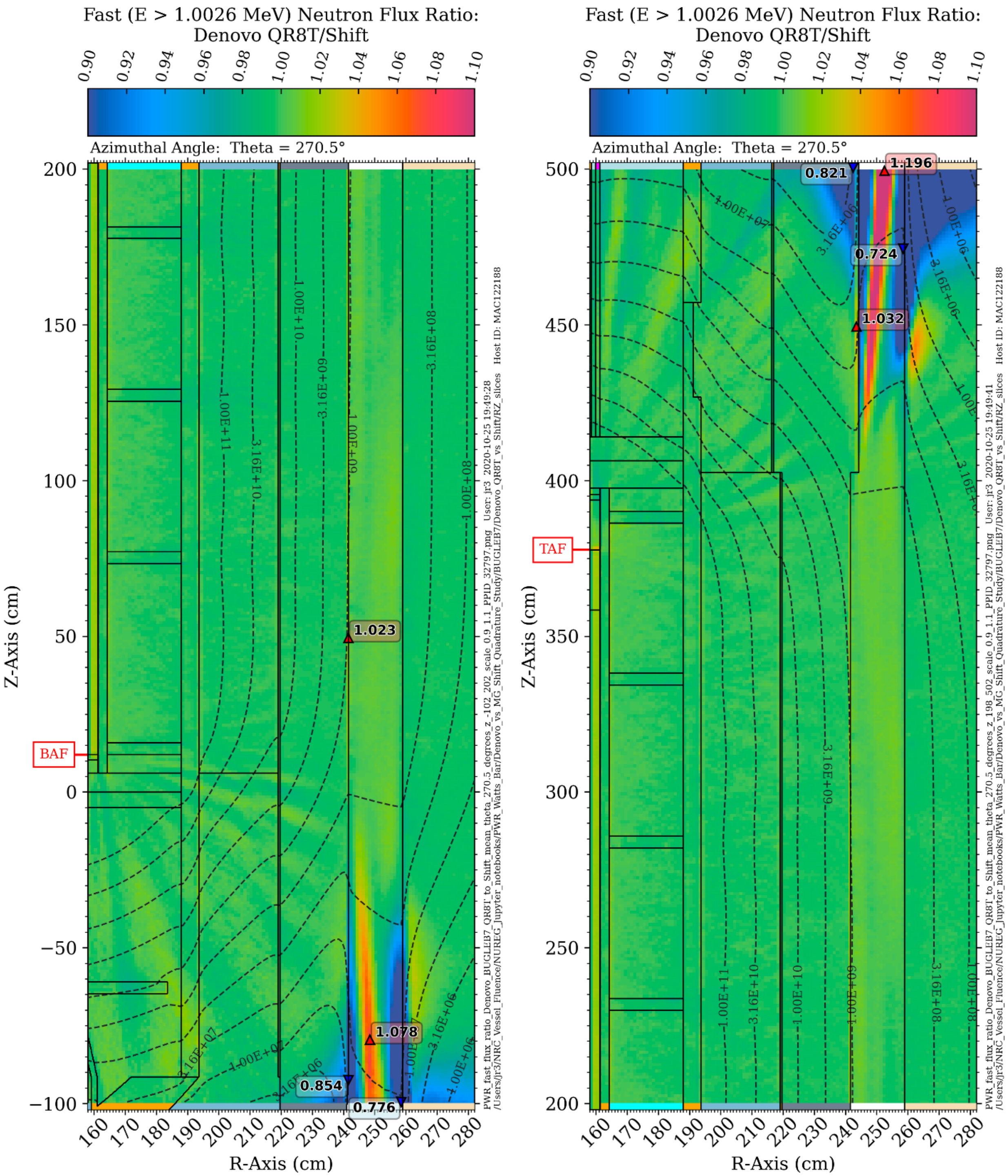

Figure 6-63 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR8T quadrature to a BUGLE-B7 Shift solution with the PWR model. Elevation view at an azimuthal angle of 270.5°

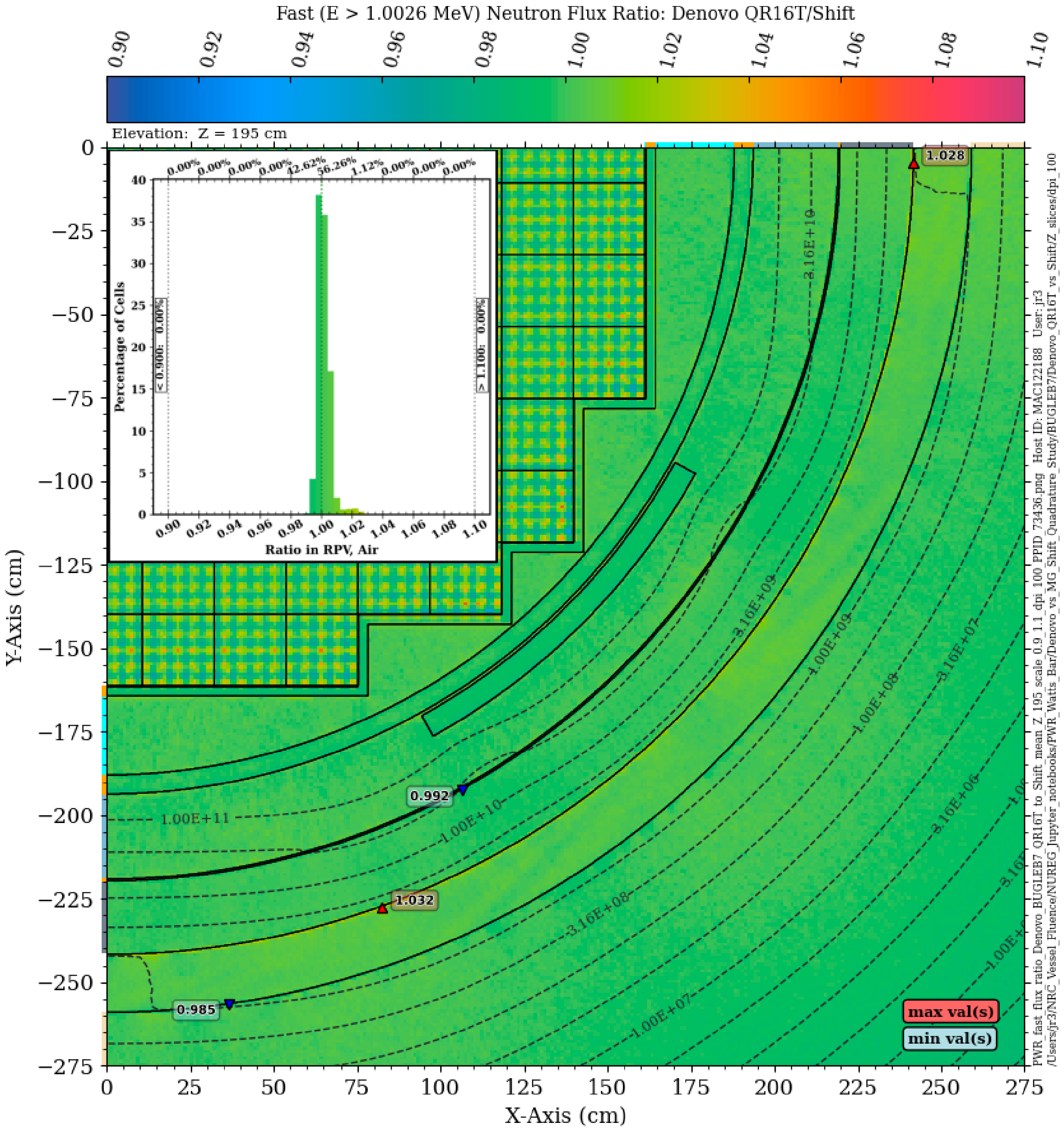

Figure 6-64 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR16T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 195 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap

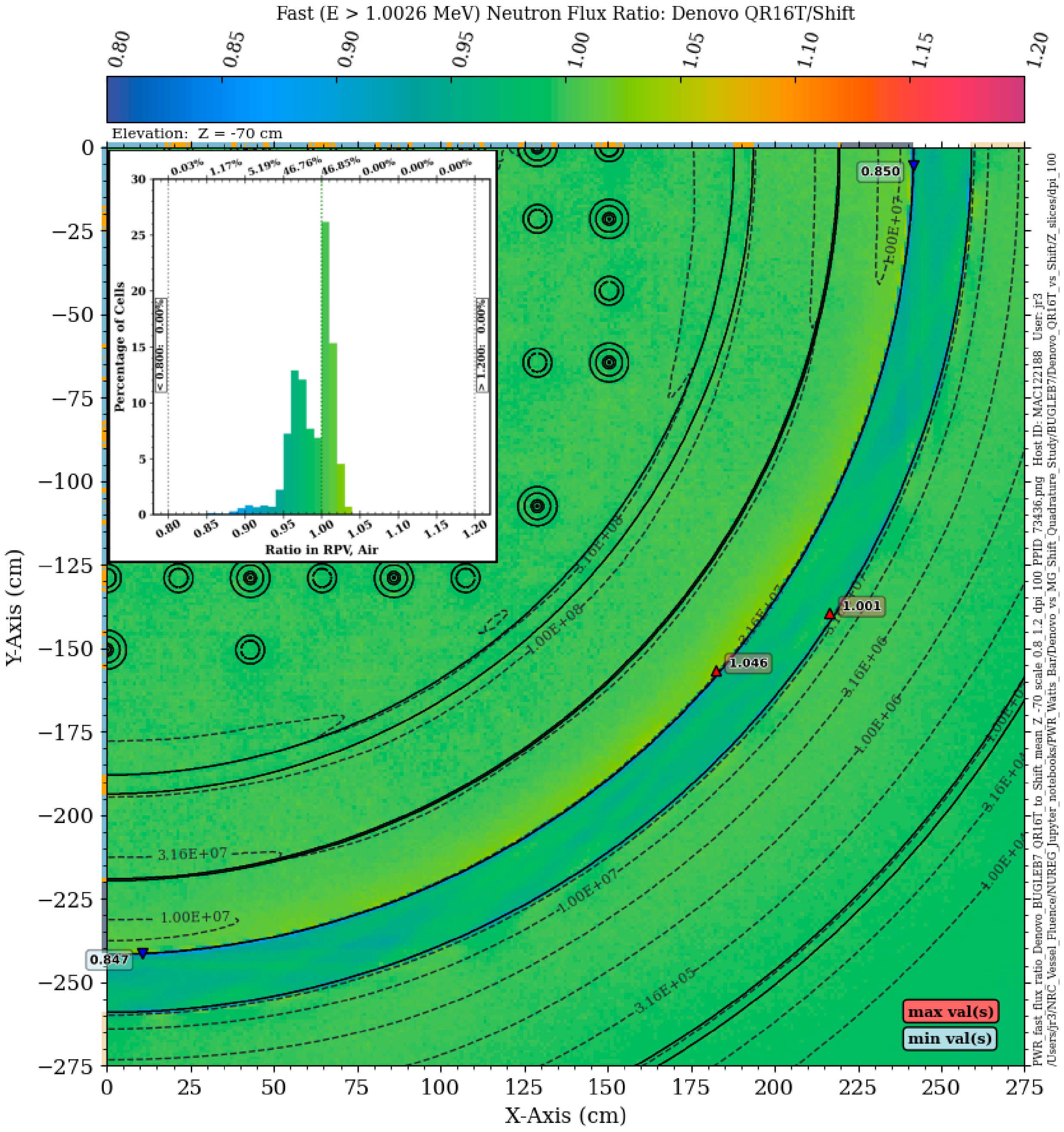

Figure 6-65 **Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR16T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = -70 cm. The histogram plot shows the distribution of ratio values within the RPV and cavity gap. Note the change in scale relative to Figure 6-64**

