C. STATISTICAL TABLES

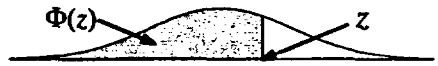

Table C.1. Standard normal cumulative distribution function, Φ.

z	Φ(z)	z	Φ(z)	z	Φ(z)	z	Φ(z)
-4.00	3.2E-5	-2.52	0.0059	-2.02	0.0217	-1.52	0.0643
-3.50	2.3E-4	-2.51	0.0060	-2.01	0.0222	-1.51	0.0655
-3.00	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793
-2.90	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951
-2.80	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056
-2.74	0.0031	-2.24	0.0125	-1.74	0.0409	-1.24	0.1075
-2.73	0.0032	-2.23	0.0129	-1.73	0.0418	-1.23	0.1093
-2.72	0.0033	-2.22	0.0132	-1.72	0.0427	-1.22	0.1112
-2.71	0.0034	-2.21	0.0136	-1.71	0.0436	-1.21	0.1131
-2.70	0.0035	-2.20	0.0139	-1.70	0.0446	-1.20	0.1151
-2.69	0.0036	-2.19	0.0143	-1.69	0.0455	-1.19	0.1170
-2.68	0.0037	-2.18	0.0146	-1.68	0.0465	-1.18	0.1190
-2.67	0.0038	-2.17	0.0150	-1.67	0.0475	-1.17	0.1210
-2.66	0.0039	-2.16	0.0154	-1.66	0.0485	-1.16	0.1230
-2.65	0.0040	-2.15	0.0158	-1.65	0.0495	-1.15	0.1251
-2.64	0.0041	-2.14	0.0162	-1.64	0.0505	-1.14	0.1271
-2.63	0.0043	-2.13	0.0166	-1.63	0.0516	-1.13	0.1292
-2.62	0.0044	-2.12	0.0170	-1.62	0.0526	-1.12	0.1314
-2.61	0.0045	-2.11	0.0174	-1.61	0.0537	-1.11	0.1335
-2.60	0.0047	-2.10	0.0179	-1.60	0.0548	-1.10	0.1357
-2.59	0.0048	-2.09	0.0183	-1.59	0.0559	-1.09	0.1379
-2.58	0.0049	-2.08	0.0188	-1.58	0.0571	-1.08	0.1401
-2.57	0.0051	-2.07	0.0192	-1.57	0.0582	-1.07	0.1423
-2.56	0.0052	-2.06	0.0197	-1.56	0.0594	-1.06	0.1446
-2.55	0.0054	-2.05	0.0202	-1.55	0.0606	-1.05	0.1469
-2.54	0.0055	-2.04	0.0207	-1.54	0.0618	-1.04	0.1492
-2.53	0.0057	-2.03	0.0212	-1.53	0.0630	-1.03	0.1515

Table C.1 Standard normal cumulative distribution function, Φ (continued).

z	Φ(z)	z	Φ(z)	z	Φ(z)	z	Φ(z)
-1.02	0.1539	-0.51	0.3050	0.00	0.5000	0.51	0.6950
-1.01	0.1562	-0.50	0.3085	0.01	0.5040	0.52	0.6985
-1.00	0.1587	-0.49	0.3121	0.02	0.5080	0.53	0.7019
-0.99	0.1611	-0.48	0.3156	0.03	0.5120	0.54	0.7054
-0.98	0.1635	-0.47	0.3192	0.04	0.5160	0.55	0.7088
-0.97	0.1660	-0.46	0.3228	0.05	0.5199	0.56	0.7123
-0.96	0.1685	-0.45	0.3264	0.06	0.5239	0.57	0.7157
-0.95	0.1711	-0.44	0.3300	0.07	0.5279	0.58	0.7190
-0.94	0.1736	-0.43	0.3336	0.08	0.5319	0.59	0.7224
-0.93	0.1762	-0.42	0.3372	0.09	0.5359	0.60	0.7257
-0.92	0.1788	-0.41	0.3409	0.10	0.5398	0.61	0.7291
-0.91	0.1814	-0.40	0.3446	0.11	0.5438	0.62	0.7324
-0.90	0.1841	-0.39	0.3483	0.12	0.5478	0.63	0.7357
-0.89	0.1867	-0.38	0.3520	0.13	0.5517	0.64	0.7389
-0.88	0.1894	-0.37	0.3557	0.14	0.5557	0.65	0.7422
-0.87	0.1922	-0.36	0.3594	0.15	0.5596	0.66	0.7454
-0.86	0.1949	-0.35	0.3632	0.16	0.5636	0.67	0.7486
-0.85	0.1977	-0.34	0.3669	0.17	0.5675	0.68	0.7517
-0.84	0.2005	-0.33	0.3707	0.18	0.5714	0.69	0.7549
-0.83	0.2033	-0.32	0.3745	0.19	0.5753	0.70	0.7580
-0.82	0.2061	-0.31	0.3783	0.20	0.5793	0.71	0.7611
-0.81	0.2090	-0.30	0.3821	0.21	0.5832	0.72	0.7642
-0.80	0.2119	-0.29	0.3859	0.22	0.5871	0.73	0.7673
-0.79	0.2148	-0.28	0.3897	0.23	0.5910	0.74	0.7703
-0.78	0.2177	-0.27	0.3936	0.24	0.5948	0.75	0.7734
-0.77	0.2206	-0.26	0.3974	0.25	0.5987	0.76	0.7764
-0.76	0.2236	-0.25	0.4013	0.26	0.6026	0.77	0.7794
-0.75	0.2266	-0.24	0.4052	0.27	0.6064	0.78	0.7823
-0.74	0.2296	-0.23	0.4090	0.28	0.6103	0.79	0.7852
-0.73	0.2327	-0.22	0.4129	0.29	0.6141	0.80	0.7881
-0.72	0.2358	-0.21	0.4168	0.30	0.6179	0.81	0.7910
-0.71	0.2389	-0.20	0.4207	0.31	0.6217	0.82	0.7939
-0.70	0.2420	-0.19	0.4247	0.32	0.6255	0.83	0.7967
-0.69	0.2451	-0.18	0.4286	0.33	0.6293	0.84	0.7995
-0.68	0.2483	-0.17	0.4325	0.34	0.6331	0.85	0.8023
-0.67	0.2514	-0.16	0.4364	0.35	0.6368	0.86	0.8051
-0.66	0.2546	-0.15	0.4404	0.36	0.6406	0.87	0.8078
-0.65	0.2578	-0.14	0.4443	0.37	0.6443	0.88	0.8106
-0.64	0.2611	-0.13	0.4483	0.38	0.6480	0.89	0.8133
-0.63	0.2643	-0.12	0.4522	0.39	0.6517	0.90	0.8159
-0.62	0.2676	-0.11	0.4562	0.40	0.6554	0.91	0.8186
-0.61	0.2709	-0.10	0.4602	0.41	0.6591	0.92	0.8212
-0.60	0.2743	-0.09	0.4641	0.42	0.6628	0.93	0.8238
-0.59	0.2776	-0.08	0.4681	0.43	0.6664	0.94	0.8264
-0.58	0.2810	-0.07	0.4721	0.44	0.6700	0.95	0.8289
-0.57	0.2843	-0.06	0.4761	0.45	0.6736	0.96	0.8315
-0.56	0.2877	-0.05	0.4801	0.46	0.6772	0.97	0.8340
-0.55	0.2912	-0.04	0.4840	0.47	0.6808	0.98	0.8365
-0.54	0.2946	-0.03	0.4880	0.48	0.6844	0.99	0.8389
-0.53	0.2981	-0.02	0.4920	0.49	0.6879	1.00	0.8413
-0.52	0.3015	-0.01	0.4960	0.50	0.6915	1.01	0.8438

Table C.1 Standard normal cumulative distribution function, Φ (continued).

z	Φ(z)	z	Φ(z)	z	Φ(z)	z	Φ(z)
1.02	0.8461	1.52	0.9357	2.02	0.9783	2.52	0.9941
1.03	0.8485	1.53	0.9370	2.03	0.9788	2.53	0.9943
1.04	0.8508	1.54	0.9382	2.04	0.9793	2.54	0.9945
1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
1.06	0.8554	1.56	0.9406	2.06	0.9803	2.56	0.9948
1.07	0.8577	1.57	0.9418	2.07	0.9808	2.57	0.9949
1.08	0.8599	1.58	0.9429	2.08	0.9812	2.58	0.9951
1.09	0.8621	1.59	0.9441	2.09	0.9817	2.59	0.9952
1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
1.11	0.8665	1.61	0.9463	2.11	0.9826	2.61	0.9955
1.12	0.8686	1.62	0.9474	2.12	0.9830	2.62	0.9956
1.13	0.8708	1.63	0.9484	2.13	0.9834	2.63	0.9957
1.14	0.8729	1.64	0.9495	2.14	0.9838	2.64	0.9959
1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
1.16	0.8770	1.66	0.9515	2.16	0.9846	2.66	0.9961
1.17	0.8790	1.67	0.9525	2.17	0.9850	2.67	0.9962
1.18	0.8810	1.68	0.9535	2.18	0.9854	2.68	0.9963
1.19	0.8830	1.69	0.9545	2.19	0.9857	2.69	0.9964
1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
1.21	0.8869	1.71	0.9564	2.21	0.9864	2.71	0.9966
1.22	0.8888	1.72	0.9573	2.22	0.9868	2.72	0.9967
1.23	0.8907	1.73	0.9582	2.23	0.9871	2.73	0.9968
1.24	0.8925	1.74	0.9591	2.24	0.9875	2.74	0.9969
1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
1.26	0.8962	1.76	0.9608	2.26	0.9881	2.76	0.9971
1.27	0.8980	1.77	0.9616	2.27	0.9884	2.77	0.9972
1.28	0.8997	1.78	0.9625	2.28	0.9887	2.78	0.9973
1.29	0.9015	1.79	0.9633	2.29	0.9890	2.79	0.9974
1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
1.31	0.9049	1.81	0.9649	2.31	0.9896	2.81	0.9975
1.32	0.9066	1.82	0.9656	2.32	0.9898	2.82	0.9976
1.33	0.9082	1.83	0.9664	2.33	0.9901	2.83	0.9977
1.34	0.9099	1.84	0.9671	2.34	0.9904	2.84	0.9977
1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
1.36	0.9131	1.86	0.9686	2.36	0.9909	2.86	0.9979
1.37	0.9147	1.87	0.9693	2.37	0.9911	2.87	0.9979
1.38	0.9162	1.88	0.9699	2.38	0.9913	2.88	0.9980
1.39	0.9177	1.89	0.9706	2.39	0.9916	2.89	0.9981
1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
1.41	0.9207	1.91	0.9719	2.41	0.9920	2.91	0.9982
1.42	0.9222	1.92	0.9726	2.42	0.9922	2.92	0.9982
1.43	0.9236	1.93	0.9732	2.43	0.9925	2.93	0.9983
1.44	0.9251	1.94	0.9738	2.44	0.9927	2.94	0.9984
1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
1.46	0.9279	1.96	0.9750	2.46	0.9931	2.96	0.9985
1.47	0.9292	1.97	0.9756	2.47	0.9932	2.97	0.9985
1.48	0.9306	1.98	0.9761	2.48	0.9934	2.98	0.9986
1.49	0.9319	1.99	0.9767	2.49	0.9936	2.99	0.9986
1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987
1.51	0.9345	2.01	0.9778	2.51	0.9940	3.50	0.99977
						4.00	0.999968

Table C.2. Percentiles of the chi-squared distribution.