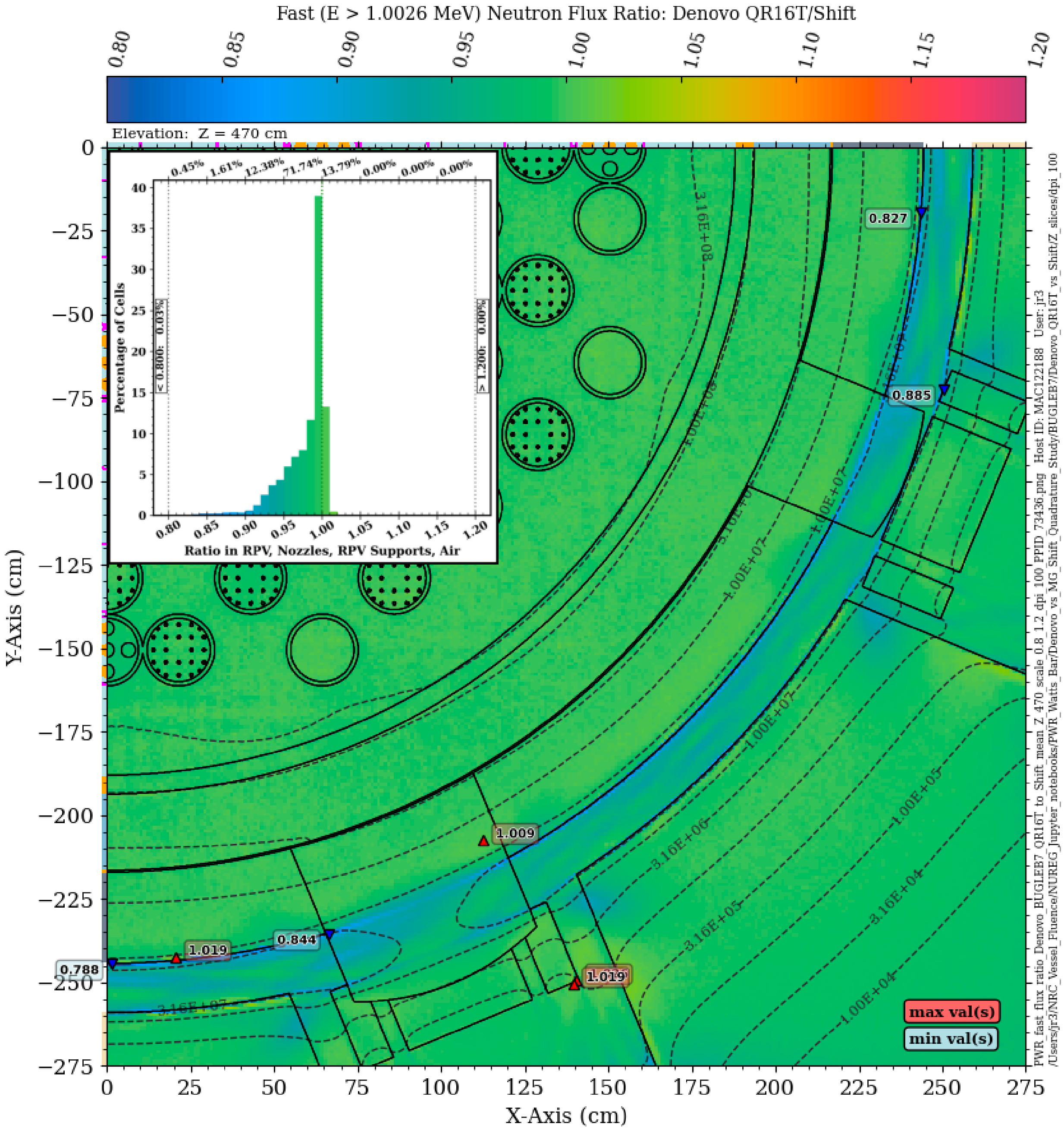

Figure 6-66 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR16T quadrature to a BUGLE-B7 Shift solution with the PWR model. Plan view at Z = 470 cm. The histogram plot shows the distribution of ratio values within the RPV, nozzle, vessel supports, and cavity gap

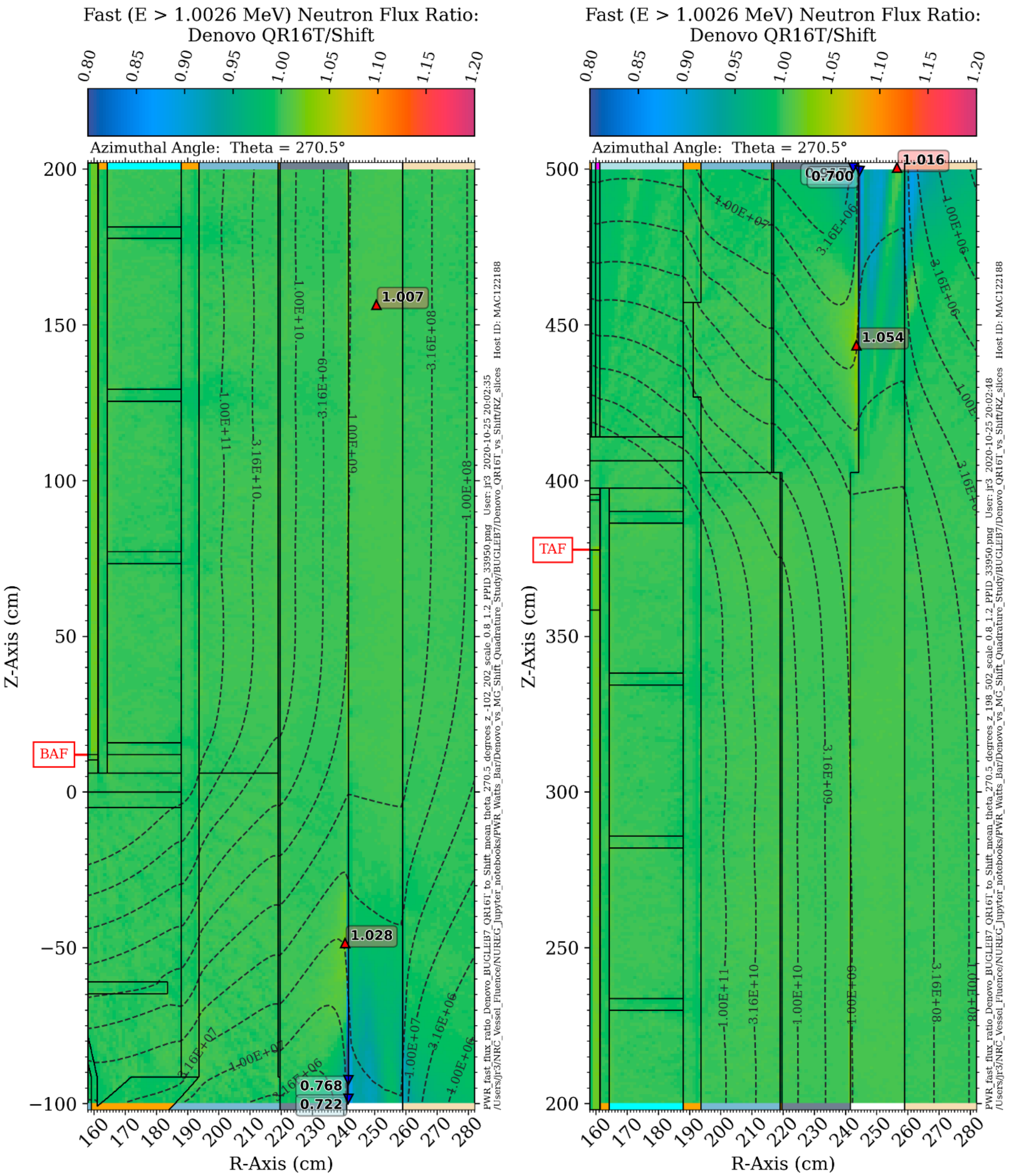

Figure 6-67 Ratio of the fast neutron flux from a BUGLE-B7 Denovo solution with QR16T quadrature to a BUGLE-B7 Shift solution with the PWR model. Elevation view at an azimuthal angle of 270.5°

6.6 Summary of quadrature studies

Angular quadrature sensitivity has long been a challenge in discrete ordinates radiation transport calculations, particularly with complex 3D models. Numerous studies have been performed over many years in attempts to provide optimal quadrature sets for RPV fluence calculations. The most widely used sets are still the level symmetric sets, particularly S8 and S16. However, their suitability for RPV fluence calculations in the extended beltline region has not been clearly established.

This section addresses three fundamental questions:

1. Are fluence calculations in the extended beltline region more sensitive to angular quadrature relative to calculations within the beltline region?
2. Are discrete ordinates calculations of dosimetry reaction rates for reactions with a high threshold energy, as in the ^{27}Al (n,α) reaction, more sensitive to quadrature than fast fluence calculations?
3. If there are significant solution differences with different quadrature sets, can one set be demonstrated to be more accurate than another?

The first two questions were addressed by performing parametric studies using Denovo PWR and BWR models. This approach is consistent with typical quadrature parameter studies that would be performed to determine whether a discrete ordinates solution has converged with respect to the angular quadrature.

The results of these studies demonstrate significant quadrature sensitivity in the extended beltline region of both models. In addition, they raise questions about the adequacy of the widely used S8 set for calculations, even within the traditional beltline region. The primary consideration with the S8 solutions may be with regard to benchmark calculations in which discrete ordinates calculations are compared to measured dosimetry data. Because of the azimuthal and axial sensitivity seen in portions of the S8 solutions, it is possible that a relatively minor shift in the location of a dosimetry capsule could have a nontrivial impact on the comparison of calculated and measured activities.

The question of sensitivity for high-energy threshold reactions was addressed by evaluating reaction rates for the ^{27}Al (n,α) reaction, which has a threshold energy of 3.25 MeV and an energy response range of 6.45–11.9 MeV. Results presented in Section 6.3.4 demonstrate that the calculation of this type of threshold reaction rate can be significantly more sensitive to quadrature effects compared with fast fluence calculations. Note that this is a concern not only in extended beltline locations, but even at locations well within the traditional beltline region.

The third question was addressed by comparing MG Shift calculations with Denovo calculations using the PWR model. These calculations used the same models and the same MG cross-section data (BUGLE-B7). Results of these calculations, which are presented in Section 6.5, suggest the following:

1. QR quadrature sets appear to be superior to level symmetric quadratures for RPV fluence calculations. A QR8T quadrature, which has the same number of angles as the high-order S16 level symmetric set, consistently provides closer agreement than S16 when compared with the Shift solution. This is not surprising, as at least some of the ray effects seen with the level symmetric sets are associated with material interfaces along a

coordinate axis or with streaming near the Z-axis. The QR quadrature sets were developed specifically to provide improved accuracy in those types of situations.

2. Given the increases in computing resources in recent years, it may be prudent to recommend a minimum quadrature order of QR8T. In the traditional beltline region, QR8T solutions provide generally good agreement with QR16T solutions, which have 136 angles per octant. However, in the extended beltline region, quadrature orders higher than QR8T should be considered.

7 SUMMARY AND CONCLUSIONS

The primary objectives of this report are to identify transport phenomena that are important in calculation of RPV fluence levels in the extended beltline region and to evaluate radiation transport methodologies that are best suited to such analyses. This work makes extensive use of large 3D transport calculations employing the Denovo discrete ordinates code and the Shift Monte Carlo code. The Shift calculations all employed the hybrid transport method, which utilizes both discrete ordinates and Monte Carlo calculations and is the current state of the art in radiation transport applications.

Both PWR and BWR models were utilized in an extensive set of parametric studies. Particular emphasis was placed on identifying aspects of current methodologies that may be appropriate for traditional beltline fluence analyses, but not for extended beltline applications. Understanding those issues provides guidance on changes that might be appropriate for extended beltline analyses, either with regard to parameter guidance with discrete ordinates calculations, or with recommendations on the use of improved transport methods that have come into use since the issuance of many of the existing guidelines for RPV fluence analyses.

One of the most significant changes in radiation transport analysis methodology over the past 10 to 20 years has been the increasing use of hybrid methods These methods provide improved accuracy in modeling of the systems being analyzed and in the physics of particle transport compared with discrete ordinates methods. The hybrid methods are capable of producing well-converged, spatially detailed Monte Carlo solutions with reasonable run times (e.g., overnight solutions on computing clusters with on the order of a hundred CPUs).

The sensitivities of extended beltline fluence calculations to physical aspects of RPV models were addressed and are summarized in Section 7.1. The selection of appropriate quadrature sets and MG cross-section libraries, which are critical parameters in discrete ordinates calculations, were addressed and are summarized in Section 7.2. The MG library studies, in particular, raise important questions about the level of accuracy that can be obtained for not only fast fluence evaluations, but also for calculations used to benchmark a transport methodology against measured dosimetry data.

7.1 Sensitivity analyses of selected physical parameters

Because neutron transport paths in the extended beltline region can be significantly different from those in the traditional beltline region, it is possible that fluence calculations in the extended beltline region may be more sensitive to variations in physical parameters (e.g., coolant density, changes in the fission spectrum with increasing burnup) than are fluence calculations in the traditional beltline region.

Hybrid radiation transport calculations were performed using PWR and BWR models to address the following physical parameters:

1. Changes in the fission spectrum from BOL to EOL in the PWR and BWR models
2. Changes in the coolant temperature (and hence density) in the PWR model
3. Changes in the axially dependent VFs in the BWR model
4. Changes in the cavity gap width in the PWR model
5. Changes in the concrete composition in the PWR model
6. The presence of a steel bioshield liner in the PWR model
7. The presence of thermal insulation in the PWR model

8. Use of homogenized core geometries in the PWR and BWR models rather than explicit ones

The results of these parameter studies, which are detailed in Section 5, are summarized below. The dimensional and material parameter studies of items 4–7 were conducted for the PWR model only. This selection was made based on the significantly higher fast neutron fluence levels at end of plant life in PWRs relative to BWRs (Section 1).

7.1.1 Fission spectrum effects

It is well known that changes in the fraction of fissions that occur in fissile isotopes in the fuel—primarily ^{235}U, ^{238}U, ^{239}Pu, and ^{241}Pu—can lead to significant changes in the fast flux in the RPV. There are two primary causes of this effect: changes in the energy spectrum of the fission neutrons, and changes in the average number of neutrons emitted per fission ($\bar{\nu}$). There are also differences in the energy release per fission (K) among the isotopes, but those differences are relative minor compared with the spectrum and $\bar{\nu}$ changes. The Pu isotopes have fission spectra that are shifted toward higher neutron energies, as well as higher values of $\bar{\nu}$, relative to the U isotopes. Because the ^{239}Pu and ^{241}Pu fission fractions' increase with increasing burnup, the fast neutron flux in the RPV tends to increase with increasing burnup.

For the PWR model used in this study, the fast flux in the RPV at the core midplane increases by ~23 to ~34% from BOL to EOL. The fission fractions by isotope for the BOL and EOL sources are noted in Section 5.2. At extended beltline locations, where there is increased neutron flux attenuation resulting from longer path lengths in the coolant between the core and the inner surface of the RPV, the EOL/BOL fast flux increase ranges from ~30 to ~50%.

For the BWR model used in this study, the fast flux in the RPV at the core midplane increases by ~20 to ~24% due to changes in the fission spectrum between BOL and EOL. At extended beltline locations, the fast flux increase ranges from ~22 to ~30%. It should be noted that the EOL fission fractions for ^{239}Pu and ^{241}Pu are significantly lower than in the PWR model. This is based on available fission fraction data as a function of burnup; it is likely that longer lived BWR fuel assemblies will have higher Pu isotopes than this model. In that case, the EOL/BOL fast flux ratios would be expected to increase at the core midplane elevation, and even more at the extended beltline locations.

These results suggest that accurate distributions of fissions by isotope throughout core life are particularly important for fast fluence predictions in the extended beltline region. It should also be noted that while RPV fluence levels in the traditional beltline region are dominated by the outermost pins in the outer fuel assemblies, sources further in from the peripheral edge and near the upper and lower limits of the fuel assemblies have increased importance for extended beltline locations.

7.1.2 Coolant temperature variation in the PWR model

The parametric study of coolant temperature changes in the PWR model is described in Section 5.3. Reductions in coolant temperature result in a decrease in the fast flux in the RPV, nozzles, and supports due to increased neutron attenuation. In extended beltline locations, there is a more significant reduction at the inner surface of the RPV due to the longer neutron transport paths from the core to the inner surface of the RPV, as noted above for the fission spectrum study. At extended beltline locations near the outer surface of the RPV and in the nozzles and vessel supports, the temperature-related reductions are less pronounced, as the cavity streaming neutrons that dominate the flux in those regions are driven by neutrons that escape the RPV well within the traditional beltline region. Increases in the coolant temperature in the PWR model result in fast flux changes that are essentially the inverse of those seen with temperature reductions. Because RPV locations in the extended beltline region are more sensitive to coolant temperature changes, accurate modeling of coolant temperatures throughout the RPV as a function of a plant's operating history is particularly important.

7.1.3 Void fractions in the BWR model

The BWR model used in this study is based on the GE14 10×10 assembly design with seven axial fuel zones. Boiling can occur in the coolant channels, resulting in changes in the coolant VF as a function of elevation and of time during a cycle. This effect was evaluated by comparing results from minimum, maximum, and average VFs by axial zone.

Changes in the RPV fast flux at the core midplane and lower elevations resulting from variation in the axial VFs are described in Section 5.4. The effects are relatively minor at elevations near or below the lower portion of the core, as the VFs in the two lowest axial zones are small and do not have significant variation between the minimum and maximum values.

Differences in the VFs between the minimum and maximum cases increase significantly compared to the two lower axial zones. This results in changes in the RPV fast flux being more significant in the upper extended beltline region than in the lower extended beltline region.

7.1.4 Reactor cavity gap width

The PWR model used in this study has cavity gap widths of 17.38 and 14.75 cm at elevations below and above the elevation where the RPV thickness increases (Table 4-1 and Figure 4-1). Because cavity streaming neutrons dominate fast flux levels in the outer portion of the RPV and in the nozzles and vessel supports, changes in the cavity geometry may be expected to have a significant effect on neutron flux levels in those locations. This is particularly important for the vessel supports in the extended beltline region. The cavity gap parameter study (Section 5.5) modeled increases of 10, 20, and 30 cm in the gap width. The increased gap widths have a very minor effect in the traditional beltline region, but they lead to significant fast flux increases in extended beltline locations.

7.1.5 Concrete composition

The reactor cavity gap width study confirmed the potential for cavity gap changes to have significant impacts on fast fluence levels in the extended beltline region. A related consideration is the composition of the concrete bioshield.

The sensitivity of fast flux changes in the extended beltline region was assessed by comparing the fast flux levels for four concrete compositions. The hydrogen density, which is of primary importance for neutron scattering behavior in concrete, was varied by nearly a factor of six (Table 5-6).

Reductions in the concrete hydrogen content result in increased scatter of neutrons from the bioshield back into the cavity gap. This in turn leads to increased fast neutron flux levels in the outer portion of the RPV in the extended beltline region, in the nozzles, and particularly in the vessel supports. Conversely, higher hydrogen content in concrete results in reduced scatter into the cavity gap, with reductions in the fast flux in the regions above and below the active core.

Based on these results, it is apparent that any change in hydrogen content in the bioshield (particularly in the portion of the bioshield nearest the inner radius) during the lifetime of an NPP should be considered in calculations of fast fluence in the extended beltline region, particularly for the nozzle supports.

7.1.6 Steel bioshield liner

The baseline PWR and BWR models in this study have bioshields constructed of Type 04 concrete (Section 5.6) with no liner on the inner surface of the concrete (i.e., the cylindrical surface facing the RPV). Some reactor plant designs include a steel liner on the inner surface of the bioshield. The presence of a liner will affect the scattering from the bioshield into the cavity gap, as the angular distribution and average energy loss of scattered neutrons are different in steel than they are in the lighter elements—particularly hydrogen—that are the dominant constituents of the concrete.