Deg. of freedom	$\chi^2_{0.005}$	$\chi^2_{0.01}$	$\chi^2_{0.025}$	$\chi^2_{0.05}$	$\chi^2_{0.10}$	$\chi^2_{0.20}$	$\chi^2_{0.30}$	$\chi^2_{0.40}$
0.25	4.8E-19	1.2E-16	1.9E-13	4.8E-11	1.24E-8	3.17E-6	8.12E-5	8.11E-4
0.5	8.4E-10	1.35E-8	5.27E-7	8.44E-6	1.35E-4	2.16E-3	0.0110	0.0350
1	3.93E-5	1.57E-4	9.82E-4	3.93E-3	0.0158	0.0642	0.148	0.275
2	0.0100	0.0201	0.0506	0.103	0.211	0.446	0.713	1.022
3	0.0717	0.115	0.216	0.352	0.584	1.005	1.424	1.869
4	0.207	0.297	0.484	0.711	1.064	1.649	2.195	2.753
5	0.412	0.554	0.831	1.145	1.610	2.343	3.000	3.655
6	0.676	0.872	1.237	1.635	2.204	3.070	3.828	4.570
7	0.989	1.239	1.690	2.167	2.833	3.822	4.671	5.493
8	1.344	1.646	2.180	2.733	3.490	4.594	5.527	6.423
9	1.735	2.088	2.700	3.325	4.168	5.380	6.393	7.357
10	2.156	2.558	3.247	3.940	4.865	6.179	7.267	8.295
11	2.603	3.053	3.816	4.575	5.578	6.989	8.148	9.237
12	3.074	3.571	4.404	5.226	6.304	7.807	9.034	10.18
13	3.565	4.107	5.009	5.892	7.042	8.634	9.926	11.13
14	4.075	4.660	5.629	6.571	7.790	9.467	10.82	12.08
15	4.601	5.229	6.262	7.261	8.547	10.31	11.72	13.03
16	5.142	5.812	6.908	7.962	9.312	11.15	12.62	13.98
17	5.697	6.408	7.564	8.672	10.09	12.00	13.53	14.94
18	6.265	7.015	8.231	9.390	10.86	12.86	14.44	15.89
19	6.844	7.633	8.907	10.12	11.65	13.72	15.35	16.85
20	7.434	8.260	9.591	10.85	12.44	14.58	16.27	17.81
21	8.034	8.897	10.28	11.59	13.24	15.44	17.18	18.77
22	8.643	9.542	10.98	12.34	14.04	16.31	18.10	19.73
23	9.260	10.20	11.69	13.09	14.85	17.19	19.02	20.69
24	9.886	10.86	12.40	13.85	15.66	18.06	19.94	21.65
25	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62
26	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58
27	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54
28	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51
29	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48
30	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44
35	17.19	18.51	20.57	22.47	24.80	27.84	30.18	32.28
40	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13
45	24.31	25.90	28.37	30.61	33.35	36.88	39.58	42.00
50	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86
55	31.73	33.57	36.40	38.96	42.06	46.04	49.06	51.74
60	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62
70	43.25	45.42	48.75	51.74	55.33	59.90	63.35	66.40
80	51.14	53.52	57.15	60.39	64.28	69.21	72.92	76.19
90	59.17	61.74	65.64	69.13	73.29	78.56	82.52	85.60
100	67.30	70.05	74.22	77.93	82.36	87.95	92.13	95.81
125	88.01	91.17	95.94	100.2	105.2	111.5	116.3	120.4
150	109.1	112.7	118.0	122.7	128.3	135.3	140.5	145.0

For large degrees of freedom d, use $\chi_p^2 = \left(z_p + \sqrt{2d-1}\right)^2 / 2$, where z_p is the corresponding percentile of a standard normal distribution.

Table C.2 Percentiles of the chi-squared distribution (continued).

$\chi^2_{0.50}$	$\chi^2_{0.60}$	$\chi^2_{0.70}$	$\chi^2_{0.80}$	$\chi^2_{0.90}$	$\chi^2_{0.95}$	$\chi^2_{0.975}$	$\chi^2_{0.99}$	$\chi^2_{0.995}$	Deg. of freedom
4.84E-3	0.0210	0.0737	0.229	0.716	1.419	2.269	3.543	4.585	0.25
0.0873	0.188	0.375	0.726	1.501	2.420	3.433	4.868	6.004	0.5
0.455	0.708	1.074	1.642	2.706	3.841	5.024	6.635	7.879	1
1.386	1.833	2.408	3.219	4.605	5.991	7.378	9.210	10.56	2
2.366	2.946	3.665	4.642	6.251	7.815	9.348	11.34	12.84	3
3.357	4.045	4.878	5.989	7.779	9.488	11.14	13.28	14.86	4
4.351	5.132	6.064	7.289	9.236	11.07	12.83	15.09	16.75	5
5.348	6.211	7.231	8.558	10.64	12.59	14.45	16.81	18.55	6
6.346	7.283	8.383	9.803	12.02	14.07	16.01	18.48	20.28	7
7.344	8.351	9.524	11.03	13.36	15.51	17.53	20.09	21.95	8
8.343	9.414	10.66	12.24	14.68	16.92	19.02	21.67	23.59	9
9.342	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	10
10.34	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	11
11.34	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	12
12.34	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	13
13.34	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	14
14.34	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	15
15.34	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	16
16.34	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	17
17.34	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	18
18.34	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	19
19.34	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	20
20.34	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	21
21.34	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	22
22.34	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	23
23.34	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	24
24.34	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	25
25.34	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	26
26.34	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	27
27.34	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	28
28.34	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	29
29.34	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	30
34.34	36.47	38.86	41.78	46.06	49.80	53.20	57.34	60.27	35
39.34	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	40
44.34	46.76	49.45	52.73	57.51	61.66	65.41	69.96	73.17	45
49.33	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	50
54.33	57.02	59.98	63.58	68.80	73.31	77.38	82.29	85.75	55
59.33	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	60
69.34	72.36	75.69	79.71	85.52	90.53	95.03	100.4	104.2	70
79.34	82.56	86.12	90.40	96.57	101.9	106.6	112.3	116.3	80
89.33	92.76	96.52	101.0	107.6	113.1	118.1	124.1	128.3	90
99.33	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	100
124.3	128.4	132.8	138.1	145.6	152.1	157.8	164.7	169.5	125
149.3	153.8	158.6	164.3	172.6	179.6	185.8	193.2	198.4	150

Table C.3. Percentiles of Student's t distribution.

Deg. of freedom	$t_{0.6}$	$t_{0.7}$	$t_{0.8}$	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	0.325	0.727	1.376	3.078	6.314	12.71	31.82	63.66
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
50	0.255	0.528	0.849	1.299	1.676	2.009	2.403	2.678
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
70	0.254	0.527	0.847	1.294	1.667	1.994	2.381	2.648
80	0.254	0.526	0.846	1.292	1.664	1.990	2.374	2.639
90	0.254	0.526	0.846	1.291	1.662	1.987	2.368	2.632
100	0.254	0.526	0.845	1.290	1.660	1.984	2.364	2.626
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
150	0.254	0.526	0.844	1.287	1.655	1.976	2.351	2.609
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576

For percentiles below the 50th, use the fact that the t distribution is symmetrical about zero, so $t_{1-q} = -t_q$.

Tables of the beta(α, β) distribution are given on the next pages. Because of the limited size of the paper, each table has been split into two pieces and printed on two facing pages. Each table contains a diagonal line, shown by blackened cells. The table entries below this line and to the left are the lower percentiles, such as the 10th or the 5th. The table entries above the diagonal and to the right are the upper percentiles, such as the 90th or the 95th. In this way, both sets of percentiles appear in a single table.

Only distributions with $\alpha < \beta$ are tabulated. These distributions have probability concentrated near zero, and are the distributions usually encountered in PRA work. For distributions with $\alpha > \beta$, use the fact that if X has a beta(α, β) distribution then $1 - X$ has a beta(β, α) distribution. An example is given as a footnote to each table.

The size of the page limits the number of parameter pairs (α, β) that can be tabulated. Therefore, interpolation is often necessary, which may give only rough accuracy. If greater accuracy is required, the user can find the beta distribution calculated by many commonly used computer packages and spreadsheets. Similarly, extrapolation beyond the table may sometimes be necessary. A footnote to each table gives an approximate extrapolation formula when $\beta >> \alpha$, and an example of its application. If greater accuracy is needed, use a commercially available computer program.

Table C.4. 90th and 10th percentiles of beta (α, β) distribution.