The presence of a steel liner was assessed for two liner compositions: 304 stainless steel and carbon steel. At locations away from the nozzles in the PWR model, the presence of a steel liner made of either SS-304 or carbon steel has a minor effect on fast flux levels in the outer portion of the RPV, leading to increases less than 3%. The most significant effect of a liner is a reduction in fast flux levels of up to ~25% in the vessel supports.

7.1.7 Thermal insulation

RPVs for PWR and BWR designs typically have a layer of thermal insulation between the RPV outer radius and the inner radius of the concrete bioshield. The effect of insulation on fast flux levels in the extended beltline region was evaluated by modeling a layer of reflective metallic insulation in the cavity gap of the PWR model. This insulation layer has a minor effect on fast flux levels in the RPV in the beltline region, where it can cause very slight increases (< 2%) in the fast flux levels at the outer surface of the RPV due to backscatter of neutrons from the insulation. At locations in the extended beltline region, the presence of thermal insulation can reduce fast flux levels in the RPV due to attenuation of the cavity streaming neutron flux by the thermal insulation. Because of reductions in cavity streaming flux levels due to attenuation in the insulation, the fast flux at locations where cavity streaming dominates the neutron flux is reduced by ~15–20%.

7.1.8 Use of homogenized core geometries

It is common practice in RPV fluence evaluations to homogenize the materials within the fuel assemblies into a set of mixtures rather than having an explicit geometric representation of the fuel pins, control assemblies, guide tubes, and other components. This is a reasonable modeling approximation for fast neutron flux calculations for the traditional beltline region, as the neutron transport characteristics of the homogenized fuel assemblies are essentially identical to the explicitly modeled assemblies used when calculating fast neutron flux levels in the RPV.

The validity of this approximation for extended beltline fluence calculations was assessed using homogenized-assembly versions of the PWR and BWR models. These homogenized models were also used for the Denovo discrete ordinates calculations discussed below. The results of these assessments confirm that the use of homogenized core models is appropriate for both extended and traditional beltline fluence evaluations.

7.2 Discrete ordinates quadrature sensitivities in the extended beltline region

Regulatory Guide 1.190 provides extensive guidance on the selection of discrete ordinates parameters (spatial, angular, and energy discretization) for RPV fast fluence calculations in the beltline region of LWR RPVs. No specific guidance is provided for fluence calculations in the extended beltline region. The sensitivity of discrete ordinates calculations to quadrature selection in the traditional and extended beltline region was evaluated in this study using a combination of discrete ordinates (Denovo) and hybrid transport (Denovo/Shift).

All discrete ordinates calculations were run using the ORNL Denovo code. Because other discrete ordinates transport codes have slight methodology differences, their sensitivity to the parameters studied may provide results that differ somewhat from those presented in this report. However, such differences are likely to be minor, and the overall conclusions of this parameter study are not expected to differ greatly when other codes are used.

Regulatory Guide 1.190 discusses appropriate meshing strategies for 2D/1D flux synthesis calculations. The adequacy of the Denovo mesh used in the parameter studies was confirmed based on (1) demonstrating that the solutions were converged with respect to mesh and (2) comparisons of Denovo and Shift calculations when both codes used the same MG cross-section data. Because discrete ordinates codes typically offer a number of spatial differencing schemes, the meshing deemed appropriate for Denovo using a linear discontinuous differencing scheme may not be appropriate for other transport codes.

Discrete ordinates calculations used for RPV fluence analyses typically employ level symmetric quadrature sets, including S8 and S16. While level symmetric quadratures have a long history and have been shown to be adequate for beltline fluence evaluations, more recent angular quadrature sets, including QR sets (Section 6), have been demonstrated to provide improved solution accuracy for (1) discrete ordinates models with material interfaces along the coordinate axes, like fuel assemblies and former plates and/or (2) streaming along or near coordinate axes, such as cavity streaming.

The initial quadrature comparisons focused on Denovo solutions using the S8 and S16 sets. Calculations with both the PWR and BWR models demonstrated nontrivial differences between S8 and S16 fast flux levels, even at the core midplane. The S8 solutions exhibited azimuthal ray effects associated with the corner fuel assemblies on the periphery of the core and at the neutron pad in the PWR model. The S8 solutions also exhibited ray effects at the jet pump risers in the BWR. The minimum and maximum S8/S16 fast flux ratios at the inner surface of the RPV vary by 9% in the PWR model and by nearly 19% in the BWR model. In addition, the S8 solution showed significant axial ray effects associated with the former plates in the PWR model.

Deviations between S8 and S16 solutions in the extended beltline region become more pronounced, particularly in the vicinity of the nozzles and the lower portion of the RPV near the lower hemispherical head in the PWR model. In the BWR model, significant S8/S16 differences occur in the shroud and in the RPV at elevations outside the axial extent of the core, particularly in regions below the bottom of the core.

Because ray effects tend to be more pronounced for higher energy neutrons, the quadrature sensitivity using S8 and S16 solutions in the PWR model was also evaluated for a high-energy threshold reaction—^{27}Al (n,α)—that is commonly used in dosimetry measurements. Results of this study show significant S8/S16 solution differences, some of which exceed 20%, even within the traditional beltline region. This is a potential concern for benchmarking of discrete ordinates calculations with dosimetry measurements, as relatively minor changes in the location of a dosimetry capsule could have a significant impact on calculated-to-measured ratios.

Further quadrature-to-quadrature comparisons were performed using a variety of QR quadratures, including QR8T, which has the same number of angles as S16, and QR16T, which served as a high-order solution. Details of these analyses can be found in Section 6.

A more stringent evaluation of quadrature effects was performed to demonstrate which quadrature sets provide the most accurate solution. This was done by comparing Denovo calculations with MG Shift calculations using the PWR model. Because both codes used the same MG library (BUGLE-B7), and the Denovo solutions had been demonstrated to be converged with respect to mesh, this comparison provided a means of effectively isolating the quadrature effects in the Denovo solution from MG effects. The results of these studies indicate that the QR sets provide superior fast flux estimates to the level symmetric S8 and S16 sets for RPV fluence calculations. The QR8T solution consistently provides closer agreement than S16 when compared with the Shift solution using MG data.

While the QR8T quadrature provides excellent agreement with the Shift solution in the traditional beltline region, it still exhibits ray effects that can produce differences of 15% or more compared to the MG Shift solution. The ray effects are reduced significantly using a QR16T quadrature, but even with that solution, there are some locations in the extended beltline region where the Denovo and Shift solutions differ by more than 10%.

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9 GLOSSARY

adjoint flux	The flux (see below) calculated using the adjoint form of the transport equation. The adjoint flux has the physical interpretation of representing the importance of particles to a specified response (e.g., flux or dpa rate).
discrete ordinates	A widely used method for solving the transport equation by discretizing the spatial, energy, and angular variables and solving the resulting set of algebraic equations using numerical methods. Discrete ordinates calculations are also referred to as deterministic calculations.
displacements per atom (dpa)	The mean number of times each atom in a crystal lattice structure is displaced from its lattice site as a result of radiation interactions.
fast fluence, fast flux	The fluence or flux of particles (e.g., neutrons) with energy above a specified threshold. While there is no standard definition of *fast neutron flux*, a commonly used energy cutoff for fast neutrons is 1 MeV. Within this report, the cutoff energy is either 1 MeV or 1.0026 MeV. The latter is used with MG cross-section libraries and with CE solutions that are compared directly with MG solutions.
fluence	The number of particles (e.g., neutrons) (dN) incident on a hypothetical sphere of cross-sectional area dA. *Fluence* can also be defined as the sum of the particle track lengths within the sphere. Fluence has units of inverse area (cm^{-2} or m^{-2}).
fluence rate	The number of particles entering a sphere, or the sum of the particle track lengths within a sphere per unit time.
flux	A more commonly used term for *fluence rate*.
hybrid	A class of techniques used to obtain a solution to the transport equation using a combination of determination and stochastic calculations.
lethargy	A measure of the amount of energy a neutron has lost as a result of scattering collisions. Lethargy is defined as $u = ln\left(\frac{E_0}{E}\right)$ where E is the neutron energy and E_0 is the maximum neutron energy (typically 20 MeV for neutron shielding calculations)

Monte Carlo	A stochastic method of obtaining a solution to the transport equation by simulating the behavior of a large number of particle histories.

APPENDIX A AN OVERVIEW OF THE MESH TALLIES AND PLOTTING METHODS USED IN THIS REPORT

Much of the data analysis in this report consists of plots of fast ($E > 1$ MeV) neutron flux distributions and ratios of solutions from parametric studies. The majority of the solutions are from Shift Monte Carlo calculations with cylindrical mesh tallies. The quadrature sensitivity studies in Section 6 utilized Denovo discrete ordinates calculations and Shift calculations. For those studies, the Shift calculations used Cartesian mesh tallies with the same grid spacing as the Denovo mesh and used the same multigroup (MG) cross-section data as the Denovo calculations.

This appendix briefly describes the level of detail in the Shift mesh tallies and provides examples of the types of plots that are used to present the analysis results. Examples of typical relative errors in the Shift calculations are also presented.

A.1 Cylindrical mesh tallies in the PWR and BWR models

The majority of the results presented in this report are based on cylindrical mesh tallies from continuous energy (CE) Shift calculations. The mesh tally intervals were selected to provide a high degree of spatial resolution while also providing solutions with mesh tally relative errors that are typically less than 1% in all locations of interest.

In the PWR model, the cylindrical mesh tally radial intervals are ~1 cm from the outer radius of the neutron pad to the outer radius of the RPV. In the cavity gap and the concrete bioshield the radial intervals are ~2 cm. The axial mesh intervals are ~2.5 cm over the height of the model. The azimuthal mesh intervals are uniform at 1°. Plots showing the radial, azimuthal, and axial mesh tally voxel boundaries for the PWR model are shown in Figure A-1 and Figure A-2.

In the BWR model, the cylindrical mesh tally radial intervals are ~2 cm over the radial extent from ~10 cm inboard of the core barrel to the inner radius of the RPV, ~1 cm through the RPV, ~3 cm in the cavity gap, and ~2 cm in the concrete bioshield. The axial mesh intervals are ~2.5 cm over the height of the model. The azimuthal mesh intervals are uniform at 1°. Plots showing the radial, azimuthal, and axial mesh tally voxel boundaries for the BWR model are shown in Figure A-3 and Figure A-4.

A.2 Fast neutron flux plots

Fast neutron flux solutions are plotted using a combination of material color assignments, contour lines, and flooded contours. Contour lines are typically shown over an extent of the cylindrical mesh tallies ranging from the water region inboard of the RPV radially out into the concrete bioshield. Within the RPV and other carbon steel regions (nozzles and nozzle supports), flooded contours are used to emphasize the fast flux behavior in these key components. Maximum and minimum values of the fast flux within each distinct carbon steel component (RPV, nozzles, nozzle supports) are indicated. In some plots the mesh tally voxel boundaries are shown. For elevation plots, the top of the active fuel (TAF) and bottom of the active fuel (BAF) elevations are indicated. Example fast neutron flux plots from the PWR model are shown in Figure A-5 and Figure A-6.

A.3 Ratio plots

Ratio plots are used in parametric studies to show the effect of changes in solution or model parameters. Many of the ratio plots in this report use contour lines and flooded contours only in the RPV, nozzles, and nozzle supports. Ratio plots also typically include an inset plot with a histogram of the ratio values in specified regions, which may include the RPV, nozzles, nozzle supports, and cavity gap. The abscissa labels on the histogram plots indicates which regions are included in the distribution. Numeric values on the upper edge of the ratio plot show the percentage of values that fall within each major interval of the abscissa.

An example ratio plot from a Shift parametric study is shown in Figure A-7.

Ratio plots for the Denovo parameter studies in Section 6 are somewhat different from the ratio plots for the Shift parameter studies in Section 5. In Section 6 the ratio of two solutions is shown over the entire plot extent. Showing the ratio for the entire extent is useful in pointing out the effects of quadrature selection. An example is shown in Figure A-8, where quadrature ray effects emanate from every corner on the peripheral fuel assemblies.

A.4 Mesh tally relative errors

As noted in Section A.1 relative errors for the Shift solutions in this report are typically less than 1% in all locations of interest. Example plots showing relative errors in the fast neutron flux for the PWR model are shown in Figure A-9 and Figure A-10. These correspond to the fast flux plots in Figure A-5 and Figure A-6.

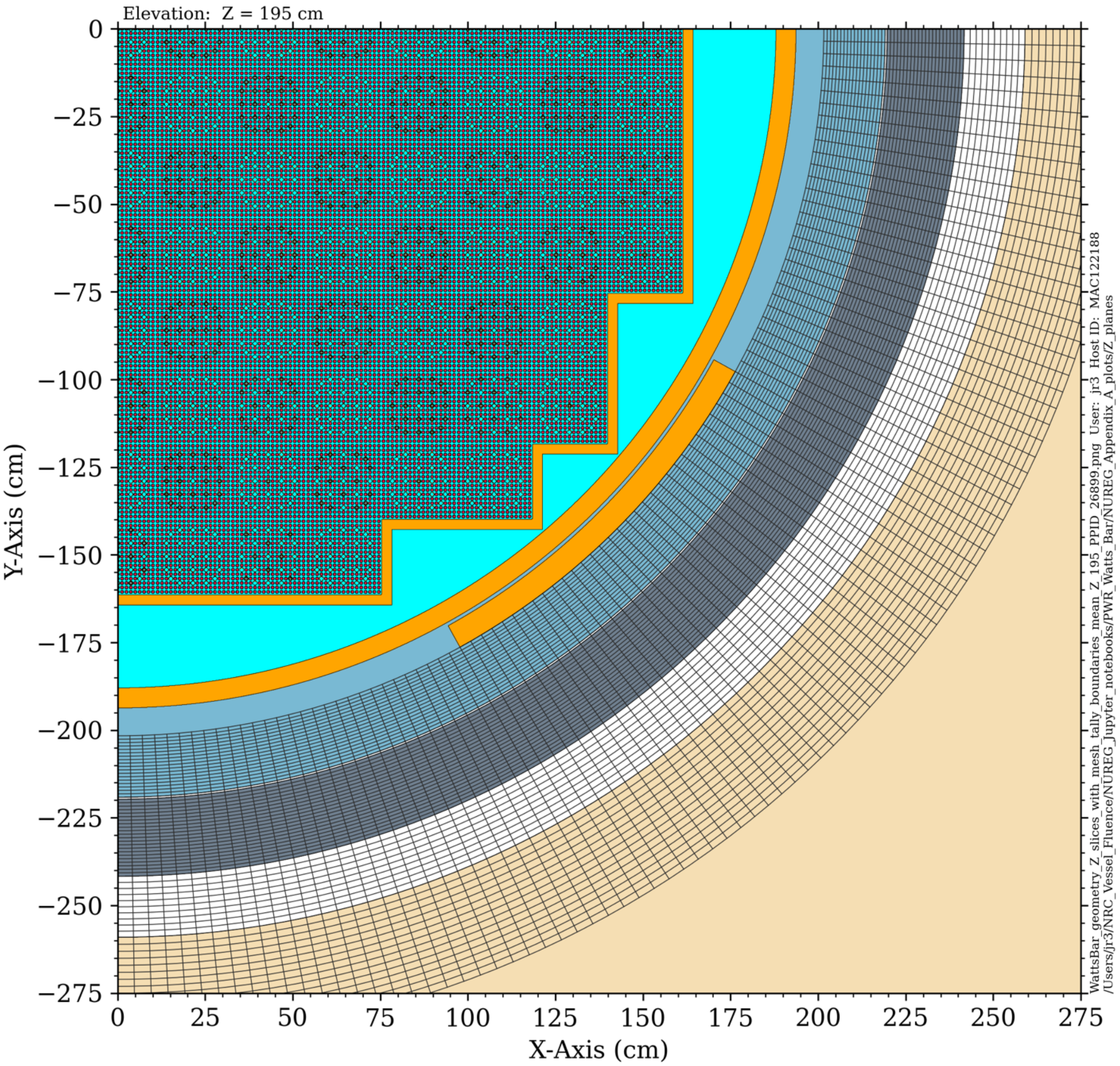

Figure A-1 Radial and azimuthal cylindrical mesh tally intervals in the PWR model. Plan view at the core midplane

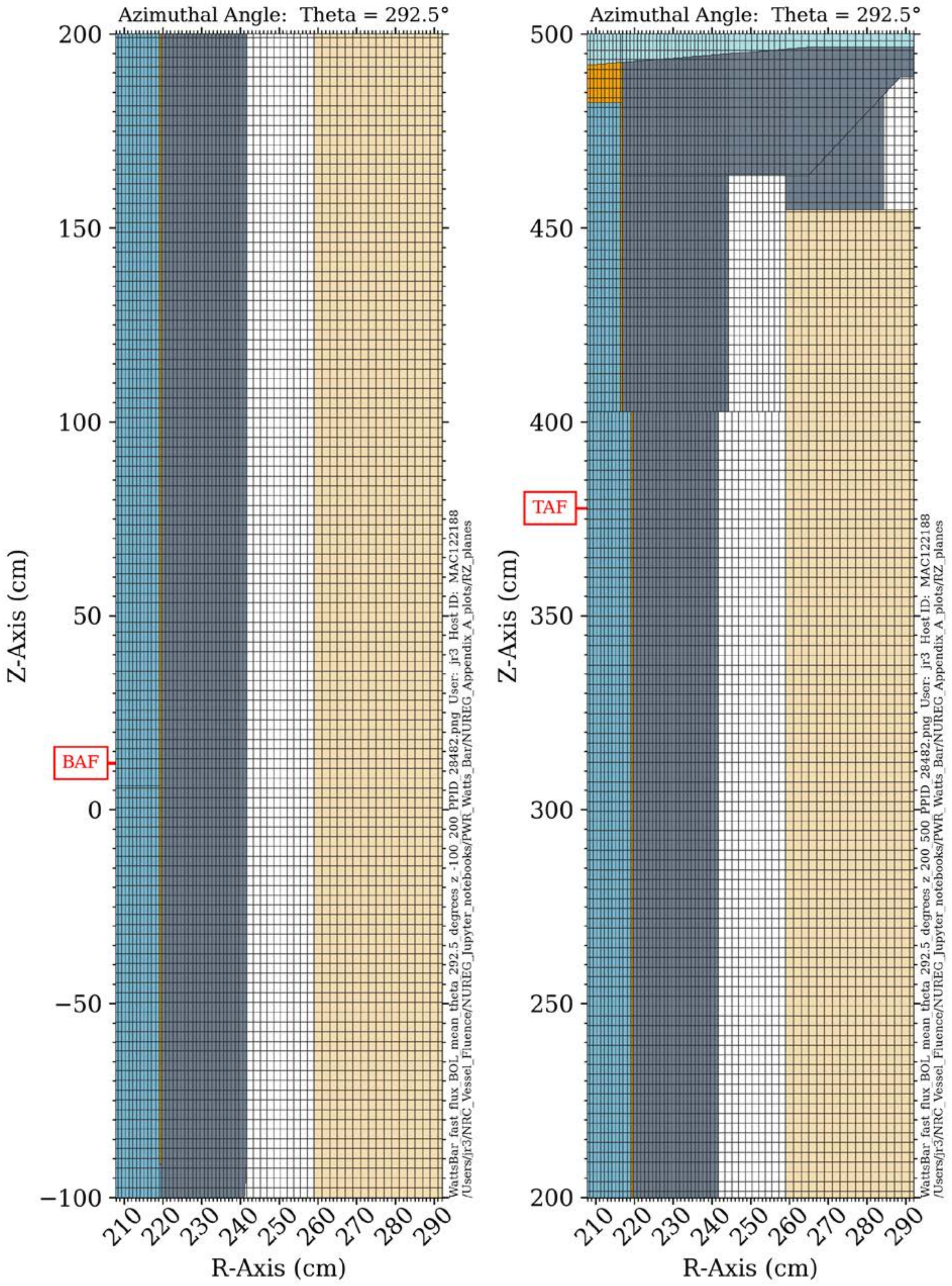

Figure A-2 **Axial cylindrical mesh tally intervals in the PWR model. Elevation view at an azimuthal angle of 292.5°**