β for 10th %iles	β for 90th percentiles											α for 90th %iles
	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
	1.00E+0	7.33E-1	3.49E-1	2.16E-1	1.55E-1	1.20E-1	9.84E-2	8.31E-2	7.20E-2	6.34E-2	5.67E-2	0.1
		9.76E-1	8.10E-1	6.49E-1	5.32E-1	4.48E-1	3.86E-1	3.39E-1	3.02E-1	2.72E-1	2.47E-1	0.5
0.1	8.87E-8		9.00E-1	7.85E-1	6.84E-1	6.02E-1	5.36E-1	4.82E-1	4.38E-1	4.01E-1	3.69E-1	1.0
0.5	3.46E-10	2.45E-2		8.44E-1	7.59E-1	6.85E-1	6.22E-1	5.68E-1	5.23E-1	4.84E-1	4.50E-1	1.5
1.0	1.00E-10	1.00E-2	1.00E-1		8.04E-1	7.38E-1	6.80E-1	6.28E-1	5.84E-1	5.45E-1	5.10E-1	2.0
1.5	5.60E-11	6.18E-3	6.78E-2	1.56E-1		7.75E-1	7.21E-1	6.73E-1	6.30E-1	5.92E-1	5.58E-1	2.5
2.0	3.86E-11	4.46E-3	5.13E-2	1.23E-1	1.96E-1		7.53E-1	7.08E-1	6.67E-1	6.30E-1	5.96E-1	3.0
2.5	2.93E-11	3.48E-3	4.13E-2	1.02E-1	1.65E-1	2.25E-1		7.36E-1	6.97E-1	6.61E-1	6.28E-1	3.5
3.0	2.37E-11	2.86E-3	3.45E-2	8.64E-2	1.43E-1	1.97E-1	2.47E-1		7.21E-1	6.87E-1	6.55E-1	4.0
3.5	1.98E-11	2.42E-3	2.97E-2	7.53E-2	1.26E-1	1.75E-1	2.21E-1	2.64E-1		7.09E-1	6.79E-1	4.5
4.0	1.71E-11	2.10E-3	2.60E-2	6.66E-2	1.12E-1	1.58E-1	2.01E-1	2.41E-1	2.79E-1		6.99E-1	5.0
4.5	1.50E-11	1.85E-3	2.31E-2	5.98E-2	1.01E-1	1.43E-1	1.84E-1	2.22E-1	2.58E-1	2.91E-1		
5.	1.33E-11	1.66E-3	2.09E-2	5.42E-2	9.26E-2	1.32E-1	1.70E-1	2.06E-1	2.40E-1	2.71E-1	3.01E-1	
6.	1.09E-11	1.37E-3	1.74E-2	4.57E-2	7.88E-2	1.13E-1	1.47E-1	1.79E-1	2.10E-1	2.40E-1	2.67E-1	
7.	9.26E-12	1.17E-3	1.49E-2	3.95E-2	6.86E-2	9.91E-2	1.29E-1	1.59E-1	1.88E-1	2.15E-1	2.41E-1	
8.	8.04E-12	1.02E-3	1.31E-2	3.48E-2	6.08E-2	8.82E-2	1.16E-1	1.43E-1	1.69E-1	1.95E-1	2.19E-1	
9.	7.10E-12	9.02E-4	1.16E-2	3.11E-2	5.45E-2	7.95E-2	1.05E-1	1.30E-1	1.54E-1	1.78E-1	2.01E-1	
10.	6.36E-12	8.09E-4	1.05E-2	2.81E-2	4.95E-2	7.23E-2	9.57E-2	1.19E-1	1.42E-1	1.64E-1	1.85E-1	
12.5	5.04E-12	6.44E-4	8.39E-3	2.27E-2	4.01E-2	5.90E-2	7.86E-2	9.82E-2	1.18E-1	1.37E-1	1.55E-1	
15.	4.17E-12	5.35E-4	7.00E-3	1.90E-2	3.37E-2	4.99E-2	6.67E-2	8.37E-2	1.01E-1	1.17E-1	1.34E-1	
20.	3.11E-12	4.00E-4	5.25E-3	1.43E-2	2.56E-2	3.81E-2	5.12E-2	6.46E-2	7.81E-2	9.16E-2	1.05E-1	
30.	2.06E-12	2.65E-4	3.51E-3	9.61E-3	1.73E-2	2.59E-2	3.49E-2	4.43E-2	5.39E-2	6.36E-2	7.33E-2	
50.	1.23E-12	1.59E-4	2.10E-3	5.80E-3	1.05E-2	1.57E-2	2.14E-2	2.73E-2	3.33E-2	3.95E-2	4.57E-2	
100.	6.10E-13	7.91E-5	1.05E-3	2.91E-3	5.28E-3	7.96E-3	1.09E-2	1.39E-2	1.70E-2	2.03E-2	2.36E-2	
	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
	α for 10th percentiles											

For example, the 10th percentile of a beta(1.5, 10) distribution is 2.81E−2. The 90th percentile of the same distribution (from the table continuation on the next page) is 2.63E−1. If X has a beta(α, β) distributions with $\alpha > \beta$, use the relation $X = 1 - Y$, where Y has a beta(β, α) distribution. Thus, for example, the 10th and 90th percentiles of a beta(10, 1.5) distribution are 1 − 2.63E−1 = 0.737 and 1 − 2.81E−2 = 0.9719, respectively.

For a beta(α, β) distribution with $\beta \gg \alpha$, the p quantile is approximated by $\chi^2_p(2\alpha) / [2\beta + \chi^2_p(2\alpha)]$. For example, the 10th percentile of a beta(2,100) distribution, shown above as 5.28E−3, is approximated by $\chi^2_{0.10}(4) / [200 + \chi^2_{0.10}(4)] = 1.064/201.064 = 5.29E{-3}$. The 90th percentile, 3.80E−2, is approximated by 7.770/207.779 = 3.74E−2.

Table C.4 90th and 10th percentiles of beta (α, β) distribution (continued).

		β for 90th percentiles											
		6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	α for 90th %iles
		4.67E-2	3.98E-2	3.46E-2	3.06E-2	2.75E-2	2.18E-2	1.81E-2	1.35E-2	8.97E-3	5.36E-3	2.67E-3	0.1
		2.09E-1	1.81E-1	1.60E-1	1.43E-1	1.29E-1	1.05E-1	8.76E-2	6.62E-2	4.45E-2	2.68E-2	1.35E-2	0.5
		3.19E-1	2.80E-1	2.50E-1	2.26E-1	2.06E-1	1.68E-1	1.42E-1	1.09E-1	7.39E-2	4.50E-2	2.28E-2	1.0
		3.94E-1	3.51E-1	3.16E-1	2.87E-1	2.63E-1	2.18E-1	1.85E-1	1.43E-1	9.82E-2	6.03E-2	3.07E-2	1.5
		4.53E-1	4.06E-1	3.68E-1	3.37E-1	3.10E-1	2.59E-1	2.22E-1	1.73E-1	1.20E-1	7.41E-2	3.80E-2	2.0
		4.99E-1	4.52E-1	4.12E-1	3.79E-1	3.50E-1	2.95E-1	2.55E-1	2.00E-1	1.40E-1	8.70E-2	4.48E-2	2.5
		5.38E-1	4.90E-1	4.50E-1	4.15E-1	3.86E-1	3.27E-1	2.84E-1	2.24E-1	1.58E-1	9.91E-2	5.13E-2	3.0
		5.71E-1	5.23E-1	4.82E-1	4.47E-1	4.17E-1	3.56E-1	3.10E-1	2.47E-1	1.75E-1	1.11E-1	5.76E-2	3.5
		5.99E-1	5.52E-1	5.11E-1	4.75E-1	4.44E-1	3.82E-1	3.34E-1	2.68E-1	1.91E-1	1.22E-1	6.37E-2	4.0
		6.24E-1	5.77E-1	5.36E-1	5.01E-1	4.69E-1	4.06E-1	3.57E-1	2.87E-1	2.07E-1	1.32E-1	6.96E-2	4.5
		6.46E-1	5.99E-1	5.59E-1	5.23E-1	4.92E-1	4.27E-1	3.78E-1	3.06E-1	2.21E-1	1.43E-1	7.54E-2	5.
		6.82E-1	6.38E-1	5.98E-1	5.63E-1	5.32E-1	4.66E-1	4.15E-1	3.40E-1	2.49E-1	1.62E-1	8.65E-2	6.
			6.69E-1	6.31E-1	5.96E-1	5.65E-1	5.00E-1	4.48E-1	3.70E-1	2.74E-1	1.80E-1	9.72E-2	7.
β for 10th %iles	6.	3.18E-1		6.58E-1	6.25E-1	5.94E-1	5.29E-1	4.77E-1	3.97E-1	2.98E-1	1.98E-1	1.08E-1	8.
	7.	2.88E-1	3.31E-1		6.50E-1	6.20E-1	5.56E-1	5.03E-1	4.22E-1	3.19E-1	2.14E-1	1.18E-1	9.
	8.	2.64E-1	3.05E-1	3.42E-1		6.42E-1	5.79E-1	5.26E-1	4.45E-1	3.40E-1	2.30E-1	1.27E-1	10.
	9.	2.43E-1	2.82E-1	3.18E-1	3.50E-1		6.27E-1	5.76E-1	4.95E-1	3.85E-1	2.66E-1	1.50E-1	12.5
	10.	2.26E-1	2.63E-1	2.97E-1	3.29E-1	3.58E-1		6.16E-1	5.36E-1	4.25E-1	2.99E-1	1.72E-1	15.
	12.5	1.91E-1	2.25E-1	2.56E-1	2.85E-1	3.12E-1	3.73E-1		6.01E-1	4.89E-1	3.56E-1	2.11E-1	20.
	15.	1.66E-1	1.96E-1	2.25E-1	2.52E-1	2.77E-1	3.34E-1	3.84E-1		5.82E-1	4.45E-1	2.79E-1	30.
	20.	1.31E-1	1.57E-1	1.81E-1	2.04E-1	2.26E-1	2.78E-1	3.23E-1	3.99E-1		5.64E-1	3.83E-1	50.
	30.	9.26E-2	1.12E-1	1.30E-1	1.48E-1	1.66E-1	2.07E-1	2.45E-1	3.12E-1	4.18E-1		5.45E-1	100.
	50.	5.83E-2	7.09E-2	8.35E-2	9.59E-2	1.08E-1	1.38E-1	1.66E-1	2.18E-1	3.06E-1	4.36E-1		
	100.	3.03E-2	3.71E-2	4.40E-2	5.09E-2	5.78E-2	7.50E-2	9.19E-2	1.25E-1	1.84E-1	2.85E-1	4.55E-1	
		6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	
		α for 10th percentiles											

Table C.5. 95th and 5th percentiles of beta (α, β) distribution.