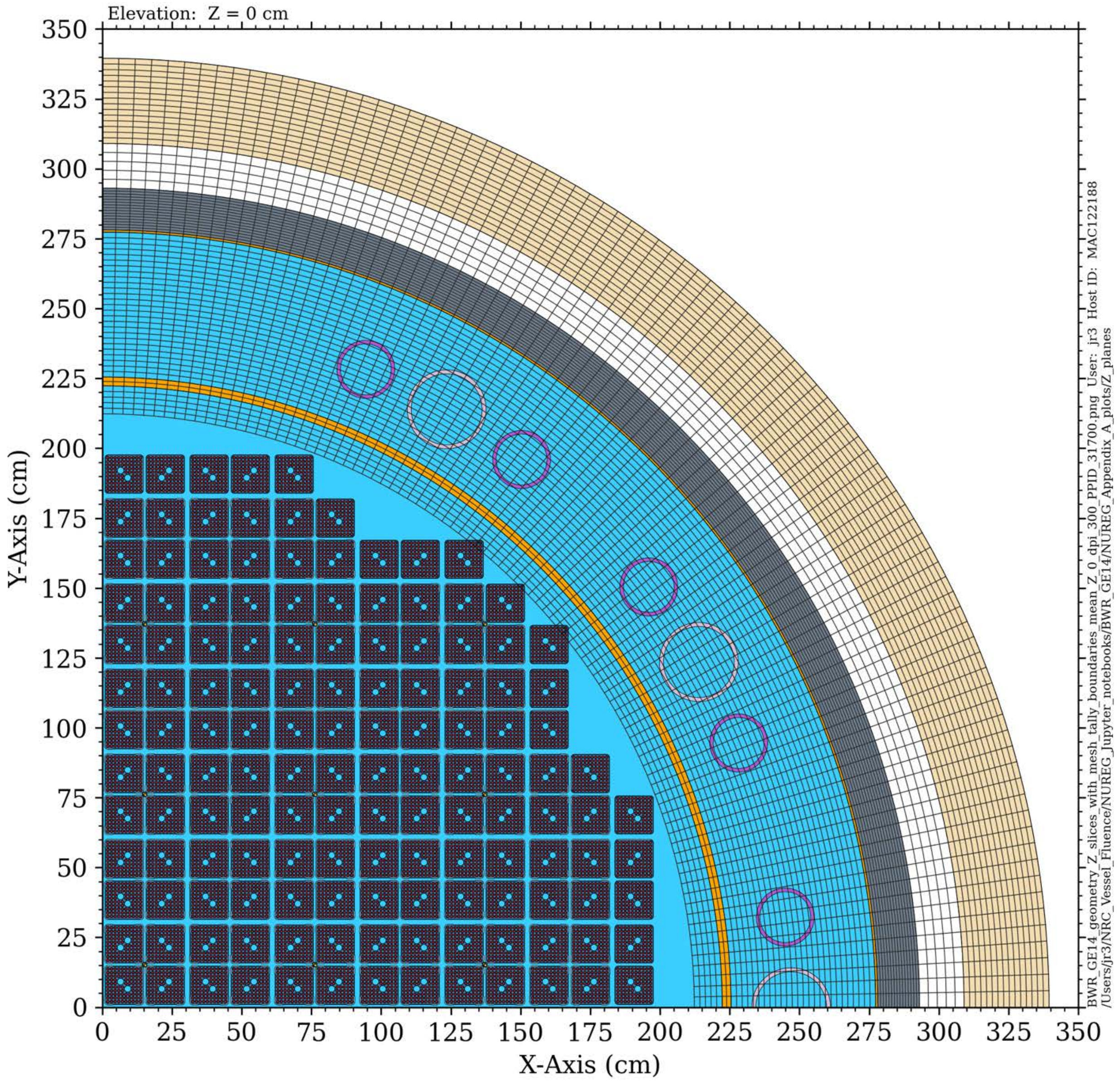

Figure A-3 Radial and azimuthal cylindrical mesh tally intervals in the BWR model. Plan view at the core midplane

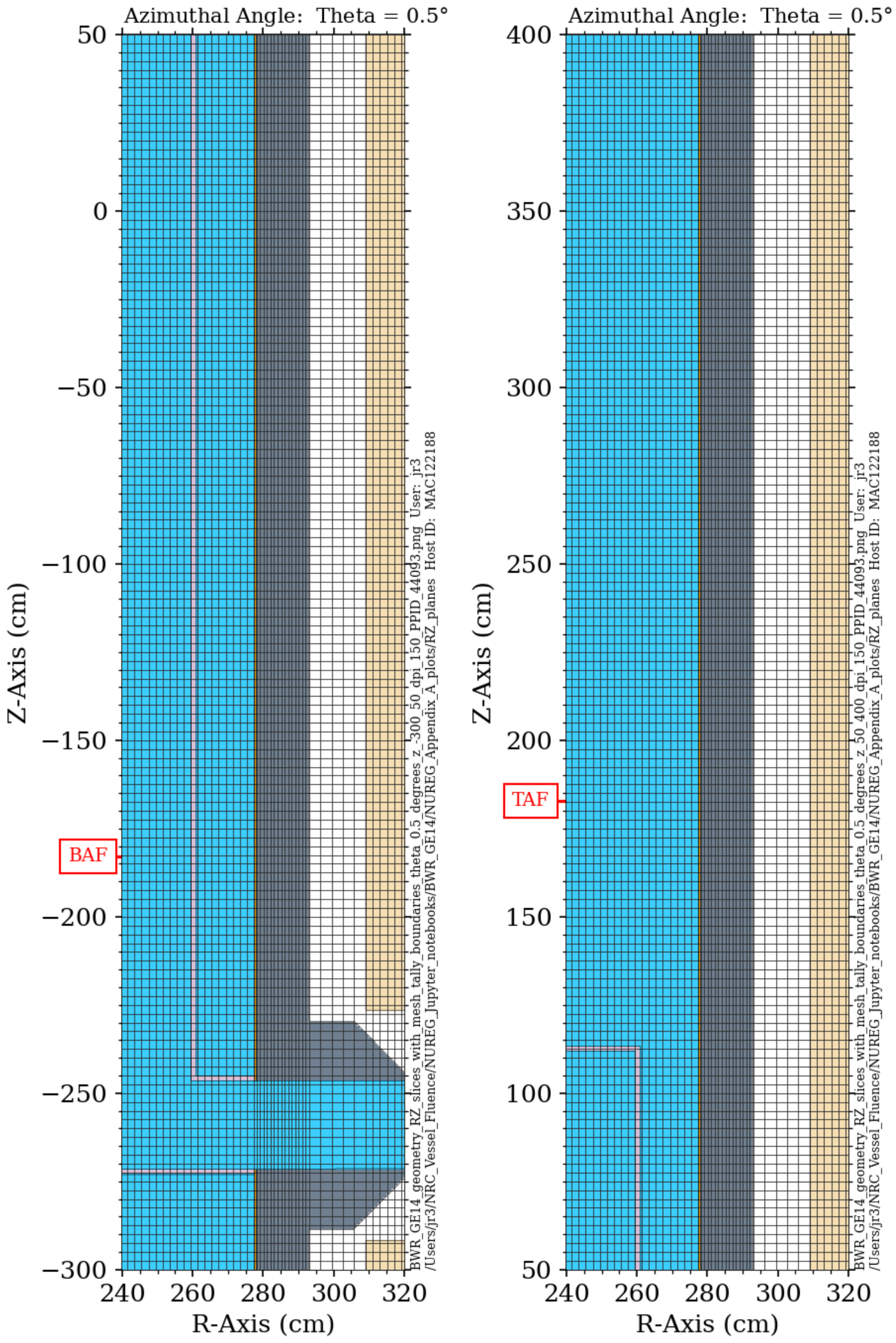

Figure A-4 Axial cylindrical mesh tally intervals in the BWR model. Elevation view at an azimuthal angle of 0.5°

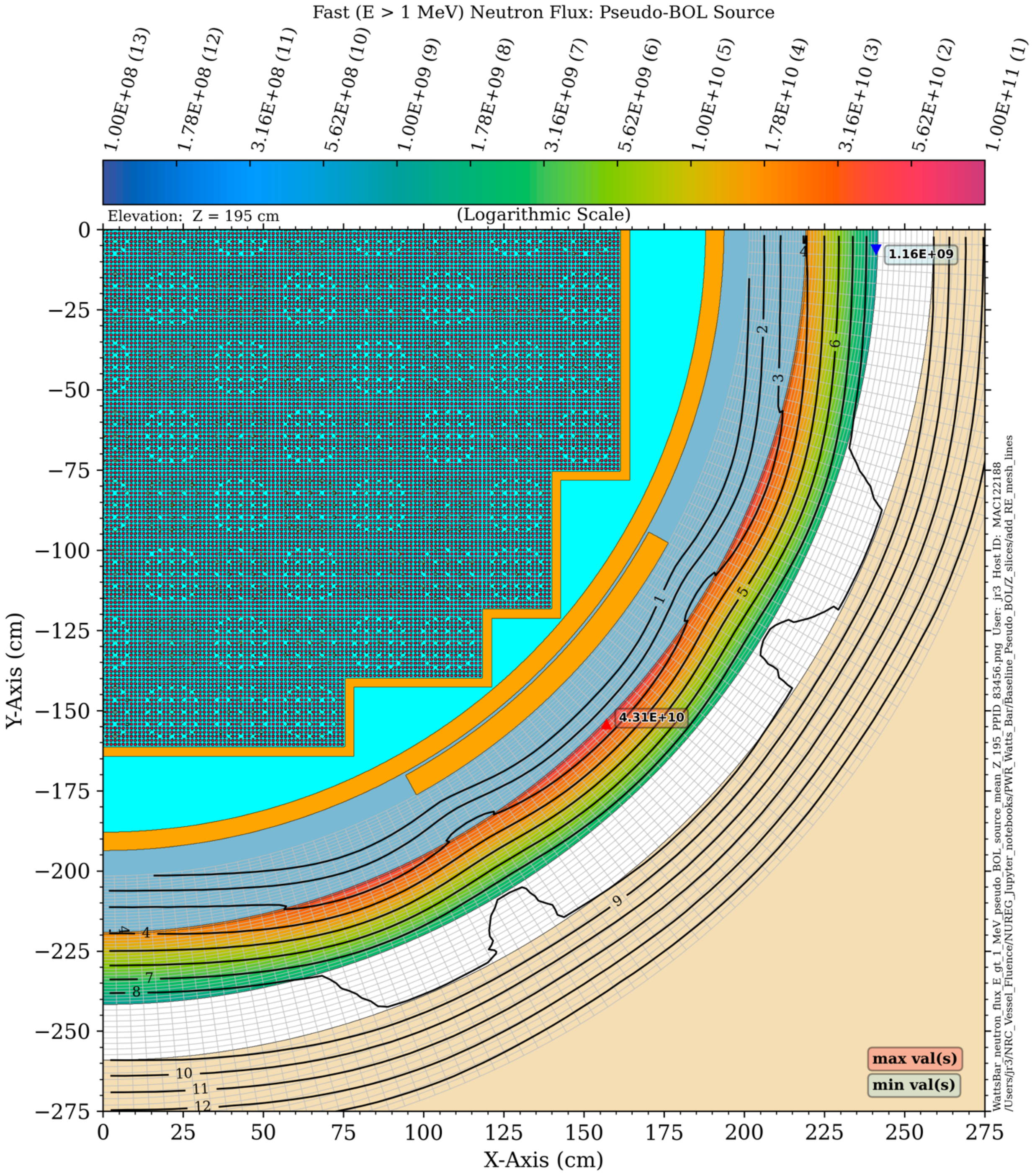

Figure A-5 Fast neutron flux at the core midplane in the PWR model. Regions other than the RPV are colored by material assignment. Contour lines show the fast neutron flux over the full extent of the cylindrical mesh tally. Flooded contours are used to highlight the fast flux in the RPV, which is the primary region of interest. Maximum and minimum fast flux values in the RPV are indicated. The cylindrical mesh tally boundaries are shown on this plot but are not present on all plots of this type

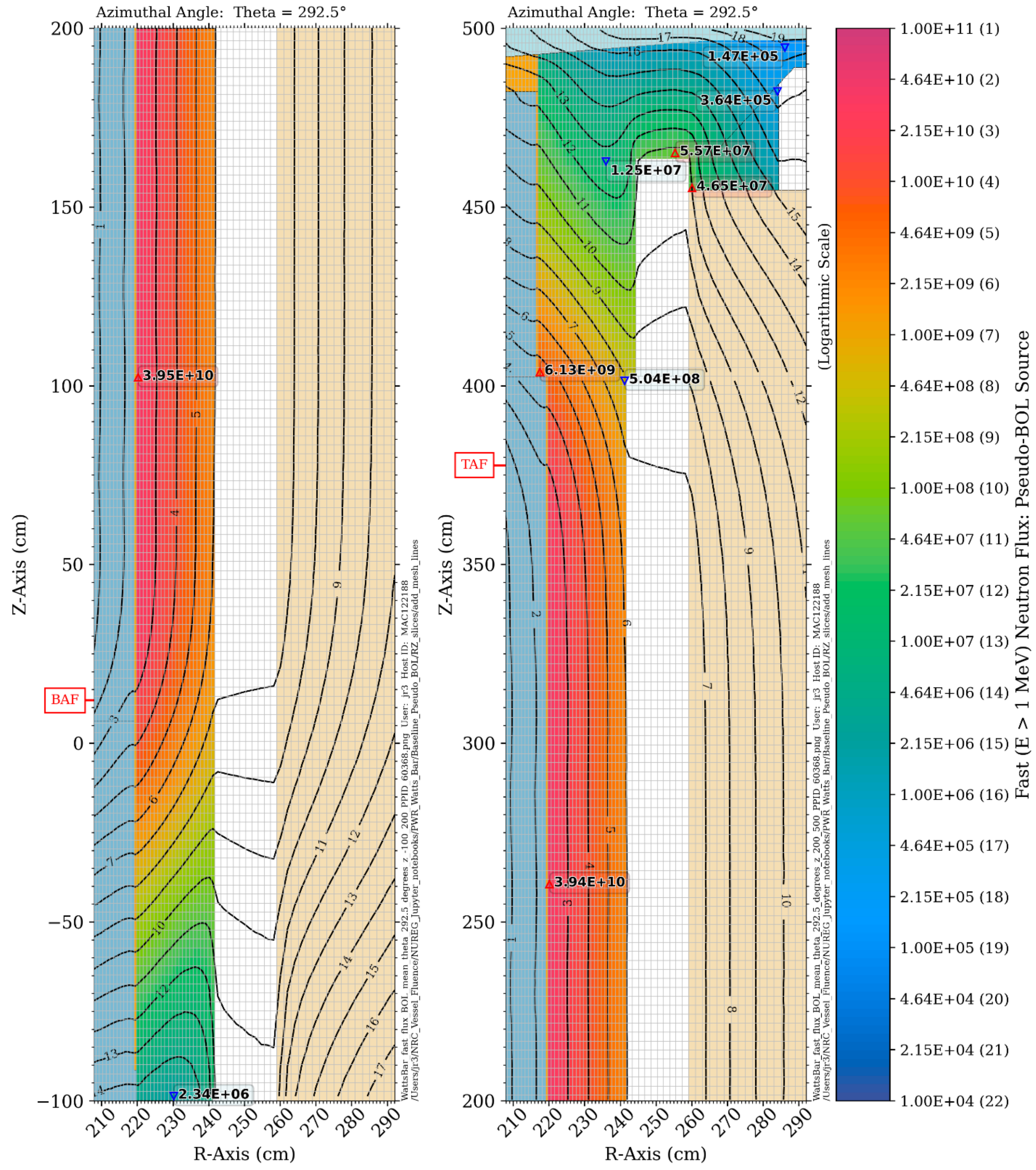

Figure A-6 **Fast neutron flux in the PWR model at an azimuthal angle of 292.5°. Regions other than the RPV are colored by material assignment. Contour lines show the fast neutron flux in all regions. Flooded contours are used to highlight the fast flux in the RPV, which is the primary region of interest. Maximum and minimum fast flux values in the RPV, RPV nozzle, and nozzle support are indicated. Maximum and minimum values in the RPV in the right-hand-side view are indicated above and below Z = 402.59 cm, where the thickness of the RPV changes. BAF and TAF are the bottom and top elevations of the active fuel**

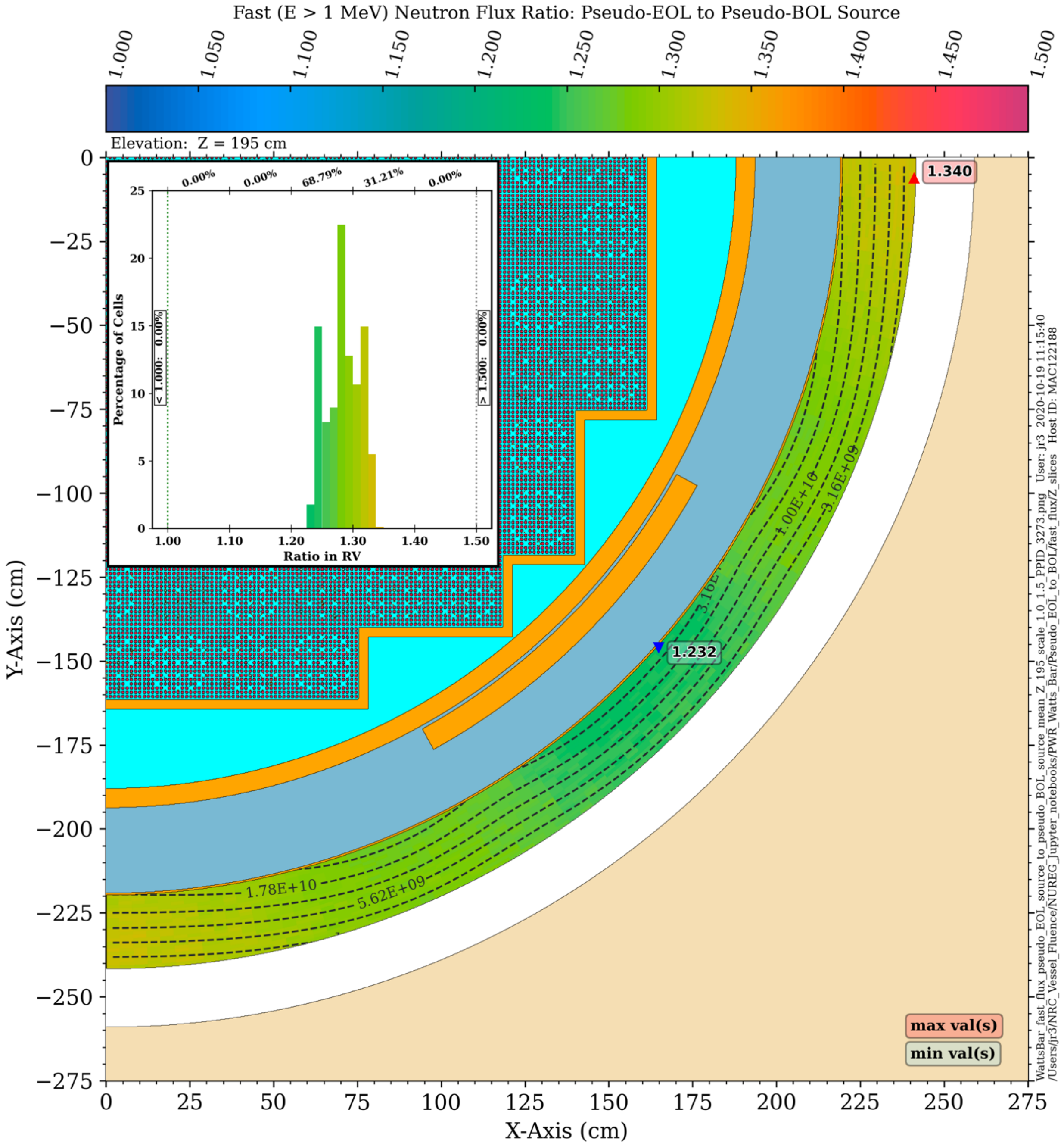

Figure A-7 Fast neutron flux ratio at the core midplane in the PWR model: pseudo-EOL source to pseudo-BOL source. Flooded contours show the fast flux ratio in the RPV. Minimum and maximum ratio values are indicated. The dashed contour lines represent the pseudo-BOL solution. The inset histogram plot shows the distribution of ratio values in the RPV