β for 5th %iles	β for 95th percentiles											α for 95th %iles
	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
	1.00E+0	9.24E-1	5.99E-1	4.09E-1	3.06E-1	2.44E-1	2.02E-1	1.72E-1	1.50E-1	1.33E-1	1.19E-1	0.1
		9.94E-1	9.02E-1	7.71E-1	6.58E-1	5.69E-1	4.99E-1	4.44E-1	3.99E-1	3.62E-1	3.32E-1	0.5
0.1	8.66E-11		9.50E-1	8.64E-1	7.76E-1	6.98E-1	6.32E-1	5.75E-1	5.27E-1	4.86E-1	4.51E-1	1.0
0.5	3.38E-13	6.16E-3		9.03E-1	8.32E-1	7.64E-1	7.04E-1	6.51E-1	6.04E-1	5.63E-1	5.27E-1	1.5
1.0	9.77E-14	2.50E-3	5.00E-2		8.65E-1	8.06E-1	7.51E-1	7.02E-1	6.57E-1	6.18E-1	5.82E-1	2.0
1.5	5.46E-14	1.54E-3	3.36E-2	9.73E-2		8.35E-1	7.85E-1	7.39E-1	6.97E-1	6.59E-1	6.24E-1	2.5
2.0	3.77E-14	1.11E-3	2.53E-2	7.60E-2	1.35E-1		8.11E-1	7.68E-1	7.29E-1	6.92E-1	6.59E-1	3.0
2.5	2.87E-14	8.68E-4	2.03E-2	6.24E-2	1.13E-1	1.65E-1		7.91E-1	7.54E-1	7.19E-1	6.87E-1	3.5
3.0	2.31E-14	7.12E-4	1.70E-2	5.30E-2	9.76E-2	1.44E-1	1.89E-1		7.75E-1	7.42E-1	7.11E-1	4.0
3.5	1.94E-14	6.03E-4	1.45E-2	4.60E-2	8.57E-2	1.28E-1	1.69E-1	2.09E-1		7.61E-1	7.31E-1	4.5
4.0	1.67E-14	5.23E-4	1.27E-2	4.07E-2	7.64E-2	1.15E-1	1.53E-1	1.90E-1	2.25E-1		7.49E-1	5.0
4.5	1.46E-14	4.62E-4	1.13E-2	3.64E-2	6.90E-2	1.04E-1	1.40E-1	1.75E-1	2.08E-1	2.39E-1		
5.	1.30E-14	4.13E-4	1.02E-2	3.30E-2	6.28E-2	9.55E-2	1.29E-1	1.61E-1	1.93E-1	2.23E-1	2.51E-1	
6.	1.07E-14	3.42E-4	8.51E-3	2.78E-2	5.34E-2	8.18E-2	1.11E-1	1.40E-1	1.69E-1	1.96E-1	2.22E-1	
7.	9.05E-15	2.91E-4	7.30E-3	2.40E-2	4.64E-2	7.15E-2	9.77E-2	1.24E-1	1.50E-1	1.75E-1	2.00E-1	
8.	7.85E-15	2.54E-4	6.39E-3	2.11E-2	4.10E-2	6.36E-2	8.73E-2	1.11E-1	1.35E-1	1.58E-1	1.81E-1	
9.	6.93E-15	2.25E-4	5.68E-3	1.89E-2	3.68E-2	5.72E-2	7.88E-2	1.01E-1	1.23E-1	1.45E-1	1.66E-1	
10.	6.21E-15	2.02E-4	5.12E-3	1.70E-2	3.33E-2	5.20E-2	7.19E-2	9.22E-2	1.13E-1	1.33E-1	1.53E-1	
12.5	4.92E-15	1.60E-4	4.10E-3	1.37E-2	2.70E-2	4.24E-2	5.89E-2	7.60E-2	9.33E-2	1.11E-1	1.28E-1	
15.	4.08E-15	1.33E-4	3.41E-3	1.15E-2	2.27E-2	3.57E-2	4.99E-2	6.47E-2	7.97E-2	9.48E-2	1.10E-1	
20.	3.03E-15	9.95E-5	2.56E-3	8.65E-3	1.72E-2	2.72E-2	3.82E-2	4.98E-2	6.17E-2	7.37E-2	8.59E-2	
30.	2.01E-15	6.61E-5	1.71E-3	5.80E-3	1.16E-2	1.85E-2	2.60E-2	3.41E-2	4.25E-2	5.11E-2	5.98E-2	
50.	1.20E-15	3.95E-5	1.03E-3	3.49E-3	7.01E-3	1.12E-2	1.59E-2	2.09E-2	2.62E-2	3.16E-2	3.72E-2	
100.	5.96E-16	1.97E-5	5.13E-4	1.75E-3	3.53E-3	5.67E-3	8.06E-3	1.06E-2	1.34E-2	1.62E-2	1.91E-2	
	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
	α for 5th percentiles											

For example, the 5th percentile of a beta(1.5, 10) distribution is 1.70E−2. The 95th percentile of the same distribution (from the table continuation on the next page) is 3.17E−1. If X has a beta(α, β) distributions with $\alpha > \beta$, use the relation $X = 1 - Y$, where Y has a beta(β, α) distribution. Thus, for example, the 5th and 95th percentiles of a beta(10, 1.5) distribution are 1 − 3.17E−1 = 0.683 and 1 − 1.70E−2 = 0.9830, respectively.

For a beta(α, β) distribution with $\beta >> \alpha$, the p quantile is approximated by $\chi^2_p(2\alpha) / [2\beta + \chi^2_p(2\alpha)]$. For example, the 5th percentile of a beta(2,100) distribution, shown here as 3.53E−3, is approximated by $\chi^2_{0.05}(4) / [200 + \chi^2_{0.05}(4)]$ = 0.711/200.711 = 3.54E−3. The 95th percentile, 4.61E−2, is approximated by 9.488/209.488 = 4.53E−2.

Table C.5 95th and 5th percentiles of beta (α, β) distribution (continued).

		β for 95th percentiles										
	6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	α for 95th %iles
	9.91E-2	8.47E-2	7.39E-2	6.56E-2	5.89E-2	4.70E-2	3.91E-2	2.92E-2	1.94E-2	1.16E-2	5.81E-3	0.1
	2.83E-1	2.47E-1	2.19E-1	1.97E-1	1.79E-1	1.45E-1	1.22E-1	9.27E-2	6.25E-2	3.79E-2	1.91E-2	0.5
	3.93E-1	3.48E-1	3.12E-1	2.83E-1	2.59E-1	2.13E-1	1.81E-1	1.39E-1	9.50E-2	5.82E-2	2.95E-2	1.0
	4.66E-1	4.17E-1	3.78E-1	3.45E-1	3.17E-1	2.64E-1	2.26E-1	1.76E-1	1.21E-1	7.48E-2	3.82E-2	1.5
	5.21E-1	4.71E-1	4.29E-1	3.94E-1	3.64E-1	3.06E-1	2.64E-1	2.07E-1	1.44E-1	8.97E-2	4.61E-2	2.0
	5.64E-1	5.14E-1	4.71E-1	4.35E-1	4.04E-1	3.42E-1	2.97E-1	2.34E-1	1.65E-1	1.03E-1	5.35E-2	2.5
	6.00E-1	5.50E-1	5.07E-1	4.70E-1	4.38E-1	3.74E-1	3.26E-1	2.59E-1	1.84E-1	1.16E-1	6.04E-2	3.0
	6.30E-1	5.80E-1	5.38E-1	5.01E-1	4.68E-1	4.02E-1	3.53E-1	2.82E-1	2.02E-1	1.28E-1	6.71E-2	3.5
	6.55E-1	6.07E-1	5.64E-1	5.27E-1	4.95E-1	4.28E-1	3.77E-1	3.04E-1	2.19E-1	1.40E-1	7.36E-2	4.0
	6.77E-1	6.30E-1	5.88E-1	5.51E-1	5.18E-1	4.51E-1	3.99E-1	3.23E-1	2.34E-1	1.51E-1	7.98E-2	4.5
	6.96E-1	6.50E-1	6.09E-1	5.73E-1	5.40E-1	4.72E-1	4.19E-1	3.42E-1	2.49E-1	1.62E-1	8.59E-2	5.
	7.29E-1	6.85E-1	6.45E-1	6.10E-1	5.77E-1	5.10E-1	4.56E-1	3.75E-1	2.77E-1	1.82E-1	9.75E-2	6.
		7.13E-1	6.75E-1	6.40E-1	6.09E-1	5.42E-1	4.87E-1	4.05E-1	3.03E-1	2.01E-1	1.09E-1	7.
β for 5th %iles: 6.	2.71E-1		7.00E-1	6.67E-1	6.36E-1	5.70E-1	5.15E-1	4.32E-1	3.26E-1	2.18E-1	1.19E-1	8.
7.	2.45E-1	2.87E-1		6.89E-1	6.59E-1	5.94E-1	5.40E-1	4.57E-1	3.48E-1	2.35E-1	1.30E-1	9.
8.	2.24E-1	2.64E-1	3.00E-1		6.80E-1	6.16E-1	5.63E-1	4.79E-1	3.68E-1	2.51E-1	1.40E-1	10.
9.	2.06E-1	2.44E-1	2.79E-1	3.11E-1		6.62E-1	6.10E-1	5.27E-1	4.13E-1	2.88E-1	1.63E-1	12.5
10.	1.91E-1	2.27E-1	2.60E-1	2.91E-1	3.20E-1		6.48E-1	5.67E-1	4.52E-1	3.21E-1	1.85E-1	15.
12.5	1.61E-1	1.93E-1	2.23E-1	2.52E-1	2.78E-1	3.38E-1		6.29E-1	5.15E-1	3.77E-1	2.25E-1	20.
15.	1.40E-1	1.68E-1	1.96E-1	2.22E-1	2.46E-1	3.03E-1	3.52E-1		6.05E-1	4.65E-1	2.94E-1	30.
20.	1.10E-1	1.34E-1	1.57E-1	1.79E-1	2.00E-1	2.50E-1	2.95E-1	3.71E-1		5.82E-1	3.98E-1	50.
30.	7.74E-2	9.50E-2	1.12E-1	1.29E-1	1.46E-1	1.86E-1	2.23E-1	2.89E-1	3.95E-1		5.58E-1	100.
50.	4.86E-2	6.02E-2	7.18E-2	8.34E-2	9.49E-2	1.23E-1	1.50E-1	2.01E-1	2.88E-1	4.18E-1		
100.	2.52E-2	3.14E-2	3.77E-2	4.42E-2	5.06E-2	6.68E-2	8.29E-2	1.14E-1	1.73E-1	2.71E-1	4.42E-1	
	6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	
	α for 5th percentiles											

Table C.6. 97.5th and 2.5th percentiles of beta (α, β) distribution.

β for 2.5th %iles	β for 97.5th percentiles: 0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	α for 97.5th %iles
	1.00E+0	9.80E-1	7.76E-1	5.84E-1	4.58E-1	3.74E-1	3.16E-1	2.72E-1	2.40E-1	2.14E-1	1.93E-1	0.1
		9.98E-1	9.51E-1	8.53E-1	7.53E-1	6.67E-1	5.95E-1	5.36E-1	4.86E-1	4.45E-1	4.10E-1	0.5
0.1	8.46E-14		9.75E-1	9.15E-1	8.42E-1	7.71E-1	7.08E-1	6.51E-1	6.02E-1	5.59E-1	5.22E-1	1.0
0.5	3.30E-16	1.54E-3		9.39E-1	8.82E-1	8.23E-1	7.67E-1	7.16E-1	6.70E-1	6.29E-1	5.91E-1	1.5
1.0	9.54E-17	6.25E-4	2.50E-2		9.06E-1	8.55E-1	8.06E-1	7.59E-1	7.16E-1	6.77E-1	6.41E-1	2.0
1.5	5.34E-17	3.86E-4	1.67E-2	6.08E-2		8.77E-1	8.33E-1	7.91E-1	7.51E-1	7.14E-1	6.79E-1	2.5
2.0	3.68E-17	2.78E-4	1.26E-2	4.73E-2	9.43E-2		8.53E-1	8.14E-1	7.77E-1	7.42E-1	7.10E-1	3.0
2.5	2.80E-17	2.17E-4	1.01E-2	3.87E-2	7.87E-2	1.23E-1		8.33E-1	7.98E-1	7.65E-1	7.34E-1	3.5
3.0	2.26E-17	1.78E-4	8.40E-3	3.28E-2	6.76E-2	1.07E-1	1.47E-1		8.16E-1	7.85E-1	7.55E-1	4.0
3.5	1.89E-17	1.51E-4	7.21E-3	2.85E-2	5.92E-2	9.44E-2	1.31E-1	1.67E-1		8.01E-1	7.73E-1	4.5
4.0	1.63E-17	1.31E-4	6.31E-3	2.51E-2	5.27E-2	8.47E-2	1.18E-1	1.52E-1	1.84E-1		7.88E-1	5.0
4.5	1.43E-17	1.15E-4	5.61E-3	2.25E-2	4.75E-2	7.68E-2	1.08E-1	1.39E-1	1.69E-1	1.99E-1		
5.	1.27E-17	1.03E-4	5.05E-3	2.04E-2	4.33E-2	7.02E-2	9.90E-2	1.28E-1	1.57E-1	1.85E-1	2.12E-1	
6.	1.04E-17	8.53E-5	4.21E-3	1.71E-2	3.67E-2	6.00E-2	8.52E-2	1.11E-1	1.37E-1	1.62E-1	1.87E-1	
7.	8.83E-18	7.27E-5	3.61E-3	1.48E-2	3.19E-2	5.24E-2	7.49E-2	9.81E-2	1.22E-1	1.45E-1	1.67E-1	
8.	7.67E-18	6.33E-5	3.16E-3	1.30E-2	2.81E-2	4.65E-2	6.67E-2	8.78E-2	1.09E-1	1.31E-1	1.52E-1	
9.	6.77E-18	5.61E-5	2.81E-3	1.16E-2	2.52E-2	4.18E-2	6.02E-2	7.95E-2	9.92E-2	1.19E-1	1.39E-1	
10.	6.06E-18	5.03E-5	2.53E-3	1.05E-2	2.28E-2	3.80E-2	5.49E-2	7.27E-2	9.09E-2	1.09E-1	1.28E-1	
12.5	4.81E-18	4.01E-5	2.02E-3	8.43E-3	1.85E-2	3.09E-2	4.49E-2	5.98E-2	7.52E-2	9.08E-2	1.07E-1	
15.	3.98E-18	3.33E-5	1.69E-3	7.05E-3	1.55E-2	2.61E-2	3.80E-2	5.08E-2	6.41E-2	7.77E-2	9.15E-2	
20.	2.96E-18[a]	2.49E-5	1.27E-3	5.31E-3	1.17E-2	1.98E-2	2.91E-2	3.90E-2	4.95E-2	6.03E-2	7.13E-2	
30.	1.96E-18	1.65E-5	8.44E-4	3.56E-3	7.91E-3	1.34E-2	1.98E-2	2.67E-2	3.40E-2	4.17E-2	4.95E-2	
50.	1.17E-18	9.87E-6	5.06E-4	2.14E-3	4.78E-3	8.16E-3	1.21E-2	1.64E-2	2.09E-2	2.58E-2	3.08E-2	
100.	5.82E-19[a]	4.92E-6	2.53E-4	1.08E-3	2.41E-3	4.12E-3	6.11E-3	8.31E-3	1.07E-2	1.32E-2	1.58E-2	
	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
	α for 2.5th percentiles											

a. May be inaccurate. Calculation had not converged after 100 iterations.

For example, the 2.5th percentile of a beta(1.5, 10) distribution is 1.05E−2. The 97.5th percentile of the same distribution (from the table continuation on the next page) is 3.67E−1. If X has a beta(α, β) distributions with $\alpha > \beta$, use the relation $X = 1 - Y$, where Y has a beta(β, α) distribution. Thus, for example, the 2.5th and 97.5th percentiles of a beta(10, 1.5) distribution are 1 − 3.67E−1 = 0.633 and 1 − 1.05E−2 = 0.9895, respectively.

For a beta(α, β) distribution with $\beta >> \alpha$, the p quantile is approximated by $\chi^2_p(2\alpha)/[2\beta + \chi^2_p(2\alpha)]$. For example, the 2.5th percentile of a beta(2,100) distribution, shown here as 2.41E−3, is approximated by $\chi^2_{0.025}(4)/[200 + \chi^2_{0.025}(4)]$ = 0.484/200.484 = 2.41E−3. The 97.5th percentile, 5.39E−2, is approximated by 11.14/211.14 = 5.28E−2.

Table C.6. 97.5th and 2.5th percentiles of beta (α, β) distribution (continued).

β for 2.5th %iles	β for 97.5th percentiles											α for 97.5th %iles
	6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	
	1.61E-1	1.38E-1	1.21E-1	1.08E-1	9.73E-2	7.79E-2	6.50E-2	4.88E-2	3.25E-2	1.95E-2	9.78E-3	0.1
	3.53E-1	3.10E-1	2.77E-1	2.49E-1	2.27E-1	1.85E-1	1.57E-1	1.19E-1	8.10E-2	4.92E-2	2.49E-2	0.5
	4.59E-1	4.10E-1	3.69E-1	3.36E-1	3.08E-1	2.56E-1	2.18E-1	1.68E-1	1.16E-1	7.11E-2	3.62E-2	1.0
	5.28E-1	4.76E-1	4.33E-1	3.97E-1	3.67E-1	3.07E-1	2.64E-1	2.06E-1	1.43E-1	8.88E-2	4.56E-2	1.5
	5.79E-1	5.27E-1	4.82E-1	4.45E-1	4.13E-1	3.49E-1	3.02E-1	2.38E-1	1.67E-1	1.04E-1	5.39E-2	2.0
	6.19E-1	5.67E-1	5.23E-1	4.84E-1	4.51E-1	3.85E-1	3.35E-1	2.66E-1	1.88E-1	1.19E-1	6.17E-2	2.5
	6.51E-1	6.00E-1	5.56E-1	5.18E-1	4.84E-1	4.16E-1	3.64E-1	2.92E-1	2.08E-1	1.32E-1	6.90E-2	3.0
	6.78E-1	6.28E-1	5.85E-1	5.47E-1	5.13E-1	4.44E-1	3.91E-1	3.15E-1	2.26E-1	1.45E-1	7.60E-2	3.5
	7.01E-1	6.52E-1	6.10E-1	5.72E-1	5.38E-1	4.68E-1	4.14E-1	3.36E-1	2.43E-1	1.57E-1	8.28E-2	4.0
	7.20E-1	6.74E-1	6.32E-1	5.94E-1	5.61E-1	4.91E-1	4.36E-1	3.56E-1	2.59E-1	1.68E-1	8.93E-2	4.5
	7.38E-1	6.92E-1	6.51E-1	6.14E-1	5.81E-1	5.11E-1	4.56E-1	3.74E-1	2.75E-1	1.79E-1	9.56E-2	5.
	7.66E-1	7.23E-1	6.84E-1	6.49E-1	6.16E-1	5.47E-1	4.91E-1	4.07E-1	3.03E-1	2.00E-1	1.08E-1	6.
		7.49E-1	7.11E-1	6.77E-1	6.46E-1	5.78E-1	5.22E-1	4.36E-1	3.28E-1	2.19E-1	1.19E-1	7.
6.	2.34E-1		7.34E-1	7.01E-1	6.71E-1	6.04E-1	5.49E-1	4.63E-1	3.52E-1	2.37E-1	1.30E-1	8.
7.	2.11E-1	2.51E-1		7.22E-1	6.92E-1	6.27E-1	5.73E-1	4.87E-1	3.73E-1	2.54E-1	1.41E-1	9.
8.	1.92E-1	2.30E-1	2.66E-1		7.11E-1	6.48E-1	5.94E-1	5.08E-1	3.93E-1	2.70E-1	1.51E-1	10.
9.	1.77E-1	2.13E-1	2.47E-1	2.78E-1		6.90E-1	6.39E-1	5.55E-1	4.38E-1	3.07E-1	1.75E-1	12.5
10.	1.63E-1	1.98E-1	2.30E-1	2.60E-1	2.89E-1		6.75E-1	5.93E-1	4.76E-1	3.40E-1	1.97E-1	15.
12.5	1.38E-1	1.68E-1	1.97E-1	2.24E-1	2.50E-1	3.10E-1		6.52E-1	5.38E-1	3.96E-1	2.38E-1	20.
15.	1.19E-1	1.46E-1	1.72E-1	1.97E-1	2.21E-1	2.76E-1	3.25E-1		6.25E-1	4.83E-1	3.07E-1	30.
20.	9.36E-2	1.16E-1	1.38E-1	1.59E-1	1.79E-1	2.28E-1	2.72E-1	3.48E-1		5.97E-1	4.10E-1	50.
30.	6.56E-2	8.19E-2	9.83E-2	1.14E-1	1.30E-1	1.69E-1	2.05E-1	2.70E-1	3.75E-1		5.69E-1	100.
50.	4.11E-2	5.18E-2	6.26E-2	7.35E-2	8.44E-2	1.11E-1	1.38E-1	1.87E-1	2.73E-1	4.03E-1		
100.	2.13E-2	2.70E-2	3.28E-2	3.88E-2	4.49E-2	6.02E-2	7.56E-2	1.06E-1	1.63E-1	2.60E-1	4.31E-1	
	6.	7.	8.	9.	10.	12.5	15.	20.	30.	50.	100.	
	α for 2.5th percentiles											

Table C.7. Acceptance limits for the Kolmogorov test of goodness of fit.