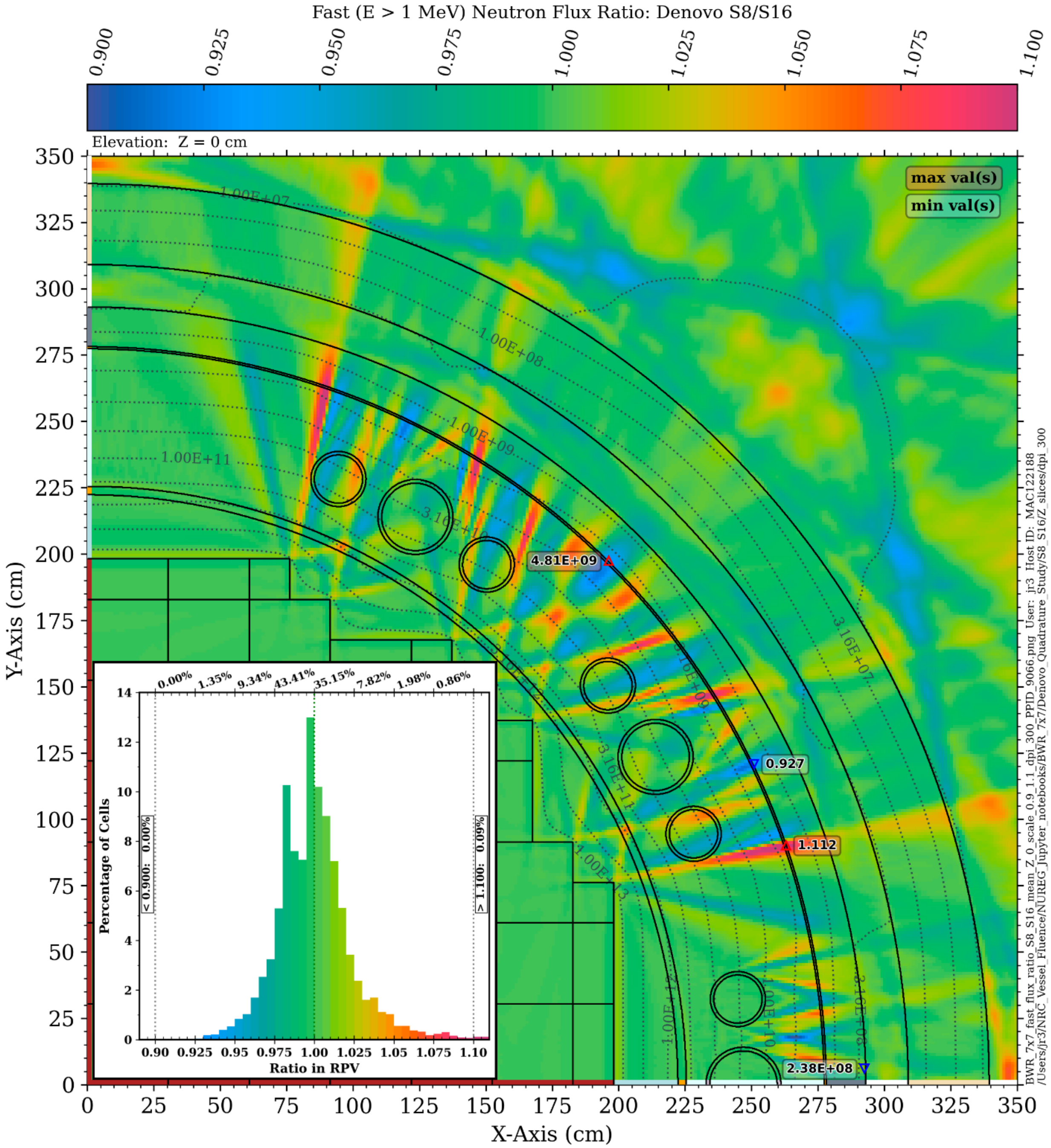

Figure A-8 Fast neutron flux ratio at the core midplane in the BWR model. The flooded contours show the ratio of an S8 Denovo solution to an S16 Denovo solution. The dashed contour lines represent the fast flux from the S16 solution. Maximum and minimum values of the S16 fast flux and the S8/S16 ratio in the RPV are shown. The inset histogram plot shows the distribution of ratio values in the RPV

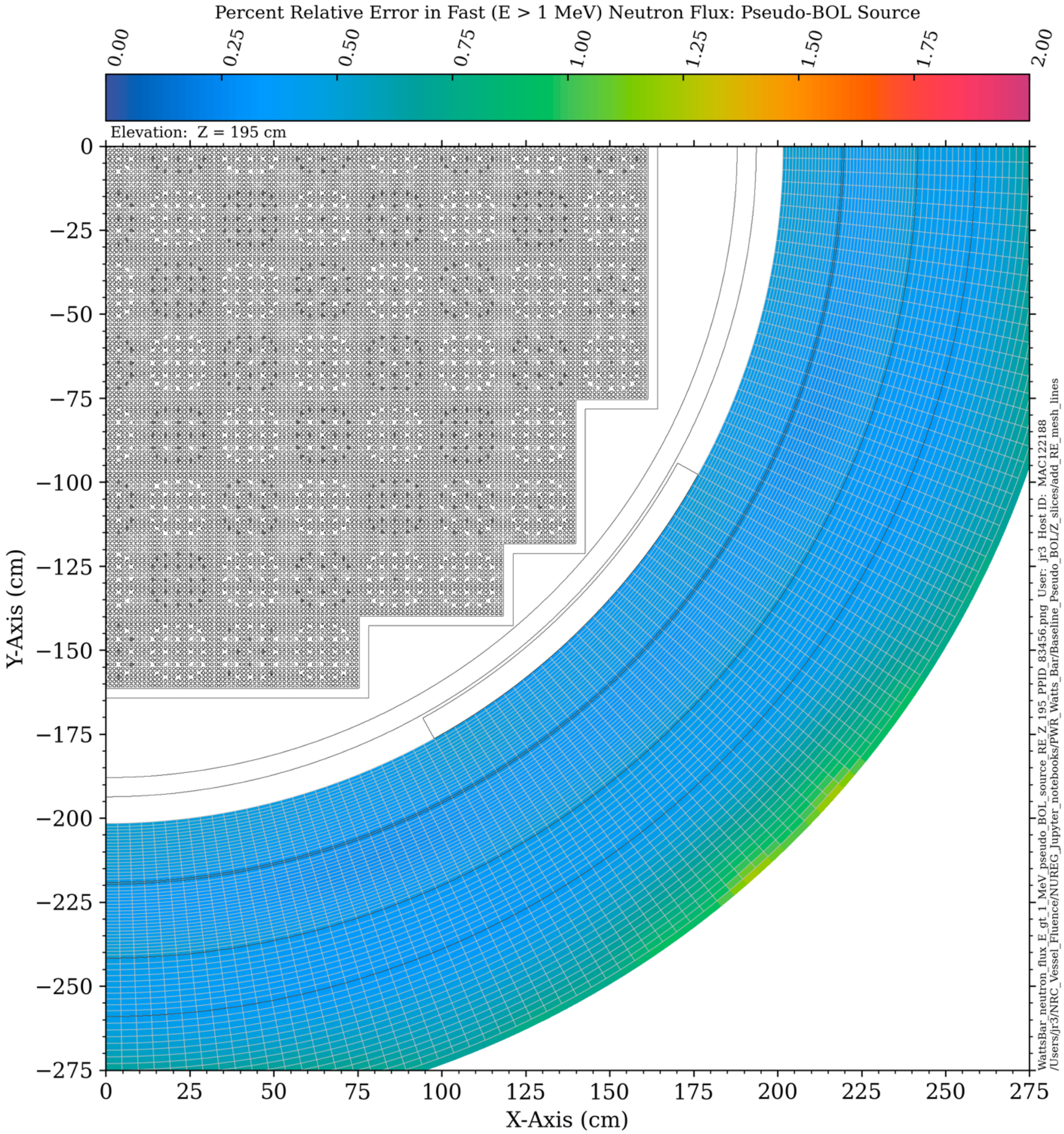

Figure A-9 Relative error in the fast neutron flux at the core midplane in the PWR model (see Figure A-5)

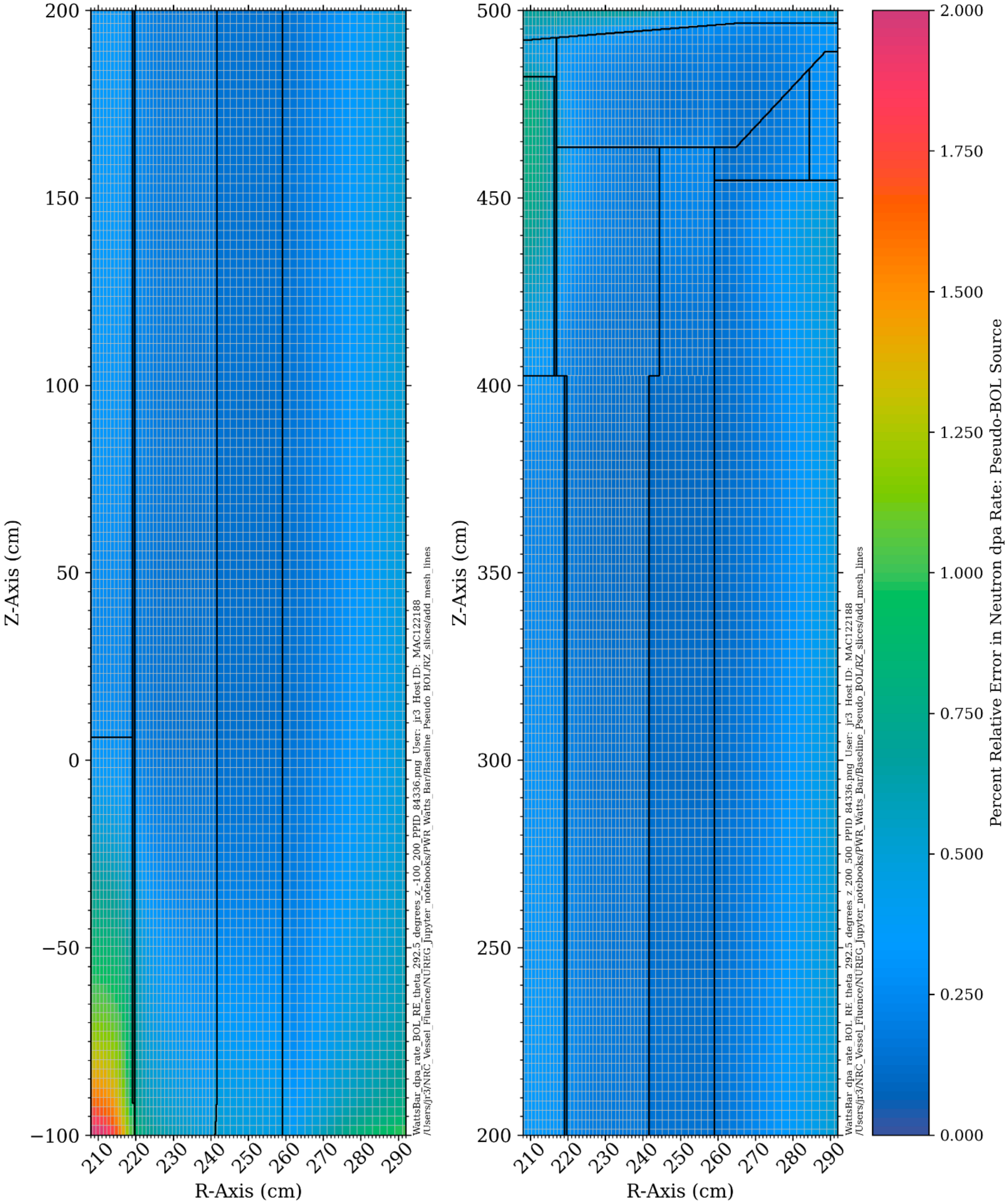

Figure A-10 Relative error in the fast neutron flux in the PWR model at an azimuthal angle of 292.5° (see Figure A-6)