Sample size (n)	Significance level				
	0.20	0.15	0.10	0.05	0.01
1	0.900	0.925	0.950	0.975	0.995
2	0.684	0.726	0.776	0.842	0.929
3	0.565	0.596	0.636	0.708	0.829
4	0.493	0.525	0.565	0.624	0.734
5	0.447	0.474	0.509	0.563	0.669
6	0.410	0.435	0.468	0.519	0.617
7	0.381	0.405	0.436	0.483	0.576
8	0.358	0.381	0.410	0.454	0.542
9	0.339	0.360	0.387	0.430	0.513
10	0.323	0.343	0.369	0.409	0.489
11	0.308	0.327	0.352	0.391	0.468
12	0.296	0.314	0.338	0.375	0.449
13	0.285	0.302	0.325	0.361	0.432
14	0.275	0.292	0.314	0.349	0.418
15	0.266	0.282	0.304	0.338	0.404
16	0.258	0.274	0.295	0.327	0.392
17	0.250	0.266	0.286	0.318	0.381
18	0.244	0.259	0.279	0.309	0.371
19	0.237	0.252	0.271	0.301	0.361
20	0.232	0.246	0.265	0.294	0.352
25	0.208	0.221	0.238	0.264	0.317
30	0.190	0.202	0.218	0.242	0.290
35	0.177	0.187	0.202	0.224	0.269
40	0.165	0.176	0.189	0.210	0.252
50	0.148	0.158	0.170	0.188	0.226
60	0.136	0.144	0.155	0.172	0.207
70	0.126	0.134	0.144	0.160	0.192
80	0.118	0.125	0.135	0.150	0.179
Large n	$1.07/\sqrt{n}$	$1.14/\sqrt{n}$	$1.22/\sqrt{n}$	$1.36/\sqrt{n}$	$1.63/\sqrt{n}$

Reject the hypothesized distribution $F(x)$ if $D = \max|F_n(x) - F(x)|$ exceeds the tabulated value.

The asymptotic formula gives values that are slightly too high — by 1% to 2% for $n = 80$.

Table C.8. Parameters of constrained noninformative prior for binomial p.

p_0	b	α
0.50	0.	0.5000
0.40	-0.8166	0.4168
0.30	-1.748	0.3590
0.20	-3.031	0.3243
0.15	-4.027	0.3211
0.10	-5.743	0.3424
0.09	-6.295	0.3522
0.08	-6.958	0.3648
0.07	-7.821	0.3802
0.06	-8.978	0.3980
0.05	-10.61	0.4171
0.04	-13.08	0.4358
0.03	-17.22	0.4531
0.02	-25.53	0.4693
0.01	-50.52	0.4848
0.005	-100.5	0.4925
0.001	-500.5	0.4985
0	$-\infty$	0.5000

The table gives parameters of the constrained noninformative prior for a binomial parameter p, when the assumed prior mean is p_0. The exact constrained noninformative prior has the form

$$f_{prior}(p) \propto e^{bp} p^{-1/2}(1 - p)^{-1/2} ,$$

with b tabulated above.

The tabulated value α is the first parameter of a beta distribution that has the same mean and variance as the constrained noninformative prior. The second parameter of that beta distribution is obtained by solving $\alpha/(\alpha + \beta) = p_0$. This results in the formula $\beta = \alpha(1 - p_0)/p_0$. Then a beta($\alpha$, β) distribution with these parameters approximates the exact constrained noninformative prior.

If $p_0 > 0.50$, the exact constrained noninformative prior has positive b. For example, if $p_0 = 0.70$, look in the table at the row for 1 - 0.70, and see that b there is -1.748. Therefore, the exact constrained noninformative prior is

$$f_{prior}(p) \propto e^{bp} p^{-1/2}(1 - p)^{-1/2} ,$$

with $b = +1.748$.

If $p_0 > 0.50$, the beta approximation is obtained by interchanging the roles of p_0 and $1 - p_0$ and of α and β in the above formulas. For example, if p_0 were 0.30, the beta approximation would have $\alpha = 0.3590$ and $\beta = 0.3590 \times 0.70/0.30 = 0.8377$. If instead $p_0 = 0.70$, the beta approximation has $\alpha = 0.8377$ and $\beta = 0.3590$.

D. GLOSSARY

Arithmetic mean. See **mean**.

Bathtub curve. A plot of the failure rate as a function of system age, showing a high failure rate when the system is new (the **burn-in** period), then a low, roughly constant failure rate, followed by a rising failure rate as the system becomes old (the **wear-out** period). See **burn-in failure**.

Bayesian inference. Statistical inference involving the use of Bayesian methods. Bayesian inference uses probability distributions to model uncertainty in unknown quantities. Thus, unknown parameters are treated formally as if they were random variables. See also **frequentist inference and statistical inference.**

Bias. The difference between the expected value of an estimator and the true quantity being estimated. For example, if Y is a function of the data that estimates a parameter θ, the bias of Y is $E(Y) - \theta$.

Bin. A group of values of a continuous variable, used to partition the data into subsets. For example, event dates can be grouped so that each year is one bin, and all the events during a single year form a subset of the data.

Burn-in failure. Failures associated with the early time-frame of a component's life cycle, during which the failure rate often starts from a maximum value and decreases rapidly. The high failure rate early in the component's life cycle can be caused by poor quality control practices and a natural wear-in or debugging period. See **bathtub curve**.

c.d.f. See **cumulative distribution function.**

Cell. When the data are expressed in a table of counts, a cell is the smallest element of the table. Each cell has an observed count and, under some null hypothesis, an expected count. Each cell can be analyzed on its own, and then compared to the other cells to see if the data show trends, patterns, or other forms of nonhomogeneity. In a $1 \times J$ table, as with events in time, each cell corresponds to one subset of the data. In a $2 \times J$ table, as with failures on demand, each data subset corresponds to two cells, one cell for failures and one for successes.

Central moment. See **moment.**

Coefficient of variation. See **relative standard deviation.**

Common-cause failure. A single event that causes failure of two or more components at the same time (also referred to as common-mode failure).

Confidence interval. In the frequentist approach, a $100p\%$ confidence interval has a probability p of containing the true unknown parameter. This is a property of the procedure, not of any one particular interval. Any one interval either does or does not contain the true parameter. However, any random data set leads to a confidence interval, and $100p\%$ of these contain the true parameter. Compare with **credible interval.**

Conjugate. A family of prior distributions is conjugate, for data from a specified distribution, if a prior distribution in the family results in the posterior distribution also being in the family. A prior distribution in the conjugate family is called a conjugate prior. For example, the gamma distributions are conjugate for Poisson data, and the beta distributions are conjugate for binomial data.

Credible interval. In the Bayesian approach, a $100p\%$ credible interval contains $100p\%$ of the Bayesian probability distribution. For example, if λ has been estimated by a posterior distribution, the 5th and 95th percentiles of this distribution contain 90% of the probability, so they form a (posterior) 90% credible interval. It is not required to have equal probability in the two tails (5% in this example), although it is very common. For example, the interval bounded by 0 and the 90th percentile would also be a 90% credible interval, a one-sided interval. Bayes credible intervals have the same intuitive purpose as frequentist confidence intervals, but their definitions and interpretations are different.

Cumulative distribution function (c.d.f.). This function gives the probability that the random variable does not exceed a given value x. For a random variable X, the c.d.f. $F(x) = \Pr(X \le x)$. If X is discrete, such as a count of events, the c.d.f. is a step function, with a jump at each possible value of X. If X is continuous, such as a duration time, the c.d.f. is continuous. See also **probability density function.** Do not confuse the statistics acronym c.d.f. with the PRA acronym CDF, denoting core damage frequency!

Density. See **probability density function.**

Disjoint. See **mutually exclusive.**

Duration. The time until something of interest happens. The thing of interest may be failure to run, recovery from a failure, restoration of offsite power, etc.

Error factor. A representation of one of the parameters of the lognormal distribution, defined as the 95th percentile divided by the median. The error factor is a measure of the spread of the distribution, and is denoted by EF.

Estimate, estimator. In the frequentist approach, an **estimator** is a function of random data, and an **estimate** is the particular value taken by the estimator for a particular data set. That is, the term **estimator** is used for the random variable, and **estimate** is used for a number. The usual convention of using upper-case letters for random variables and lower-case letters for numbers is often ignored in this setting, so the context must be used to show whether a random variable or a number is being discussed.

Event rate. See **failure rate** for repairable systems, and replace the word "failure" by "event."

Expected value. If X is discrete with p.d.f. f, the expected value of X, denoted $E(X)$, is $\Sigma x_i f(x_i)$. If instead X is continuously distributed with density f, the expected value is $\int x f(x) dx$. The expected value of X is also called the **mean of** X. It is a measure of the center of the distribution of X.

Exposure time. The length of time during which the events of interest can possibly occur. The units must be specified, such as reactor-critical-years, site-calendar-hours, or system-operating-hours. Also called **time at risk.**

Failure on demand. Failure when a standby system is demanded, even though the system was apparently ready to function just before the demand. It is modeled as a random event, having some probability, but unpredictable on any one specific demand. Compare **standby failure.**

Failure rate. For a repairable system, the failure rate, λ, is such that $\lambda\Delta t$ is approximately the expected number of failures in a short time period from t to $t + \Delta t$. If simultaneous failures do not occur, $\lambda\Delta t$ is also approximately the probability that a failure will occur in the period from t to $t + \Delta t$. In this setting, λ is also called a **failure frequency.** For a nonrepairable system, $\lambda\Delta t$ is approximately the probability that an unfailed system at time t will fail in the time period from t to $t + \Delta t$. In this setting, λ is also called the **hazard rate.**

Fractile. See **quantile.**

Frequency. For a repairable system, **frequency** and **rate** are two words with the same meaning, and are used interchangeably. If simultaneous events do not occur, the frequency $\lambda(t)$ satisfies $\lambda(t)\Delta t \approx \Pr$ (an event occurs between t and $t + \Delta t$), for small Δt.

Frequentist inference. Statistical inference that interprets the probability of an event as the long-term relative frequency of occurrence of the event, in many repetitions of an experiment when the event may or may not occur. Unknown parameters are regarded as fixed numbers, not random. See also **Bayesian inference** and **statistical inference.**

Geometric mean. The geometric mean is an estimator of the location or center of a distribution. It is applicable only for positive data. The geometric mean, say $\tilde{t}$, for $t_1, t_2, ..., t_n$, is defined as

$$\tilde{t} = \exp\left[(1/n)\Sigma \ln t_i\right].$$

It is always less than or equal to the arithmetic mean.

Goodness of fit. This term refers to a class of nonparametric methods that are used to study whether or not a given set of data follows a hypothesized distribution. Both hypothesis tests and graphical methods are used to investigate goodness of fit.

Hazard rate. For a nonrepairable system, **hazard rate** and **failure rate** are two phrases with the same meaning, used interchangeably. The hazard rate $h(t)$ satisfies $h(t)\Delta t \approx \Pr(t < T \le t + \Delta t \mid T > t)$, where Δt is small and T denotes the duration time of interest. The hazard rate is also called the **hazard function.**

Hypothesis. A statement about the model that generated the data. If the evidence against the null hypothesis, H_0, is strong, H_0 is rejected in favor of the alternative hypothesis, H_1. If the evidence against H_0 is not strong, H_0 is "accepted"; that is, it is not necessarily believed, but it is given the benefit of the doubt and is not rejected.

Improper distribution. A function that is treated as a probability distribution function (p.d.f.), but which is not a p.d.f. because it does not have a finite integral. For example, a uniform distribution (constant p.d.f.) on an infinite range is improper. Improper distributions are sometimes useful prior distributions, as long as the resulting posterior distribution is a proper distribution.

Independent. See **statistical independence**.

Inference. See **statistical inference**.

Initiating event. Any event, either internal or external to the plant, that triggers a sequence of events that challenge plant control and safety systems, whose failure could potentially lead to core damage or large early release.

Interval. The notation (a, b) denotes the interval of all points from a to b. This is enough for all the applications in this handbook. However, sometimes an additional refinement is added, giving a degree of mathematical correctness that most readers may ignore: The standard notation in mathematics is that (a, b) includes the points between a and b, but not the two end points. In set notation, it is $\{x \mid a < x < b\}$. Square brackets show that the end points are included. Thus, $(a, b]$ includes b but not a, $\{x \mid a < x \le b\}$.

Interval estimate. One way of estimating a parameter is to identify that it falls in some interval (L, U) with a specified degree of certainty, or confidence. The interval (L, U) is referred to as an interval estimate of the parameter. L and U are calculated from the random data. The frequentist interval estimate is referred to as a confidence interval. It does not give a probability statement about the true parameter value. Rather, the interpretation of a $100(1 - \alpha)\%$ confidence interval is that, if the random data were drawn many times, $100(1 - \alpha)\%$ of the resulting interval estimates would contain the true value. A Bayesian interval estimate is referred to as a subjective probability interval, or credible interval, and can be interpreted as giving a subjective probability statement about the true parameter value being contained in the interval. Compare with **point estimate**. See also **confidence interval, credible interval**.

Inverse c.d.f. algorithm. An algorithm for generating random numbers (presented in Section 6.3.2.5.4).

Latin hypercube sampling (LHS). See **Monte Carlo simulation**.

Likelihood. For discrete data, the likelihood is the probability of the observations. For continuous data, the likelihood is the joint density of the observations, which is the product of the densities of the individual observations if the observations are independent. When some of the observations are discrete and some are continuous, the likelihood is the product of the two types. The likelihood is typically treated as a function of the parameters, with the data regarded as fixed.

Markov Chain Monte Carlo (MCMC). See **Monte Carlo simulation**.

Maximum likelihood estimator. For data generated from a distribution with one unknown parameter, say θ, the maximum likelihood estimate (MLE) of θ is the parameter value that maximizes the likelihood of the data. It is a function of the data, and is commonly denoted $\hat{\theta}$. The MLE is a popular frequentist estimator for two reasons. (1) In commonly used models, the MLE is an intuitively natural function of the data. (2) Under certain, commonly valid, conditions, as the number of observations becomes large the MLE is approximately unbiased with approximately the minimum possible variance, and is approximately normally distributed.

Mean. The mean, μ, of a random variable X is the weighted average of the outcomes, where the weights are the probabilities of the outcomes. More precisely, the mean of X is the expected value $E(X)$, $\Sigma x_i f(x_i)$ if X is discrete with p.d.f. f, and $\int x f(x)dx$ if X is continuously distributed with density f. See also **expected value**.

Mean square error or mean squared error. The expected squared difference between an estimator and the true quantity being estimated. For example, if Y is a function of the data that estimates a parameter θ, the mean squared error (MSE) of Y is $E[(Y - \theta)^2]$. It can be shown that the $\text{MSE}(Y) = \text{var}(Y) + [\text{bias}(Y)]^2$.

Median. For a random variable X with a continuous distribution, the median is that value m for which $\Pr(X < m) = 0.5$, and thus also $\Pr(X > m) = 0.5$. For a sample of data values, or for a discrete random variable X taking a finite number of values with equal probability, the median is the middle value in the ordered set of values. The median m is the 50th percentile, $x_{0.50}$. See **percentile** for the general definition.

Mode. A mode of a distribution is a local maximum value of the probability density or probability distribution function (p.d.f.). A normal distribution has a single mode, which measures the center of the distribution.

Moment. The kth moment about a of a random variable X is the expected value of $(X - a)^k$. If X is discrete with p.d.f. f, this is $\Sigma(x_i - a)^k f(x_i)$. If X is continuous with density f, the kth moment about a is $\int(x-a)^k f(x)dx$. The moments about 0 are sometimes called simply "the" moments. Moments about the mean are called **central moments.** The first moment is the mean, often denoted μ. The second central moment is the variance.

Monte Carlo Sampling. See **Monte Carlo simulation.**

Monte Carlo simulation. Generally referred to as Monte Carlo Sampling by probabilistic risk assessment (PRA) analysts, Monte Carlo simulation uses a sample of values from an uncertainty distribution to approximate the distribution. Moments and percentiles of the distribution are approximated by the corresponding moments and percentiles of the sample.

In the usual PRA application, the initiating event frequencies and basic event probabilities have Bayesian distributions. They are sampled by simple random sampling or by Latin hypercube sampling (LHS). The parameter values are propagated through a fault-tree/event-tree model to produce a simulation of the uncertainty distribution of a quantity of interest, such as core damage frequency.

Monte Carlo simulation is also used to approximate the posterior distributions of parameters, when direct calculations are not feasible. In situations with a single parameter and a nonconjugate prior, a simple random sample from the posterior distribution can usually be constructed. In more complicated situations, such as those with multiple interrelated parameters, the unknown parameters can be simulated by Markov chain Monte Carlo (MCMC) to produce a sequence, or chain, of values of each parameter. The values are not independent. However, if the initial portion of the chain is discarded, the remainder of the terms mimic the posterior distribution of the parameter.

Mutually exclusive. Events are mutually exclusive, or **disjoint,** if no two of them have any elements in common. The intersection of any two of the events is the empty set.

Nonparametric. In parametric inference, the data are assumed to come from a known distributional form, with only the parameters unknown. In nonparametric inference, no distributional form is assumed. Not only are the values of the parameters unknown, but the form of the distribution is unknown as well. See **parametric.**

Nonrepairable system. A system that can only fail once, after which data collection stops. An example is a standby safety system, if the failure to run cannot be recovered during the mission of the system. Data from a nonrepairable system consist of data from identical copies of the system. For example, data from a safety system may be collected, with each run starting with the system nominally operable, and the system either running to completion of the mission or failing before that time. The successive demands to run are regarded as demands on identical copies of the system. See **repairable system.**

Null hypothesis. See **hypothesis.**

Order statistics. The random values arranged from smallest to largest. For example, suppose that three times are observed, with $t_1 = 8.4$, $t_2 = 3.0$, and $t_3 = 5.1$. The order statistics are $t_{(1)} = 3.0$, $t_{(2)} = 5.1$, and $t_{(3)} = 8.4$. Before the data are observed, one can consider the order statistics as random variables, $T_{(1)}, T_{(2)}, ..., T_{(n)}$.

Outage, outage time. An outage is an event when a system is unavailable, that is, out of service for some reason. The outage time is the duration of the event. Compare with **unavailability.**

Parameter. A parametric family of distributions is a collection of distributions that is indexed by one or more quantities called parameters. For example, suppose that $f(t; \lambda) = \lambda e^{-\lambda t}$, where t; $\lambda > 0$. For each value of λ, $f(t; \lambda)$ is a probability density function. Here λ is the parameter that identifies the particular density in the family of exponential density functions. The normal family has two parameters, the mean and the variance.

Parametric. Parametric statistical inference is concerned with learning the values of unknown parameters (and their associated properties) from sample data for a given or assumed family of distributions. See **nonparametric.**

p.d.f. See **probability density function** and **probability distribution function.**

Percentile. Consider a continuous distribution with density (p.d.f.) f and cumulative distribution function (c.d.f.) F. The 100qth percentile is the value x such that $F(x) = q$, or equivalently

$$\int_{-\infty}^{x} f(u)du = q\,.$$

If the distribution is concentrated on the positive line, the lower limit of integration may be replaced by 0. The 100qth percentile is equal to the qth quantile. For example, the 95th percentile equals the 0.95 quantile. If X has a discrete distribution, a percentile may not be unique. The 100qth percentile is defined in this case as x such that $\Pr(X \le x) \ge 100q\%$ and $\Pr(X \ge x) \ge 100(1 - q)\%$.

Similarly, for a finite sample, the 100qth percentile is defined as x such that at least 100q% of the values in the sample are x or smaller, and at least 100(1 − q)% are x or larger. For example, if a sample is a set of three numbers, {1.2, 2.5, 5.9}, the median (corresponding to q = 0.5) is 2.5, because at least half of the numbers are 2.5 or smaller and at least half are 2.5 or larger. If the sample has four numbers, {1.2, 2.5, 2.8, 5.9}, then any number from 2.5 to 2.8 can be considered a median. In this case, the average, (2.5 + 2.8)/2, is often chosen.

Point estimate. An estimate of a parameter in the form of a single number is called a point estimate of the parameter. For example, the mean of a sample of values of a random variable X is a commonly used point estimate of the mean of the distribution. Compare with **interval estimate**.

Poisson process. A process in which events (such as failures) occur in a way such that the number of events X in total time t is described by a Poisson distribution. See Section 2.2.2, Section 7.2, or Appendix A.6.2 for more details.

Pool. To combine data from distinct sources, ignoring possible differences between the sources. Data are sometimes pooled from distinct time periods, components, trains, and/or power plants.

Population. In the PRA setting, population refers to the random distribution that generates data. Population attributes, such as the population mean or population median, are those attributes of the probability distribution. Compare with **sample**.

Posterior credible interval. See **credible interval**.

Posterior distribution. A distribution that quantifies, in a Bayesian way, the belief about a parameter after data have been observed. It reflects both the prior belief and the observed data.

Power of a test. The probability that the test will reject H_0 when H_0 is false. If many possible alternatives to H_0 are considered, the power depends on the particular alternative. See **hypothesis**.

Prior. A colloquial abbreviation for **prior distribution.**

Prior distribution. A distribution that quantifies, in a Bayesian way, the belief about a parameter before any data have been observed.

Probability model. A term for the set of mathematical relationships which are used to define both cumulative distribution functions and either probability distribution functions (discrete case) or probability density functions (continuous case).

Probability density function (p.d.f.). For a continuous random variable X, the probability density function f satisfies

$$\Pr(a \le X \le b) = \int_a^b f(x)dx .$$

Properties of the density are

$f(x) \ge 0$ for all x

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$f(x)\Delta x \approx \Pr(x < X \le x + \Delta x)$ for small Δx.

The p.d.f. is related to the c.d.f. by

$f(x) = F'(x)$, the derivative,

and

$$F(x) = \int_{-\infty}^{x} f(u)du .$$

See **cumulative distribution function.**

Probability distribution function (p.d.f.). For a discrete random variable X, the p.d.f. $f(x) = \Pr(X = x)$.

p-value. In the context of testing, the p-value is the significance level at which the data just barely cause H_0 to be rejected. H_0 is rejected when a test statistic is extreme, and the p-value is the probability (under H_0) that the random test statistic would be at least as extreme as actually observed.

Quantile. Consider a continuous distribution with density (p.d.f.) f and cumulative distribution function (c.d.f.) F. The qth quantile is the value x such that $F(x) = q$, or equivalently:

$$\int_{-\infty}^{x} f(u)du = q$$

If the distribution is concentrated on the positive line, the lower limit of integration may be replaced by 0. The qth quantile is equal to the ($100q$)th percentile. For example, the 0.95 quantile equals the 95th percentile. If X has a discrete distribution, a quantile may not be unique. Some authors use the term **fractile** instead of quantile. See **percentile** for a fuller explanation.

Random sample. $x_1, ..., x_n$ are a random sample if they are the observed values of $X_1, ..., X_n$, where the X_is are statistically independent of each other and all have the same distribution.

Random variable. A rule that assigns a number to every outcome in a sample space. For example, if a pump was demanded to start n times, the sample space consists of all the possible outcomes, with their probabilities. A random variable of interest might be the *number* of failures to start. If a stuck valve is repaired, the sample space consists of all the possible outcomes of the repair process, with their probabilities. A random variable of interest might be the time required for repair, a *number*.

Range. The difference between the largest and smallest values of a sample is called the range of the sample.

Rate. See **frequency**.

Reactor critical year. 8760 hours during which a reactor is critical.

Rejection-method algorithm. An algorithm for generating a random sample from a particular distribution. Its general form is given in Section 6.2.2.6, and applied in several places there and in Section 6.3.2.4.

Relative standard deviation. The standard deviation, expressed as a fraction of the mean. The relative standard deviation of X is st.dev.$(X)/E(X)$. Some authors call it the **coefficient of variation**, and express it as a percent.

Relative variance. The square of the **relative standard deviation.** The relative variance of X is $\text{var}(X)/[E(X)]^2$.

Renewal process. A process in which events (such as failures or restorations) occur in a way such that the times between events are independent and identically distributed. For example, if the process consists of failures and nearly instantaneous repairs, each repair restores the system to good-as-new condition.

Repairable system. A system that can fail repeatedly. Each failure is followed by repair, and the possibility of another failure sooner or later. An example is a power plant, with initiating events counted as the "failures." After such an event, the plant is brought back up to its operating condition, and more initiating events can eventually occur. See **nonrepairable system.**

Residual. When a model is fitted to data, the residual for a data point is the data value minus the fitted value (the estimated mean). The residuals together can be used to quantify the overall scatter of the data around the fitted model. If the assumed model assigns different variances to different data points, the standardized residuals are sometimes constructed. A **standardized residual** is the ordinary residual divided by its estimated standard deviation.

Return-to-service test. A test performed at the end of maintenance, which must be successful. If the system does not perform successfully on the test, the maintenance is resumed and the test is not counted as a return-to-service test. A return-to-service test can demonstrate that no latent failed conditions exist (see **standby failure**), but it provides absolutely no information about the probability of failure on a later demand (see **failure on demand**).

Sample. This term refers to data that are generated randomly from some distribution. Sample attributes, such as the sample mean or sample median, are those attributes calculated from the sample. They may be used as estimators of the corresponding population attributes. The sample may be thought of as random, before the data are generated, or as fixed, after the data are generated. See also **population, random sample, sample mean, sample median,** and **sample variance.**

Sample mean. The arithmetic average of the numbers in a random sample. If the numbers are $x_1, ..., x_n$, the sample mean is often denoted $\bar{x}$. It is an estimate of the **population mean,** that is, of the expected value $E(X)$.

Sample median. Let $x_{(1)}, ... x_{(n)}$ be the order statistics from a random sample. The sample median is the middle value. If n is odd, the sample median is $x_{((n+1)/2)}$. If n is even, the sample median is any number between $x_{(n/2)}$ and $x_{(n/2+1)}$, although usually the average of these two numbers is used.

Sample variance. Let $x_1, ..., x_n$ be a random sample, with sample mean $\bar{x}$. The sample variance, often denoted s^2, is

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 .$$

It is an estimate of the **population variance**, var(X).

Significance level of a test. The probability of making a Type I error, that is, of rejecting H_0 when H_0 is true. (If H_0 includes a number of possibilities, so that the probability of rejecting H_0 varies, the significance level is defined as the maximum of those probabilities.) The significance level is denoted by α. Compare with **p-value** and **statistically significant**.

Skewed distribution. A distribution that is not symmetrical. A distribution that is restricted to the range from 0 to ∞ is typically skewed to the right, or positively skewed. Its mean is larger than its median, and the 95th percentile is farther from the median than the 5th percentile is. The Poisson, gamma, and lognormal distributions are a few examples of positively skewed distributions.

Standard deviation. The standard deviation of a distribution is the square root of the variance. The standard deviation and variance are two measures of how much spread or dispersion there is in a distribution.

Standard error. The estimated standard deviation of the estimator of a parameter, in the frequentist approach. For example, suppose that λ is the parameter to be estimated, and $\hat{\lambda}$ is the estimator. The estimator depends on random data, and therefore is random, with a standard deviation, s.d.($\hat{\lambda}$). The estimated value of this standard deviation is the standard error for λ.

Standardized residual. See **residual**.

Standby failure. For a standby system, failure to start resulting from an existing, or latent, failed condition. The system is in this failed condition for some time, but the condition is not discovered until the demand. Compare **failure on demand**.

Statistic. A function of the data, such as the sample mean or the Pearson chi-squared statistic. Before the data are observed, the statistic is a random variable which can take many values, depending on the random data. The observed value of a statistic is a number.

Statistical independence. Two events are statistically independent if the probability of both occurring is the product of their marginal (or individual) probabilities: $\Pr(E_1 \cap E_2) = \Pr(E_1) \times \Pr(E_2)$. Three or more events are statistically independent if the probability of any set of the events is equal to the product of the probabilities of those events. Two or more random variables are statistically independent if their joint p.d.f. equals the product of the marginal (or individual) p.d.f.s. For brevity, the word *statistically* is often dropped.

It can be shown that two random variables are statistically independent if and only if any event defined in terms of one random variable is statistically independent of any event defined in terms of the other random variable. (A similar statement holds for more than two random variables.) For example, suppose that X and Y are independent continuously distributed random variables, with joint density

$$f_{X,Y}(x, y) = f_X(x) f_Y(y) .$$

Let A be the event $a \le X \le b$, and let B be the event $c \le Y \le d$. Then

$$\Pr(A \cap B) = \Pr(a \le X \le b \text{ and } c \le Y \le d)$$

$$= \int_a^b \int_c^d f_{X,Y}(x, y)dydx$$

by the definition of a joint density

$$= \int_a^b \int_c^d f_X(x) f_Y(y)dydx$$

because X and Y are independent

$$= \int_a^b f_X(x)dx \int_c^d f_Y(y)dy$$

evaluating the integral

$$= \Pr(a \le X \le b)\Pr(c \le Y \le d)$$

by definition of the marginal densities

$$= \Pr(A) \times \Pr(B).$$

Statistical inference. The area of statistics concerned with using sample data to answer questions and make statements about the distribution of a random variable from which the sample data were obtained.

Statistically significant. A departure from a null hypothesis is called statistically significant if the hypothesis is rejected with some small significance level, customarily set to 0.05. See **p-value** and **significance level of a test**.

Stochastic. Referring to a random, rather than a deterministic, process. This is an elevated word for *random.*

System. In this handbook, system is the general word used to denote a collection of hardware for which data are collected. The term can apply to a specific system typically found in a nuclear power plant, such as the auxiliary feedwater system, or to a train, or a component, or even a small piece part, as long as data for the system are reported.

Time at risk. See **exposure time.**

Type I error. A rejection of the null hypothesis when it is true.

Type II error. "Acceptance" of the null hypothesis when it is false, that is, failure to reject the null hypothesis when it is false.

Unavailability. For a standby system, the probability that the system is unavailable, out of service, when demanded. This may be divided into different causes — unavailability from planned maintenance and unavailability from unplanned maintenance. Unavailability is distinct from failure to start of a nominally available system. Compare **outage.**

Uncertainty. The imprecisions in the analyst's knowledge or available information about the input parameters to PRA models, the PRA models themselves, and the outputs from such models.

Variance. The variance of a random variable X, denoted by σ^2, is the second moment about the mean, the average of the squared deviations from the mean, $E[(X-\mu)^2]$. It measures the dispersion in the distribution. Compare **standard deviation.**

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11. ABSTRACT *(200 words or less)*

Probabilistic risk assessment (PRA) is a mature technology that can provide a quantitative assessment of the risk from accidents in nuclear power plants. It involves the development of models that delineate the response of systems and operators to accident initiating events. Additional models are generated to identify the component failure modes required to cause the accident mitigating systems to fail. Each component failure mode is represented as an individual "basic event" in the systems models. Estimates of risk are obtained by propagating the uncertainty distributions for each of the parameters through the PRA models.

The data analysis portion of a nuclear power plant PRA provides estimates of the parameters used to determine the frequencies and probabilities of the various events modeled in a PRA. This handbook provides guidance on sources of information and methods for estimating the parameters used in PRA models and for quantifying the uncertainties in the estimates. This includes determination of both plant-specific and generic estimates for initiating event frequencies, component failure rates and unavailabilities, and equipment non-recovery probabilities.

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